Minijet transverse spectrum in high-energy hadron-nucleus collisions

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Hadron-nucleus collisions at CERN LHC energies are studied by including explicitly semihard parton rescatterings in the collision dynamics. Under rather general conditions, we obtain explicit formulas for the semihard cross section and the inclusive minijet transverse spectrum. As an effect of the rescatterings the spectrum is lowered at small p_t and is enhanced at relatively large transverse momenta, the deformation being more pronounced at increasing rapidity. Its study allows to test the proposed interaction mechanisms and represents an important baseline to examine nucleus-nucleus collisions.

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I. INTRODUCTION

Given the rapid growth of the hard cross section in hadronic and nuclear collisions [1], a typical inelastic event will be dominated by the perturbative regime at very high energies so that, at the CERN Large Hadron Collider (LHC), one may expect to be able to derive global features of the inelastic interaction by perturbative methods. When the perturbative regime dominates a physical observable which represents global features of the inelastic interaction, the hadron (or nuclear) scale should therefore also appear in the corresponding perturbative calculation, presumably introduced through the nonperturbative input. At LHC energies the parton densities involved in the typical interaction are so high that one has to deal with processes initiated by many partons. The non-perturbative input in this case is given by the multiparton distribution function, which is a dimensional quantity, and may therefore introduce the above mentioned scale in the perturbative computations, which would otherwise be scale free.

By introducing interactions initiated by many partons one may therefore gain the capability of describing, by means of perturbative QCD, at least a few general properties of the typical interaction at the energy of the LHC. To pursue such a program one should then (i) evaluate perturbative QCD processes involving many partons in the initial state, (ii) face the problem of the unknown non perturbative input and develop a strategy in that respect, and (iii) study the infrared problem by finding observable quantities which are infrared stable. This last step represents the final achievement of the whole program.

An explicit approach to semihard interactions in heavy ion collisions at the LHC, on the lines previously described, has been accomplished, at least partially, with the help of a few simplifying hypotheses. The program was implemented in Refs. [2–5], and various physical quantities have been evaluated in Refs. [6,7].

The purpose of the present paper is to discuss the case of hadron-nucleus interactions (hA, for brevity). Being intermediate between hadron-hadron (hh) and nucleus-nucleus (AA) interactions, hadron-nucleus interactions allow several simplifications in the formalism developed to discuss heavy-ion collisions. In fact, as will be shown hereafter and differently with respect to the latter case, in the hadron-nucleus instance we were able to obtain closed analytical expressions for the semihard cross section under rather general conditions. We will then study the inclusive minijet transverse spectrum, which is related in a direct way to the underlying dynamics, and is therefore an important baseline for the study of nucleus-nucleus collisions.

Besides its intrinsic interest, inclusion of semihard rescatterings in the computation of the transverse spectrum has been advocated by many authors [8–12] as the basic mechanism underlying the Cronin effect [13], namely the deformation of the hadron p_t spectra in nuclear collisions as compared with the expectations of a single large-p_t production mechanism. Multiple parton collisions have also been related to higher-twist parton distributions [14–16]. A nonperturbative study of the transverse spectrum in hA collisions in the framework of the McLerran-Venugopalan model for nuclear and hadronic collisions was presented in Ref. [17].

Another reason for the interest in hadron-nucleus collisions is that theoretical models can be tested against experimental data in a situation where further nuclear effects are absent, like, e.g., the formation of a hot and dense medium which can further modify the transverse spectrum via energy loss [18,19]. Therefore a detailed understanding of hA collisions represents an important baseline for the generalization to AA collisions [20,21] and for the discovery of novel physical effects [22].

In Sec. II we discuss the semihard hA cross section and recall the main ideas and tools needed in the present approach. Section III is devoted to a discussion of the inclusive minijet transverse spectrum, with particular emphasis on the mechanism of subtraction of infrared divergences, which is explicitly implemented in our approach. Results of numerical evaluations of the inclusive spectra of minijets in hadron-nucleus collisions are presented in Sec. IV. Section V is devoted to a concluding summary.

II. SEMIHARD HADRON-NUCLEUS CROSS SECTION

To face the problem of unitarity corrections to the computation of the hA cross section, we make use of the self-

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shadowing property of the hard component of the interaction [23], which we recall briefly in Sec. II A. Because of self-shadowing all unitarity corrections to the semihard cross-section will be expressed by means of semihard partonic cross section only [see Eq. (2.3)], so that one does not need to make any commitment on the soft component when only the semihard part of the nuclear cross section is of interest. Self-shadowing allows, moreover, to also control the soft component of the interaction by perturbative means, since that contribution is limited to a fraction of the cross section proportional to the probability of not having any hard interaction at all [see Eq. (2.4)].

While Ref. [23] considered colliding nucleons as basic degrees of freedom, we want to represent the semihard hA cross section in an analogous way, but considering partons instead of nucleons as elementary objects. Indeed, the semihard component of the interaction satisfies the requirements of the self-shadowing cross sections if one assumes that a parton which has undergone interactions with a large momentum exchange can always be recognized in the final state. To represent the interaction between hadrons and nuclei in terms of partonic interactions, each one with a relatively large momentum exchange, one needs to write the cross section for a given nonperturbative input, namely for a definite partonic configuration of the two interacting objects. Then, as a perturbative input, one needs to write the probability of having at least one semihard interaction between the two configurations of partons. We discuss the latter in Sec. II B, and in Sec. II C we introduce a functional formalism to deal with multiparton distributions [4] and we combine them with partonic interaction probabilities to obtain the semihard hA cross section [24].

A. Self-shadowing

Let us consider the inelastic hadron-nucleus cross-section $(\sigma_{in})_A$, whose expression may be expanded, in the Glauber approach, as a binomial probability distribution of inelastic hadron-nucleon collisions:

$$(\sigma_{in})_A=\int d^2\beta[1-(1-\sigma_{in}T(\beta))^A]$$
$$=\int d^2\beta\sum_{n=1}^{A}\binom{A}{n}[\sigma_{in}T(\beta)]^n[1-\sigma_{in}T(\beta)]^{A-n}. \tag{2.1}$$

In Eq. (2.1) $T(\beta)$ is the nuclear thickness function, which depends on the impact parameter β and is normalized to 1, A is the atomic mass number and σ_{in} is the inelastic hadron-nucleon cross-section. One may classify all events according to a given selection criterion, which we call $\mathcal{C}$, while we use $\mathcal{N}$ to refer to events that are not of type $\mathcal{C}$. In particular, $\mathcal{C}$ may represent hard hadron-nucleon interactions. We assume that in a hadron-nucleon collision all events of type $\mathcal{C}$ contribute to $\sigma_{\mathcal{C}}$; all other events contribute to $\sigma_{\mathcal{N}}$, so that the inelastic hadron-nucleon cross-section may be written as

$$\sigma_{in}=\sigma_{\mathcal{C}}+\sigma_{\mathcal{N}}. \tag{2.2}$$

One may then ask for an expression of the cross section $(\sigma_{\mathcal{C}})_A$ to produce events of type $\mathcal{C}$ in a collision of a hadron against a nuclear target. Then, using Eq. (2.2) in Eq. (2.1) and disregarding the terms that do not contain $\sigma_{\mathcal{C}}$ one obtains [23]

$$(\sigma_{\mathcal{C}})_A=\int d^2\beta[1-(1-\sigma_{\mathcal{C}}T(\beta))^A]$$
$$=\int d^2\beta\sum_{n=1}^{A}\binom{A}{n}[\sigma_{\mathcal{C}}T(\beta)]^n[1-\sigma_{\mathcal{C}}T(\beta)]^{A-n}. \tag{2.3}$$

Note that, in spite of the fact that we included superpositions of elementary events of type $\mathcal{C}$ with events both of kind $\mathcal{C}$ and of kind $\mathcal{N}$, the nuclear cross section $(\sigma_{\mathcal{C}})_A$ is obtained by summing all possible multiple hadron-nucleon interactions of type $\mathcal{C}$ alone with a binomial probability distribution, precisely as $(\sigma_{in})_A$ is obtained by a binomial distribution of hadron-nucleon inelastic interactions.

The only part of the nuclear interaction still missing is the cross section for elementary events of type $\mathcal{N}$ alone. It can be obtained by considering the following difference:

$$\frac{d(\sigma_{in})_A}{d^2\beta}-\frac{d(\sigma_{\mathcal{C}})_A}{d^2\beta}$$
$$=[1-\sigma_{\mathcal{C}}T(\beta)]^A\times\left\{1-\left[1-\frac{\sigma_{\mathcal{N}}T(\beta)}{1-\sigma_{\mathcal{C}}T(\beta)}\right]^A\right\}$$
$$=[1-\sigma_{\mathcal{C}}T(\beta)]^A\times\sum_{k=1}^{A}\binom{A}{k}\left(\frac{\sigma_{\mathcal{N}}T(\beta)}{1-\sigma_{\mathcal{C}}T(\beta)}\right)^k$$
$$\times\left(1-\frac{\sigma_{\mathcal{N}}T(\beta)}{1-\sigma_{\mathcal{C}}T(\beta)}\right)^{A-k}, \tag{2.4}$$

which is therefore bounded by $[1-\sigma_{\mathcal{C}}T(\beta)]^A$, namely by the probability of not having any interaction of type $\mathcal{C}$ at a given impact parameter β. The ratio $\sigma_{\mathcal{N}}T(\beta)/[1-\sigma_{\mathcal{C}}T(\beta)]$ may be understood as the probability of a hadron-nucleon interaction at a given impact parameter, under the condition that no event of type $\mathcal{C}$ takes place. Hence after removing all events of type $\mathcal{C}$ the interaction is expressed by a binomial distribution of events of type $\mathcal{N}$.

B. Semihard rescatterings

When the kinematics of the collision allows a high density of target partons, namely at a high center of mass energy and large atomic numbers, a single projectile parton may interact with several targets with large momentum exchange in different directions in transverse space. The simplest possibility of such an interaction was discussed in Ref. [5], where the forward amplitude of the process and all the cuts were derived in the case of a pointlike projectile against two pointlike targets, in the limit of an infinite number of colors and for $t/s\to 0$. In this case one finds that the different cuts of the $3\to 3$ forward amplitude are all proportional to one another, and the proportionality factors are the AGK weights

[25]. A consequence is that one may express the three-body interaction as a product of two-body interaction probabilities. The results obtained in this simple case may indicate a convenient approximation of the many-parton interaction probability: one can in fact argue that the many-parton interaction process may be approximated by a product of two-parton interactions, so that one can call the process *reinteraction* or *rescattering*.

The whole interaction is therefore expressed in terms of two-parton interaction probabilities, precisely as the interaction between two nuclei has been expressed in terms of hadron-nucleon collisions in Sec. II A. Hence, given a configuration with n partons of the projectile and m partons of the target, we introduce the probability $\mathcal{P}_{n,m}$ of having at least one partonic collision, in a way analogous to the expression of the inelastic nucleus-nucleus cross-section [26],

$$\mathcal{P}_{n,m}=\left[1-\prod_{i=1}^{n}\prod_{j=1}^{m}(1-\hat{\sigma}_{ij})\right], \tag{2.5}$$

where $\hat{\sigma}_{ij}$ is the probability of interaction of a given pair of partons i and j. Since the distance over which the hard interactions are localized is much smaller than the soft interaction scale, one may approximate $\hat{\sigma}(x_ix_j;b_i-b_j)\approx\sigma(x_ix_j)\delta^{(2)}(b_i-b_j)$, where x_i and x_j are the momentum fractions of the colliding partons, b_i and b_j their transverse coordinates, and $\sigma(x_ix_j)$ is the partonic cross section, whose infrared divergence is cured by introducing a regulator p_0. For example, p_0 may be the lower cutoff on the momentum exchange in each partonic collision, or a small mass introduced in the transverse propagator to prevent the divergence of the cross section at zero momentum exchange. The expression for $\mathcal{P}_{n,m}$ is the analogue of Eq. (2.3) and represents the explicit implementation of self-shadowing for the interaction of two partonic configurations.

C. Hadron-nucleus cross section

At a given resolution, provided by the regulator p_0, one may find the nuclear (or hadronic) system in various partonic configurations. We call $P^{(n)}(u_1\cdots u_n)$ the probability of a configuration with n partons (the *exclusive n-parton distribution*) where $u_i\equiv(b_i,x_i)$ represents the transverse coordinate of the ith parton b_i and its longitudinal fractional momentum x_i. The distributions are symmetric in the variables u_i, and can be obtained from a generating functional defined with the help of auxiliary functions $J(u)$ as follows [4]:

$$P^{(n)}(u_1,\ldots,u_n)=\frac{\delta}{\delta J(u_1)}\cdots\frac{\delta}{\delta J(u_n)}\mathcal{Z}[J]_{|J=0},$$

where

$$\mathcal{Z}[J]=\exp\left[\int\Gamma(u)[J(u)-1]du+\sum_{n=2}^{\infty}\frac{1}{n!}\int C^{(n)}(u_1\cdots u_n)\times[J(u_1)-1]\cdots[J(u_n)-1]du_1\cdots du_n\right].$$

$\Gamma(u)$ is the single parton distribution and $C^{(n)}$ are the *n-parton correlations*.

The general expression of the inelastic semihard cross section at a fixed impact parameter may be obtained by folding the interaction probability [Eq. (2.5)] with the exclusive multiparton distributions of the two colliding systems (in our case a hadron h and a nucleus of atomic number A):

$$\frac{d\sigma_H}{d^2\beta}=\int\sum_{m,n=1}^{\infty}\left[\frac{1}{n!}P_h^{(n)}(u_1,\ldots,u_n)\right]\times\mathcal{P}_{n,m}\left[\frac{1}{m!}P_A^{(m)}(u_1',\ldots,u_m')\right]\prod_{i=1}^{n}du_i\prod_{j=1}^{m}du_j', \tag{2.6}$$

where β is the impact parameter between h and A. In the case of hadron-nucleus interactions one may be allowed to neglect rescatterings of the partons of the nucleus. Indeed, even at very high center of mass energies the average number of scattering per incoming parton is smaller than the average number of nucleons along the parton trajectory, except in the very forward rapidity region [6]. With this assumption one can obtain a closed formula for the cross section [4]:

$$\frac{d\sigma_H}{d^2\beta}=\{1-\exp[\delta\cdot(e^{-\delta'\cdot\hat{\sigma}}-1)]\}\times\mathcal{Z}_h[J+1]\mathcal{Z}_A[J'+1]|_{J=J'=0}, \tag{2.7}$$

where the following notation is used:

$$\delta_i=\int du_i\frac{\delta}{\delta J(u_i-\beta)};\quad \delta_j'=\int du_i\frac{\delta}{\delta J'(u_j')}.$$

A meaningful approximation is to consider the nuclear partons uncorrelated, and if we neglect also the correlations inside the projectile hadron we get an explicit expression:

$$\frac{d\sigma_H}{d^2\beta}=1-\exp\left\{-\int du\Gamma_h(u-\beta)[1-e^{-\int\hat{\sigma}(u,u')\Gamma_A(u')du'}]\right\}. \tag{2.8}$$

Note that the cross section is a function of

$$W_h(u,\beta)=\Gamma_h(u-\beta)[1-e^{-\int\hat{\sigma}(u,u')\Gamma_A(u')du'}]=\Gamma_h(u-\beta)\mathcal{P}_A(u), \tag{2.9}$$

which represents the number of projectile partons that have interacted with the target, i.e., the projectile *wounded partons* [2,4]; we call them *minijets*, even if they did not yet hadronize. $\mathcal{P}_A(u)$ represents the probability that a projectile parton with a given $u=(x,b)$ has at least one semihard interaction with the target; hence the cross section is obtained by summing all events with at least one interaction.

One would obtain the same expression for the average number of wounded partons [Eq. (2.9)] under more general hypotheses by working out directly from Eq. (2.6) the average number of projectile partons which have undergone at least one semihard interaction. The only assumption needed is that all the target partons are uncorrelated [2,4]. Therefore, $\int du W(u,\beta) = \langle n \rangle d\sigma/d^2\beta$ represents the integrated inclusive cross section required to detect all scattered projectile partons, and takes into account the correlations of the projectile partons at all orders.

III. INCLUSIVE MINIJET TRANSVERSE SPECTRUM

After the introduction of semihard parton rescatterings, integrated quantities like the semihard cross section and the minijet multiplicity show a weak dependence on the infrared cutoff needed to regularize the infrared divergences arising in perturbative computations [2,6]. Conversely, it will be shown that differential quantities like the minijet p_t-spectrum are more sensitive to the detailed dynamics of the interaction, and show a stronger dependence on the cutoff, if only logarithmic. To reduce this dependence on the cutoff one needs to improve further the picture of the dynamics by also including gluon radiation in the interaction process. Some steps along this line in the case of deep inelastic electron-nucleus scattering have been presented in Ref. [16]. In this paper, however, we neglect the problem of gluon radiation and concentrate on the effects of elastic rescatterings.

The deformation of the high-p_t hadron spectra which leads to the Cronin effect was studied in terms of semihard parton rescatterings in Refs. [8–11], where partons that suffered up to two scatterings were included. This leads to a good description of the data for pA collisions up to $\sqrt{s} = 39$ GeV in the hadron-nucleon center of mass frame. However, the two-scattering approximation breaks down at higher energies, except at very high p_t, and the whole wounded parton transverse spectrum is needed. More phenomenological approaches [12,20] model the effects of multiple scattering as Gaussian p_t broadening for each rescattering suffered by a parton. A random-walk model of the multiple scatterings was proposed in Ref. [21].

A. Transverse spectrum

We can expand the average number of projectile wounded partons [Eq. (2.9)], at a given x and b in a collision with impact parameter β, in the following way:

$$W_h(x,b,\beta) = \Gamma_h(x,b-\beta) \sum_{\nu=1}^{\infty} \frac{\langle n_A(x,b) \rangle^{\nu}}{\nu!} e^{-\langle n_A(x,b) \rangle}, \tag{3.1}$$

where $\langle n_A(x,b) \rangle \equiv \int dx' \Gamma_A(x',b) \sigma(xx')$ is the average number of scatterings of a projectile parton at a given x and b [4]. The average number of wounded partons is then given by the average number of incoming partons Γ_h, multiplied by the probability of having at least one semihard scattering, which is given by a Poisson distribution in the number of scatterings, ν, with average number $\langle n_A(x,b) \rangle$. Therefore, we can obtain the inclusive differential distribution in p_t by introducing a constraint in the transverse momentum integrals that give the integrated parton-parton cross sections in the expression above:

$$\begin{aligned} \frac{dW_h}{d^2p_t}(x,b,\beta) = \Gamma_h(x,b-\beta) \sum_{\nu=1}^{\infty} \frac{1}{\nu!} \int \Gamma_A(x_1',b) \cdots \Gamma_A(x_\nu',b) \\ \times e^{-\int dx' \Gamma_A(x',b)\sigma(xx')} \frac{d\sigma}{d^2k_1} \cdots \frac{d\sigma}{d^2k_\nu} \\ \times \delta^{(2)}(\mathbf{k}_1 + \cdots + \mathbf{k}_\nu - \mathbf{p_t}) \\ \times d^2k_1 \cdots d^2k_\nu dx_1' \cdots dx_\nu'. \end{aligned} \tag{3.2}$$

The limits of integration on x_i' and x' are $xx_i's \geqslant 4k_i^2$ and $xx's \geqslant 4p_0^2$, respectively, and all the distribution functions are evaluated for simplicity at a fixed scale.

By using the above formula one can study the p_t broadening of a wounded parton, in particular the square root of the average transverse momentum squared acquired through its path across the nucleus. Consider a single projectile parton with fixed x and b. The probability that it acquires a certain p_t after the collision is given by Eq. (3.2) divided by the number $\Gamma_h(x,b-\beta)$ of incoming partons:

$$\frac{d\mathcal{P}_A(x,b)}{d^2p_t} = \frac{dW_h}{d^2p_t}(x,b,\beta) \frac{1}{\Gamma_h(x,b-\beta)}.$$

Then the average transverse momentum squared of a wounded parton is given by $\langle p_t^2(x,b) \rangle_A = \langle\langle p_t^2 \rangle\rangle / \langle\langle 1 \rangle\rangle$, where $\langle\langle f(p_t) \rangle\rangle = \int d^2p_t f(p_t) d\mathcal{P}_A / d^2p_t$. By exploiting the azimuthal symmetry of the differential parton-parton cross-sections, and the symmetry of Eq. (3.2) under exchanges of k_i, it is easy to see that

$$\begin{aligned} \langle p_t^2(x,b) \rangle_A &= \frac{1}{\mathcal{P}_A} \int d^2p_t dx' p_t^2 \frac{d\sigma}{d^2p_t}(xx') \Gamma_A(x',b) \\ &= \langle p_t^2(x,b) \rangle_1 \frac{\langle n_A(x,b) \rangle}{\mathcal{P}_A(x,b)}, \end{aligned} \tag{3.3}$$

where

$$\langle p_t^2(x,b) \rangle_1 = \frac{\int d^2p_t dx' p_t^2 \frac{d\sigma(xx')}{d^2p_t} \Gamma_A(x',b)}{\int dx' \sigma(xx') \Gamma_A(x',b)}$$

is the average transverse momentum squared in a single parton-parton collision. The p_t broadening of the wounded partons in a hA collision is then given by the p_t broadening in a single collision multiplied by the average number of rescatterings suffered by a wounded parton. A similar result for the p_t broadening of a fast parton traversing a nuclear medium was derived in Ref. [27]. Two interesting limits can be considered:

$$\langle p_t^2(x,b)\rangle_A \sim \begin{cases} \langle p_t^2(x,b)\rangle_1 & \text{as } p_0\to\infty \\ \langle p_t^2(x,b)\rangle_1 \langle n_A(x,b)\rangle & \text{as } p_0\to 0. \end{cases} \tag{3.4}$$

Since the minijet yield is dominated by transverse momenta of the order of the cutoff, these two limits say roughly that the minijets at high p_t [i.e., high p_0 in Eq. (3.4)], suffer mainly one scattering. On the contrary, at low p_t [i.e., low p_0 in Eq. (3.4)] they undergo a random walk in the transverse momentum plane and the broadening is proportional to the average number of steps in the random walk $\langle n_A\rangle$. This picture will be studied in more detail in Sec. IV A.

An explicit formula for the transverse spectrum can be obtained by studying its Fourier transform, since all the convolutions in Eq. (3.2) turn into products and the sum over ν may be explicitly performed. To this end, we introduce the Fourier transform of the parton-parton scattering cross section:

$$\tilde{\sigma}(v;xx') = \int d^2k e^{i\mathbf{k}\cdot\boldsymbol{v}} \frac{d\sigma}{d^2k}(xx').$$

Note that $\tilde{\sigma}(0;xx')=\sigma(xx')$, and that due to the azimuthal symmetry of $d\sigma/d^2k$, its Fourier transform depends only on the modulus, v, of $\boldsymbol{v}$. Then the transverse spectrum [Eq. (3.2)] may be written as

$$\frac{dW_h}{d^2p_t}(x,b,\beta)=\Gamma_h(x,b-\beta)\int \frac{d^2v}{(2\pi)^2} e^{-i\mathbf{p_t}\cdot\boldsymbol{v}} \tilde{W}_h(v;x,b), \tag{3.5}$$

where

$$\begin{aligned}\tilde{W}_h(v;x,b) &= \sum_{\nu=1}^{\infty} \frac{1}{\nu!}\left[\int dx' \Gamma_A(x',b)\tilde{\sigma}(v;xx')\right]^{\nu} \\ &\quad \times e^{-\int dx' \Gamma_A(x',b)\tilde{\sigma}(0;xx')} \\ &= e^{\int dx' \Gamma_A(x',b)\{\tilde{\sigma}(v;xx')-\tilde{\sigma}(0;xx')\}} \\ &\quad - e^{-\int dx' \Gamma_A(x',b)\tilde{\sigma}(0;xx')}. \end{aligned} \tag{3.6}$$

An immediate consequence is that the transverse spectrum has a finite limit as $p_t\to 0$, even when a cutoff on the momentum exchange is used:

$$\left.\frac{dW_h}{d^2p_t}\right|_{\mathbf{p_t}=0}(x,b,\beta)=\Gamma_h(x,b-\beta)\int \frac{d^2v}{(2\pi)^2}\tilde{W}_h(v;x,b).$$

B. Expansion in the number of scatterings

We can obtain an expansion of $\tilde{W}_h$ in the number of the rescatterings suffered by the incoming parton by expanding Eq. (3.6) in powers of $\tilde{\sigma}$:

$$\begin{aligned}\tilde{W}_h(v;x,b) &= \sum_{\nu=1}^{\infty} \tilde{W}_h^{(\nu)}(v;x,b) \\ &= \sum_{\nu=1}^{\infty} \frac{1}{\nu!}\Bigg[\Bigg(\int dx' \Gamma_A(x',b) \\ &\quad \times[\tilde{\sigma}(v;xx')-\tilde{\sigma}(0;xx')]\Bigg)^{\nu} \\ &\quad - \left(-\int dx' \Gamma_A(x',b)\tilde{\sigma}(0;xx')\right)^{\nu}\Bigg]. \end{aligned} \tag{3.7}$$

Coming back to the p_t space, the expansion of the transverse spectrum in number of scatterings reads

$$\begin{aligned}\frac{dW_h}{d^2p_t}(x,b,\beta) &= \sum_{\nu=1}^{\infty} \frac{dW_h^{(\nu)}}{d^2p_t}(x,b,\beta) \\ &= \sum_{\nu=1}^{\infty} \Gamma_h(x,b-\beta)\int \frac{d^2v}{(2\pi)^2} e^{-i\mathbf{p}_t\cdot\boldsymbol{v}} \\ &\quad \times \tilde{W}_h^{(\nu)}(v;x,b). \end{aligned} \tag{3.8}$$

The series [Eq. (3.7)] can be obtained also by expanding $\tilde{W}(v)$ around $v=0$. Since the variable v is Fourier-conjugated to p_t, the expansion of the transverse spectrum [Eq. (3.8)], will be valid at high p_t, and we expect a breakdown of any truncation at sufficiently low momentum. Note that we can obtain this high-p_t expansion of the spectrum directly in p_t space by expanding the exponential in Eq. (3.2) and collecting the terms of the same order in σ. As an example, the first three terms Eqs. (A1), (A2), and (A6), can be found in the Appendix A. The study of this series is the subject of Sec. III C; numerical results up to $n=3$ scatterings will be discussed in Sec. IV A, and compared to the whole spectrum. In Appendix A we will discuss the symmetrization of the terms of the series.

C. Cancellation of the divergences

All terms of expansion (3.8) are divergent in the infrared region, so that we need to cure them with the regulator p_0. Nevertheless, the infrared divergences are already regularized to a large extent by the subtraction terms originated by the expansion of $\exp[-\langle n_A(x,b)\rangle]$ appearing in Eq. (3.2); namely, by the constraint of probability conservation. This cancellation mechanism was observed also in Ref. [9] for the two-scattering term and in Ref. [19] in a different context.

It is instructive to examine in detail how the subtraction works for the lower order terms of the expansion. We start by considering the case of a single rescattering ($\nu=2$). To simplify the notation we write the elementary differential cross section $d\sigma/d^2k$ as $\sigma(\mathbf{k})$, and notice that it depends only on the modulus k of the momentum. By expressing the semihard cross section as $\sigma=\int d^2k\sigma(\mathbf{k})$ the term of order σ^2 may be written as

$$\frac{dW_h^{(2)}}{d^2p_t}(x,b,\beta)=\Gamma_h(x,b-\beta)\int \Gamma_A(x_1',b)\Gamma_A(x_2',b)\times dx_1'dx_2'd^2k_1d^2k_2\frac{\sigma(\mathbf{k}_1)\sigma(\mathbf{k}_2)}{2}\times[\delta^{(2)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p}_t)-\delta^{(2)}(\mathbf{k}_1-\mathbf{p}_t)-\delta^{(2)}(\mathbf{k}_2-\mathbf{p}_t)], \quad (3.9)$$

where the first term in the square brackets represents two successive scatterings with no absorption. The two negative terms are the corrections induced by the expansion of the absorption factor $\exp[-\langle n_A(x,b)\rangle]$ of the single-scattering term, $\nu=1$ in Eq. (3.2), and correspond to a single-scattering along with the effects of absorption in the initial or final state. The expression we obtained is symmetric in the integration variables $\mathbf{k}_1$ and $\mathbf{k}_2$.

The cutoff dependence is originated by the singular behavior of the integrand for $\mathbf{k}_1\approx 0$ or for $\mathbf{k}_2\approx 0$, since the δ functions in the square brackets prevent the possibility of $\mathbf{k}_1$ and $\mathbf{k}_2$ are both zero at the same time. Because of the symmetry under the exchange $\mathbf{k}_1\leftrightarrow\mathbf{k}_2$, to study the cutoff dependence of Eq. (3.9) it is enough to discuss the integration around $\mathbf{k}_1=0$. In the region $\mathbf{k}_1\approx 0$ the term $\delta^{(2)}(\mathbf{k}_1-\mathbf{p}_t)$ does not contribute, as long as $\mathbf{p}_t$ is finite. The integration in $\mathbf{k}_2$ is done with the help of the δ functions, and one obtains

$$\int d^2k_1\sigma(\mathbf{k}_1)[\sigma(\mathbf{p_t}-\mathbf{k}_1)-\sigma(\mathbf{p_t})].$$

On the other hand, for $\mathbf{k}_1\approx 0$, one may use the expansion $\sigma(\mathbf{p_t}-\mathbf{k}_1)\simeq\sigma(\mathbf{p_t})-\sigma'(\mathbf{p_t})\mathbf{p}_t\cdot\mathbf{k}_1/p_t$, where $\mathbf{p}_t\cdot\mathbf{k}_1$ represents the scalar product of the two vectors, and $\sigma'(\mathbf{p_t})=(d/d|\mathbf{p_t}|)\sigma(\mathbf{p_t})$ depends only on the modulus of $\mathbf{p_t}$. One is left with

$$-\frac{\sigma'(\mathbf{p_t})}{p_t}\int \mathbf{p}_t\cdot\mathbf{k}_1\sigma(\mathbf{k}_1)d^2k_1=0,$$

where the vanishing result is due to the azimuthal symmetry of $\sigma(\mathbf{k}_1)$. The dominant contribution to the integral comes therefore from the next term in the expansion of $\sigma(\mathbf{k}_1-\mathbf{p_t})$, which goes as k_1^2. Hence the resulting singularity is only logarithmic in p_0, since $\sigma(\mathbf{k})\sim k^{-4}$ as $k\to 0$. The subtraction terms, originated by the absorption factor $\exp[-\langle n_A(x,b)\rangle]$ in Eq. (3.9), have canceled the singularity of the rescattering term almost completely. This feature is common to all the terms of the expansion (3.9) as it is discussed briefly at the end of Appendix B, where the three-scattering term is discussed in detail.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section we discuss in detail, both qualitatively and quantitatively, the modifications induced by the rescatterings on the minijet inclusive transverse spectrum. We consider a proton-lead collision with center of mass energy $\sqrt{s}=6$ TeV in the nucleon-nucleon center of mass frame and impact parameter $\beta=0$. In the numerical computations we used the leading order perturbative parton-parton cross section with a mass regulator $m\equiv p_0$,

$$\frac{d\sigma}{d^2p}(xx')=k\frac{9\pi\alpha_s(Q)^2}{(p^2+m^2)^2}\theta(xx's-4(p^2+m^2))\theta(1-x)\theta(1-x'),$$

where k is the k factor that simulates next-to-leading order corrections (we chose $k=2$). The single-parton nuclear distribution function has been taken to be factorized in x and b,

$$\Gamma_A(x,b)=\tau_A(b)G(x,Q),$$

where τ_A is the nuclear thickness function normalized to A and G is the proton distribution function. We evaluated the strong coupling constant and the nuclear distribution functions at a fixed scale $Q=m$. In the computations we used a hard-sphere geometry

$$\tau_A(b)=A\frac{3}{2\pi R^3}\sqrt{R^2-b^2}\,\theta(R^2-b^2),$$

where $R=1.12A^{1/3}$ is the nuclear radius measured in fm. For G we used the 1998 Glück-Reya-Vogt (GRV98) leading order (LO) parametrization [28]. At low p_t the spectrum is obtained by computing numerically the Fourier transform in Eq. (3.5), but at high p_t the result begins to oscillate too much, and in that region the spectrum was computed by using the expansion in the number of scattering up to the three-scattering term [the formulas actually used, Eqs. (A1), (A4), and (A7), are discussed in Appendix A]. We checked that the spectrum obtained by Fourier transformation matched the expansion smoothly.

A. Effects of rescatterings

In this section we discuss the projectile and the target transverse spectrum averaged over a given rapidity interval,

$$\frac{dW_h}{d^2p_t}(\beta,\eta_{min},\eta_{max})=\frac{1}{\eta_{max}-\eta_{min}}\int_{\eta\in[\eta_{min},\eta_{max}]}dx d^2b\times\frac{dW_h}{d^2p_t}(x,b,\beta), \quad (4.1)$$

where we approximated the pseudorapidity by $\eta=\log(x\sqrt{s}/p_0)$. The target spectrum dW_A/d^2p_t is obtained by interchanging h and A in Eq. (4.1). Note that now we are taking into account all possible rescatterings of the target as well.

In Fig. 1 we compare the full transverse spectrum (solid line) with its expansion in the number of scatterings up to three scatterings (dotted and dashed lines). We show both the projectile and target minijet spectrum in a pseudorapidity region $\eta\in[3,4]$ for the projectile and $\eta\in[-4,-3]$ for the target. Note that the rapidity is defined with reference to the projectile hadron direction of motion. The choice of a forward region (backward for the target) is done to enhance the effect of the rescatterings and to better discuss the deformation induced in the spectrum. Indeed, in those regions the

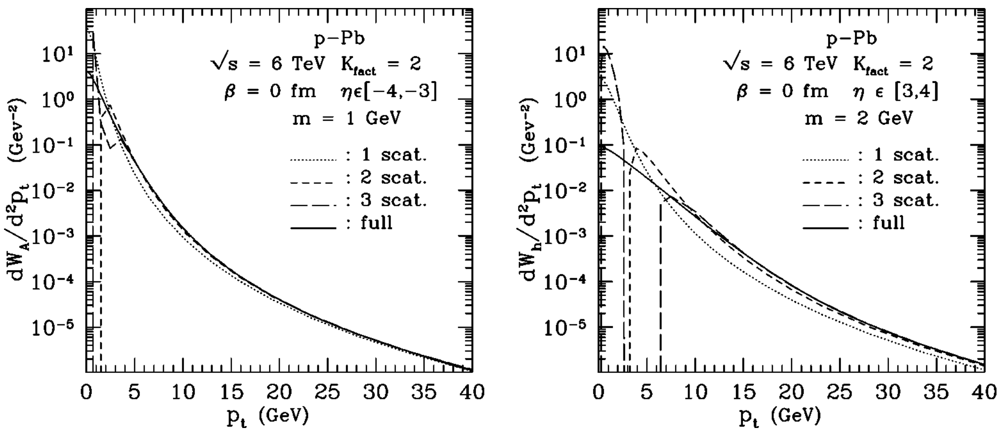

FIG. 1. *Left:* Target p_t spectrum for $\eta \in [-4,-3]$. *Right:* Projectile p_t spectrum for $\eta \in [3,4]$. The full transverse spectrum (solid line) is compared with the one-, two- and three-scattering approximations (viz. dotted, short-dashed, and long-dashed lines).

average fractional momentum of an incoming parton is large, so that the number of available target partons is large and the probability of rescattering becomes large.

First we look at the projectile spectrum. At high p_t the spectrum is enhanced with respect to the single scattering approximation because of the p_t broadening induced by the rescatterings. As p_t is further increased it approaches the single-scattering spectrum, as expected on general grounds when the p_t distribution of the elementary scattering follows a power law. This can be understood qualitatively by looking at the path in p_t space followed by the incoming parton. Given a final large p_t, due to the leading divergences in Eq. (3.9), the leading processes to obtain that p_t with two semihard scatterings are a first scattering with momentum transfer $q_1 \approx p_0$ followed by a second one with $q_2 \approx p_t$, and vice versa. For an analogous reason, the leading configuration to reach the final p_t with three scatterings is $q_1 \approx p_t$ plus $q_2 \approx q_3 \approx p_0$ and permutations. This sequence of three scatterings is less probable than the process with two scatterings as p_t increases because the fraction of phase-space volume that this process occupies decreases much faster with p_t than in the two-scattering case. For an analogous reason the relative importance of the two-scattering term with respect to the single-scattering term also decreases as p_t increases. In conclusion as p_t increases the average number of scatterings per parton decreases, and eventually the spectrum is well described by the single-scattering approximation.

At intermediate p_t the average number of scatterings per parton increases and the shape of the spectrum is more and more distorted with respect to the single-scattering case. In fact, the fraction of phase space available to the leading configuration of a multiple scattering process ($q_1 \approx p_t$, $q_2 \approx \cdots \approx q_n \approx p_0$ and permutations) increases as p_t decreases. However, this is not the only mechanism at work. Indeed, in our computation each wounded parton is counted as one minijet in the final state, independently of the number of rescatterings. On the other hand, in the single-scattering approximation one identifies the number of minijets in the final state with the number of parton-parton collision. This leads to an overestimate of the jet multiplicity and to a divergence of the spectrum at $p_t = 0$ as p_0 goes to zero. Therefore at low p_t the minijet yield is more and more suppressed with respect to the single scattering approximation.

At very low transverse momentum $p_t \lesssim p_0$ a parton undergoes a large number of rescatterings, all with $q_i \approx p_0$. Hence the parton performs a random walk in the transverse plane and the spectrum becomes flat as $p_t \to 0$ because the phase space becomes isotropically populated. This shows that at very low p_t multiple semihard scatterings are consistent with the random-walk model of Ref. [21], while at moderate and high p_t the physical picture is rather different.

By comparing the results for the projectile and target transverse spectrum one sees that a projectile parton is traversing a very dense target and the effects of the rescatterings are large. Conversely, a target parton "sees" a rather dilute system, and its minijet spectrum does not differ too much from the single-scattering result, except at very low p_t. Moreover the changes induced by the rescatterings on integrated quantities, like those entering in the expression of the hadron-nucleus cross section, are minimal. This is consistent with our approximation of not including rescatterings for the target partons to obtain analytical formulas for the hadron-nucleus cross section. One can also see that the three-scattering approximation describes well the projectile spectrum for $p_t \gtrsim 15$ GeV, while it breaks down completely at $p_t \lesssim 7$ GeV, where it becomes negative. For the target spectrum the three-scattering approximation is not accurate for $p_t \lesssim 4$ GeV.

B. Minijet inclusive transverse spectrum

In this section we study the minijet transverse spectrum resulting from the sum of the transverse spectra of the projectile and target wounded partons:

$$\frac{dW_{hA}}{d^2p_t}(\beta,\eta_{min},\eta_{max}) = \frac{1}{\eta_{max}-\eta_{min}}\int_{\eta\in[\eta_{min},\eta_{max}]} dx d^2b \times\left(\frac{dW_h}{d^2p_t}(x,b,\beta)+\frac{dW_A}{d^2p_t}(x,b,\beta)\right). \quad (4.2)$$

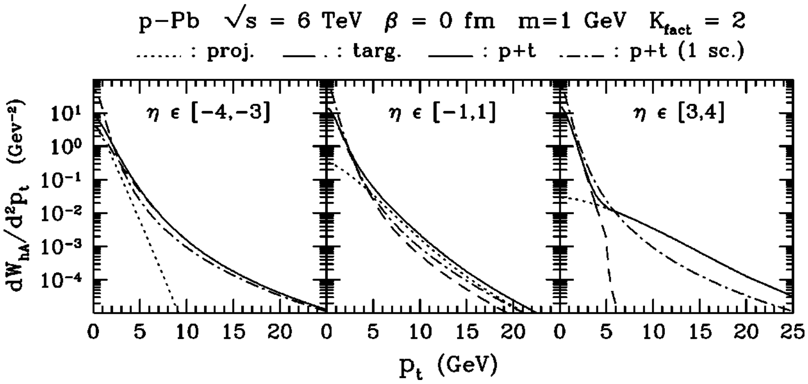

FIG. 2. Projectile plus target p_t spectrum (solid line) at different rapidities compared to the result of the one-scattering approximation (dot-dashed line). Also shown are the contributions of the projectile minijets (dotted line) and the target minijets (dashed line).

We analyze the spectrum in three rapidity regions, namely $\eta\in[-4,-3]$, $\eta\in[-1,1]$, and $\eta\in[3,4]$ (respectively "backward," "central" and "forward" with reference to the projectile direction of motion). While the target partons basically do not suffer any rescattering in all three regions, the projectile partons undergo many rescatterings in the forward region, some in the central region and basically no one backwards.

In Fig. 2 we show the spectrum (4.2) (solid line) and the contributions of the projectile and of the target (dotted and dashed lines, respectively). For comparison the total spectrum obtained in the one-scattering approximation is also plotted (dot-dashed line). The spectra are computed with a regulating mass $m=1$ GeV.

In the backward region both the projectile and the target suffer mainly one scattering over all the p_t range except at $p_t\sim 0$, and the spectrum is dominated almost everywhere by target minijets.

In central and forward regions the target jets still suffer basically one scattering over all the p_t range. Conversely, the projectile crosses a denser and denser target and undergoes an average number of rescatterings that increases with pseudorapidity. This means that at low p_t the projectile spectrum is very reduced with respect to the one-scattering approximation, and the minijet yield may become negligible with respect to the minijet yield from the target. The overall effect is that at low p_t the spectrum is dominated by minijet production from the target while at intermediate and high p_t it is dominated by minijet production from the projectile.

At very forward rapidities this effect becomes quite dramatic and the spectrum acquires a structured shape: it follows the inverse power behavior of the single-scattering term at high p_t, it is concave at intermediate p_t because of the suppression of the projectile minijets, and it becomes convex again at low p_t, where the target begins to dominate.

In Fig. 3 we study the dependence of the spectrum on the choice of the cutoff, and plot the result for $m=1$, 2, and 3 GeV. The deformation of the spectrum decreases as the regulator increases (indeed, the average number of rescattering decreases) and for $m\gtrsim 3$ GeV it begins to become negligible.

The effects of the rescatterings are better displayed by studying the ratio of the full transverse spectrum and the single-scattering approximation,

$$R_\beta(p_t)=\frac{dW_{hA}/d^2p_t}{dW_{hA}^{(1)}/d^2p_t}=\frac{dW_{hA}/d^2p_t}{A_\beta\ \ dW_{pp}^{(1)}/d^2p_t}, \qquad (4.3)$$

where $A_\beta=\int d^2b\,\tau_h(b-\beta)\tau_A(b)$ is the number of target nucleons interacting with the projectile at a given impact parameter.

In Fig. 4 we plotted the ratio $R_\beta(p_t)$, which measures the Cronin effect for minijet production, computed with three different regulators $m=1$, 2, and 3 GeV. At $m=3$ GeV the effect of the rescatterings is rather small in all the three rapidity intervals, except at very low p_t, and does not affect the integrated quantities like the average number of minijets. As the regulating mass is decreased the rescatterings begin to show up, and lead to a large effect in the forward region.

The ratio $R_\beta(p_t)$ is characterized by three quantities: the momentum $p_\times$ where the R_β crosses 1, the momentum p_M where it reaches the maximum and the height R_M of the maximum. The sensitivity of $p_\times$ on the cutoff decreases as the pseudorapidity increases. Loosely speaking, when the av-

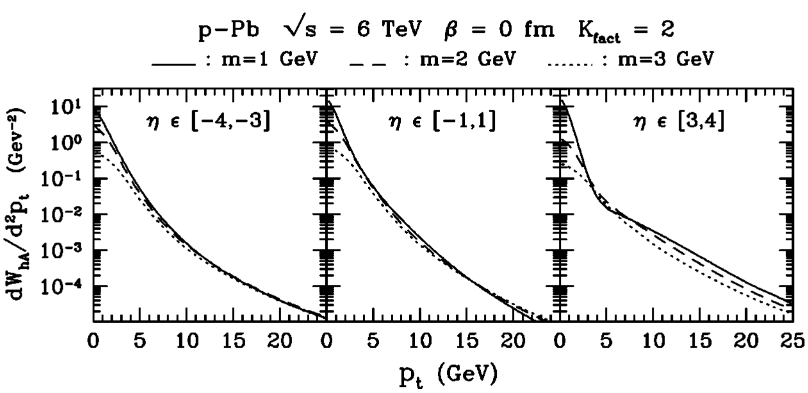

FIG. 3. Regulator dependence of the projectile plus target p_t spectrum at different rapidities for $m=1$, 2, and 3 GeV (viz. solid, dashed and dotted lines).

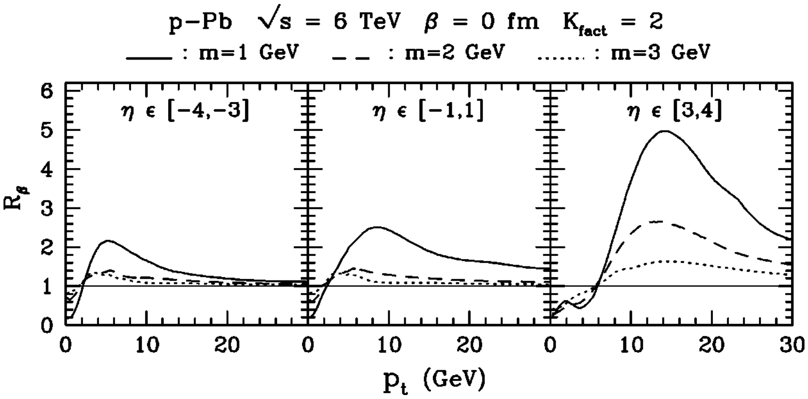

FIG. 4. Ratio of the full projectile plus parton p_t spectrum to the one-scattering approximation at different rapidities and for $m=1$, 2, and 3 GeV (viz. solid, dashed, and dotted lines).

erage number of scatterings is high, as it is the case at $p_t \simeq p_\times$, the jets lose memory of p_0, which gives the order of magnitude of the typical momentum exchanged in each collision. p_M shows a slightly larger sensitivity on the regulator, since it lies in a region where the average number of scatterings is smaller. This behavior is very different from the conclusions drawn by considering only the expansion up to two scatterings, where both $p_\times$ and p_M are proportional to p_0 [11]. In fact, at low center of mass energies the two-scattering is a good approximation in all rapidity ranges, except may be very forward. However, it breaks down in any case at transverse momenta comparable to the regulator p_0. Therefore, while most of the spectrum is well described by the two-scattering approximation, the behavior of $p_\times$ and p_M is not.

On the other hand, the height of the peak is much more sensitive to the cutoff, since its leading term is roughly proportional to some power of the logarithm of the regulator:

$$\left[\frac{dW_{hA}}{d^2p_t}-\frac{dW_{hA}^{(1)}}{d^2p_t}\right]_{p_t=p_M}\underset{p_0\to 0}{\sim}\left[\log\left(\frac{p_M^2}{p_0^2}\right)\right]^{\langle n_{resc}(p_M)\rangle}.$$

Since p_M is not very large, the average number of rescatterings at that value of the transverse momentum, $\langle n_{resc}(p_M)\rangle$, is much greater than 1, and the sensitivity of R_M on p_0 is high. At high p_t the average number of rescatterings tends to zero, so the sensitivity of the R_β on p_0 decreases and disappears at very large transverse momenta.

Note that the peak is located in a p_t region, where soft interactions (which have been disregarded in our approach) are expected to be negligible; therefore, in that region our perturbative computations should describe the spectrum almost completely. Following Ref. [11] we might interpret p_0 as the momentum scale at which the interaction deviates from the perturbative computations. With this interpretation p_0 would acquire a physical meaning: though introducing p_0 to separate hard and soft interactions is only a theoretical device and physical phenomena do not depend on p_0, it is a well defined question to ask up to what scale the perturbative computations are good. If the collision dynamics would be determined by parton multiple elastic scatterings alone, then the measure of the height of the peak would be a way of measuring p_0.

On the other hand, the sensitivity of R_β on p_0 signals a weakness in our description of the dynamics underlying the hadron-nucleus collision. We expect that such a sensitivity will be considerably reduced when also including in the dynamics the gluon radiation emitted by the multiply scattering partons. Some of the effects of the radiation on the transverse spectrum might however be described by the parameter p_0 in our model, in which radiation is neglected. Since the inclusion of gluon radiation in the dynamics would introduce new physical scales, such as the radiation formation time, related to the energy of the collision and the nuclear size, we would expect in any case that the value of p_0 will depend on $\sqrt{s}$ and A.

V. CONCLUSIONS

The purpose of the present paper is to draw attention to some of the advantages of studying hadron-nucleus semihard interactions at the LHC. As in the case of lower energies, hA interactions represent an important intermediate step to relate hh and AA reactions, being much simpler to understand as compared with the latter. Moreover, even at higher energies, such as those obtainable at the BNL Relativistic Heavy Ion Collider (RHIC) and LHC, in hA collisions we do not expect the formation of a dense and hot system, like the quark-gluon plasma, so that one can study directly the nuclear modification of the dynamics without the need of disentangling the effects of the structure of the target and those due to the formation and evolution of the dense system. Hadron-nucleus interactions represent therefore the baseline for the detection and the study of the new phenomena peculiar to AA collisions.

We faced the problem of unitarity corrections to the semihard cross section by including explicitly semihard parton rescatterings in the collision dynamics, and exploiting the self-shadowing property of the semihard interactions. In the interaction mechanism we took into account just elastic parton-parton collisions, while we neglected the production processes at the partonic level (e.g., all $2\rightarrow 3$ etc. elementary partonic processes), whose inclusion represents a nontrivial step in our approach and deserves further study.

Contrary to the case of AA collisions, it is possible to obtain closed analytical expressions for the semihard hA cross section; see Eq. (2.8). To that end a crucial assumption has been to consider the hadron as a dilute system, so that rescatterings of nuclear partons can be neglected while rescatterings of the projectile are fully taken into account. In our expressions we have disregarded correlations in the nuclear multiparton distributions, whose effect may nevertheless be studied in a straightforward way within the present functional approach.

We have then focused on the inclusive minijet transverse spectrum at fixed impact parameter, [Eq. (3.5)], which is influenced in a more direct way by the rescatterings. The modifications of the transverse spectrum induced by the semihard rescatterings of the projectile partons is emphasized in the ratio $R_\beta(p_t)$ [Eq. (4.3)], defined as our p_t spectrum divided by the impulse approximation. In particular, we have evaluated it at $\beta=0$ for different values of the regulator p_0. The results are described by the values of $p_\times$ [defined by $R_\beta(p_\times)=1$], p_M (which is the value of p_t that maximizes the ratio) and R_M (which is the maximum of R_β). We obtain that both $p_\times$ and p_M depend weakly on p_0, while R_M has, on the contrary, a strong dependence on p_0 when the regulator is rather small. Therefore, the results for the spectrum also allows us to identify the limits of the picture of the dynamics considered in this paper. Analogously to the average transverse energy and the number of minijets in AA collisions [6], some of the features of R_β, such as $p_\times$ and p_M, show a tendency toward a limiting value at small p_0. All these quantities depend therefore only marginally on details of the dynamics which have not been taken into account in the present approach. Conversely, the limits of the simplified picture of the interaction show up in R_M. Because of its strong dependence on p_0, in order to describe the spectrum one needs in fact to fix experimentally the value of p_0 by measuring R_M. This feature might be not so unpleasant, because if one limits the analysis to the inclusive transverse spectrum of minijets in hA collisions, all the effects which are not taken into account in the interaction (like the gluon radiation in the elementary collision process) are summarized by the value of a single phenomenological parameter. However this feature will not hold any further if one had to evaluate more differential properties of the produced state, which can be properly discussed only after explicitly introducing further details into the description of the elementary interaction process.

The experimental measure of the Cronin effect in minijet production in hA collisions would therefore be of major importance: it would allow one to establish the correctness of the whole approach described here, and it would represent the basis for a deeper insight in the semi hard interaction dynamics both for hA and AA collisions.

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APPENDIX A: SYMMETRIZATION OF THE EXPANSION IN THE NUMBER OF SCATTERINGS

For a numerical computation of the high-p_t expansion of the minijet spectrum in the number of scatterings suffered by a projectile parton it is convenient to implement the subtraction of the IR divergences directly in the integrand. In this way the Monte Carlo integrations, which we use because of the high dimensionality of the phase space (in particular for three or more scatterings), work at their best. In fact, Eqs. (3.9) and (B1) are not suited for numerical implementation due to the delta functions. The basic property that allowed the cancellation of the divergence in the integrand was the symmetry under exchanges of the integration variables. Unfortunately after using the delta functions to perform the integrals, one obtains in general nonsymmetric expressions.

The goal of this appendix is to study how to symmetrize each term of the expansion of the transverse spectrum. We will discuss them in detail up to the three-scattering term, but the techniques discussed can also be applied to the generic term in the expansion. For simplicity, we will use the following notation, already introduced in the main text:

$$\sigma(\mathbf{k})=\frac{d\sigma}{d^2k}(xx').$$

1. One-scattering term

The one-scattering term does not include any subtraction term, so that we do not need to symmetrize it. It is simply given by

$$\frac{dW_h{}^{(1)}}{d^2p_t}(x,b,\beta)=\Gamma_h(x,b-\beta)\int dx'\Gamma_A(x',b)\sigma(\mathbf{p_t}), \tag{A1}$$

and corresponds to the result one obtains by considering only disconnected parton collisions and neglecting parton rescatterings. It also corresponds to modeling the hadron-nucleus collision as a superposition of hadron-nucleus collisions.

2. Two-scattering term

The two-scattering term is given by Eq. (3.9), and we need to perform one integration over $\mathbf{k}_1$ or over $\mathbf{k}_2$ to dispose of the δ functions. By simply calling $\mathbf{q}$ the remaining integration variable we obtain

$$\frac{dW_h^{(2)}}{d^2p_t}(x,b,\beta)=\Gamma_h(x,b-\beta)\int dx_1'dx_2'\Gamma_A(x_1',b)\Gamma_A(x_2',b)$$
$$\times\int d^2q[\sigma(\mathbf{q})\sigma(\mathbf{p_t}-\mathbf{q})-2\sigma(\mathbf{q})\sigma(\mathbf{p_t})]. \tag{A2}$$

As discussed in Sec. III C, the negative term in the expression above subtracts the leading inverse power divergence in the integrand leaving only a logarithmic divergence. However, the cancellation happens only after performing the integration over $\mathbf{q}$, which may be a difficult result to achieve numerically (actually this is not a problem for the two-scattering term, due to the low dimensionality of the integral, but becomes a large issue from three scatterings on).

There are two divergences to be subtracted: one in $\mathbf{q}\sim 0$ and the other in $\mathbf{q}\sim\mathbf{p_t}$, but the subtraction term is divergent just in $\mathbf{q}\sim 0$, and the cancellation of the inverse power singularities is obtained only after performing the integration over $\mathbf{q}$. To allow the numerical integration to do a better and faster job, we require that the divergences in the convolution term and in the subtraction term be canceled directly in the integrand. This is obtained by symmetrizing the integrand with respect to an interchange of the two singularities in the convolution term. Let us introduce therefore an operator that performs the interchange of the two singularities:

$$\mathbb{T}\colon \mathbf{q}\to\mathbf{p_t}-\mathbf{q},$$

so that

$$\mathbb{T}\int d^2q f(\mathbf{q})=\int d^2q f(\mathbf{p_t}-\mathbf{q}).$$

Note that the change of variables operated by $\mathbb{T}$ has a unit Jacobian and that $\mathbb{T}^2=\mathbb{I}$. Then we define the symmetrized two-scattering term as

$$\left.\frac{dW_A^{(2)}}{d^2p_t}\right|_{sym}=\mathbb{S}^{(2)}\frac{dW_h^{(2)}}{d^2p_t},$$

where we introduced the symmetrization operator

$$\mathbb{S}^{(2)}=\frac{1}{2}(\mathbb{I}+\mathbb{T}). \tag{A3}$$

The result is

$$\begin{aligned}\left.\frac{dW_A^{(2)}}{d^2p_t}\right|_{sym}(x,b,\beta)={}&\Gamma_h(x,b-\beta)\int dx_1'dx_2'\Gamma_A(x_1',b)\\&\times\Gamma_A(x_2',b)\int d^2q[\sigma(\mathbf{q})\sigma(\mathbf{p_t}-\mathbf{q})\\&-\sigma(\mathbf{q})\sigma(\mathbf{p_t})-\sigma(\mathbf{p_t}-\mathbf{q})\sigma(\mathbf{p_t})].\end{aligned} \tag{A4}$$

Note that the first term in Eq. (A4) describes two subsequent scatterings with total transverse momentum p_t and is the naive perturbative QCD result. The two negative terms are the absorption terms induced by probability conservation. The two IR divergences of the first term are canceled by these two subtraction terms: as $\mathbf{q}\to\mathbf{0}$ by the first one and as $\mathbf{q}\to\mathbf{p_t}$ by the second one. The remaining linear singularity gives a zero contribution because it is odd in a neighborhood of $\mathbf{q}=0$ and $\mathbf{q}=\mathbf{p_t}$, so that only the logarithmic divergence remain. Note that now the two divergences are subtracted directly in the integrand, which was the goal of the symmetrization procedure.

Equation (A4) is the expression that we use in the numerical computations of the transverse spectrum at high p_t. It could have been guessed directly from Eq. (A2), but the use of symmetrization operator (A3) will facilitate the discussion of the more complicated three scattering term.

3. Three-scattering term

To prepare the ground for the treatment of the three-scattering term, we note that $\mathbb{T}$ generates the group of the permutations of the two singularities $\mathbf{q}\sim 0$ and $\mathbf{q}\sim\mathbf{p_t}$; this is called the symmetric group of order 2, and is indicated as $S_2=\langle\mathbb{T}\rangle=\{\mathbb{I},\mathbb{T}\}$, where $\langle\mathbb{T}\rangle$ means "generated by $\mathbb{T}$." It is then easy to see that we can construct the symmetrizing operator Eq. (A3) by summing all the elements of S_2 and by dividing by its cardinality.

From Eq. (B1), after exploiting the δ functions, the three-scattering term reads

$$\begin{aligned}\frac{dW_h^{(3)}}{d^2p_t}(x,b,\beta)={}&\Gamma_h(x,b-\beta)\int dx_1'dx_2'dx_3'\Gamma_A(x_1',b)\\&\times\Gamma_A(x_2',b)\Gamma_A(x_3',b)\\&\times\frac{1}{3!}\int d^2q d^2r[\sigma(\mathbf{q})\sigma(\mathbf{r})\sigma(\mathbf{p_t}-\mathbf{q}-\mathbf{r})\\&-3\sigma(\mathbf{q})\sigma(\mathbf{p_t}-\mathbf{q})\sigma(\mathbf{p_t})\\&+3\sigma(\mathbf{q})\sigma(\mathbf{r})\sigma(\mathbf{p_t})].\end{aligned} \tag{A5}$$

Following the general analysis previously done at the end of the last paragraph, we observe that in Eq. (A5) in absence of the cutoff we would have four divergences, i.e.,

$$\mathbf{q}\sim 0,\quad \mathbf{r}\sim 0,\quad \mathbf{p_t}-\mathbf{q}-\mathbf{r}\sim 0,\quad \mathbf{p_t}-\mathbf{q}\sim 0. \tag{A6}$$

Then to write the symmetrized three-scattering term, we need to consider the group S_4 of the permutations of these four divergences, which has $4!=24$ elements,

$$\mathcal{P}_{Bsym}^{(3)}=\mathbb{S}^{(3)}\mathcal{P}_B^{(2)},$$

where

$$\mathbb{S}^{(3)}=\frac{1}{24}\sum_{\mathbb{T}\in S_4}\mathbb{T}.$$

When applying this operator to the three-scattering term the resulting expression has 49 terms and is too long to be discussed here. To obtain an idea of the result, we will consider only the subgroup S_3 given by the permutations of the first three divergences in Eq. (A6), which are the divergences that appear in the first term of Eq. (A5), i.e., the naive three-scattering term. After symmetrization it will be immediate to check that all the "single" divergences cancel explicitly in the integrand, while "double" divergences cancel only after performing the integrations over the transverse momenta. We call a "single" divergence a point $(\mathbf{q},\mathbf{r})$, such that only one of the expressions in Eq. (A6) is near zero, and a "double" divergence a point such that two of these terms are nearly zero. For example $\{\mathbf{q}\sim 0;\ \mathbf{r}\not\sim 0,\mathbf{p_t},\mathbf{p_t}-\mathbf{q}\}$ and $\{\mathbf{q}\sim 0;\ \mathbf{r}\sim\mathbf{p_t}\}$ are a single divergence and a double divergence, respectively.

The first step is the definition of the operators that exchange the three singularities:

$$\mathbb{T}_1:\begin{cases}\mathbf{q}\to\mathbf{r}\\ \mathbf{r}\to\mathbf{q},\end{cases}\quad \mathbb{T}_2:\begin{cases}\mathbf{q}\to\mathbf{p_t}-\mathbf{q}-\mathbf{r}\\ \mathbf{r}\to\mathbf{r},\end{cases}\quad \mathbb{T}_3:\begin{cases}\mathbf{q}\to\mathbf{q}\\ \mathbf{r}\to\mathbf{p_t}-\mathbf{q}-\mathbf{r}.\end{cases}$$

Note that they are idempotent: $\mathbb{T}_i=\mathbb{I}$. Next we observe that the group S_3 of the permutations of the three singularities is made of $3!=6$ objects, and that

$$S_3=\langle \mathbb{T}_1,\mathbb{T}_2,\mathbb{T}_3\rangle=\{\mathbb{T}_0,\mathbb{T}_1,\mathbb{T}_2,\mathbb{T}_3,\mathbb{T}_4,\mathbb{T}_5\},$$

where $\mathbb{T}_0=\mathbb{I}$, $\mathbb{T}_4=\mathbb{T}_1\mathbb{T}_2$ and $\mathbb{T}_5=\mathbb{T}_1\mathbb{T}_3$, so that the reduced symmetrizing operator is

$$\mathbb{S}^{(3)}_{red}=\frac{1}{3!}\sum_{i=0}^{5}\mathbb{T}_i\,.$$

Finally one can write the partially symmetrized three-scattering probability:

$$\begin{aligned}\left.\frac{dW_A^{(3)}}{d^2p_t}\right|_{sym}(x,b,\beta)&=\mathbb{S}^{(3)}_{red}\frac{dW_A^{(3)}}{d^2p_t}(x,b,\beta)\\
&=\Gamma_h(x,b-\beta)\int \Gamma_A(x_1',b)\Gamma_A(x_2',b)\Gamma_A(x_3',b)dx_1'dx_2'dx_3'd^2k_1d^2k_2d^2k_3\frac{1}{3!}\int \mathrm{d}^2q\,\mathrm{d}^2r\\
&\times\Big[\sigma(\mathbf{q})\sigma(\mathbf{r})\sigma(\mathbf{p_t}-\mathbf{q}-\mathbf{r})-\frac12\sigma(\mathbf{q})\sigma(\mathbf{r})\sigma(\mathbf{p_t}-\mathbf{q})+\frac12\sigma(\mathbf{q})\sigma(\mathbf{p_t}-\mathbf{q})\sigma(\mathbf{p_t})-\frac12\sigma(\mathbf{q})\sigma(\mathbf{r})\sigma(\mathbf{p_t}-\mathbf{r})\\
&+\frac12\sigma(\mathbf{p_t}-\mathbf{r})\sigma(\mathbf{q})\sigma(\mathbf{p_t})-\frac12\sigma(\mathbf{p_t}-\mathbf{q}-\mathbf{r})\sigma(\mathbf{q})\sigma(\mathbf{p_t}-\mathbf{q})+\frac12\sigma(\mathbf{p_t}-\mathbf{r})\sigma(\mathbf{p_t}-\mathbf{q})\sigma(\mathbf{p_t})\\
&-\frac12\sigma(\mathbf{p_t}-\mathbf{q}-\mathbf{r})\sigma(\mathbf{r})\sigma(\mathbf{p_t}-\mathbf{r})+\frac12\sigma(\mathbf{p_t}-\mathbf{r})\sigma(\mathbf{p_t}-\mathbf{q}-\mathbf{r})\sigma(\mathbf{p_t})-\frac12\sigma(\mathbf{r})\sigma(\mathbf{p_t}-\mathbf{q}-\mathbf{r})\sigma(\mathbf{q}+\mathbf{r})\\
&+\frac12\sigma(\mathbf{r})\sigma(\mathbf{q}-\mathbf{r})\sigma(\mathbf{p_t})-\frac12\sigma(\mathbf{q})\sigma(\mathbf{p_t}-\mathbf{q}-\mathbf{r})\sigma(\mathbf{q}+\mathbf{r})+\frac12\sigma(\mathbf{q})\sigma(\mathbf{q}-\mathbf{r})\sigma(\mathbf{p_t})\Big].\end{aligned}\tag{A7}$$

Analogously to what has been done for the two-scattering term, one can see by inspection that the four single divergences [Eq. (A6)] explicitly cancel in the integrand, while double divergences cancel only after performing the integrations over q and r. By considering all four singularities, and by using the whole S_4 group we obtained explicit cancellation of both "single" and "double" divergences directly in the integrand, as is discussed in Appendix B. Nonetheless, the partial symmetrization is enough to get satisfactory numerical results.

In conclusion, to compute numerically the expansion of the transverse minijet spectrum in the number of scatterings one has to fully exploit the symmetry properties of each term, in such a way that all the divergences get cancelled directly in the integrand. This is crucial to obtain a good numerical precision and to speed up the computation of the terms with three or more scatterings. In this appendix we developed a general technique to perform such a symmetrization.

APPENDIX B: CANCELLATION OF THE DIVERGENCES IN THE THREE-SCATTERING TERM

Hereafter we consider in detail the cancellation of the divergences in the term with three scatterings:

$$\begin{aligned}\frac{dW_h^{(3)}}{d^2p_t}(x,b,\beta)=&\,\Gamma_h(x,b-\beta)\int dx_1'dx_2'dx_3'd^2k_1d^2k_2d^2k_3\\
&\times\Gamma_A(x_1',b)\Gamma_A(x_2',b)\Gamma_A(x_3',b)\\
&\times\frac{\sigma(\mathbf{k}_1)\sigma(\mathbf{k}_2)\sigma(\mathbf{k}_3)}{6}[\delta^{(2)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{p}_t)\\
&-\delta^{(2)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p}_t)-\delta^{(2)}(\mathbf{k}_2+\mathbf{k}_3-\mathbf{p}_t)\\
&-\delta^{(2)}(\mathbf{k}_3+\mathbf{k}_1-\mathbf{p}_t)+\delta^{(2)}(\mathbf{k}_1-\mathbf{p}_t)\\
&+\delta^{(2)}(\mathbf{k}_2-\mathbf{p}_t)+\delta^{(2)}(\mathbf{k}_3-\mathbf{p}_t)].\end{aligned}\tag{B1}$$

The different δ functions in Eq. (B1) correspond to all the terms of order σ^3 in Eq. (3.2), and represent the triple scattering term together with all subtraction terms induced by the expansion of the absorption factor $\exp[-\langle n_A(x,b)\rangle]$ of the double- and single-scattering terms. The expression has been symmetrized with respect to $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ and is singular for $\mathbf{k}_1=0$, $\mathbf{k}_2=0$, and $\mathbf{k}_3=0$. The δ functions in Eq. (B1) prevent the tree momenta from being close to zero at the same time, then we start by discussing the most singular configuration corresponding to two integration variables both close to zero. Given the symmetry of the integrand it is enough to study the integration region with $\mathbf{k}_1 \approx 0$ and $\mathbf{k}_2 \approx 0$. In this region the terms $\delta^{(2)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p}_t)$, $\delta^{(2)}(\mathbf{k}_1-\mathbf{p}_t)$, and $\delta^{(2)}(\mathbf{k}_2-\mathbf{p}_t)$ do not contribute. The integrals on the transverse momenta are therefore written as

$$\int d^2k_1 d^2k_2 \sigma(\mathbf{k}_1)\sigma(\mathbf{k}_2)[\sigma(\mathbf{p}_t-\mathbf{k}_1-\mathbf{k}_2)-\sigma(\mathbf{p}_t-\mathbf{k}_1) -\sigma(\mathbf{p}_t-\mathbf{k}_2)+\sigma(\mathbf{p_t})]. \tag{B2}$$

In the region where $\mathbf{k}_1 \approx 0$ and $\mathbf{k}_2 \approx 0$ one may use the expansion

$$\sigma(\mathbf{p_t}-\mathbf{k}) \simeq \sigma(\mathbf{p_t}) - \sigma'(\mathbf{p_t})\frac{\mathbf{p_t}\cdot\mathbf{k}}{p_t} + \frac{1}{2}\left[\sigma''(\mathbf{p_t})\frac{(\mathbf{p_t}\cdot\mathbf{k})^2}{p_t^2} - \sigma'(\mathbf{p_t})\frac{(\mathbf{p_t}\times\mathbf{k})^2}{p_t^3}\right], \tag{B3}$$

where $\mathbf{p_t}\times\mathbf{k}$ represents the vector product of $\mathbf{p_t}$ and $\mathbf{k}$ and $\sigma''(\mathbf{p_t}) = (d^2/d|\mathbf{p_t}|^2)\sigma(\mathbf{p_t})$ depends only on the modulus of $\mathbf{p_t}$. All terms proportional to $\sigma(\mathbf{p_t})$ cancel and all the terms linear in $\mathbf{k}$ integrate to zero thanks to the azimuthal symmetry of $\sigma(\mathbf{k})$. Then one is left with

$$\int d^2k_1 d^2k_2 \sigma(\mathbf{k}_1)\sigma(\mathbf{k}_2)\left\{\frac{\sigma''(\mathbf{p_t})}{2p_t^2}[(\mathbf{p}_t\cdot(\mathbf{k}_1+\mathbf{k}_2))^2 -(\mathbf{p}_t\cdot\mathbf{k}_1)^2-(\mathbf{p}_t\cdot\mathbf{k}_2)^2] - \frac{\sigma'(\mathbf{p_t})}{2p_t^3}[(\mathbf{p}_t\times(\mathbf{k}_1+\mathbf{k}_2))^2 -(\mathbf{p}_t\times\mathbf{k}_1)^2-(\mathbf{p}_t\times\mathbf{k}_2)^2]\right\},$$

which simplifies to

$$\int d^2k_1 d^2k_2 \sigma(\mathbf{k}_1)\sigma(\mathbf{k}_2)\left\{\frac{\sigma''(\mathbf{p_t})}{p_t^2}(\mathbf{p}_t\cdot\mathbf{k}_1)(\mathbf{p}_t\cdot\mathbf{k}_2) - \frac{\sigma'(\mathbf{p_t})}{p_t^3}(\mathbf{p}_t\times\mathbf{k}_1)(\mathbf{p}_t\times\mathbf{k}_2)\right\} = 0.$$

The result is again zero because of the azimuthal symmetry of $\sigma(\mathbf{k})$. Hence all terms of the expansion (B3) up to the second order in k do not contribute. All other terms linear in $\mathbf{k}_1$ or $\mathbf{k}_2$, which are obtained from the first terms in the square brackets in Eq. (B2), do not contribute for the same reason, so the first term different from zero is at least of order $k_1^2 k_2^2$, and originates a square-logarithm singularity as a function of the regulator p_0.

One may repeat the argument for the regions where only one of the integration variables is close to zero. We consider in detail the case where $\mathbf{k}_1 \approx 0$ and $\mathbf{k}_2$ and $\mathbf{k}_3$ are both finite. In this region the term $\delta^{(2)}(\mathbf{p_t}-\mathbf{k})$ does not contribute to Eq. (B1). The transverse momentum integrals are therefore

$$\int d^2k_1 d^2k_2 \sigma(\mathbf{k}_1)\sigma(\mathbf{k}_2)\{\sigma(\mathbf{p}_t-\mathbf{k}_1-\mathbf{k}_2)-\sigma(\mathbf{p}_t-\mathbf{k}_1) -\sigma(\mathbf{p}_t-\mathbf{k}_2)+\sigma(\mathbf{p_t})\} + \int d^2k_1 d^2k_3 \sigma(\mathbf{k}_1)\sigma(\mathbf{k}_3) \times\{-\sigma(\mathbf{p}_t-\mathbf{k}_1)+\sigma(\mathbf{p_t})\}.$$

To study the singularity it is sufficient to keep the first two terms in the expansion of $\sigma(\mathbf{k})$ in Eq. (B3), the remaining ones leading to a logarithmic divergence. One obtains

$$\int d^2k_1 d^2k_2 \sigma(\mathbf{k}_1)\sigma(\mathbf{k}_2)\left\{\sigma(\mathbf{p}_t-\mathbf{k}_2) -\sigma'(\mathbf{p}_t-\mathbf{k}_2)\frac{(\mathbf{p}_t-\mathbf{k}_2)\cdot\mathbf{k}_1}{\mathbf{p}_t-\mathbf{k}_2} - \sigma(\mathbf{p_t}) + \sigma'(\mathbf{p_t})\frac{\mathbf{p}_t\cdot\mathbf{k}_1}{p_t} -\sigma(\mathbf{p}_t-\mathbf{k}_2)+\sigma(\mathbf{p_t})\right\} + \int d^2k_1 d^2k_3 \sigma(\mathbf{k}_1)\sigma(\mathbf{k}_3) \times\left\{-\sigma(\mathbf{p_t}) + \sigma'(\mathbf{p_t})\frac{\mathbf{p}_t\cdot\mathbf{k}_1}{p_t} - \sigma(\mathbf{p_t})\right\},$$

which simplifies to

$$\int d^2k_1 d^2k_2 \sigma(\mathbf{k}_1)\sigma(\mathbf{k}_2)\left\{-\sigma'(\mathbf{p}_t-\mathbf{k}_2)\frac{(\mathbf{p}_t-\mathbf{k}_2)\cdot\mathbf{k}_1}{\mathbf{p}_t-\mathbf{k}_2} + 2\sigma'(\mathbf{p_t})\frac{\mathbf{p}_t\cdot\mathbf{k}_1}{p_t}\right\} = 0.$$

As in the previous case one obtains a vanishing result thanks to the azimuthal symmetry of $\sigma(\mathbf{k})$. In summary, all integrations in the singular points of the three-scattering term [Eq. (B1)] induce at most a square-logarithm singularity as a function of the cutoff.

The reduction of the divergences from powerlike to logarithmic is a common feature of all the terms of the expansion of the transverse spectrum in the number of scatterings, as one may see by looking at its Fourier transform [Eq. (3.6)]. Indeed, to study the dependence of the inclusive spectrum on the regulator p_0 at a given p_t different from zero one needs to consider the first term in the square brackets only. The cutoff enters into the difference

$$\tilde{\sigma}(v) - \tilde{\sigma}(0) = \int \frac{d\sigma}{d^2k}[e^{i\mathbf{k}\cdot\boldsymbol{v}}-1]d^2k = -v^2\frac{\pi}{2}\int_{p_0}^{\infty} k^3 \frac{d\sigma}{d^2k}dk + \text{finite terms},$$

so that, also in this case, the divergence for $p_0 \to 0$ is only logarithmic.

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