Bounds on the unitarity triangle, $\sin 2\beta$ and $K\to\pi\nu\bar{\nu}$ decays in models with minimal flavor violation

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(Received 2 August 2001; published 9 November 2001)

We present a general discussion of the unitarity triangle from ε_K, $\Delta M_{d,s}$ and $K\to\pi\nu\bar{\nu}$ in models with minimal flavor violation (MFV), allowing for arbitrary signs of the generalized Inami-Lim functions F_{tt} and X relevant for $(\varepsilon_K,\Delta M_{d,s})$ and $K\to\pi\nu\bar{\nu}$, respectively. In the models in which F_{tt} has a sign opposite to the one in the standard model, i.e. $F_{tt}<0$, the data for $(\varepsilon_K,\Delta M_{d,s})$ imply an absolute lower bound on the $B_d\to\psi K_S$ CP asymmetry $a_{\psi K_S}$ of 0.69, which is substantially stronger than 0.42 arising in the case of $F_{tt}>0$. An important finding of this paper is the observation that for given $Br(K^+\to\pi^+\nu\bar{\nu})$ and $a_{\psi K_S}$ only *two* values for $Br(K_L\to\pi^0\nu\bar{\nu})$, corresponding to the two signs of X, are possible in the full class of MFV models, independently of any new parameters arising in these models. This provides a powerful test for this class of models. Moreover, we derive *absolute* lower and upper bounds on $Br(K_L\to\pi^0\nu\bar{\nu})$ as functions of $Br(K^+\to\pi^+\nu\bar{\nu})$. Using the present experimental upper bounds on $Br(K^+\to\pi^+\nu\bar{\nu})$ and $|V_{ub}/V_{cb}|$, we obtain the absolute upper bound $Br(K_L\to\pi^0\nu\bar{\nu})<7.1\times10^{-10}$ (90% C.L.).

DOI: 10.1103/PhysRevD.64.115010 PACS number(s): 12.60.−i, 11.30.Er, 12.15.Hh, 13.20.Eb

I. INTRODUCTION

The exploration of CP violation in $B_d\to\psi K_S$ decays and the related determination of the angle β in the usual unitarity triangle of the Cabibbo-Kobayashi-Maskawa (CKM) matrix are hot topics in present particle physics [1–17]. The corresponding time-dependent CP asymmetry takes the following general form:

$$a_{\psi K_S}(t)\equiv\frac{\Gamma(B_d^0(t)\to\psi K_S)-\Gamma(\overline{B_d^0}(t)\to\psi K_S)}{\Gamma(B_d^0(t)\to\psi K_S)+\Gamma(\overline{B_d^0}(t)\to\psi K_S)} =\mathcal{A}_{CP}^{\rm dir}\cos(\Delta M_d t)+\mathcal{A}_{CP}^{\rm mix}\sin(\Delta M_d t), \quad (1)$$

where the rates correspond to decays initially, i.e. at time $t=0$, present B_d^0 or $\overline{B_d^0}$ mesons, and $\Delta M_d>0$ denotes the mass difference between the mass eigenstates of the B_d^0-$\overline{B_d^0}$ system. The quantities $\mathcal{A}_{CP}^{\rm dir}$ and $\mathcal{A}_{CP}^{\rm mix}$ are usually referred to as "direct" and "mixing-induced" CP-violating observables, respectively. In the standard model (SM), Eq. (1) simplifies as follows [18]:

$$a_{\psi K_S}(t)=-\sin 2\beta\sin(\Delta M_d t)\equiv-a_{\psi K_S}\sin(\Delta M_d t), \quad (2)$$

thereby allowing the extraction of $\sin 2\beta$. It should be noted that a measurement of a nonvanishing value of $\mathcal{A}_{CP}^{\rm dir}$ at the level of 10% would be a striking indication for new physics, as emphasized in a recent analysis of the $B\to\psi K$ system [13]. However, for the particular kind of physics beyond the SM considered in the present paper, direct CP violation in $B_d\to\psi K_S$ decays is negligible.

In the future, $\sin 2\beta$ can also be determined through the measurement of the branching ratios for the rare decays $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$ [19]. In the SM, we have, to an excellent approximation,

$$\sin 2\beta=\frac{2r_s}{1+r_s^2}, \quad (3)$$

with

$$r_s=\sqrt{\sigma}\,\frac{\sqrt{\sigma(B_1-B_2)}-P_c(\nu\bar{\nu})}{\sqrt{B_2}}. \quad (4)$$

Here B_1 and B_2 are the following "reduced" branching ratios:

$$B_1=\frac{Br(K^+\to\pi^+\nu\bar{\nu})}{4.42\times10^{-11}},\quad B_2=\frac{Br(K_L\to\pi^0\nu\bar{\nu})}{1.93\times10^{-10}}, \quad (5)$$

the quantity $P_c(\nu\bar{\nu})=0.40\pm0.06$ [20] describes the internal charm-quark contribution to $K^+\to\pi^+\nu\bar{\nu}$, and

$$\sigma\equiv\frac{1}{(1-\lambda^2/2)^2}, \quad (6)$$

with λ being one of the Wolfenstein parameters [21]. In writing Eq. (3), we have assumed that $\sin 2\beta>0$, as expected in the SM. The numerical values in Eq. (5) and the value for $P_c(\nu\bar{\nu})$ differ slightly from those given in [19,20] due to $\lambda=0.222$ used here instead of $\lambda=0.22$ used in these papers. We will return to this point below.

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The strength of formulas (2) and (3) is their theoretical cleanness, allowing a precise determination of $\sin 2\beta$ free of hadronic uncertainties that is independent of other parameters like $|V_{cb}|$, $|V_{ub}/V_{cb}|$ and m_t. Therefore the comparison of these two determinations of $\sin 2\beta$ with each other is particularly well suited for tests of CP violation in the SM, and offers a powerful tool to probe the physics beyond it [19,22].

The simplest class of extensions of the SM are those models with "minimal flavor violation" (MFV) in which the contributions of any new operators beyond those present in the SM are negligible. In these models, all flavor-changing transitions are still governed by the CKM matrix, with no new complex phases beyond the CKM phase [23,24]. If one assumes, in addition, that all new-physics contributions which are not proportional to $V_{td(s)}$ are negligible [24], then all the SM expressions for the decay amplitudes and particle-antiparticle mixing can be generalized to the MFV models by simply replacing the m_t-dependent Inami-Lim functions [25] by the corresponding functions F_i in the extensions of the SM. The latter functions now acquire additional dependences on the parameters present in these extensions. Examples are the two-Higgs-doublet model II (THDM) and the constrained minimal supersymmetric standard model (MSSM) if $\tan\overline{\beta} = v_2/v_1$ is not too large. For MFV models, direct CP violation in $B_d \to \psi K_S$ is negligible and the $\cos(\Delta M_d t)$ term in Eq. (1) vanishes.

Let us consider the off-diagonal element of the $B_q^0 - \overline{B_q^0}$ mixing matrix as an example ($q \in \{d,s\}$). In the SM, we have (for a detailed discussion, see [26]),

$$M_{12}^{(q)} = \frac{G_F^2 M_W^2}{12\pi^2}\, \eta_B m_{B_q} \hat{B}_{B_q} F_{B_q}^2 (V_{tq}^* V_{tb})^2 S_0(x_t) e^{i(\pi - \phi_{CP}(B_q))}, \tag{7}$$

where $\hat{B}_{B_q}$ is a non-perturbative parameter, F_{B_q} the B_q-meson decay constant, and $\eta_B = 0.55$ a perturbative QCD factor [27,28], which is common to $M_{12}^{(d)}$ and $M_{12}^{(s)}$. Finally, the convention-dependent phase $\phi_{CP}(B_q)$ is defined through

$$(\mathcal{C}P)|B_q^0\rangle = e^{i\phi_{CP}(B_q)}|\overline{B_q^0}\rangle. \tag{8}$$

In the MFV models, we just have to replace the Inami-Lim function $S_0(x_t)$ resulting from box diagrams with $(t, W^\pm)$ exchanges through an appropriate new function, which we denote by F_{tt} [5,24]

$$S_0(x_t) \to F_{tt}. \tag{9}$$

Expression (7) plays a key role for Eq. (2), as $\Delta M_d = 2|M_{12}^{(d)}|$, and 2β results from the difference of $\arg(M_{12}^{(d)})$ and the weak phase of the $B_d \to \psi K_S$ decay amplitude, where the convention-dependent quantity $\phi_{CP}(B_q)$ cancels.

Two interesting properties of the MFV models have recently been pointed out [24,12]:

(1) There exists a universal unitarity triangle (UUT) [24] common to all these models and the SM that can be constructed by using measurable quantities that depend on the CKM parameters but are not polluted by the new parameters present in the extensions of the SM. These quantities simply do not depend on the functions F_i.

(2) There exists an absolute lower bound on $\sin 2\beta$ [12] that follows from the interplay of ΔM_d and ε_K, measuring "indirect" CP violation in the neutral kaon system. It depends only on $|V_{cb}|$ and $|V_{ub}/V_{cb}|$, as well as on the non-perturbative parameters $\hat{B}_K$, $F_{B_d}\sqrt{\hat{B}_d}$ and ξ entering the standard analysis of the unitarity triangle.

The UUT can be constructed, for instance, by using $\sin 2\beta$ from Eq. (2) and the ratio $\Delta M_s/\Delta M_d$. At later stages also formula (3) should become useful in this respect. While the error in the determination of $\sin 2\beta$ from Eq. (2) should decrease down to ± 0.05 around the year 2005 and further to ± 0.01 during the CERN Large Hadron Collider (LHC) era, the error of ± 0.05 for $\sin 2\beta$ from Eq. (3) requires the measurements of B_1 and B_2 with an accuracy of $\pm 10\%$. This should be possible for B_1 around the year 2005 but will take longer for B_2. The relevant formulas for these determinations of the UUT can be found in [24], where also other quantities suitable for this purpose are discussed.

Concerning the lower bound on $\sin 2\beta$, a conservative scanning of all relevant input parameters gives [12,15],

$$(\sin 2\beta)_{\min} = 0.42, \tag{10}$$

corresponding to $\beta \geqslant 12°$. This bound could be considerably improved when the values of $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $\hat{B}_K$, $F_{B_d}\sqrt{\hat{B}_d}$, ξ and—in particular of ΔM_s—will be better known [12,15]. A handy approximate formula for $\sin 2\beta$ as a function of these parameters has recently been given in [17]. Using less conservative ranges of parameters, these authors find $(\sin 2\beta)_{\min} = 0.52$.

There is also an upper bound on $\sin 2\beta$, which is valid for the standard model and the full class of MFV models. It is simply given by [29]

$$(\sin 2\beta)_{\max} = 2R_b^{\max}\sqrt{1-(R_b^{\max})^2} \approx 0.82, \tag{11}$$

where

$$R_b \equiv \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{\overline{\varrho}^2 + \overline{\eta}^2} = \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| \tag{12}$$

is one side of the unitarity triangle. Here [29],

$$\overline{\varrho} \equiv \varrho(1-\lambda^2/2), \qquad \overline{\eta} \equiv \eta(1-\lambda^2/2), \tag{13}$$

where λ, ϱ and η are Wolfenstein parameters [21]. In obtaining the numerical value in Eq. (11), which corresponds to $\beta \lesssim 28°$, we have used $R_b^{\max} = 0.46$.

In this paper, we would like to point out that the analyses of the MFV models performed in [24,12,15,17] have implicitly assumed that the new functions F_i, summarizing the SM and new-physics contributions to ε_K, $\Delta M_{d,s}$ and $K \to \pi\nu\overline{\nu}$, have the same sign as the standard Inami-Lim functions. This assumption is certainly correct in the THDM and the MSSM. On the other hand, it cannot be excluded at present that there exist MFV models in which the functions F_i relevant for ε_K, ΔM_s and $K\to\pi\nu\bar{\nu}$ have a sign *opposite* to the corresponding SM Inami-Lim functions. In fact, in the case of the $B\to X_s\gamma$ decay, such a situation is even possible in the MSSM if particular values of the supersymmetric parameters are chosen. Beyond MFV, scenarios in which the new-physics contributions to neutral meson mixing and rare K decays were larger than the SM contributions and had opposite sign have been considered in [30]. Due to the presence of new complex phases in these general scenarios and new sources of flavor violation, the predictive power of the corresponding models is much smaller than of the MFV models considered here.

In the following, we would like to generalize the existing formula for the MFV models to arbitrary signs of the generalized Inami-Lim functions F_i and investigate the implications of the sign reversal in question for the determination of $\sin 2\beta$ and the unitarity triangle (UT) through $a_{\psi K_S}$, ε_K, $\Delta M_{d,s}$ and $K\to\pi\nu\bar{\nu}$. In this context, we will also discuss strategies, allowing a direct determination of the sign of F_{tt}. However, the major findings of this paper deal with the rare kaon decays $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$. In particular, we point out that—for given $Br(K^+\to\pi^+\nu\bar{\nu})$ and $a_{\psi K_S}$—only *two* values for $Br(K_L\to\pi^0\nu\bar{\nu})$, corresponding to the two possible signs of the generalized Inami-Lim function X, are possible in the full class of MFV models, independently of any new parameters present in these models. This feature provides an elegant strategy to check whether a MFV model is actually realized in nature and—if so—to determine the sign of X. Moreover, we derive *absolute* lower and upper bounds on the branching ratio $Br(K_L\to\pi^0\nu\bar{\nu})$ as a function of $Br(K^+\to\pi^+\nu\bar{\nu})$, and emphasize the utility of $B\to X_s\nu\bar{\nu}$ decays to obtain further constraints. The branching ratio $Br(K^+\to\pi^+\nu\bar{\nu})$ and the CP asymmetry $a_{\psi K_S}$ should be known rather accurately prior to the measurement of $Br(K_L\to\pi^0\nu\bar{\nu})$.

Our paper is organized as follows: in Sec. II, we analyze the unitarity triangle and $\sin 2\beta$ using $\Delta M_{d,s}$, ε_K and $a_{\psi K_S}$. Section III is devoted to the $K\to\pi\nu\bar{\nu}$ decays, and our conclusions are summarized in Sec. IV.

II. $\sin 2\beta$ AND THE UT FROM $\Delta M_{d,s}$, ε_K AND $a_{\psi K_S}$

A. $\sin 2\beta$ from $\Delta M_{d,s}$ and ε_K

In MFV models, the new-physics contributions to $\Delta M_{d,s}$ can be parametrized by a single function F_{tt}, as we have noted in Eq. (9). The same "universal" function also enters the observable ε_K [5,12,24]. In the SM, it reduces to the Inami-Lim function $S_0(x_t)\approx 2.38$.

An important quantity for our discussion is the length of one side of the unitarity triangle, R_t, defined by

$$R_t\equiv\frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|}=\sqrt{(1-\bar{\varrho})^2+\bar{\eta}^2}=\frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|. \tag{14}$$

From ΔM_d and $\Delta M_d/\Delta M_s$, one finds [24,12,15],

$$R_t=1.10\frac{R_0}{A}\frac{1}{\sqrt{|F_{tt}|}}\quad\text{with}$$

$$R_0\equiv\sqrt{\frac{\Delta M_d}{0.50/\text{ps}}}\left[\frac{230\text{ MeV}}{\sqrt{\hat{B}_d}F_{B_d}}\right]\sqrt{\frac{0.55}{\eta_B}} \tag{15}$$

and

$$R_t=0.83\xi\sqrt{\frac{\Delta M_d}{0.50/\text{ps}}}\sqrt{\frac{15.0/\text{ps}}{\Delta M_s}}\quad\text{with}\quad\xi\equiv\frac{F_{B_s}\sqrt{\hat{B}_{B_s}}}{F_{B_d}\sqrt{\hat{B}_{B_d}}}, \tag{16}$$

respectively. The corresponding hadronic parameters were introduced after Eq. (7). The Wolfenstein parameter A is defined by $|V_{cb}|=A\lambda^2$. These formulas show very clearly that the sign of F_{tt} is immaterial for the analysis of $\Delta M_{d,s}$.

On the other hand, the constraint from ε_K reads [15]

$$\bar{\eta}[(1-\bar{\varrho})A^2\eta_2F_{tt}+P_c(\varepsilon)]A^2\hat{B}_K=0.204, \tag{17}$$

where $\eta_2=0.57$ is a perturbative QCD factor [27], and $P_c(\varepsilon)=0.30\pm0.05$ [31] summarizes the contributions not proportional to $V_{ts}^*V_{td}$.

Following [12], but not assuming F_{tt} to be positive, we find, from Eqs. (15) and (17),

$$\sin 2\beta=\text{sgn}(F_{tt})\frac{1.65}{R_0^2\eta_2}\left[\frac{0.204}{A^2B_K}-\bar{\eta}P_c(\varepsilon)\right], \tag{18}$$

where the first term in the parenthesis is typically by a factor 2–3 times larger than the second term. We observe that the sign of F_{tt} determines the sign of $\sin 2\beta$. Moreover, as Eq. (17) implies $\bar{\eta}<0$ for $F_{tt}<0$, also the sign of the second term in the parenthesis is changed. This means that, for a given set of input parameters, not only the sign of $\sin 2\beta$, but also its magnitude is affected by a reversal of the sign of F_{tt}.

At this point the following remark is in order. When using analytic formulas like Eqs. (15), (16), and (17) one should remember that the numerical constants given there are sensitive functions of λ. Consequently, varying λ but keeping these values fixed would result in errors. On the other hand, for fixed $|V_{cb}|$ any change of λ modifies the parameter A and consequently the impact of the variation of λ within its uncertainties on $\sin 2\beta$ and the unitarity triangle is very small. The numerical values in Eqs. (15), (16) and (17) and the value for $P_c(\varepsilon)$ differ slightly from those given in [12,15] due to $\lambda=0.222$ used here instead of $\lambda=0.22$ used in these papers. Moreover, we have redefined R_0. This increase of λ in question is made in order to be closer to the experimental value of $|V_{ud}|$ [6].

The lower bound in Eq. (10) has been obtained by varying over all positive values of F_{tt} consistent with the experimental values of $\Delta M_{d,s}$, $|V_{ub}/V_{cb}|$ and $|V_{cb}|$, and scanning all the relevant input parameters in the ranges given in Table I. Repeating this analysis for $F_{tt}<0$, we find

TABLE I. The ranges of the input parameters.

Quantity	Central	Error
λ	0.222	
$\lvert V_{cb}\rvert$	0.041	± 0.002
$\lvert V_{ub}/V_{cb}\rvert$	0.085	± 0.018
$\lvert V_{ub}\rvert$	0.00349	± 0.00076
$\hat{B}_K$	0.85	± 0.15
$\sqrt{\hat{B}_d}F_{B_d}$	230 MeV	± 40 MeV
m_t	166 GeV	± 5 GeV
$(\Delta M)_d$	0.487/ps	± 0.014/ps
$(\Delta M)_s$	>15.0/ps	
ξ	1.15	± 0.06

$$(-\sin 2\beta)_{\rm min}=0.69. \tag{19}$$

This result is rather sensitive to the minimal value of $\sqrt{\hat{B}_{B_d}}F_{B_d}$. Taking $(\sqrt{\hat{B}_{B_d}}F_{B_d})_{\rm min}=170$ MeV instead of 190 MeV used in Eq. (19), we obtain the bound of 0.51. For the same choice, the bound in Eq. (10) is decreased to 0.35. For $(\sqrt{\hat{B}_{B_d}}F_{B_d})_{\rm min}\geqslant 195$ MeV there are no solutions for $\sin 2\beta$ for the ranges of parameters given in Table I. Finally, only for $\hat{B}_K\geqslant 0.96$, $\lvert V_{cb}\rvert\geqslant 0.0414$ and $\lvert V_{ub}/V_{cb}\rvert\geqslant 0.094$ solutions for $\sin 2\beta$ exist.

We conclude that in the case of $F_{tt}<0$ the lower bound on $\lvert\sin 2\beta\rvert$ is substantially stronger than for a positve F_{tt}. This is not surprising because in this case the contributions to ε_K proportional to $V_{ts}^*V_{td}$ interfere destructively with the charm contribution. Consequently, $\lvert\sin 2\beta\rvert$ has to be larger to fit ε_K. Our discussion in the preceeding paragraph shows that the decrease in the uncertainties of the parameters in Table I could well soon shift $\lvert\sin 2\beta\rvert$ above the upper bound in Eq. (11) and consequently exclude all MFV models with $F_{tt}<0$.

B. $a_{\psi K_S}$

Concerning $a_{\psi K_S}$, the situation is a bit more involved. As we have noted after Eq. (9), the angle 2β in Eq. (2) originates from

$$2\beta=\arg(M_{12}^{(d)})-\phi_{\rm D}(B_d\to\psi K_S), \tag{20}$$

where $\phi_{\rm D}(B_d\to\psi K_{\rm S})$ denotes a characteristic weak phase of the $B_d\to\psi K_{\rm S}$ amplitude. In the SM expression (2), it has been taken into account that $S_0(x_t)>0$, and it has been assumed implicitly that the bag parameter $\hat{B}_{B_d}$ is positive. As emphasized in [32], for $\hat{B}_{B_d}<0$, the sign in Eq. (2) would flip. However, this case appears very unlikely to us. Indeed, all existing non-perturbative methods give $\hat{B}_{B_d}>0$, which we shall also assume in our analysis. A similar comment applies to $\hat{B}_K$. However, since $S_0(x_t)$ is replaced by the new parameter F_{tt} in the case of the MFV models, which need not be positive, the following phase ϕ_d is actually probed by the CP asymmetry of $B_d\to\psi K_{\rm S}$:

$$\phi_d=2\beta+\arg(F_{tt}). \tag{21}$$

Consequently, formula (2) is generalized as follows:

$$a_{\psi K_{\rm S}}=\sin\phi_d={\rm sgn}(F_{tt})\sin 2\beta. \tag{22}$$

On the other hand, if we use Eq. (18) to predict $a_{\psi K_{\rm S}}$, the sign of the resulting CP asymmetry is unaffected:

$$a_{\psi K_{\rm S}}=\frac{1.65}{R_0^2\eta_2}\left[\frac{0.204}{A^2B_K}-\bar{\eta}P_c(\varepsilon)\right]. \tag{23}$$

However, its absolute value will generally be larger for $F_{tt}<0$.

This analysis demonstrates that in the MFV models $\sin 2\beta$ can either be positive, as in the SM, or negative. This implies that, in addition to the universal unitarity triangle proposed in [24], there exists another universal unitarity triangle with $\sin 2\beta<0$, which is valid for MFV models with $F_{tt}<0$. This also means that the "true" CKM angle β in the MFV models can only be determined from $a_{\psi K_{\rm S}}$ and $\Delta M_s/\Delta M_d$ up to a sign that depends on the sign of F_{tt}. In the spirit of [24], one can distinguish these two cases by studying simultaneously ε_K and ΔM_d. If the data on $a_{\psi K_{\rm S}}$ should violate the bound in Eq. (19) but satisfy Eq. (10), the full class of MFV models with $F_{tt}<0$ would be excluded by the measurement of $a_{\psi K_{\rm S}}(t)$ alone. If also the bound (10) should be violated, all MFV models would be excluded. The present experimental situation is given as follows:

$$a_{\psi K_{\rm S}}=\begin{cases}0.59\pm 0.14\pm 0.05 & \text{(BaBar [1])}\\ 0.99\pm 0.14\pm 0.06 & \text{(Belle [2])}\\ 0.79^{+0.41}_{-0.44} & \text{[Collider Detector at Fermilab (CDF) [3]].}\end{cases} \tag{24}$$

Combining these results with the earlier measurement by ALEPH $(0.84^{+0.82}_{-1.04}\pm 0.16)$ [4] gives the grand average

$$a_{\psi K_{\rm S}}=0.79\pm 0.10, \tag{25}$$

which does not yet allow us to draw any definite conclusions. In particular, the most recent B-factory results in Eq. (24) are no longer in favor of a small value of $a_{\psi K_S}$, so that not even the case corresponding to negative F_{tt} can be excluded. On the other

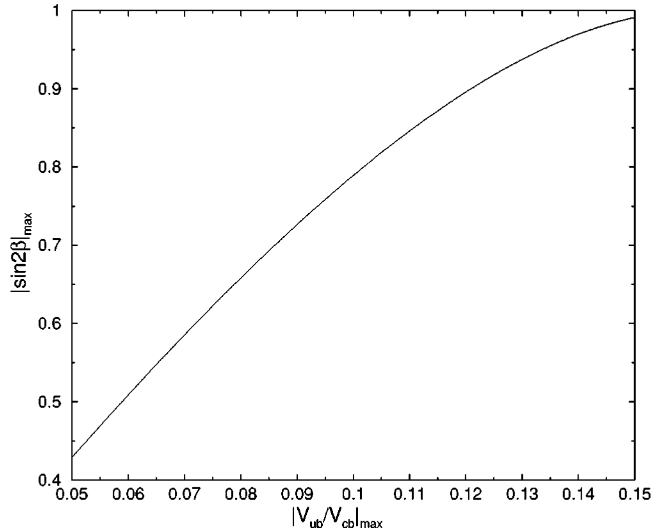

FIG. 1. $|\sin 2\beta|_{\max}$ as a function of $|V_{ub}/V_{cb}|_{\max}$.

hand, in view of the Belle result [2], the upper bound given in Eq. (11) may play an important role to search for new physics in the future. We observe that whereas the BaBar result [1] is fully consistent with $|\sin 2\beta|_{\max}=0.82$, corresponding to $|V_{ub}/V_{cb}|_{\max}=0.105$, the Belle result violates this bound. This can also be seen in Fig. 1, where we show $|\sin 2\beta|_{\max}$ as a function of $|V_{ub}/V_{cb}|_{\max}$. Only for values of $|V_{ub}/V_{cb}|$ that are substantially higher than the ones given in Table I could the Belle result be valid within the MFV models. Finally, as seen from Eq. (19) and Fig. 1, a decrease of $|V_{ub}/V_{cb}|_{\max}$ down to 0.085 would put the MFV models with $F_{tt}<0$ into difficulties, independently of other input parameters in Table I.

C. Direct determination of sgn(F_{tt})

It would of course be important to measure the sign of the parameter F_{tt} directly and to check the consistency with the bounds discussed above. Let us, in order to illustrate how this can be done, assume for a moment that $a_{\psi K_S}=0.75$ has been measured, corresponding to $\phi_d=48.6°$ or $131.4°$. Taking into account the data on $|V_{ub}/V_{cb}|$, requiring $\sqrt{\bar{\varrho}^2+\bar{\eta}^2}\lesssim 0.5$ [see our discussion below Eq. (13)], it is an easy exercise to convince ourselves that $\phi_d=48.6°$ corresponds to $\beta=24.3°$ and $\arg(F_{tt})=0$, whereas $131.4°$ is related to $\beta=-24.3°$ and $\arg(F_{tt})=180°$. Both cases can be distinguished through the unambiguous determination of ϕ_d. Several strategies were proposed to accomplish this goal [33].

The key element for the resolution of the twofold ambiguity in the extraction of ϕ_d from $a_{\psi K_S}=\sin\phi_d$ is the determination of $\cos\phi_d$. For the example given in the previous paragraph, $\cos\phi_d=+0.66$ would imply MFV models with $F_{tt}>0$, containing also the standard model, whereas $\cos\phi_d=-0.66$ would imply unambiguously the presence of new physics, corresponding to $F_{tt}<0$ in MFV scenarios. The quantity $\cos\phi_d$ can be probed through the angular distribution of $B_d\to\psi K^*[\to\pi^0 K_S]$ decays [34], allowing us to extract

$$\cos\delta_f \cos\phi_d . \tag{26}$$

Here δ_f is a strong phase corresponding to a given final-state configuration f of the ψK^* system. Theoretical tools, such as "factorization," may be sufficiently accurate to determine sgn($\cos\delta_f$), thereby allowing the direct extraction of $\cos\phi_d$. In the case of B_s decays, even information on the sign of F_{tt} can be obtained in a direct way, as the SM "background" is negligibly small in

$$\phi_s=-2\lambda^2\eta+\arg(F_{tt})\approx\arg(F_{tt}). \tag{27}$$

In analogy to the $B_d\to\psi K^*[\to\pi^0 K_S]$ case, the quantity

$$\cos\tilde{\delta}_f\cos\phi_s=\cos\tilde{\delta}_f \mathrm{sgn}(F_{tt}) \tag{28}$$

can be probed through the observables of the $B_s\to\psi\phi$ angular distribution [35]. These modes are very accessible at hadron machines. Using again a theoretical input, such as "factorization," to determine sgn($\cos\tilde{\delta}_f$), the sign of F_{tt} can be extracted. If ϕ_d is known unambiguously, $SU(3)$ flavor-symmetry arguments can be used to fix sgn($\cos\tilde{\delta}_f$) from $B_d\to\psi K^*$ decays [35]; alternative ways to determine $\cos\phi_s$=sgn(F_{tt}) from B_s decays were also noted in that paper.

D. UUT from $a_{\psi K_S}$ and $\Delta M_s/\Delta M_d$

In [36,24], a construction of the UUT by means of $a_{\psi K_S}$ and R_t following from $\Delta M_s/\Delta M_d$ has been presented. Generally, for given values of $(a_{\psi K_S}, R_t)$, there are eight solutions for $(\bar{\varrho},\bar{\eta})$. However, only two solutions are consistent with the bound in Eq. (11), corresponding to the two possible signs of F_{tt}.

For the derivation of explicit expressions for $\bar{\varrho}$ and $\bar{\eta}$, it is useful to consider

$$\mathrm{sgn}(F_{tt})\mathrm{ctg}\beta=\frac{1-\bar{\varrho}}{|\bar{\eta}|}\equiv f(\beta), \tag{29}$$

as Eq. (14) implies

$$R_t^2=(1-\bar{\varrho})^2+\bar{\eta}^2=[f(\beta)^2+1]\bar{\eta}^2. \tag{30}$$

Consequently, admitting also negative F_{tt}, we obtain

$$\bar{\eta}=\mathrm{sgn}(F_{tt})\left[\frac{R_t}{\sqrt{f(\beta)^2+1}}\right], \quad \bar{\varrho}=1-f(\beta)|\bar{\eta}|. \tag{31}$$

If we take into account the constraint from $|V_{ub}/V_{cb}|$, yielding $\bar{\varrho}<1$, we conclude that $f(\beta)$ is always positive. Moreover, as $a_{\psi K_S}=\mathrm{sgn}(F_{tt})\sin 2\beta$, we may write

$$f(\beta)=\frac{1\pm\sqrt{1-a_{\psi K_S}^2}}{a_{\psi K_S}}=\mathrm{sgn}(F_{tt})\left[\frac{1\pm|\cos 2\beta|}{\sin 2\beta}\right]. \tag{32}$$

Now the upper bound $|\beta|\lesssim 28°$ [see Eq. (11)] implies $|\mathrm{ctg}\beta|=f(\beta)\gtrsim 1.9$. As $0<a_{\psi K_S}<1$, the "$-$" solution in Eq.

(32) is hence ruled out, and the measurement of $a_{\psi K_S}$ determines $f(\beta)$ *unambiguously* through

$$f(\beta)=\frac{1+\sqrt{1-a_{\psi K_S}^2}}{a_{\psi K_S}}. \tag{33}$$

Finally, with the help of Eq. (31), we arrive at

$$\bar{\eta}=\mathrm{sgn}(F_{tt})R_t\sqrt{\frac{1-\sqrt{1-a_{\psi K_S}^2}}{2}},$$

$$\bar{\varrho}=1-\left[\frac{1+\sqrt{1-a_{\psi K_S}^2}}{a_{\psi K_S}}\right]|\bar{\eta}|. \tag{34}$$

The function $f(\beta)$ also plays a key role for the analysis of the $K\to\pi\nu\bar{\nu}$ system, which is the topic of Sec. III.

E. Lower and upper bounds on J_{CP} and $\mathrm{Im}\lambda_t$

The areas A_Δ of all unitarity triangles are equal and related to the measure of CP violation J_{CP} [37]:

$$|J_{CP}|=2A_\Delta=\lambda\left(1-\frac{\lambda^2}{2}\right)|\mathrm{Im}\lambda_t|, \tag{35}$$

where $\lambda_t=V_{ts}^*V_{td}$. The cleanest measurement of $\mathrm{Im}\lambda_t$ is offered by $Br(K_L\to\pi^0\nu\bar{\nu})$ [19], which is discussed in the following section. The importance of the measurement of J_{CP} has been stressed in particular in [38].

From ε_K and $\Delta M_{d,s}$, we find the following absolute upper and lower bounds on $|\mathrm{Im}\lambda_t|$ in the MFV models:

$$|\mathrm{Im}\lambda_t|_{\max}=\begin{cases}1.74\times10^{-4}, & F_{tt}>0,\\ 1.70\times10^{-4}, & F_{tt}<0,\end{cases} \tag{36}$$

and

$$|\mathrm{Im}\lambda_t|_{\min}=\begin{cases}0.55\times10^{-4}, & F_{tt}>0,\\ 1.13\times10^{-4}, & F_{tt}<0,\end{cases} \tag{37}$$

with $\mathrm{sgn}(\mathrm{Im}\lambda_t)=\mathrm{sgn}(F_{tt})$. In the SM, $0.94\times10^{-4}\leqslant\mathrm{Im}\lambda_t\leqslant1.60\times10^{-4}$, and the unitarity of the CKM matrix implies $|\mathrm{Im}\lambda_t|_{\max}=1.83\times10^{-4}$.

III. sin 2β AND UT FROM $K\to\pi\nu\bar{\nu}$ IN MFV MODELS

A. Preface

In MFV models, the short-distance contributions to $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$ proportional to $V_{ts}^*V_{td}$ are described by a function X, resulting from Z^0 penguin and box diagrams. In evaluating $\sin2\beta$ in terms of the branching ratios for $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$, the function X drops out [19]. Being determined from two branching ratios, there is a four-fold ambiguity in $\sin2\beta$ that is reduced to a two-fold ambiguity if $\bar{\varrho}<1$, as required by the size of $|V_{ub}/V_{cb}|$. The left over solutions correspond to two signs of $\sin2\beta$ that can be adjusted to agree with the analysis of ε_K. In the SM, the THDM and the MSSM, the functions F_{tt} and X are both positive, resulting in $\sin2\beta$ given by Eqs. (3)–(5). We would now like to generalize this discussion and the SM formulas for $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$ to MFV models with arbitrary signs of F_{tt} and X. As one of our major findings, we point out the interesting feature that—for given $Br(K^+\to\pi^+\nu\bar{\nu})$ and $a_{\psi K_S}$—only *two* values for $Br(K_L\to\pi^0\nu\bar{\nu})$, corresponding to the two signs of X, are possible in the full class of MFV models, independently of any new parameters arising in these models.

B. $K^+\to\pi^+\nu\bar{\nu}$

The reduced branching ratio B_1 defined in Eq. (5) is given by

$$B_1=\left[\frac{\mathrm{Im}\lambda_t}{\lambda^5}|X|\right]^2+\left[\frac{\mathrm{Re}\lambda_c}{\lambda}\mathrm{sgn}(X)P_c(\nu\bar{\nu})+\frac{\mathrm{Re}\lambda_t}{\lambda^5}|X|\right]^2, \tag{38}$$

where $\lambda_t=V_{ts}^*V_{td}$ with

$$\mathrm{Im}\lambda_t=\eta A^2\lambda^5,\quad \mathrm{Re}\lambda_t=-\left(1-\frac{\lambda^2}{2}\right)A^2\lambda^5(1-\bar{\varrho}), \tag{39}$$

and $\lambda_c=-\lambda(1-\lambda^2/2)$. Therefore, the standard analysis of the unitarity triangle by means of $K^+\to\pi^+\nu\bar{\nu}$ [19,29] can be generalized to arbitrary signs of X and F_{tt} through the replacements

$$X\to|X|,\quad P_c(\nu\bar{\nu})\to\mathrm{sgn}(X)P_c(\nu\bar{\nu}),\quad \bar{\eta}\to\mathrm{sgn}(F_{tt})|\bar{\eta}|. \tag{40}$$

We find then that the measured value of $Br(K^+\to\pi^+\nu\bar{\nu})$ determines an ellipse in the $(\bar{\varrho},\bar{\eta})$ plane,

$$\left(\frac{\bar{\varrho}-\varrho_0}{\bar{\varrho}_1}\right)^2+\left(\frac{\bar{\eta}}{\bar{\eta}_1}\right)^2=1, \tag{41}$$

centered at $(\varrho_0,0)$ with

$$\varrho_0=1+\mathrm{sgn}(X)\frac{P_c(\nu\bar{\nu})}{A^2|X|}, \tag{42}$$

and having the squared axes

$$\bar{\varrho}_1^2=r_0^2,\quad \bar{\eta}_1^2=\left(\frac{r_0}{\sigma}\right)^2\quad\text{with}\quad r_0^2=\frac{\sigma B_1}{A^4|X|^2}. \tag{43}$$

The ellipse (41) intersects with the circle (12). This allows us to determine $\bar{\varrho}$ and $\bar{\eta}$:

$$\bar{\varrho}=\frac{1}{1-\sigma^2}\left[\varrho_0\mp\sqrt{\sigma^2\varrho_0^2+(1-\sigma^2)(r_0^2-\sigma^2R_b^2)}\right], \tag{44}$$

$$\bar{\eta}=\mathrm{sgn}(F_{tt})\sqrt{R_b^2-\bar{\varrho}^2},$$

and consequently

$$R_t^2=1+R_b^2-2\bar{\varrho}. \tag{45}$$

Given $\bar{\varrho}$ and $\bar{\eta}$, one can determine V_{td}

$$V_{td}=A\lambda^3(1-\bar{\varrho}-i\bar{\eta}),\quad |V_{td}|=A\lambda^3 R_t. \tag{46}$$

The deviation of ϱ_0 from unity measures the relative importance of the internal charm contribution. For $X>0$, we have, as usual, $\varrho_0>1$ so that the "+" solution in Eq. (44) is excluded because of $\varrho<1$. On the other hand, for $X<0$, the center of the ellipse is shifted to $\varrho_0<1$, and for $|X| \leqslant P_c(\nu\bar{\nu})/A^2$ can even be at $\varrho_0\leqslant 0$.

Once $Br(K^+\to\pi^+\nu\bar{\nu})$ will be accurately measured, it will be important to check whether the values for $\bar{\varrho}$ and $\bar{\eta}$ in Eq. (44) obtained using $X=X_{\rm SM}$ agree with those obtained by means of Eq. (34). However, even in the case of agreement it will be impossible to claim the absence of new physics in $K^+\to\pi^+\nu\bar{\nu}$ as the same values for $(\bar{\varrho},\bar{\eta})$ can be obtained for $\mathrm{sgn}(X)<0$ with a suitably increased $|X|$. As we will discuss below the removal of this ambiguity will be possible with the help of $K_{\rm L}\to\pi^0\nu\bar{\nu}$.

In the case of disagreement between Eqs. (34) and (44) the assumption of MFV would necessarily imply some new physics contributions to $K^+\to\pi^+\nu\bar{\nu}$. On the other hand, such a conclusion could turn out to be premature in the case of significant contributions of new operators to ΔM_s and no such contributions to ΔM_d and $K^+\to\pi^+\nu\bar{\nu}$. In this case the relation between R_t and $\Delta M_s/\Delta M_d$, as given in Eq. (16), would be modified [39], resulting in different values for $(\bar{\varrho},\bar{\eta})$ obtained using Eq. (34). The latter could then in principle agree with the ones obtained by means of Eq. (44) within the SM. This is precisely the situation in the MSSM at large $\tan\bar{\beta}=v_2/v_1$ [39].

C. $K_{\rm L}\to\pi^0\nu\bar{\nu}$, $K^+\to\pi^+\nu\bar{\nu}$ and the unitarity triangle

The reduced branching ratio B_2 defined in Eq. (5) is given by

$$B_2=\left[\frac{\mathrm{Im}\lambda_t}{\lambda^5}|X|\right]^2. \tag{47}$$

Following [19], but admitting both signs of X and F_{tt}, we find

$$\bar{\varrho}=1+\left[\frac{\pm\sqrt{\sigma(B_1-B_2)}+\mathrm{sgn}(X)P_c(\nu\bar{\nu})}{A^2|X|}\right],$$

$$\bar{\eta}=\mathrm{sgn}(F_{tt})\frac{\sqrt{B_2}}{\sqrt{\sigma}A^2|X|}, \tag{48}$$

where σ was defined in Eq. (6). Introducing

$$r_s\equiv\frac{1-\bar{\varrho}}{\bar{\eta}}=\mathrm{ctg}\beta, \tag{49}$$

we then find

$$r_s=\mathrm{sgn}(F_{tt})\sqrt{\sigma}\left[\frac{\mp\sqrt{\sigma(B_1-B_2)}-\mathrm{sgn}(X)P_c(\nu\bar{\nu})}{\sqrt{B_2}}\right], \tag{50}$$

with Eqs. (3) and (5) unchanged. We observe that r_s is independent of $|X|$ but the sign of the interference between the $V_{ts}^*V_{td}$ contribution and the charm contribution $P_c(\nu\bar{\nu})$ to $K^+\to\pi^+\nu\bar{\nu}$ matters.

In order to deal with the ambiguities present in Eq. (50), we consider

$$\mathrm{sgn}(F_{tt})r_s=\sqrt{\sigma}\left[\frac{\mp\sqrt{\sigma(B_1-B_2)}-\mathrm{sgn}(X)P_c(\nu\bar{\nu})}{\sqrt{B_2}}\right]=f(\beta), \tag{51}$$

where $f(\beta)$ was introduced in Eq. (29). As we have noted after Eq. (31), $f(\beta)$ has to be positive. Consequently, for $X>0$, only the "+" solution is allowed. On the other hand, in the case of $X<0$, the "−" solution gives also a positive value of $f(\beta)$ if

$$B_1-B_2<\frac{P_c(\nu\bar{\nu})^2}{\sigma}\approx 0.15. \tag{52}$$

Numerical studies show that both $Br(K^+\to\pi^+\nu\bar{\nu})$ and $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$ have to be below 1×10^{-11} to satisfy Eq. (52). As such low values are extremely difficult to measure, we will not consider this possibility further, which leaves us with the "+" solution in Eq. (50).

In Table II, we show the resulting values of $\mathrm{sgn}(F_{tt})\sin 2\beta=a_{\psi K_{\rm S}}$ for several choices of $Br(K^+\to\pi^+\nu\bar{\nu})$ and $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$, setting $P_c(\nu\bar{\nu})=0.40$. We observe that the sign of X is important; we also note that certain values violate the bounds in Eqs. (10) and (11). This implies that certain combinations of the two branching ratios are excluded within the MFV models. Let us then find out which combinations are still allowed.

D. $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$ from $a_{\psi K_S}$ and $Br(K^+\to\pi^+\nu\bar{\nu})$

As $a_{\psi K_S}$ and $Br(K^+\to\pi^+\nu\bar{\nu})$ will be known rather accurately prior to the measurement of $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$, it is of interest to calculate $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$ as a function of $a_{\psi K_S}$ and $Br(K^+\to\pi^+\nu\bar{\nu})$. From Eq. (51), we obtain

$$B_1=B_2+\left[\frac{f(\beta)\sqrt{B_2}+\mathrm{sgn}(X)\sqrt{\sigma}P_c(\nu\bar{\nu})}{\sigma}\right]^2. \tag{53}$$

The important virtue of Eq. (53) when compared with Eq. (50) is the absence of the ambiguity due to the $\mp$ in front of $\sqrt{\sigma(B_1-B_2)}$.

TABLE II. $\mathrm{sgn}(F_{tt})\sin 2\beta = a_{\psi K_S}$ in MFV models for specific values of $Br(K_L\to\pi^0\nu\bar{\nu})\equiv Br(K_L)$ and $Br(K^+\to\pi^+\nu\bar{\nu})\equiv Br(K^+)$ for $\mathrm{sgn}(X)=+1$ (-1) and $P_c(\nu\bar{\nu})=0.40$.

$Br(K_L)$ $[10^{-11}]$	$Br(K^+)=8.0$ $[10^{-11}]$	$Br(K^+)=16$ $[10^{-11}]$	$Br(K^+)=24$ $[10^{-11}]$
2.0	0.60 (0.35)	0.40 (0.27)	0.31 (0.22)
3.0	0.71 (0.43)	0.48 (0.32)	0.38 (0.27)
4.0	0.79 (0.49)	0.55 (0.37)	0.43 (0.32)
5.0	0.86 (0.54)	0.60 (0.42)	0.48 (0.35)
6.0	0.91 (0.59)	0.65 (0.45)	0.52 (0.38)
7.0	0.94 (0.64)	0.70 (0.49)	0.56 (0.41)
8.0	0.97 (0.68)	0.73 (0.52)	0.60 (0.44)

As we have seen in Eq. (33), the measurement of $a_{\psi K_S}$ determines $f(\beta)$ unambiguously. This finding, in combination with Eq. (53), implies the following interesting property of the MFV models:

For given $a_{\psi K_S}$ and $Br(K^+\to\pi^+\nu\bar{\nu})$ only two values of $Br(K_L\to\pi^0\nu\bar{\nu})$, corresponding to the two possible signs of X, are possible in the full class of MFV models, independently of any new parameters present in these models.

Consequently, measuring $Br(K_L\to\pi^0\nu\bar{\nu})$ will either select one of these two possible values or rule out all MFV models. We would like to emphasize that the latter possibility could take place even if the lower bound on $|\sin 2\beta|$ [12] is satisfied by the data on $a_{\psi K_S}$, which is favored by the most recent B-factory results given in Eq. (24).

In Table III, we show values of $Br(K_L\to\pi^0\nu\bar{\nu})$ in the MFV models for specific values of $a_{\psi K_S}$ and $Br(K^+\to\pi^+\nu\bar{\nu})$ and the two signs of X. Note that the second column gives the *absolute* lower bound on $Br(K_L\to\pi^0\nu\bar{\nu})$ in the MFV models as a function of $Br(K^+\to\pi^+\nu\bar{\nu})$. This bound follows simply from the lower bound in Eq. (10). On the other hand, the last column gives the corresponding *absolute* upper bound. This bound is the consequence of the upper bound in Eq. (11). The third column gives the lower bound on $Br(K_L\to\pi^0\nu\bar{\nu})$ corresponding to the bound in Eq. (19) that applies for a negative F_{tt}.

A more detailed presentation is given in Figs. 2 and 3. In Fig. 2, we show $Br(K_L\to\pi^0\nu\bar{\nu})$ as a function of $Br(K^+\to\pi^+\nu\bar{\nu})$ for chosen values of $a_{\psi K_S}$ and $\mathrm{sgn}(X)=+1$. The corresponding plot for $\mathrm{sgn}(X)=-1$ is shown in Fig. 3. It should be emphasized that the plots shown in Figs. 2 and 3 are universal for all MFV models. Table III and Figs. 2 and 3 make it clear that the measurements of $Br(K_L\to\pi^0\nu\bar{\nu})$, $Br(K^+\to\pi^+\nu\bar{\nu})$ and $a_{\psi K_S}$ will easily allow the distinction between the two signs of X. The uncertainty due to $P_c(\nu\bar{\nu})$ is non-negligible but it should be decreased with the improved knowledge of the charm-quark mass.

TABLE III. Values of $Br(K_L\to\pi^0\nu\bar{\nu})$ in the MFV models in units of 10^{-11} for specific values of $a_{\psi K_S}$ and $Br(K^+\to\pi^+\nu\bar{\nu})$ and $\mathrm{sgn}(X)=+1$ (-1). We set $P_c(\nu\bar{\nu})=0.40$.

$Br(K^+\to\pi^+\nu\bar{\nu})$ $[10^{-11}]$	$a_{\psi K_S}=0.42$	$a_{\psi K_S}=0.69$	$a_{\psi K_S}=0.82$
5.0	0.45 (2.0)	1.4 (5.8)	2.2 (8.6)
10.0	1.2 (3.5)	3.8 (10.0)	5.9 (15.0)
15.0	2.1 (4.8)	6.3 (14.0)	9.9 (21.1)
20.0	3.0 (6.2)	9.0 (17.9)	14.1 (27.0)
25.0	3.9 (7.5)	11.8 (21.7)	18.4 (32.8)
30.0	4.9 (8.7)	14.6 (25.4)	22.7 (38.6)

We would like to emphasize that the upper bound on $Br(K_L\to\pi^0\nu\bar{\nu})$ in the last column of Table III is substantially stronger than the model-independent bound following from isospin symmetry [40]

$$Br(K_L\to\pi^0\nu\bar{\nu}) < 4.4\times Br(K^+\to\pi^+\nu\bar{\nu}). \tag{54}$$

Indeed, taking the experimental bound $Br(K^+\to\pi^+\nu\bar{\nu}) \leqslant 5.9\times 10^{-10}$ (90% C.L.) from AGS E787 [41], we find

$$Br(K_L\to\pi^0\nu\bar{\nu})_{\mathrm{MFV}} \leqslant \begin{cases} 4.9\times 10^{-10}, & \mathrm{sgn}(X)=+1, \\ 7.1\times 10^{-10}, & \mathrm{sgn}(X)=-1. \end{cases} \tag{55}$$

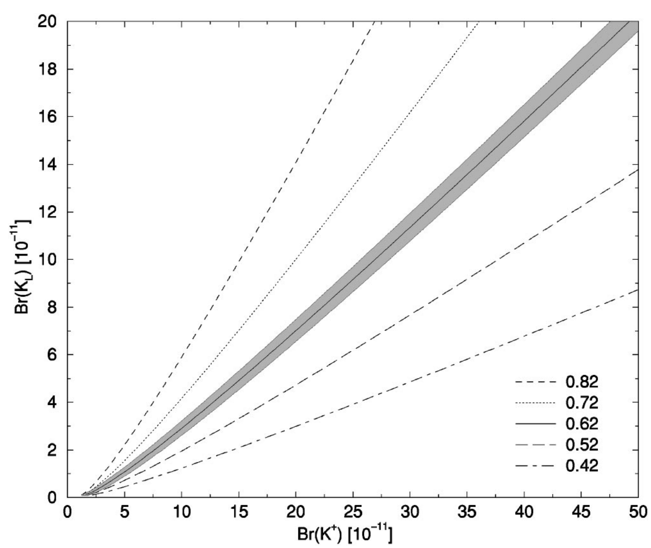

FIG. 2. $Br(K_L\to\pi^0\nu\bar{\nu})$ as a function of $Br(K^+\to\pi^+\nu\bar{\nu})$ for several values of $a_{\psi K_S}$ in the case of $\mathrm{sgn}(X)=+1$. For $a_{\psi K_S}=0.62$, also the uncertainty due to $P_c(\nu\bar{\nu})=0.40\pm 0.06$ has been shown.

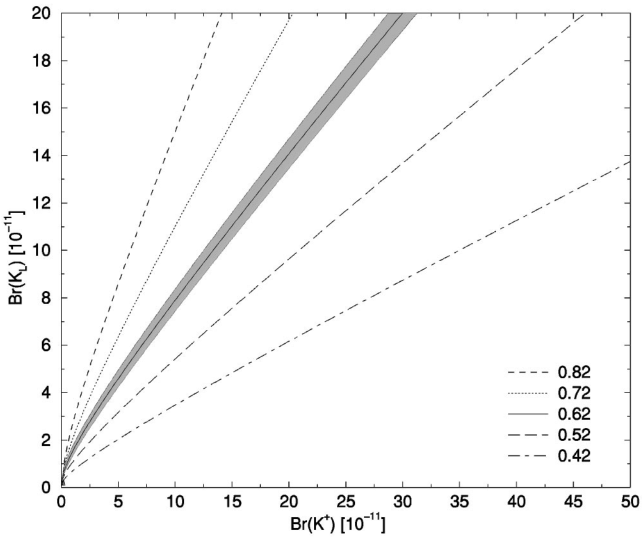

FIG. 3. $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$ as a function of $Br(K^+\to\pi^+\nu\bar{\nu})$ for several values of $a_{\psi K_S}$ in the case of $\mathrm{sgn}(X)=-1$. For $a_{\psi K_S}=0.62$, also the uncertainty due to $P_c(\nu\bar{\nu})=0.40\pm0.06$ has been shown.

This should be compared with $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})<26\times10^{-10}$ (90% C.L.) following from Eq. (54), and with the present upper bound from the KTeV experiment at Fermilab [42], yielding $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})<5.9\times10^{-7}$. The corresponding predictions within the SM read [15]

$$Br(K^+\to\pi^+\nu\bar{\nu})=(7.5\pm2.9)\times10^{-11},$$
$$Br(K_{\rm L}\to\pi^0\nu\bar{\nu})=(2.6\pm1.2)\times10^{-11}. \tag{56}$$

As can be seen in Table III and in Figs. 2 and 3, the bounds in Eq. (55) will be considerably improved when $Br(K^+\to\pi^+\nu\bar{\nu})$ and $a_{\psi K_S}$ will be known better. The experimental outlook for both decays has recently been reviewed by Littenberg [43]. The existing measurement [41]

$$Br(K^+\to\pi^+\nu\bar{\nu})=(1.5^{+3.4}_{-1.2})\times10^{-10}, \tag{57}$$

should be considerably improved already this year.

E. An upper bound on $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$ from $Br(B\to X_s\nu\bar{\nu})$

The branching ratio for the inclusive rare decay $B\to X_s\nu\bar{\nu}$ can be written in the MFV models as follows [15]:

$$Br(B\to X_s\nu\bar{\nu})=1.57\times10^{-5}\left[\frac{Br(B\to X_c e\bar{\nu})}{0.104}\right]\times\left|\frac{V_{ts}}{V_{cb}}\right|^2\left[\frac{0.54}{f(z)}\right]X^2, \tag{58}$$

where $f(z)=0.54\pm0.04$ is the phase-space factor for $B\to X_c e\bar{\nu}$ with $z=m_c^2/m_b^2$, and $Br(B\to X_c e\bar{\nu})=0.104\pm0.004$.

Formulas (47) and (58) imply an interesting relation between the decays $K_{\rm L}\to\pi^0\nu\bar{\nu}$ and $B\to X_s\nu\bar{\nu}$:

$$Br(K_{\rm L}\to\pi^0\nu\bar{\nu})=42.3\times(\mathrm{Im}\lambda_t)^2\left[\frac{0.104}{Br(B\to X_c e\bar{\nu})}\right]\times\left|\frac{V_{cb}}{V_{ts}}\right|^2\left[\frac{f(z)}{0.54}\right]Br(B\to X_s\nu\bar{\nu}), \tag{59}$$

which is valid in all MFV models. Equation (59) constitutes still another connection between K- and B-meson decays, in addition to those discussed already in this paper and in [19,20,22,17,44].

Now, the experimental upper bound on $Br(B\to X_s\nu\bar{\nu})$ reads [45]

$$Br(B\to X_s\nu\bar{\nu})<6.4\times10^{-4}\quad(90\%\ \text{C.L.}). \tag{60}$$

Using this bound and setting $\mathrm{Im}\lambda_t=1.74\times10^{-4}$ [see Eq. (36)], $|V_{ts}|=|V_{cb}|$, $f(z)=0.58$ and $Br(B\to X_c e\bar{\nu})=0.10$, we find from Eq. (59) the upper bound

$$Br(K_{\rm L}\to\pi^0\nu\bar{\nu})<9.2\times10^{-10}\quad(90\%\ \text{C.L.}), \tag{61}$$

which is not much weaker than the bound in Eq. (55). As the bound in Eq. (60) should be improved in the B-factory era, also the latter bound should be improved in the next years.

F. Determination of X

The knowledge of the function X would be very important information, providing constraints on the MFV models. In the SM, we have $X\approx1.5$. Present bounds on the function X from $K^+\to\pi^+\nu\bar{\nu}$ and $B\to X_s\nu\bar{\nu}$ within MFV models were recently discussed in [17]. In particular, from Eqs. (58) and (60) we find

$$|X|<6.8, \tag{62}$$

which agrees well with [17].

In the future, a theoretically clean determination of X will be made possible by determining $\bar{\eta}$ and $\bar{\varrho}$ by means of $\Delta M_s/\Delta M_d$ and $a_{\psi K_S}$ [see Eq. (16) and (34)], and inserting them into (39) and (38). In this manner, we may calculate $Br(K^+\to\pi^+\nu\bar{\nu})$ as a function of X. The measurement of this branching ratio yields then two values of $|X|$, corresponding to $\mathrm{sgn}(X)=\pm1$. We illustrate this in Fig. 4, where we plot $Br(K^+\to\pi^+\nu\bar{\nu})$ as a function of $|X|$ for $\mathrm{sgn}(X)=\pm1$. Here we have assumed, as an example, $A=0.83$, $(\bar{\varrho},\bar{\eta})=(0.23,0.35)$, which corresponds to $a_{\psi K_S}=0.75$, and $P_c(\nu\bar{\nu})=0.40$. As expected, $Br(K^+\to\pi^+\nu\bar{\nu})$ is substantially smaller in the case of a negative X.

Direct access to $|X|$ will also be provided by $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$, as can be seen from Eq. (47). If a MFV model is realized in nature, both determinations have to give the same value of $|X|$. This requirement allows us to distinguish between the two branches in Fig. 4, thereby offering another way to fix the sign of X.

However, the strategy presented in Sec. III D, which is based on Figs. 2 and 3 and involves just $a_{\psi K_S}$, $Br(K^+\to\pi^+\nu\bar{\nu})$ and $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$, is much more elegant to

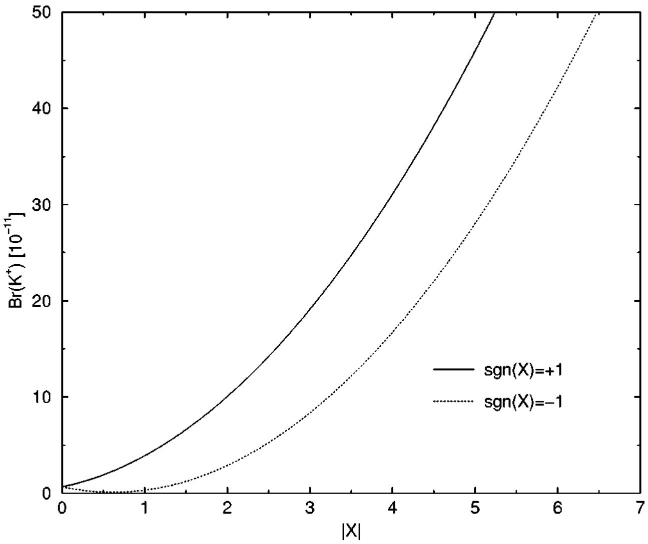

FIG. 4. $Br(K^+\to\pi^+\nu\bar{\nu})$ as a function of $|X|$ for $\mathrm{sgn}(X)=\pm 1$ in the case of $A=0.83$, $(\bar{\varrho},\bar{\eta})=(0.23{,}0.35)$ and $P_c(\nu\bar{\nu})=0.40$.

check whether a MFV model is realized in the $K\to\pi\nu\bar{\nu}$ system and—if so—to determine $\mathrm{sgn}(X)$. In order to determine also $|X|$, $\Delta M_s/\Delta M_d$ is needed as an additional input, as we have seen above.

IV. CONCLUSIONS

In this paper, we have explored the determination of $\sin 2\beta$ through the standard analysis of the unitarity triangle, the *CP* asymmetry $a_{\psi K_S}$, and the decays $K\to\pi\nu\bar{\nu}$ in MFV models, admitting new-physics contributions that reverse the sign of the corresponding generalized Inami-Lim functions F_{tt} and X. Our findings can be summarized as follows:

There are bounds on $\sin 2\beta$, which can be translated into lower bounds on $a_{\psi K_S}$. For $F_{tt}>0$, $(a_{\psi K_S})_{\min}=0.42$ [12], whereas we obtain a stronger bound of $(a_{\psi K_S})_{\min}=0.69$ in the case of $F_{tt}<0$. Consequently, for $0.42<a_{\psi K_S}<0.69$, the full class of MFV models with $F_{tt}<0$ would be excluded; for $a_{\psi K_S}<0.42$, even all MFV models would be ruled out. The reduction of the uncertainties of the relevant input parameters could improve these bounds in the future. We have also discussed strategies to determine the sign of F_{tt} directly, allowing interesting consistency checks of the MFV models.

The most recent *B*-factory data are no longer in favor of small values of $a_{\psi K_S}$, and the present world average of 0.79 ± 0.10 does not even allow us to exclude the case corresponding to $F_{tt}<0$. Consequently, an important role may be played in the future by the upper bound on $a_{\psi K_S}$ that is implied by $|V_{ub}/V_{cb}|$. Since the BaBar and Belle results are not fully consistent with each other, the measurement of $a_{\psi K_S}$ will remain a very exciting issue. Let us hope that the situation will be clarified soon.

We have generalized the SM analysis of the unitarity triangle through $K\to\pi\nu\bar{\nu}$ to MFV models, allowing negative values of X. In particular, we have explored the behavior of $Br(K_{\mathrm{L}}\to\pi^0\nu\bar{\nu})$ as a function of $a_{\psi K_S}$ and $Br(K^+\to\pi^+\nu\bar{\nu})$ for the general MFV model. This is an important exercise, since the latter two quantities will be known rather precisely before $Br(K_{\mathrm{L}}\to\pi^0\nu\bar{\nu})$ will be accessible. In this context, we have pointed out that for given $Br(K^+\to\pi^+\nu\bar{\nu})$ and $a_{\psi K_S}$, only two values for $Br(K_{\mathrm{L}}\to\pi^0\nu\bar{\nu})$ are possible in the full class of MFV models, which correspond just to the two signs of X and are independent of any new parameters present in these models. Consequently, the measurement of this branching ratio will either select one particular class of MFV models, or will exclude all of them.

At present, the existing lower and upper bounds on $a_{\psi K_S}$ in the MFV models allow us to find *absolute* lower and upper bounds on the branching ratio $Br(K_{\mathrm{L}}\to\pi^0\nu\bar{\nu})$ as a function of $Br(K^+\to\pi^+\nu\bar{\nu})$. We find that the present upper bounds on $Br(K^+\to\pi^+\nu\bar{\nu})$ and $|V_{ub}/V_{cb}|$ imply an absolute upper bound $Br(K_{\mathrm{L}}\to\pi^0\nu\bar{\nu})<7.1\times 10^{-10}$ (90% C.L.), which is substantially stronger than the bound following from isospin symmetry. On the other hand, the experimental upper bound on $Br(B\to X_s\nu\bar{\nu})$ implies $Br(K_{\mathrm{L}}\to\pi^0\nu\bar{\nu})<9.2\times 10^{-10}$ (90% C.L.).

The present paper, in conjunction with earlier analyses [24,12,15,17], demonstrates the simplicity of the MFV models, allowing transparent and general tests of these models without the necessity of assuming particular values for their new parameters.

It will be exciting to follow the development in the experimental values of $a_{\psi K_S}$, $Br(K^+\to\pi^+\nu\bar{\nu})$, $Br(K_{\mathrm{L}}\to\pi^0\nu\bar{\nu})$, $Br(B\to X_s\nu\bar{\nu})$ and $\Delta M_s/\Delta M_d$. Possibly already before the LHC era we will know whether any of the MFV models survive all tests discussed here and in [19,22,24,12,15,17], or whether new operators and/or new complex phases are required to describe the data.

ACKNOWLEDGMENTS

A.J.B. would like to thank Piotr Chankowski and Łucja Sławianowska for discussions. This work has been supported in part by the German Bundesministerium für Bildung and Forschung under the contract 05HT9WOA0.

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