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New analytic running coupling in spacelike and timelike regions

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The new model for the QCD analytic running coupling, proposed recently, is extended to the timelike region. This running coupling naturally arises under unification of the analytic approach to QCD and the renormalization group (RG) formalism. A new method for determining the coefficients of the ''analytized'' RG equation is elaborated. It enables one to take into account the higher loop contributions to the new analytic running coupling (NARC) in a consistent way. The expression for the new analytic running coupling, independent of the normalization point, is obtained by invoking the asymptotic freedom condition. It is shown that the difference between the values of the NARC in respective spacelike and timelike regions is rather valuable for intermediate energies. This is essential for the correct extracting of the running coupling from experimental data. The new analytic running coupling is applied to the description of the inclusive τ lepton decay. The consistent estimation of the parameter $\Lambda_{\rm QCD}$ is obtained here.

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I. INTRODUCTION

The description of hadron dynamics in the infrared (IR) region remains an urgent problem of contemporary elementary particle theory. The asymptotic freedom in quantum chromodynamics (QCD) enables one to apply the standard perturbative approach at large momenta transferred. However, a number of phenomena (for example, quark confinement, nonvanishing vacuum expectation values, etc.) are beyond such calculations. Moreover, the use of perturbation theory results in known unphysical singularities in the IR region (for example, the ghost pole appears in the expression for the running coupling).

Recently a new model for the QCD analytic running coupling has been proposed [1]. The basic idea of this model is unification of the analytic approach to QCD [2] with the renormalization group (RG) formalism for recovering proper analytic properties of the theory. An obvious advantage of the new analytic running coupling (NARC) is that it incorporates IR enhancement with asymptotic freedom behavior in a single expression and does not contain additional parameters. In Refs. [1,3,4] it was shown explicitly that the new analytic running coupling, *without invoking any additional assumptions*, leads to the quark-antiquark potential rising at large distances. In addition, a reasonable estimation of the parameter $\Lambda_{\rm QCD}$ was obtained there. The absence of unphysical singularities in the physical region of positive[1] q^2 is also an appealing feature of NARC.

The objective of this paper is to construct a consistent continuation of the new analytic running coupling to the timelike region and to elaborate the method for determination of the coefficients of ''analytized'' RG equation (i.e., RG equation with recovered proper analytic properties, see Sec. II). The latter enables one to take consistently into account the higher loop contributions to the new analytic running coupling in the spacelike region.

The layout of the paper is as follows. In Sec. II the new analytic running coupling is considered in the spacelike region. The method for determining the coefficients of the analytized RG equation is proposed that enables one to take consistently into account the higher loop contributions to NARC. By invoking the asymptotic freedom condition the expression for the new analytic running coupling, independent of the normalization point, is obtained. In Sec. III the continuation of the new analytic running coupling to the timelike region is constructed. It is shown that distinction between the corresponding values of NARC in the spacelike and timelike regions becomes valuable in the intermediate energy region. This fact may be important when extracting the QCD running coupling from experimental data. Further, it is shown explicitly that for the timelike NARC the respective β function is proportional to the spectral density, thus confirming the hypothesis due to Schwinger. In Sec. IV the results on studies of the new model for the QCD analytic running coupling are briefly summarized. In the framework of the approach developed the description of the inclusive τ lepton decay is performed. The obtained value of the parameter $\Lambda_{\rm QCD}$ fairly well agrees with its previous estimations. This implies the applicability of the new analytic running coupling to description of the both typical perturbative and intrinsically nonperturbative processes of quantum chromodynamics. In the Conclusion (Sec. V) the obtained results are formulated in a compact way, and further studies in this approach are outlined.

II. NEW ANALYTIC RUNNING COUPLING IN THE SPACELIKE REGION

In paper [1] a new model for the QCD analytic running coupling has been proposed. The model is based on the unification of the so-called analytic approach to QCD [2] with the RG formalism. Obvious advantage of the new analytic running coupling is incorporation, in a single expression, of

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[1]In this paper a metric with the signature $(-1,1,1,1)$ is used, so that $q^2>0$ corresponds to a spacelike momentum transfer.

IR enhancement[2] and asymptotic freedom. Remarkably, the model contains no additional parameters. Similarly to the standard case, $\Lambda_{\rm QCD}$ remains the only characterizing parameter of the theory. By making use of the NARC in Refs. [1,3,4] the confining interquark potential was explicitly derived without invoking any additional assumptions, the acceptable estimation of the parameter $\Lambda_{\rm QCD}$ being obtained. It is also to be noted that NARC is free of unphysical singularities in the physical region $q^2>0$.

According to Ref. [1], in the spacelike (Euclidean) region $q^2>0$ the new analytic running coupling at the l-loop level, ${}^{\rm N}\alpha_{\rm an}^{(l)}(q^2)$, is defined as the solution of the analytized RG equation

$$\frac{d\ln[{}^{\rm N}\tilde{\alpha}_{\rm an}^{(l)}(q^2)]}{d\ln q^2}=-\left\{\sum_{j=0}^{l-1}B_j[\tilde{\alpha}_{\rm s}^{(l)}(q^2)]^{j+1}\right\}_{\rm an}. \tag{1}$$

Here $\tilde{\alpha}_{\rm s}^{(l)}(q^2)$ is the l-loop perturbative running coupling, $\tilde{\alpha}(q^2)\equiv\alpha(q^2)\beta_0/(4\pi)$. The curly brackets $\{S(q^2)\}_{\rm an}$ mean the "analytization" (i.e., the recovering of proper analytic properties in the q^2 variable) of the function $S(q^2)$ by making use of the Källén-Lehmann spectral representation

$$\{S(q^2)\}_{\rm an}\equiv\int_0^\infty\frac{\varrho(\sigma)}{\sigma+q^2}d\sigma, \tag{2}$$

where the spectral density $\varrho(\sigma)$ is determined by the initial (perturbative) expression for $S(q^2)$:

$$\varrho(\sigma)=\frac{1}{2\pi i}\lim_{\varepsilon\to 0_+}[S(-\sigma-i\varepsilon)-S(-\sigma+i\varepsilon)]. \tag{3}$$

In addition, according to the model proposed in [1] the solution of the RG equation (1) before analytization must be the perturbative running coupling at the respective loop level. In other words, at any loop level the relationship

$$\frac{d\ln[\tilde{\alpha}_{\rm s}^{(l)}(q^2)]}{d\ln q^2}=-\sum_{j=0}^{l-1}B_j[\tilde{\alpha}_{\rm s}^{(l)}(q^2)]^{j+1} \tag{4}$$

holds. It is this condition that enables one to determine consistently the coefficients B_j on the right-hand side of Eq. (1).

For this purpose let us consider the standard RG equation for the invariant charge $g(\mu)$ (see, e.g., Refs. [6,7])

$$\frac{1}{g(\mu)}\frac{dg(\mu)}{d\ln\mu}=\beta(g(\mu)). \tag{5}$$

For the β function the perturbative expansion

$$\beta(g(\mu))=-\left\{\beta_0\left[\frac{g^2(\mu)}{16\pi^2}\right]+\beta_1\left[\frac{g^2(\mu)}{16\pi^2}\right]^2+\cdots\right\} \tag{6}$$

takes place, where $\beta_0=11-2n_{\rm f}/3$, $\beta_1=102-38n_{\rm f}/3$, and $n_{\rm f}$ is the number of active quarks. Further, Eq. (5) can be rewritten in the following form:

$$\frac{d\ln[g^2(\mu)]}{d\ln\mu^2}=\beta(g(\mu)). \tag{7}$$

Introducing the standard notations $\alpha_{\rm s}(\mu^2)=g^2(\mu)/(4\pi)$ and $\tilde{\alpha}_{\rm s}(\mu^2)=\alpha_{\rm s}(\mu^2)\beta_0/(4\pi)$, Eq. (7) at the l-loop level can be reduced to Eq. (4)

$$\frac{d\ln[\tilde{\alpha}_{\rm s}^{(l)}(\mu^2)]}{d\ln\mu^2}=-\sum_{j=0}^{l-1}\beta_j\left[\frac{\tilde{\alpha}_{\rm s}^{(l)}(\mu^2)}{\beta_0}\right]^{j+1}, \tag{8}$$

if one puts $B_j=\beta_j/(\beta_0)^{j+1}$. Actually, in this case Eq. (4) is nothing but the standard perturbative RG equation that defines the invariant charge.

Thus, in the spacelike region the new analytic running coupling at the l-loop level ${}^{\rm N}\alpha_{\rm an}^{(l)}(q^2)$ is defined as the solution of the equation

$$\frac{d\ln[{}^{\rm N}\tilde{\alpha}_{\rm an}^{(l)}(q^2)]}{d\ln q^2}=-\left\{\sum_{j=0}^{l-1}B_j[\tilde{\alpha}_{\rm s}^{(l)}(q^2)]^{j+1}\right\}_{\rm an},\quad B_j=\frac{\beta_j}{\beta_0^{j+1}}. \tag{9}$$

At the one-loop level Eq. (9) can be integrated explicitly with the result [1]

$${}^{\rm N}\alpha_{\rm an}^{(1)}(q^2)=\frac{4\pi}{\beta_0}\frac{z-1}{z\ln z},\quad z=\frac{q^2}{\Lambda^2}. \tag{10}$$

At the higher loop levels there is only the integral representation for the new analytic running coupling. One should note from the very beginning, that the solution of Eq. (9) is determined up to a constant factor due to the logarithmic derivative on its left-hand side. In our previous studies this problem has been eliminated by normalization of the solution to Eq. (9) on its value at a point q_0^2 [see, e.g., Eq. (2) in Ref. [8]]. In this paper we propose to remove this ambiguity in a more physical way. Indeed, this can be easily achieved by invoking the condition of the asymptotic freedom, namely ${}^{\rm N}\alpha_{\rm an}^{(l)}(q^2)\to\alpha_{\rm s}^{(l)}(q^2)$ when $q^2\to\infty$ (in fact, it has already been employed at the one-loop level, see Eq. (10)). Then, involving the similar condition ${}^{\rm N}\alpha_{\rm an}^{(l)}(q^2)\to{}^{\rm N}\alpha_{\rm an}^{(1)}(q^2)$ when $q^2\to\infty$, we obtain the following integral representation for the l-loop new analytic running coupling (see Ref. [9] for the details):

$${}^{\rm N}\alpha_{\rm an}^{(l)}(q^2)=\frac{4\pi}{\beta_0}\frac{z-1}{z\ln z}\exp\left[\int_0^\infty\Delta\mathcal{R}^{(l)}(\sigma)\ln\left(1+\frac{\sigma}{z}\right)\frac{d\sigma}{\sigma}\right], \tag{11}$$

where $\Delta\mathcal{R}^{(l)}(\sigma)=\mathcal{R}^{(l)}(\sigma)-\mathcal{R}^{(1)}(\sigma)$, and

[2] It is worth noting here that such a behavior of invariant charge follows from the analysis of the Schwinger-Dyson equations [5].

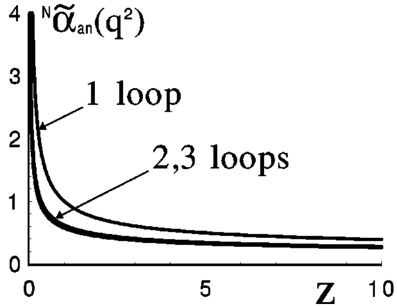

FIG. 1. The new analytic running coupling ${}^{\rm N}\tilde{\alpha}_{\rm an}(q^2)$ [Eq. (11)] in the spacelike region at the one-, two-, and three-loop levels, $z = q^2/\Lambda^2$.

$$\mathcal{R}^{(l)}(\sigma)=\frac{1}{2\pi i}\lim_{\varepsilon\to 0_+}\sum_{j=0}^{l-1} B_j\{[\tilde{\alpha}_{\rm s}^{(l)}(-\sigma-i\varepsilon)]^{j+1} -[\tilde{\alpha}_{\rm s}^{(l)}(-\sigma+i\varepsilon)]^{j+1}\}, \quad \sigma\geqslant 0. \tag{12}$$

It is worth noting here that for the NARC the Källén-Lehmann representation holds

$${}^{\rm N}\alpha_{\rm an}^{(l)}(q^2)=\frac{4\pi}{\beta_0}\int_0^\infty \frac{{}^{\rm N}\tilde{\rho}^{(l)}(\sigma)}{\sigma+z}\,d\sigma, \quad z=\frac{q^2}{\Lambda^2}, \tag{13}$$

where the spectral density at the one-loop level is

$${}^{\rm N}\tilde{\rho}^{(1)}(\sigma)=\left(1+\frac{1}{\sigma}\right)\frac{1}{\ln^2\sigma+\pi^2}, \tag{14}$$

and the l-loop spectral density ${}^{\rm N}\tilde{\rho}^{(l)}(\sigma)$ is defined below [see Eq. (18)]. Figure 1 shows the new analytic running coupling (11) computed at the one-, two-, and three-loop levels. It is clear from these curves that NARC possesses a quite good loop stability (for the properties of the new analytic running coupling see Refs. [8,9]).

An important merit of the new analytic running coupling is that it allows one to obtain the confining quark-antiquark $(q\bar{q})$ potential without invoking any additional assumptions [1,3]. Besides, the interquark potential contains no adjustable parameters (it depends solely on the distance r and the parameter $\Lambda_{\rm QCD}$), and at small distances it has the standard behavior prescribed by the asymptotic freedom. Comparison of the $q\bar{q}$ potential generated by the NARC [1] with the phenomenological (Cornell) potential and with the lattice simulation data [10], as well as the estimation of the gluon condensate value on the base of NARC give a consistent value[3] of the parameter $\Lambda_{\rm QCD}$: $\Lambda \simeq 600\pm 90$ MeV. This can be considered as an evidence of the fact that the new analytic running coupling adequately takes into account the nonperturbative nature of quantum chromodynamics (see also Ref. [11]).

[3]This estimation corresponds to the one-loop level with three active quarks.

III. NEW ANALYTIC RUNNING COUPLING IN THE TIMELIKE REGION

In the previous section the definition of the new analytic running coupling in the spacelike region has been given. However, for consistent description of a number of QCD processes (for example, τ lepton decay or e^+e^- annihilation to hadrons) one has to use the continuation of the running coupling to the timelike region.

In Ref. [12] the procedure of continuation of the invariant charge from the spacelike region to the timelike region (and vise versa) was elaborated by making use of the dispersion relation for the Adler D function. In particular, if the running coupling in the spacelike region is $\alpha(q^2)$, then its consistent continuation to the timelike region is defined by the integral

$$\hat{\alpha}(s)=\frac{1}{2\pi i}\int_{s+i\varepsilon}^{s-i\varepsilon}\frac{d\zeta}{\zeta}\alpha(-\zeta), \quad s=-q^2>0, \tag{15}$$

where the integration contour goes from the point $s+i\varepsilon$ to the point $s-i\varepsilon$ and lies in the region of analyticity of the function $\alpha(-\zeta)$. Here and further the running coupling in the spacelike region is denoted by $\alpha(q^2)$ and in the timelike region by $\hat{\alpha}(s)$.

In order to simplify Eq. (15) let us choose the integration contour in the following way. From the point $s+i\varepsilon$ the integration path goes in a parallel way with the real axis to infinity, then along the circle with an infinitely large radius it goes counter-clockwise to the point $(\infty-i\varepsilon)$, and then it goes in a parallel way with the real axis to the point $s-i\varepsilon$. As a result, the continuation of the one-loop new analytic running coupling ${}^{\rm N}\alpha_{\rm an}^{(1)}(q^2)$ to the timelike region is given by

$${}^{\rm N}\hat{\alpha}_{\rm an}^{(1)}(s)=\frac{4\pi}{\beta_0}\int_{s/\Lambda^2}^\infty {}^{\rm N}\tilde{\rho}^{(1)}(\zeta)\,\frac{d\zeta}{\zeta} \tag{16}$$

with the spectral density ${}^{\rm N}\tilde{\rho}^{(1)}(\zeta)$ defined in Eq. (14).

The account of higher loop corrections leads to significant technical complications. So, at the l-loop level for the new analytic running coupling in the timelike region the integral representation of the same form takes place:

$${}^{\rm N}\hat{\alpha}_{\rm an}^{(l)}(s)=\frac{4\pi}{\beta_0}\int_{s/\Lambda^2}^\infty {}^{\rm N}\tilde{\rho}^{(l)}(\zeta)\,\frac{d\zeta}{\zeta}, \tag{17}$$

where

$${}^{\rm N}\tilde{\rho}^{(l)}(\zeta)={}^{\rm N}\tilde{\rho}^{(1)}(\zeta)\exp\left[\int_0^\infty \Delta\mathcal{R}^{(l)}(\sigma)\ln\left|1-\frac{\sigma}{\zeta}\right|\frac{d\sigma}{\sigma}\right] \times\left[\cos\psi^{(l)}(\zeta)+\frac{\ln\zeta}{\pi}\sin\psi^{(l)}(\zeta)\right], \tag{18}$$

$\mathcal{R}^{(l)}(\sigma)$ is defined in Eq. (12), and

$$\psi^{(l)}(\zeta)=\pi\int_\zeta^\infty \Delta\mathcal{R}^{(l)}(\sigma)\,\frac{d\sigma}{\sigma}. \tag{19}$$

In Eq. (18) the principle value of the integral is assumed.

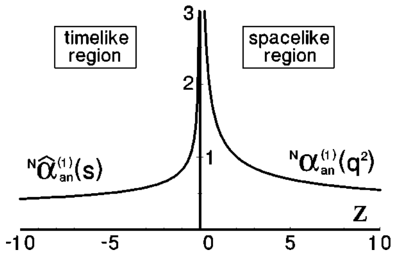

FIG. 2. The one-loop new analytic running coupling in the spacelike ($q^2>0$) and timelike ($s=-q^2>0$) regions [Eqs. (10) and (16), respectively], $z=q^2/\Lambda^2$.

It is interesting to note here that, in our approach, a simple relation holds between the β function, corresponding to the running coupling in the timelike region [Eq. (17)], and the relevant spectral density:

$$\beta(\hat{\alpha}(s))\equiv\frac{d[{}^{\mathrm{N}}\hat{\alpha}_{\mathrm{an}}^{(l)}(s)]}{d[\ln s]}=-\frac{4\pi}{\beta_0}\,{}^{\mathrm{N}}\tilde{\rho}^{(l)}(s). \tag{20}$$

Thus the β function in question proves to be proportional to the spectral density ${}^{\mathrm{N}}\tilde{\rho}^{(l)}(s)$. Obviously, this result completely agrees with the attempts, originated by Schwinger [13], to find a direct physical interpretation of the β function (the so-called Schwinger hypothesis, see Refs. [12,14]).

The plots of the functions ${}^{\mathrm{N}}\alpha_{\mathrm{an}}^{(1)}(q^2)$ and ${}^{\mathrm{N}}\hat{\alpha}_{\mathrm{an}}^{(1)}(s)$ are shown in Fig. 2. At large values of the arguments these expressions have identical behavior prescribed by the asymptotic freedom $\alpha(q^2)\sim 1/\ln|z|$. However, for intermediate energies the difference between these couplings is rather valuable. The ratio of the one-loop new analytic running coupling in the spacelike region ${}^{\mathrm{N}}\alpha_{\mathrm{an}}^{(1)}(q^2)$ to its continuation to the timelike region ${}^{\mathrm{N}}\hat{\alpha}_{\mathrm{an}}^{(1)}(-q^2)$ is presented in Fig. 3. It follows from this figure that the deviation increases when approaching the IR region, and, in particular, rises up to about 10% when $\sqrt{s}\simeq 10$ GeV. Apparently, this circumstance should be taken into account when extracting the QCD running coupling from experimental data.

IV. DISCUSSION

The basic idea of the analytic approach to quantum field theory can be treated as an attempt to recover explicitly the proper analytic properties in the q^2 variable for the relevant physical quantities. A concrete realization of this approach may be different. For example, in the original model by Shirkov and Solovtsov [2] the analytization procedure is applied to the perturbative invariant charge. Unlike this proposal, our new model for the QCD analytic running coupling [1] seeks to recover the proper analytic properties of the RG equation. More precisely, the new analytic running coupling is derived as the solution of the RG equation with the β function "improved" by the analytization procedure (see also Refs. [1,11] for the details). From the general point of view, in this case one can anticipate new properties of the invariant charge. Indeed, in our model there is the IR enhancement of the new analytic running coupling, while the invariant charge due to Shirkov and Solovtsov tends to a finite value in the infrared limit (the IR freezing of the running coupling). It is this property of our model that ultimately leads to the confining quark-antiquark potential. Further, the behavior of the NARC in the UV region is completely determined by the asymptotic freedom in QCD. Thus, in our model [1] the IR enhancement and the UV asymptotic freedom are involved in a single expression for the new analytic running coupling without introducing any additional parameters into the theory. It is important to emphasize here that such a behavior of the invariant charge is in agreement with the Schwinger-Dyson equations (see Ref. [5]).

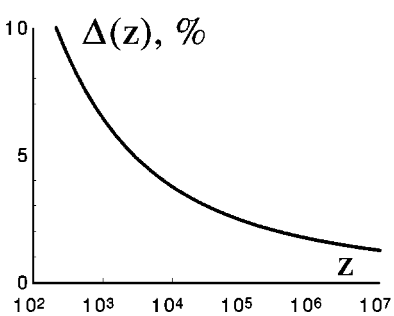

FIG. 3. The relative difference $\Delta(z)=[{}^{\mathrm{N}}\alpha_{\mathrm{an}}^{(1)}(q^2)/{}^{\mathrm{N}}\hat{\alpha}_{\mathrm{an}}^{(1)}(-q^2)-1]\times 100\%$ between the values of the one-loop new analytic running coupling in the spacelike and timelike regions, $z=q^2/\Lambda^2$.

Let us turn now to the perturbative phenomena. One of the QCD processes, the most sensitive to the low energy behavior of the invariant charge, is the inclusive semileptonic branching ratio of the τ lepton decay, R_τ. Therefore, we address the consideration of this process restricting ourselves, for simplicity, to the one loop level.

We proceed from the nonstrange part of the R_τ ratio associated with the vector quark currents

$$R_{\tau,\mathrm{V}}=\frac{N_{\mathrm{c}}}{2}|V_{\mathrm{ud}}|^2 S_{\mathrm{EW}}(1+\delta_{\mathrm{QCD}}), \tag{21}$$

where $N_{\mathrm{c}}=3$ is the number of quark colors, $|V_{\mathrm{ud}}|=0.9735\pm 0.0008$ denotes the Cabibbo-Kobayashi-Maskawa matrix element [15], $S_{\mathrm{EW}}=1.0194\pm 0.0040$ is the electroweak factor [16], and δ_{QCD} is the QCD correction (see, e.g., Refs. [17,18] and references therein). A recent experimental measurement of the ratio (21) by ALEPH Collaboration gave [19] $R_{\tau,\mathrm{V}}=1.775\pm 0.017$.

In accordance with the standard prescription the one-loop QCD correction in our approach is determined by the integral

$$\delta_{\mathrm{QCD}}=\frac{2}{\pi}\int_{4m^2}^{M_\tau^2}\frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+2\frac{s}{M_\tau^2}\right){}^{\mathrm{N}}\hat{\alpha}_{\mathrm{an}}^{(1)}(s), \tag{22}$$

where ${}^{\mathrm{N}}\hat{\alpha}_{\mathrm{an}}^{(1)}(s)$ is the new analytic running coupling in the timelike region (16), and m is the light quark mass, with its world average value $m=(4.25\pm 1.75)$ MeV [15]. It is worth noting here that there is no need to involve the contour integration in Eq. (22), since NARC (16) does not contain unphysical singularities in the region $s>0$. In other words, the integration in Eq. (22) can be performed in a straightforward way. By introducing the notations $x=s/M_\tau^2$, $x_0=4m^2/M_\tau^2$, and $c_0=-(2x_0-2x_0^3+x_0^4)$, one can represent Eq. (22) in a form convenient for integration

$$\delta_{\mathrm{QCD}}=\frac{4\pi}{\beta_0}\int_{x_0}^{1}(2x-2x^3+x^4+c_0)\,{}^{\mathrm{N}}\tilde{\rho}^{(1)}\left(x\frac{M_\tau^2}{\Lambda^2}\right)\frac{dx}{x}+\frac{4\pi}{\beta_0}(1+c_0)\int_{M_\tau^2/\Lambda^2}^{\infty}{}^{\mathrm{N}}\tilde{\rho}^{(1)}(\sigma)\frac{d\sigma}{\sigma}, \tag{23}$$

where the spectral density ${}^{\mathrm{N}}\tilde{\rho}^{(1)}(\sigma)$ is defined in Eq. (14).

For the value of $R_{\tau,\mathrm{V}}$ given above [19] we obtain the estimation $\Lambda=(560\pm70)$ MeV for two active quarks (the uncertainty accounts the errors in the values of $R_{\tau,\mathrm{V}}$, $|V_{\mathrm{ud}}|$, S_{EW}, and m). However, in order to compare this result with earlier estimations, one should continue it to the region of three active quarks. This gives the value $\Lambda=(517\pm70)$ MeV that perfectly agrees with the previous estimations. We remind that this value of the parameter Λ_{QCD} corresponds to the one-loop level with three active quarks.

Thus, in the framework of the model under consideration one succeeds in obtaining a consistent, with its previous estimations, value of the parameter Λ_{QCD} by making use of the experimental data on the inclusive τ lepton decay, the world average value of light quark mass being employed.

From all this we may infer that the new model for the QCD analytic running coupling substantially incorporates, in a consistent way, both perturbative (high energy) and nonperturbative (low energy) behavior of quantum chromodynamics.

V. CONCLUSION

This paper is a further development of the new model for the QCD analytic running coupling proposed in [1]. This running coupling naturally arises under unification of the analytic approach to QCD with the RG formalism. The elaborated method for determination of the coefficients of the analytized RG equation enables one to take consistently into account the higher loop contributions to the new analytic running coupling. The invoking of the asymptotic freedom condition allows one to derive the expression for the NARC independent of the normalization point. The continuation of the new analytic running coupling to the timelike region is constructed. It is demonstrated that the difference between the respective values of the new analytic running coupling in the spacelike and timelike regions is considerable for intermediate energies. This fact seems to be important for extracting the running coupling from experimental data. It is shown that for the timelike new analytic running coupling the hypothesis due to Schwinger is confirmed. In the framework of the new model for the QCD analytic running coupling the description of the inclusive τ lepton decay is performed. The obtained value of the parameter Λ_{QCD} is in a good agreement with its previous estimations.

Thus, the application of the new analytic running coupling to the description of various physical phenomena (both, perturbative and intrinsically nonperturbative ones) gives a consistent estimation of the parameter Λ_{QCD}. This fact, as well as the agreement of the IR behavior of the NARC with Schwinger–Dyson equations, and ability to obtain, without invoking any additional assumptions, the confining quark-antiquark potential, implies that the new model for the QCD analytic running coupling substantially incorporates, in a consistent way, both perturbative (high energy) and nonperturbative (low energy) behavior of quantum chromodynamics.

In further studies it would be useful to obtain simple explicit expressions which approximate the new analytic running coupling in the spacelike and timelike regions with account of the higher loop corrections.

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