

# One-loop corrections to the chargino and neutralino mass matrices in the on-shell scheme

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We present a consistent procedure for the calculation of the one-loop corrections to the charginos and neutralinos by using their *on-shell* mass matrices. The on-shell gaugino mass parameters  $M$  and  $M'$  and the Higgsino mass parameter  $\mu$  are defined by the elements of these on-shell mass matrices. The on-shell mass matrices are different by finite one-loop corrections from the tree-level ones given in terms of the on-shell input parameters. When the on-shell  $M$  and  $\mu$  are determined by the chargino sector, the neutralino masses receive corrections up to 4%. This must be taken into account in precision measurements at future  $e^+e^-$  linear colliders.

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## I. INTRODUCTION

For the comparison of the Standard Model predictions with the precision experiments at the CERN  $e^+e^-$  collider LEP, calculations at the tree-level were no longer sufficient, making the inclusion of radiative corrections necessary. They provided important information on the top quark mass, the Higgs boson mass, and the unification condition for the gauge couplings.

The future generation of colliders, the Fermilab Tevatron, CERN Large Hadron Collider (LHC), and an  $e^+e^-$  linear collider will explore an energy range where one expects the appearance of supersymmetric particles. It will be possible to measure cross sections, branching ratios, masses, etc. At an  $e^+e^-$  linear collider it will even be possible to perform precision experiments for the production and decay of supersymmetric (SUSY) particles [1]. This allows one to test the underlying SUSY model. For instance, for the mass determination of charginos and neutralinos an accuracy of  $\Delta m_{\tilde{\chi}^{\pm,0}} = 0.1\text{--}1$  GeV might be reached by performing threshold scans [2]. The precision measurements of their couplings will also be possible [1,3–6]. Therefore, for a comparison with experiment, higher-order effects have to be included in the theoretical calculations.

For calculations of radiative corrections to the masses, production cross sections, decay rates, etc. of charginos and neutralinos, a proper renormalization of the chargino and neutralino mixing matrices is needed. One-loop corrections to the masses were given in [7,8] in the scale-dependent dimensional reduction (DR) scheme. Chargino production at one loop was treated by several authors [9–11]. In [9], the DR scheme was used where the scale dependence was canceled by replacing the tree-level parameters by running ones. In [10], effective chargino mixing matrices were introduced, which are independent of the renormalization scale  $Q$ . A pure on-shell renormalization scheme was adopted for the full one-loop radiative corrections to chargino and neutralino production in  $e^+e^-$  annihilation [11] and squark decays into charginos and neutralinos [12]. In this paper, we study another type of the correction which has not been discussed so far.

We present a consistent procedure for the calculation of the one-loop corrections to the on-shell mass matrix of charginos,  $X$ , and that of neutralinos,  $Y$ . One has to distinguish three types of the mass matrix: the tree-level mass matrix  $\tilde{X}$  ( $\tilde{Y}$ ) given in terms of the on-shell parameters, the  $\overline{\text{DR}}$  running tree-level matrix  $X^0$  ( $Y^0$ ), and the on-shell mass matrix  $X$  ( $Y$ ) which generates physical (pole) masses and on-shell mixing matrices by diagonalization. The on-shell parameters in  $\tilde{X}$  and  $\tilde{Y}$  are given as follows: The SU(2) gaugino mass parameter  $M$  and the Higgsino mass parameter  $\mu$  are defined by the elements of the chargino on-shell mass matrix  $X$ , the U(1) gaugino mass parameter  $M'$  by the neutralino mass matrix  $Y$ , and the other parameters ( $m_W, m_Z, \sin^2 \theta_W, \tan \beta$ ) are given by the gauge and Higgs boson sectors. We calculate the finite shifts  $\Delta X = X - \tilde{X}$  and  $\Delta Y = Y - \tilde{Y}$ . Especially, the zero elements of the tree-level matrix  $\tilde{Y}$  receive nonzero corrections by  $\Delta Y$ . In the numerical analysis, we calculate the contributions of the fermion and sfermion loops, which are usually most important. We also discuss the case where the on-shell  $M'$  is defined by the SUSY grand unified theory (GUT) relation  $M' = \frac{5}{3} \tan^2 \theta_W M$ .

First we illustrate in Sec. II our renormalization procedure for the fermion field with  $n$  components. In Sec. III, we give the explicit one-loop corrections to the mass matrices of charginos and neutralinos. We work out the shifts in the matrix elements, with a discussion of the on-shell renormalization of  $M$ ,  $\mu$ , and  $M'$ . The case in which the on-shell  $M'$  is defined by  $M$  is also discussed. In Sec. IV, we present some numerical results of the corrections by fermion and sfermion loops, mainly the correction to the masses for fixed on-shell parameters. Conclusions are given in Sec. V.

## II. ON-SHELL RENORMALIZATION OF FERMIONS

In this section, we show the on-shell renormalization of the fermion field  $\psi$  with  $n$  components (each component being a four-component Dirac spinor) and its mass matrix, following the formulas in [13,14].

The mass term of the fermion in the interaction basis is

$$V = \bar{\psi}_R M \psi_L + \bar{\psi}_L M^\dagger \psi_R. \quad (1)$$

$M$  is an  $n \times n$  mass matrix, which is real assuming  $CP$  conservation. With the rotations

$$f_R = U \psi_R, \quad (2)$$

$$f_L = V \psi_L, \quad (3)$$

the mass matrix can be diagonalized:

$$V = \bar{f} M_D f = \bar{f}_L M_D f_R + \bar{f}_R M_D f_L = \sum_{i=1}^n m_{f_i} \bar{f}_i f_i, \quad (4)$$

with the diagonal matrix  $M_D = \text{diag}(m_{f_1}, m_{f_2}, \dots, m_{f_n})$ . For Majorana fermions we allow negative  $m_{f_i}$  in order to keep  $U = V$  real.  $M_D$  is related to  $M$  by

$$M_D = U M V^T. \quad (5)$$

We express the bare quantities (with superscript 0) by the renormalized ones:

$$f_{L,R}^0 = (1 + \frac{1}{2} \delta Z_{L,R}) f_{L,R}, \quad (6)$$

$$\bar{f}_{L,R}^0 = \bar{f}_{L,R} (1 + \frac{1}{2} \delta Z_{L,R}^\dagger), \quad (7)$$

$$U^0 = U + \delta U, \quad (8)$$

$$V^0 = V + \delta V, \quad (9)$$

$$M^0 = M + \delta M. \quad (10)$$

$\delta Z$ ,  $\delta U$ ,  $\delta V$ , and  $\delta M$  are  $(n \times n)$  matrices. Hence

$$\psi_R^0 = [U^T (1 + \frac{1}{2} \delta Z_R) + \delta U^T] f_R \quad (11)$$

and

$$\psi_L^0 = [V^T (1 + \frac{1}{2} \delta Z_L) + \delta V^T] f_L. \quad (12)$$

By demanding that the counterterms  $\delta U$  and  $\delta V$  cancel the antisymmetric parts of the wave-function corrections, we get the fixing conditions for  $\delta U$  and  $\delta V$ :

$$\delta U = \frac{1}{4} (\delta Z_R - \delta Z_R^T) U, \quad (13)$$

$$\delta V = \frac{1}{4} (\delta Z_L - \delta Z_L^T) V. \quad (14)$$

This is equivalent to redefining the wave-function shifts in a symmetric way,

$$f_{L,R}^0 = [1 + \frac{1}{4} (\delta Z_{L,R} + \delta Z_{L,R}^\dagger)] f_{L,R}, \quad (15)$$

$$\bar{f}_{L,R}^0 = \bar{f}_{L,R} [1 + \frac{1}{4} (\delta Z_{L,R}^\dagger + \delta Z_{L,R})], \quad (16)$$

and setting  $\delta U = \delta V = 0$ . The renormalization conditions Eqs. (13) and (14) have already been used in [13,11]. According to Eq. (5), the on-shell mass matrix  $M$  is composed of the on-shell mixing matrices ( $U, V$ ) and the pole masses,  $M_D = \text{diag}(m_{f_i}(\text{pole}))$ .

We start from the most general form of the matrix element of the one-loop renormalized two-point function for mixing fermions,

$$i\hat{\Gamma}_{ij}(k^2) = i\{\delta_{ij}(\mathbf{k} - m_{f_i}) + \mathbf{k}[P_L \hat{\Pi}_{ij}^L(k^2) + P_R \hat{\Pi}_{ij}^R(k^2)] + \hat{\Pi}_{ij}^{S,L}(k^2) P_L + \hat{\Pi}_{ij}^{S,R}(k^2) P_R\}. \quad (17)$$

$\hat{\Pi}^L$ ,  $\hat{\Pi}^R$ ,  $\hat{\Pi}^{S,L}$ , and  $\hat{\Pi}^{S,R}$  are the fermion self-energy matrices. The ‘‘hat’’ denotes the renormalized quantities. Then the mass shifts  $\delta m_{f_k}$  are given by

$$\delta m_{f_k} = \frac{1}{2} \text{Re}\{m_{f_k} [\Pi_{kk}^L(m_{f_k}^2) + \Pi_{kk}^R(m_{f_k}^2)] + \Pi_{kk}^{S,L}(m_{f_k}^2) + \Pi_{kk}^{S,R}(m_{f_k}^2)\}, \quad (18)$$

and the off-diagonal wave-function renormalization constants of  $\delta Z_R$  and  $\delta Z_L$  read ( $i \neq j$ )

$$(\delta Z_R)_{ij} = \frac{2}{m_{f_i}^2 - m_{f_j}^2} \text{Re}[\Pi_{ij}^R(m_{f_j}^2) m_{f_j}^2 + \Pi_{ij}^L(m_{f_j}^2) m_{f_i} m_{f_j} + \Pi_{ij}^{S,R}(m_{f_j}^2) m_{f_i} + \Pi_{ij}^{S,L}(m_{f_j}^2) m_{f_j}]. \quad (19)$$

$(\delta Z_L)_{ij}$  is obtained by replacing  $L \leftrightarrow R$  in Eq. (19). The counterterm for the mass matrix element  $\delta M_{ij}$  can be written as

$$\delta M_{ij} = \frac{1}{2} \sum_{k,l} U_{ki} V_{lj} \text{Re}[\Pi_{kl}^L(m_{f_k}^2) m_{f_k} + \Pi_{kl}^R(m_{f_l}^2) m_{f_l} + \Pi_{kl}^{S,L}(m_{f_k}^2) + \Pi_{lk}^{S,R}(m_{f_l}^2)]. \quad (20)$$

### III. CHARGINO AND NEUTRALINO MASS MATRICES AT THE ONE-LOOP LEVEL

In the MSSM the chargino mass matrix is given by

$$X = \begin{pmatrix} M & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \cos \beta & \mu \end{pmatrix}. \quad (21)$$

It is diagonalized by the two real  $(2 \times 2)$  matrices  $U$  and  $V$ :

$$U X V^T = M_D = \begin{pmatrix} m_{\tilde{\chi}_1^+} & 0 \\ 0 & m_{\tilde{\chi}_2^+} \end{pmatrix}, \quad (22)$$

with  $m_{\tilde{\chi}_1^+}$  and  $m_{\tilde{\chi}_2^+}$  the physical masses of the charginos (choosing  $m_{\tilde{\chi}_1^+} < m_{\tilde{\chi}_2^+}$ ). The shifts for  $U$  and  $V$  are then given by Eqs. (13) and (14):

$$\delta U = \frac{1}{4} (\delta Z_R^{\tilde{\chi}_1^+} - \delta Z_R^{\tilde{\chi}_2^+ T}) U, \quad (23)$$

$$\delta V = \frac{1}{4} (\delta Z_L^{\tilde{\chi}_1^+} - \delta Z_L^{\tilde{\chi}_2^+ T}) V. \quad (24)$$

The shift in  $X$  follows from

$$\delta X = \delta(U^T M_D V) = \delta U^T M_D V + U^T M_D \delta V + U^T \delta M_D V. \quad (25)$$

Its matrix elements are

$$(\delta X)_{ij} = \sum_{k=1}^2 [m_{\tilde{\chi}_k^+} (\delta U_{ki} V_{kj} + U_{ki} \delta V_{kj}) + \delta m_{\tilde{\chi}_k^+} U_{ki} V_{kj}], \quad (26)$$

where the elements  $\delta U_{ki}$  and  $\delta V_{kj}$  are obtained from Eqs. (23) and (24) together with Eq. (19).  $\delta m_{\tilde{\chi}_k^+}$  is given by Eq. (18). The explicit forms for the chargino self-energies are given in Eq. (A1).

Now we want to calculate the on-shell mass matrix  $X$  at the one-loop level. We first show the relation between three types of the mass matrix:  $X$ ,  $\tilde{X}$ , and  $X^0$ .  $\tilde{X}$  is the tree-level mass matrix, which has the form of Eq. (21) in terms of the on-shell parameters  $(M, \mu, m_W, \tan \beta)$ .  $\tilde{X}$  is diagonalized by the matrices  $\tilde{U}$  and  $\tilde{V}$  to give the eigenvalues  $\tilde{m}_i$ . The bare mass matrix (or the DR running tree-level matrix)  $X^0$  is related to  $\tilde{X}$  by

$$X^0 = \tilde{X} + \delta_c X, \quad (27)$$

where  $\delta_c$  means the variation of the (on-shell) parameters in  $\tilde{X}$ . The correction (25) represents the difference between  $X^0$  and the on-shell  $X$ , with  $X^0 = X + \delta X$ . This implies

$$X = \tilde{X} + \delta_c X - \delta X = \tilde{X} + \Delta X. \quad (28)$$

The one-loop corrected matrix  $X$  is the sum of the tree-level mass matrix  $\tilde{X}$  with the on-shell quantities and the ultraviolet (UV) finite shifts  $\Delta X$ .

To discuss the shifts  $\Delta X$ , we need to fix the definition of the on-shell parameters in  $\tilde{X}$ . In this paper, we define the on-shell parameters  $M$  and  $\mu$  by the elements of the on-shell mass matrix of charginos, by  $M = X_{11}$  and  $\mu = X_{22}$  respectively. This definition gives the counterterms

$$\delta M = (\delta X)_{11}, \quad (29)$$

$$\delta \mu = (\delta X)_{22}. \quad (30)$$

We comment later on the case in which  $M$  and  $\mu$  are fixed by the neutralino mass matrix. In addition, we fix the on-shell  $m_W$  as the physical (pole) mass and  $\tan \beta$  by the condition in the Higgs sector, as given in the Appendix. As a result, we have

$$\Delta X_{11} = 0, \quad (31)$$

$$\Delta X_{12} = \left( \frac{\delta m_W}{m_W} + \cos^2 \beta \frac{\delta \tan \beta}{\tan \beta} \right) X_{12} - \delta X_{12}, \quad (32)$$

$$\Delta X_{21} = \left( \frac{\delta m_W}{m_W} - \sin^2 \beta \frac{\delta \tan \beta}{\tan \beta} \right) X_{21} - \delta X_{21}, \quad (33)$$

$$\Delta X_{22} = 0, \quad (34)$$

with  $\delta X_{12}$  and  $\delta X_{21}$  given by Eq. (25) with the replacements  $U \rightarrow \tilde{U}$ ,  $V \rightarrow \tilde{V}$ , and  $m_{\tilde{\chi}_k} \rightarrow \tilde{m}_k$ . The counterterm  $\delta m_W$  is given in Eq. (A9) together with Eq. (A11).  $\delta \tan \beta$  is obtained from Eqs. (A15) and (A16). By diagonalizing the matrix  $X$ , one gets the one-loop pole masses of charginos,  $m_{\tilde{\chi}_{1,2}^+}$ , and their on-shell rotation matrices  $U$  and  $V$ , which enter in all chargino couplings.

If the chargino masses  $m_{\tilde{\chi}_{1,2}^+}$  are known from experiment (e.g., from a threshold scan), one first calculates the tree-level parameters  $\tilde{M}$ ,  $\tilde{\mu}$ ,  $\tilde{U}$ , and  $\tilde{V}$ , using Eqs. (21) and (22) together with the experimental information of chargino couplings [5,6,15].  $\delta \tilde{U}$  and  $\delta \tilde{V}$  are then obtained from Eqs. (23) and (24), depending on the sfermion parameters. This enables one to calculate  $\Delta X_{12}$  and  $\Delta X_{21}$  and the one-loop corrected mass matrix  $X$ . By requiring that  $X$  give the measured chargino masses  $m_{\tilde{\chi}_{1,2}^+}$ , one then gets the correct on-shell parameters  $M$  and  $\mu$ . The error that one starts from  $\tilde{M}$  and  $\tilde{\mu}$  is of higher order. The dependence of this procedure on sfermion parameters will be discussed in Sec. IV.

Let us now turn to the neutralino sector. The mass matrix in the interaction basis has the form

$$Y = \begin{pmatrix} M' & 0 & -m_Z \sin \theta_W \cos \beta & m_Z \sin \theta_W \sin \beta \\ 0 & M & m_Z \cos \theta_W \cos \beta & -m_Z \cos \theta_W \sin \beta \\ -m_Z \sin \theta_W \cos \beta & m_Z \cos \theta_W \cos \beta & 0 & -\mu \\ m_Z \sin \theta_W \sin \beta & -m_Z \cos \theta_W \sin \beta & -\mu & 0 \end{pmatrix}. \quad (35)$$

Since we assume  $CP$  conservation, this matrix is real and symmetric. It is diagonalized by the real matrix  $Z$ :

$$ZYZ^T = M_D = \text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}). \quad (36)$$

We allow negative values for the mass parameters  $m_{\tilde{\chi}_i^0}$ . The convention  $|m_{\tilde{\chi}_1^0}| < |m_{\tilde{\chi}_2^0}| < |m_{\tilde{\chi}_3^0}| < |m_{\tilde{\chi}_4^0}|$  is used.

The shift for the rotation matrix  $Z$  is obtained from Eqs. (13) and (14) by the substitutions  $U \rightarrow Z$  and  $V \rightarrow Z$ ,

$$\delta Z = \frac{1}{4} (\delta Z_L^{\tilde{\chi}^0} - \delta Z_L^{\tilde{\chi}^0 T}) Z, \quad (37)$$

$$\delta Z^T = \frac{1}{4} Z^T (\delta Z_R^{\tilde{\chi}^0 T} - \delta Z_R^{\tilde{\chi}^0}). \quad (38)$$

Note that  $\delta Z_L^{\tilde{\chi}^0} = \delta Z_R^{\tilde{\chi}^0}$  due to the Majorana character of the neutralinos. The shift  $\delta Y$  is given by  $\delta Y = \delta(Z^T M_D Z)$ , i.e., for the matrix elements

$$(\delta Y)_{ij} = \sum_{k=1}^4 [\delta m_{\tilde{\chi}_k^0} Z_{ki} Z_{kj} + m_{\tilde{\chi}_k^0} \delta Z_{ki} Z_{kj} + m_{\tilde{\chi}_k^0} Z_{ki} \delta Z_{kj}]. \quad (39)$$

The wave-function correction terms  $\delta Z^{\tilde{\chi}^0}$  are given by Eq. (19) and  $\delta m_{\tilde{\chi}_k^0}$  by Eq. (18). The formulas for the neutralino self-energies are shown in Eqs. (A4) and (A5).

We again start with the tree-level mass matrix  $Y^{\text{tree}} \equiv \tilde{Y}$ , which has the form of Eq. (35) in terms of the on-shell parameters  $(M, \mu, M', m_Z, \sin \theta_W, \tan \beta)$ . First one calculates the tree-level masses  $\tilde{m}_k$ , and the rotation matrix  $\tilde{Z}$  by diagonalizing  $\tilde{Y}$ . In analogy to the chargino case, the one-loop on-shell mass matrix  $Y$  is

$$Y = Y^0 - \delta Y = \tilde{Y} + \delta_c Y - \delta Y = \tilde{Y} + \Delta Y. \quad (40)$$

Here  $\delta_c$  means the variation of the parameters in  $\tilde{Y}$ . Again  $\Delta Y$  is UV-finite.

We need to fix the on-shell input parameters in  $\tilde{Y}$ . The on-shell  $M$  and  $\mu$  are already determined by the chargino sector. We define the on-shell parameter  $M'$  by the on-shell mass matrix of neutralinos as  $Y_{11} = M'$ . This condition gives

$$\delta M' = (\delta Y)_{11}. \quad (41)$$

We further fix the on-shell  $m_Z$ ,  $\sin^2 \theta_W = 1 - m_W^2/m_Z^2$ , and  $\tan \beta$  in the same way as in the case of charginos.

The 10 independent entries of the real and symmetric matrix  $\Delta Y$  are

$$\Delta Y_{11} = 0, \quad (42)$$

$$\Delta Y_{12} = -\delta Y_{12}, \quad (43)$$

$$\Delta Y_{13} = \left( \frac{\delta m_Z}{m_Z} + \frac{\delta \sin \theta_W}{\sin \theta_W} - \sin^2 \beta \frac{\delta \tan \beta}{\tan \beta} \right) Y_{13} - \delta Y_{13}, \quad (44)$$

$$\Delta Y_{14} = \left( \frac{\delta m_Z}{m_Z} + \frac{\delta \sin \theta_W}{\sin \theta_W} + \cos^2 \beta \frac{\delta \tan \beta}{\tan \beta} \right) Y_{14} - \delta Y_{14}, \quad (45)$$

$$\Delta Y_{22} = \delta M - \delta Y_{22}, \quad (46)$$

$$\Delta Y_{23} = \left( \frac{\delta m_Z}{m_Z} - \tan^2 \theta_W \frac{\delta \sin \theta_W}{\sin \theta_W} - \sin^2 \beta \frac{\delta \tan \beta}{\tan \beta} \right) Y_{23} - \delta Y_{23}, \quad (47)$$

$$\Delta Y_{24} = \left( \frac{\delta m_Z}{m_Z} - \tan^2 \theta_W \frac{\delta \sin \theta_W}{\sin \theta_W} + \cos^2 \beta \frac{\delta \tan \beta}{\tan \beta} \right) Y_{24} - \delta Y_{24}, \quad (48)$$

$$\Delta Y_{33} = -\delta Y_{33}, \quad (49)$$

$$\Delta Y_{34} = -\delta \mu - \delta Y_{34}, \quad (50)$$

$$\Delta Y_{44} = -\delta Y_{44}, \quad (51)$$

with  $\delta Y_{ij}$  given by Eq. (39) with  $Z \rightarrow \tilde{Z}$ , and  $m_{\tilde{\chi}_k^0} \rightarrow \tilde{m}_{\tilde{\chi}_k^0}$ .  $\delta \sin \theta_W$  and  $\delta m_Z$  can be calculated from Eqs. (A9)–(A13). Notice that the elements  $Y_{12} = Y_{21}$ ,  $Y_{33}$ , and  $Y_{44}$  are no longer zero. Recall that  $\delta M = \delta X_{11}$  and  $\delta \mu = \delta X_{22}$ . The corrected neutralino masses and the corrected rotation matrix  $Z$  are obtained by diagonalizing the matrix  $Y$ , Eq. (40).

So far we have treated  $M'$  as an independent parameter to be determined in the neutralino sector. If we assume a relation between gaugino masses, we may define the on-shell  $M'$  as a function of other on-shell parameters instead of  $Y_{11}$ . The shift  $\Delta Y_{11}$  is then no longer zero. For example, when the SUSY SU(5) relation  $M' = \frac{5}{3} \tan^2 \theta_W M$  holds for the DR parameters, and the on-shell  $M'$  is defined by imposing the same relation on the on-shell parameters, one has

$$\Delta Y_{11} = \left( \frac{2}{\cos^2 \theta_W} \frac{\delta \sin \theta_W}{\sin \theta_W} + \frac{\delta M}{M} \right) Y_{11} - \delta Y_{11}. \quad (52)$$

We note that Eq. (52) is also applicable in other models for gaugino masses, e.g., in the anomaly mediated SUSY breaking model [16,17] where  $M' = 11 \tan^2 \theta_W M$ .

Finally, we would like to remark that one could also first determine the on-shell values of  $M', M, \mu$  from the neutralino sector, which means  $\Delta Y_{11} = \Delta Y_{22} = \Delta Y_{34} = \Delta Y_{43} = 0$ , see Eqs. (35) and (42)–(51). This would imply corrections  $\Delta X_{11}$  and  $\Delta X_{22}$  in the chargino system.

#### IV. NUMERICAL EXAMPLES

In this section, we will give some numerical examples for the on-shell one-loop corrected mass matrices and masses of the charginos and neutralinos. We take into account the contributions from all fermions and sfermions.

For simplicity, we will take in the following (if not specified otherwise) for the soft breaking sfermion mass parameters of the first and second generation  $M_{\tilde{U}_{1,2}} = M_{\tilde{U}_{1,2}} = M_{\tilde{D}_{1,2}} = M_{\tilde{L}_{1,2}} = M_{\tilde{E}_{1,2}}$ , of the third generation  $M_{\tilde{Q}_3} = \frac{10}{9} M_{\tilde{U}_3} = \frac{10}{11} M_{\tilde{D}_3} = M_{\tilde{L}_3} = M_{\tilde{E}_3} = M_{\tilde{Q}}$ , and for the trilinear couplings  $A_t = A_b = A_\tau = A$ . We take  $m_t = 175$  GeV,  $m_b = 5$  GeV,  $m_Z = 91.2$  GeV,  $m_W = 80$  GeV, and  $m_{A^0} = 500$  GeV. Thus the input parameter set is  $\{\tan \beta, M_{\tilde{Q}_1}, M_{\tilde{Q}}, A, M, M', \mu\}$ .

We always assume that the (on-shell) values of  $M$  and  $\mu$  are obtained from the chargino sector as described in Sec. III. Then the chargino mass matrix only gets corrections in the off-diagonal elements of the matrix  $X$ . In general, the corrections to the chargino masses are small ( $< 1\%$ ). For instance, for  $\tan \beta = 7$  and  $\{M_{\tilde{Q}_1}, M_{\tilde{Q}}, A, M, \mu\} = \{300, 300, -500, 300, -400\}$  GeV one gets for  $\Delta X_{12}/X_{12} \approx 0.7/122$  and for  $\Delta X_{21}/X_{21} \approx -1.1/16$ ,  $\Delta m_{\tilde{\chi}_1^+}/m_{\tilde{\chi}_1^+} = -0.24\%$  and  $\Delta m_{\tilde{\chi}_2^+}/m_{\tilde{\chi}_2^+} = -0.14\%$ . For the same parameters but  $\tan \beta = 40$  one gets for  $\Delta X_{12}/X_{12} \approx 2.3/113$ ,  $\Delta X_{21}/X_{21} \approx -3.2/2.8$ ,  $\Delta m_{\tilde{\chi}_1^+}/m_{\tilde{\chi}_1^+} = -0.76\%$ , and  $\Delta m_{\tilde{\chi}_2^+}/m_{\tilde{\chi}_2^+} = -0.46\%$ .

As shown in Sec. III, the on-shell parameters  $M$  and  $\mu$  for

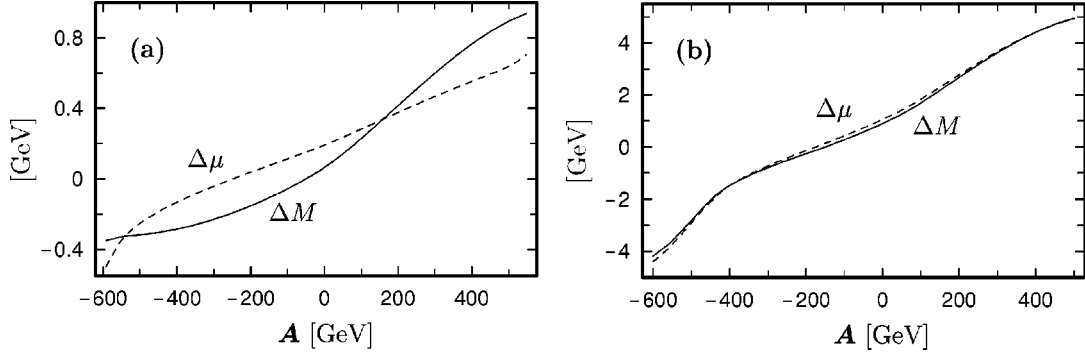


FIG. 1.  $\Delta M = M - M^{\text{eff}}$  (full lines) and  $\Delta \mu = \mu - \mu^{\text{eff}}$  (dashed lines) as functions of  $A$  with  $\tan \beta = 7$ ,  $M_{\tilde{Q}_1} = M_{\tilde{Q}_2} = 300$  GeV, and  $\text{sgn}(\mu) = -1$ . In (a)  $\{m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+}\} = \{126.7, 322\}$  GeV giving  $\{M^{\text{eff}}, \mu^{\text{eff}}\} = \{300, -130\}$  GeV for  $M > |\mu|$ . In (b)  $\{m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+}\} = \{200, 300\}$  GeV giving  $\{M^{\text{eff}}, \mu^{\text{eff}}\} = \{227.7, -255.6\}$  GeV for  $M < |\mu|$ .

given values of the pole masses  $m_{\tilde{\chi}_1^+}$  and  $m_{\tilde{\chi}_2^+}$  depend on sfermion parameters. In Fig. 1, we show the values of  $M$  and  $\mu$  as functions of  $A$ , for fixed  $m_{\tilde{\chi}_{1,2}^+}$ ,  $\tan \beta = 7$ , and  $M_{\tilde{Q}_1} = M_{\tilde{Q}_2} = 300$  GeV. For comparison we show the difference from the effective parameters used in [11],  $M^{\text{eff}}$  and  $\mu^{\text{eff}}$ . These are obtained from the pole masses  $m_{\tilde{\chi}_{1,2}^+}$  by tree-level sum rules and are therefore independent of sfermion parameters. We see that the dependence on the sfermion parameters becomes large for  $M \sim |\mu|$ , i.e., large gaugino-Higgsino mixing.

Let us now discuss the neutralino sector for the on-shell  $M$  and  $\mu$  fixed by the chargino sector. We first treat the on-shell  $M'$  as an independent parameter. Then the one-loop corrections to the mass matrix (35) are calculated by Eqs. (42)–(51).

In Fig. 2(a), we show the relative correction  $\delta m_{\tilde{\chi}_1^0}/m_{\tilde{\chi}_1^0}$  as a function of  $\mu$  for  $\tan \beta = 7$  and  $\{M_{\tilde{Q}_1}, M_{\tilde{Q}_2}, A, M, M'\} = \{300, 300, -500, 300, 149.4\}$  GeV.

One can see that the corrections to  $m_{\tilde{\chi}_1^0}$  and  $m_{\tilde{\chi}_2^0}$  can go up to 2.2% for  $|\mu| \sim 100$  GeV, where  $\tilde{\chi}_1^0$  and  $\tilde{\chi}_2^0$  are Higgsino-like. Figure 2(b) shows the same as Fig. 2(a) for  $\tan \beta = 40$  with the other parameters unchanged. The general behavior of the curves is very similar.

We also present numerical values of the mass matrix  $\tilde{Y}$

and its correction  $\Delta Y$  calculated for the same set of parameters as in Fig. 2(a), with  $\mu = 110$  GeV,

$$\tilde{Y} + \Delta Y = \begin{pmatrix} 149.4 & 0 & -6.2 & 43.3 \\ 0 & 300 & 11.3 & -79.2 \\ -6.2 & 11.3 & 0 & 110 \\ 43.3 & -79.2 & 110 & 0 \end{pmatrix} \text{ GeV} \\ + \begin{pmatrix} 0 & 0.3 & 0.0 & 0.9 \\ 0.3 & -0.1 & -0.1 & -0.2 \\ 0.0 & -0.1 & -0.0 & 0.2 \\ 0.9 & -0.2 & 0.2 & -4.3 \end{pmatrix} \text{ GeV}.$$

Notice that also the zero elements of  $\tilde{Y}$  get nonzero corrections. Especially, the most important contribution comes from the element  $\Delta Y_{44}$ , that is, from the  $\tilde{H}_2^0$  to  $\tilde{H}_2^0$  transition via a  $t\tilde{t}$  loop. The effects of  $\Delta Y_{22}$ ,  $\Delta Y_{33}$ ,  $\Delta Y_{34}$ , and  $\Delta Y_{44}$  on the masses were discussed in the limiting cases, for  $|\mu| \ll (M, M')$  in [18] and for  $M \ll (|\mu|, M')$  in [19, 17], respectively.

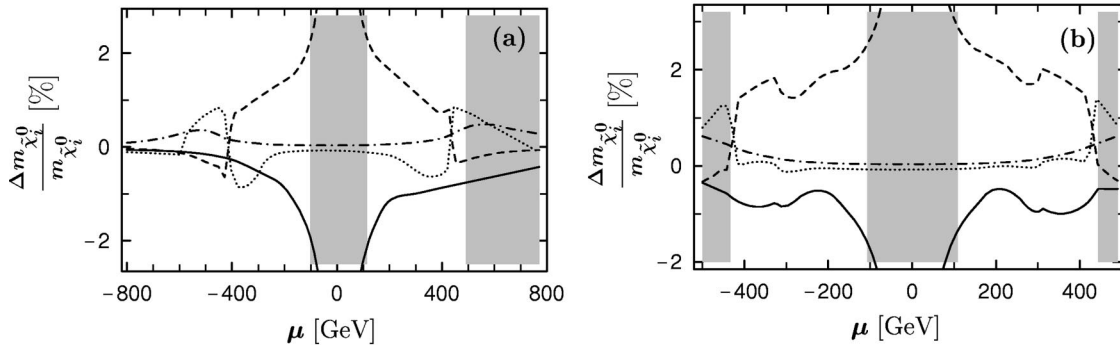


FIG. 2. Relative corrections to neutralino masses as a function of  $\mu$  for  $\tan \beta = 7$  (a) and  $\tan \beta = 40$  (b) with  $\{M_{\tilde{Q}_1}, M_{\tilde{Q}_2}, A, M, M'\} = \{300, 300, -500, 300, 149.4\}$  GeV. The full, dashed, dotted, and dash-dotted lines correspond to  $\tilde{\chi}_1^0$ ,  $\tilde{\chi}_2^0$ ,  $\tilde{\chi}_3^0$ , and  $\tilde{\chi}_4^0$  mass corrections, respectively. The gray areas are excluded by the bounds  $m_{\tilde{\chi}_1^+} \geq 100$  GeV,  $m_{h^0} > 95$  GeV.



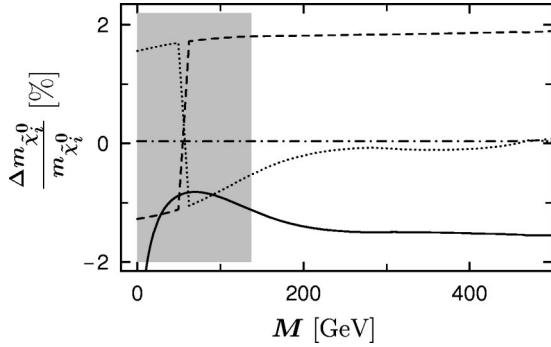


FIG. 3. Relative corrections to neutralino masses as a function of  $M$  for  $\tan\beta=7$  and  $\{M_{\tilde{\chi}_1^0}, M_{\tilde{\chi}_2^0}, A, \mu\} = \{300, 300, -500, -130\}$  GeV. The full, dashed, dotted, and dash-dotted lines correspond to  $\tilde{\chi}_1^0$ ,  $\tilde{\chi}_2^0$ ,  $\tilde{\chi}_3^0$ , and  $\tilde{\chi}_4^0$  mass corrections, respectively. The gray areas are excluded by the bound  $m_{\tilde{\chi}_1^+} \geq 100$  GeV.

In Fig. 3, we show the  $M$  dependence with  $M' = 0.498M$  for  $\mu = -130$  GeV and the other parameters as in Fig. 2(a). One sees that up to  $M \approx 200$  GeV, the  $M$  dependence of  $\delta m_{\tilde{\chi}_i^0}/m_{\tilde{\chi}_i^0}$  is rather strong and becomes weak when  $\tilde{\chi}_1^0$  becomes Higgsino-like. One also sees the various discontinuities in  $\tilde{\chi}_2^0$  and  $\tilde{\chi}_3^0$  due to “level crossings” (see, e.g., [20]).

Figure 4 exhibits the dependence on  $M_{\tilde{Q}}$  for the same parameter set as in Fig. 2(a) with  $\mu = -130$  GeV. The corrections to the masses become smaller with increasing  $M_{\tilde{Q}}$ . The dependence on  $M_{\tilde{Q}_1}$  for fixed  $M_{\tilde{Q}}$  is very small. The effects of the nondecoupling corrections [21,10] to the gaugino-Higgsino mixing elements of  $Y$  and  $X$  cannot be seen in Fig. 4.

In Fig. 5, we show the dependence on  $A_\tau$ , with  $\mu = -130$  GeV,  $A_b = A_\tau$  fixed to 500 GeV, and the other parameters as in Fig. 2(a). It is strong for the Higgsino-like states  $\tilde{\chi}_1^0$  and  $\tilde{\chi}_2^0$  being mainly due to the  $A_\tau$  dependence of  $\Delta Y_{44}$ . The sensitivity to the value of  $A_b$  and  $A_\tau$  is very weak.

Finally, we discuss the interesting case in which  $M'$  and  $M$  are related by the SUSY SU(5) GUT relation  $M' = \frac{5}{3} \tan^2 \theta_W M$  in the  $\overline{\text{DR}}$  scheme. Then it is possible to define the on-shell  $M'$  by imposing this relation on the on-shell parameters. In this case,  $\Delta Y_{11}$  gets a contribution according to Eq. (52). This correction is relatively large. For instance, for the parameter set of Fig. 2(a) with  $\mu = -130$  GeV one gets  $\Delta Y_{11}/Y_{11} \approx 5.7/149$ . This corresponds to the SUSY threshold correction to the unification condition of the gaugino masses [7,22].  $\Delta Y_{11}$  gives a large correction to the mass of a  $B$ -ino-like neutralino. This is clearly seen in Fig. 6. Figure 6(a) shows the correction to  $m_{\tilde{\chi}_3^0}$  as a function of  $M$  for  $\mu = -130$  GeV and the other parameters as in Fig. 2(a). The solid line shows the case in which the  $\overline{\text{DR}}$  parameters  $M$  and  $M'$  satisfy the SUSY GUT relation and the on-shell  $M'$  is defined by the same relation. For comparison, the dotted line shows the case in which the on-shell  $M'$  is defined by  $Y_{11}$  as an independent parameter (with  $\Delta Y_{11} = 0$ ), but its value coincides with the on-shell  $\frac{5}{3} \tan^2 \theta_W M$ .  $\tilde{\chi}_3^0$  is  $B$ -ino-like for  $M > 200$  GeV and  $\delta m_{\tilde{\chi}_3^0}/m_{\tilde{\chi}_3^0}$  goes up to 4%. In Fig. 6(b), we show  $\delta m_{\tilde{\chi}_1^0}/m_{\tilde{\chi}_1^0}$  for  $\mu = -300$  GeV. Here  $\tilde{\chi}_1^0$  is almost purely  $B$ -ino-like, hence it also gets a large correction.

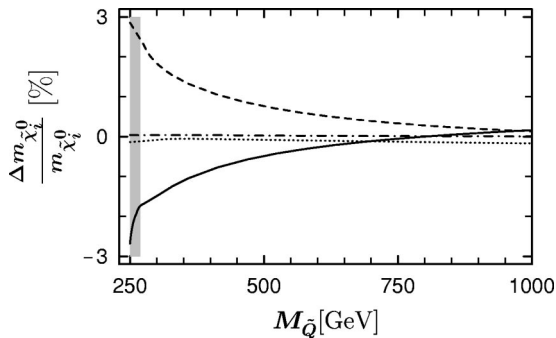


FIG. 4. Relative corrections to neutralino masses as a function of  $M_{\tilde{Q}}$  for  $\tan\beta=7$  and  $\{M_{\tilde{Q}_1}, A, M, M', \mu\} = \{300, -500, 300, 149.4, -130\}$  GeV. The full, dashed, dotted, and dash-dotted lines correspond to  $\tilde{\chi}_1^0$ ,  $\tilde{\chi}_2^0$ ,  $\tilde{\chi}_3^0$ , and  $\tilde{\chi}_4^0$  mass corrections, respectively. The gray areas are excluded by the bound  $m_{\tilde{\tau}_1} \geq 100$  GeV.

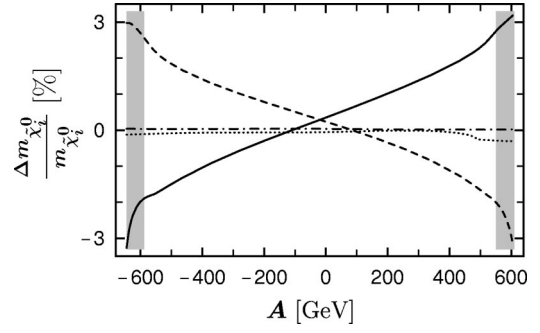


FIG. 5. Relative corrections to neutralino masses as a function of  $A$  for  $\tan\beta=7$  and  $\{M_{\tilde{\chi}_1^0}, M_{\tilde{\chi}_2^0}, M, M', \mu\} = \{300, 300, 300, 149.4, -130\}$  GeV. The full, dashed, dotted, and dash-dotted lines correspond to  $\tilde{\chi}_1^0$ ,  $\tilde{\chi}_2^0$ ,  $\tilde{\chi}_3^0$ , and  $\tilde{\chi}_4^0$  mass corrections, respectively. The gray areas are excluded by the bound  $m_{\tilde{\tau}_1} \geq 100$  GeV.

$= \frac{5}{3} \tan^2 \theta_W M$  in the  $\overline{\text{DR}}$  scheme. Then it is possible to define the on-shell  $M'$  by imposing this relation on the on-shell parameters. In this case,  $\Delta Y_{11}$  gets a contribution according to Eq. (52). This correction is relatively large. For instance, for the parameter set of Fig. 2(a) with  $\mu = -130$  GeV one gets  $\Delta Y_{11}/Y_{11} \approx 5.7/149$ . This corresponds to the SUSY threshold correction to the unification condition of the gaugino masses [7,22].  $\Delta Y_{11}$  gives a large correction to the mass of a  $B$ -ino-like neutralino. This is clearly seen in Fig. 6. Figure 6(a) shows the correction to  $m_{\tilde{\chi}_3^0}$  as a function of  $M$  for  $\mu = -130$  GeV and the other parameters as in Fig. 2(a). The solid line shows the case in which the  $\overline{\text{DR}}$  parameters  $M$  and  $M'$  satisfy the SUSY GUT relation and the on-shell  $M'$  is defined by the same relation. For comparison, the dotted line shows the case in which the on-shell  $M'$  is defined by  $Y_{11}$  as an independent parameter (with  $\Delta Y_{11} = 0$ ), but its value coincides with the on-shell  $\frac{5}{3} \tan^2 \theta_W M$ .  $\tilde{\chi}_3^0$  is  $B$ -ino-like for  $M > 200$  GeV and  $\delta m_{\tilde{\chi}_3^0}/m_{\tilde{\chi}_3^0}$  goes up to 4%. In Fig. 6(b), we show  $\delta m_{\tilde{\chi}_1^0}/m_{\tilde{\chi}_1^0}$  for  $\mu = -300$  GeV. Here  $\tilde{\chi}_1^0$  is almost purely  $B$ -ino-like, hence it also gets a large correction.

## V. CONCLUSIONS

We have presented a consistent method for the calculation of the one-loop corrections to the on-shell mass matrices of charginos and neutralinos, and hence their masses. The on-shell parameters  $M$ ,  $\mu$ , and  $M'$  are determined by the elements of the on-shell mass matrices. We have calculated the corrections to the tree-level mass matrices in terms of on-shell parameters. We have performed a detailed numerical analysis of the corrections due to fermion and sfermion loops, as a function of the SUSY parameters. When the parameters  $M$  and  $\mu$  are determined by the chargino system, one gets corrections to the neutralino masses of up to 4%. We have also treated the case in which the on-shell  $M'$  is defined by  $M$  using the SUSY GUT relation. Therefore, these corrections have to be taken into account in precision experiments at future  $e^+e^-$  linear colliders.

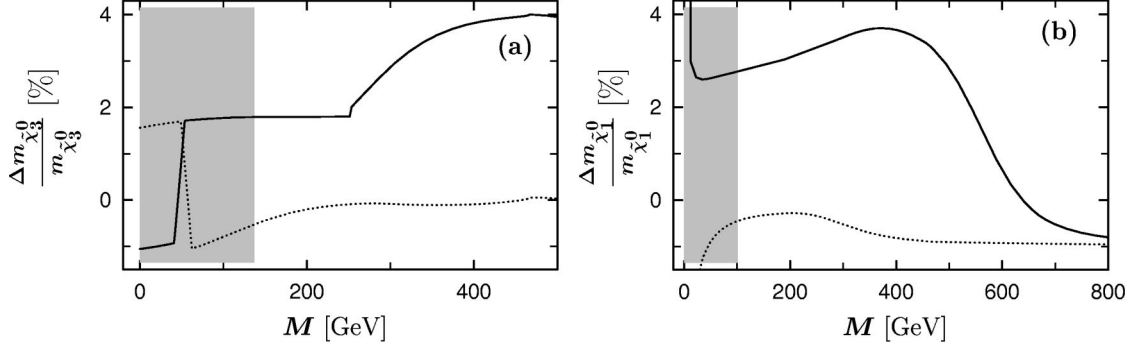


FIG. 6. Comparison of relative corrections to  $m_{\chi_3^0}$  (a) and  $m_{\chi_1^0}$  (b). The full line shows the case where the SUSY SU(5) GUT relation is assumed for the  $\overline{\text{DR}}$  parameters  $M$  and  $M'$ , and the on-shell  $M'$  is determined from  $M$  by the same relation. The dotted line corresponds to the case where the on-shell  $M'$  is an independent parameter but satisfies the SUSY GUT relation. Other parameters are  $\tan\beta=7$ ,  $\{M_{\tilde{Q}_1}, M_{\tilde{Q}_2}, A\} = \{300, 300, -500\}$  GeV, and  $\mu = [-110$  (a),  $-300$  (b)] GeV. The gray areas are excluded by the bound  $m_{\chi_1^+} \geq 100$  GeV.

### ACKNOWLEDGMENTS

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### APPENDIX

In the following, we give the formulas for the various self-energies due to fermion and sfermion one-loop contributions and the formulas for the counterterms of  $m_Z$ ,  $m_W$ ,  $\sin\theta_W$ , and  $\tan\beta$  used in this work.

#### Chargino self-energies

The chargino self-energies read

$$\begin{aligned} \Pi_{ij}^L(k^2) &= -\frac{1}{(4\pi)^2} \sum_{\text{gen}} N_C \sum_{a=1}^2 [l_{ai}^{\tilde{u}} l_{aj}^{\tilde{u}} B_1(k^2, m_d^2, m_{\tilde{u}_a}^2) \\ &\quad + k_{ai}^{\tilde{d}} k_{aj}^{\tilde{d}} B_1(k^2, m_u^2, m_d^2)], \\ \Pi_{ij}^R(k^2) &= -\frac{1}{(4\pi)^2} \sum_{\text{gen}} N_C \sum_{a=1}^2 [k_{ai}^{\tilde{u}} k_{aj}^{\tilde{u}} B_1(k^2, m_d^2, m_{\tilde{u}_a}^2) \\ &\quad + l_{ai}^{\tilde{d}} l_{aj}^{\tilde{d}} B_1(k^2, m_u^2, m_d^2)], \\ \Pi_{ij}^{S,L}(k^2) &= \frac{1}{(4\pi)^2} \sum_{\text{gen}} N_C \sum_{a=1}^2 [m_u l_{ai}^{\tilde{d}} k_{aj}^{\tilde{d}} B_0(k^2, m_u^2, m_d^2) \\ &\quad + m_d k_{ai}^{\tilde{u}} l_{aj}^{\tilde{u}} B_0(k^2, m_d^2, m_{\tilde{u}_a}^2)], \\ \Pi_{ij}^{S,R}(k^2) &= \frac{1}{(4\pi)^2} \sum_{\text{gen}} N_C \sum_{a=1}^2 [m_u l_{aj}^{\tilde{d}} k_{ai}^{\tilde{d}} B_0(k^2, m_u^2, m_d^2) \\ &\quad + m_d k_{aj}^{\tilde{u}} l_{ai}^{\tilde{u}} B_0(k^2, m_d^2, m_{\tilde{u}_a}^2)]. \end{aligned} \quad (\text{A1})$$

Here and in the following, the index  $u$  ( $d$ ) denotes an up (down) -type fermion and  $\sum_{\text{gen}}$  denotes the sum over all six fermion generations.  $N_C=3$  (1) in the quark (lepton) case. The chargino-sfermion-fermion couplings are

$$\begin{aligned} l_{ak}^{\tilde{u}} &= -g V_{k1} R_{a1}^{\tilde{u}} + h_u V_{k2} R_{a1}^{\tilde{u}}, & k_{ak}^{\tilde{u}} &= h_d U_{k2} R_{a1}^{\tilde{u}}, \\ l_{ak}^{\tilde{d}} &= -g U_{k1} R_{a1}^{\tilde{d}} + h_d U_{k2} R_{a1}^{\tilde{d}}, & k_{ak}^{\tilde{d}} &= h_u V_{k2} R_{a1}^{\tilde{d}}, \end{aligned} \quad (\text{A2})$$

where  $U, V(R^{\tilde{f}})$  are the chargino (sfermion) mixing matrices and

$$h_u = \frac{gm_u}{\sqrt{2}m_W \sin\beta}, \quad h_d = \frac{gm_d}{\sqrt{2}m_W \cos\beta}. \quad (\text{A3})$$

The two-point functions  $B_0$  and  $B_1$  [23] are given in the convention [24].

#### Neutralino self-energies

The neutralino self-energies read

$$\begin{aligned} \Pi_{ij}^{\tilde{\chi}^0 L}(k^2) &= \Pi_{ij}^{\tilde{\chi}^0 R}(k^2) \\ &= \frac{-1}{(4\pi)^2} \sum_{\text{gen}} N_C \sum_{f=u,d} \sum_{a=1}^2 (a_{ai}^{\tilde{f}} a_{aj}^{\tilde{f}} + b_{ai}^{\tilde{f}} b_{aj}^{\tilde{f}}) \\ &\quad \times B_1(k^2, m_f^2, m_{\tilde{f}_a}^2), \\ \Pi_{ij}^{\tilde{\chi}^0 S,L}(k^2) &= \Pi_{ij}^{\tilde{\chi}^0 S,R}(k^2) \\ &= \frac{1}{(4\pi)^2} \sum_{\text{gen}} N_C \sum_{f=u,d} \sum_{a=1}^2 (a_{ai}^{\tilde{f}} b_{aj}^{\tilde{f}} + a_{aj}^{\tilde{f}} b_{ai}^{\tilde{f}}) \\ &\quad \times B_0(k^2, m_f^2, m_{\tilde{f}_a}^2). \end{aligned} \quad (\text{A4})$$

The neutralino-sfermion-fermion couplings are

$$a_{ak}^{\tilde{f}} = g f_{Lk}^{\tilde{f}} R_{a1}^{\tilde{f}} + h_f Z_{kx} R_{a2}^{\tilde{f}}, \quad b_{ak}^{\tilde{f}} = g f_{Rk}^{\tilde{f}} R_{a2}^{\tilde{f}} + h_f Z_{kx} R_{a1}^{\tilde{f}}, \quad (\text{A6})$$





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