Color confinement and dual superconductivity of the vacuum. III

J. M. Carmona,[1,*] M. D'Elia,[2,†] A. Di Giacomo,[3,4,‡] B. Lucini,[5,§] and G. Paffuti[3,4,‖]
[1]Departamento de Física Teórica, Universidad de Zaragoza, 50007 Zaragoza, Spain
[2]Dipartimento di Fisica dell'Università di Genova and INFN, Sezione di Genova, Via Dodecaneso 33, I-16146 Genova, Italy
[3]Dipartimento di Fisica dell'Università, Via Buonarroti 2 Ed. B, I-56127 Pisa, Italy
[4]INFN sezione di Pisa, Via Vecchia Livornese 1291, I-56010 S. Piero a Grado (Pi), Italy
[5]Theoretical Physics, University of Oxford, 1 Keble Road, OX1 3NP Oxford, United Kingdom
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It is demonstrated that the condensation of magnetic charges in the confined phase of SU(2) and SU(3) gauge theories is independent of the specific Abelian projection used to define the monopoles. Hence the dual excitations which condense in the vacuum to produce confinement must have a magnetic U(1) charge in all the Abelian projections. Some physical implications of this result are discussed.

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I. INTRODUCTION

This paper is the third of a series in which we study the dual superconductivity of the QCD vacuum [1–3] as a mechanism for the confinement of color. In the first two papers [4,5] we detected a condensation of magnetic charges in the confining phase by means of a disorder parameter $\langle \mu \rangle$, which is the vacuum expectation value of a magnetically charged operator μ. $\langle \mu \rangle$ was determined by simulating the theory on a lattice: it is nonzero in the confining phase, and tends to zero at the deconfining transition, above which it vanishes.

The connection of $\langle \mu \rangle$ to confinement was proved by a quantitative determination of the critical indices and of the critical coupling. In Ref. [4], SU(2) gauge theory was studied, while Ref. [5] was devoted to SU(3) with similar results.

Magnetic charges in gauge theories are defined by a procedure known as Abelian projection [2]: with every local field Φ belonging to the adjoint representation (N_c-1) U(1) fields can be associated, and with each of them a conserved magnetic charge. In fact there exists a functional infinity of monopole species N_c-1 for each field Φ, which in principle can condense in the vacuum and confine the corresponding U(1) electric charge by the dual Meissner effect. It is not known *a priori* if monopole condensations in different Abelian projections are independent phenomena.

The indication obtained in Refs. [4,5] by analysis of a number of different choices of Φ was that all of them show the same behavior, so that they are equivalent to each other. The possibility that in some way all the Abelian projections could be physically equivalent was first advocated in Ref. [2]. In this paper we add strong evidence for that equivalence.

In order to explain what we do, let us first recall how magnetic charges are associated with any field Φ in the adjoint representation. We shall do this for SU(2) to simplify notation; extension to SU(N) only adds formal complications.

Let $\vec{\Phi}(x)$ be a field in the adjoint representation (color vector), and let $\hat{\Phi}(x)$ be its color orientation:

$$\hat{\Phi}(x) \equiv \frac{\vec{\Phi}(x)}{|\vec{\Phi}(x)|}. \tag{1}$$

$\hat{\Phi}(x)$ is well defined, except at zeros of $\vec{\Phi}(x)$.

Define a gauge invariant field strength $F_{\mu\nu}(x)$ [6],

$$F_{\mu\nu} = \hat{\Phi} \cdot \vec{G}_{\mu\nu} - \frac{1}{g}(D_\mu \hat{\Phi} \wedge D_\nu \hat{\Phi}) \cdot \hat{\Phi}, \tag{2}$$

where $\vec{G}_{\mu\nu} = \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g \vec{A}_\mu \wedge \vec{A}_\nu$ is the gauge field strength and $D_\mu \hat{\Phi} = (\partial_\mu + g \vec{A}_\mu \wedge) \hat{\Phi}$ is the covariant derivative of $\hat{\Phi}$.

Both terms on the right-hand side of Eq. (2) are separately gauge invariant and color singlets: their combination is chosen in such a way that bilinear terms $A_\mu A_\nu$, $A_\mu \Phi$, and $A_\nu \Phi$ cancel. Actually, by simple algebra,

$$F_{\mu\nu} = \hat{\Phi} \cdot (\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu) - \frac{1}{g}(\partial_\mu \hat{\Phi} \wedge \partial_\nu \hat{\Phi}) \cdot \hat{\Phi}. \tag{3}$$

If we transform to a gauge in which $\hat{\Phi}(x) = \text{const}$ in space-time, the last term cancels and

$$F_{\mu\nu} = \partial_\mu(\hat{\Phi} \cdot \vec{A}_\nu) - \partial_\nu(\hat{\Phi} \cdot \vec{A}_\mu) \tag{4}$$

is an Abelian field strength. Such a gauge transformation is called an Abelian projection. It is in general a singular transformation which exposes monopoles at the sites where $\vec{\Phi}(x) = 0$.

If $F^\star_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$ is the dual field to $F_{\mu\nu}$, one can define a magnetic current

$$j_\mu = \partial^\nu F^\star_{\mu\nu}; \tag{5}$$

*Email address: jcarmona@posta.unizar.es
†Email address: delia@ge.infn.it
‡Email address: digiaco@df.unipi.it
§Email address: lucini@thphys.ox.ac.uk
‖Email address: paffuti@df.unipi.it

j_μ is zero if Bianchi identities hold, but can be nonzero in compact formulations in terms of parallel transport, such as lattice formulation [7]. In any case it follows from the antisymmetry of $F^{\star}_{\mu\nu}$ that

$$\partial^\mu j_\mu = 0. \tag{6}$$

In the dual superconductor view of color confinement the symmetry [Eq. (6)] is expected to be realized in the manner of Wigner in the deconfined phase, and to be broken by the Higgs mechanism in the confined phase. An operator μ which carries magnetic charge can provide a disorder parameter to discriminate between the two possibilities. Such an operator was developed and tested in Refs. [8–10,4,5].

What was found in Refs. [4,5] was that there is indeed dual superconductivity in a number of Abelian projections. As explained in detail in Sec. II, the full identification of the projected gauge requires that one go to a gauge in which $\vec{\Phi}(x)\cdot\vec{\lambda}$ is diagonal in color indices ($\vec{\lambda}$ are the generators in the fundamental representation), with a fixed order of the eigenvalues.

One can diagonalize $\vec{\Phi}(x)\cdot\vec{\lambda}$ up to the ordering of the eigenvalues, choosing it randomly, and still define an operator μ which creates a magnetic charge in that Abelian projection. In this paper we show that the corresponding disorder parameter behaves exactly in the same way as the one with ordered eigenvalues.

We can even define a completely random Abelian projection, in which we do not diagonalize any operator Φ, but we take, e.g., for SU(2), $\hat{\Phi} = \sigma_3$, the nominal 3 axis used in the simulation, and define the corresponding μ. Again we find that μ defined in this way behaves exactly in the same way as those defined in Refs. [4,5], and scales with the same critical indices. The above Abelian projections are kind of an average over a continuous infinity of Abelian projections, and the result demonstrates, beyond any doubt, the complete independence of dual superconductivity from the choice of the Abelian projection.

Our results are compatible with Ref. [11], where our disorder parameter in the random gauge was computed by Schrödinger functional techniques. In the literature of the last years, there has been the idea that monopoles defined by a particular Abelian projection (the maximal Abelian projection) are more relevant than others to confinement [12,13]. We will discuss this issue in Sec. V, where we draw conclusions from our results.

In Sec. II the construction of the disorder parameter $\langle\mu\rangle$ will be recalled, to define the Abelian projection with random ordering (APRO) and the random Abelian projection (RAP). In Sect. III the numerical algorithms used will be discussed. The results will be described in Sec. IV. Section V will close the paper with conclusions.

II. DISORDER PARAMETER

In this section we recall the definition of the disorder parameter for confinement. Let O be an operator which transforms in the adjoint representation of the gauge group, i.e.,

$$O = \sum \lambda^a O^a, \tag{7}$$

with λ^a the generators in the fundamental representation. The Abelian projection technique [2] prescribes fixing the gauge by a gauge transformation in such a way that

$$O_{gf} = G^\dagger O G = \mathrm{diag}(o_1, \cdots, o_N),$$

with (8)

$$o_1 < o_2 < \cdots < o_N .$$

After Abelian projection, there is still a $U(1)^{N-1}$ gauge freedom left, since a transformation of the form

$$\Omega = \mathrm{diag}(e^{i\omega_1}, \ldots, e^{i\omega_n}), \quad \sum \omega_i = 0 \tag{9}$$

does not change the gauge fixing condition [Eq. (8)].

After Abelian projection, the gauge variables of SU(N) are divided in two sets: the *photons* [the $N-1$ neutral fields under the residual $U(1)^{N-1}$] and the *gluons* (charged fields with respect to the residual symmetry). Abelian magnetic monopoles can arise at points where two eigenvalues of O are degenerate [2].

Condensation of Abelian monopoles defined by Abelian projection was demonstrated numerically in Ref. [4] for SU(2) and in Ref. [5] for SU(3). This has been done by constructing an operator magnetically charged in a given Abelian projection and by studying the behavior of the vacuum expectation value of that operator across the phase transition at finite temperature. In the language of statistical mechanics we call this operator a *disorder operator* and its vacuum expectation value (VEV) a *disorder parameter*, the terminology being that the weak coupling (deconfined) phase is the ordered phase. The construction can be done in different Abelian projections: in Refs. [4,5] a number of Abelian projections were studied, and for all of them it was found that indeed monopoles condense at low temperature, while the corresponding magnetic symmetry is implemented in the manner of *Wigner* at high temperature. Moreover, the disorder parameter scales with the correct critical indices in the critical region, and is independent of the choice of the Abelian projection. These results suggest that the observed behavior of the disorder parameter is generally independent of the Abelian projection and of the Abelian operator chosen.

Let us review the construction of the disorder parameter. We introduce a time-independent external field

$$\Phi_i(\vec{n},\vec{y}) = G e^{iTb_i(\vec{n}-\hat{\imath},\vec{y})} G^\dagger, \tag{10}$$

where G is the gauge transformation that diagonalizes the operator O according to Eq. (8), $\vec{b}$ is the discretised transverse field (i.e, $\vec{\nabla}\cdot\vec{b}=0$ on the continuum) generated at lattice spatial point $\vec{n}$ by a magnetic monopole sitting at $\vec{y}$ and T is a generator of the Cartan subalgebra.

Let $U_{\mu\nu}$ be the Wilson plaquette, defined in the usual notations as

$$U_{\mu\nu}(x)=U_{\mu}(x)U_{\nu}(x+\hat{\mu})(U_{\mu}(x+\hat{\nu}))^{\dagger}(U_{\nu}(x))^{\dagger}, \quad (11)$$

where $x\equiv(\vec{n},t)$. We introduce a shift $U_{i0}(\vec{n},0)\rightarrow\tilde{U}_{i0}(\vec{n},0)$ by inserting the external field $\Phi_i(\vec{n}+\hat{i},\vec{y})$ in the path ordered product at time zero, as follows:

$$\tilde{U}_{i0}(\vec{n},0)=U_i(\vec{n},0)\Phi_i(\vec{n}+\hat{i},\vec{y})U_0(\vec{n}+\hat{i},0)\times(U_i(\vec{n},1))^{\dagger}(U_0(\vec{n},0))^{\dagger}, \quad (12)$$

and we define $\tilde{U}_{\mu\nu}(x)\equiv U_{\mu\nu}(x)$ elsewhere.

The Wilson action for SU(N) gauge theory is

$$S=\beta\sum_{\mu\nu x}\left(1-\frac{1}{2N}[U_{\mu\nu}(x)+(U_{\mu\nu}(x))^{\dagger}]\right), \quad (13)$$

where the sum extends over all the lattice points and directions.

By replacing in the previous equation the standard plaquette with the modified plaquette $\tilde{U}_{\mu\nu}(i)$, we obtain the ''monopole'' action

$$S_M(\vec{y},0)=\beta\sum_{\mu\nu x}\left(1-\frac{1}{2N}[\tilde{U}_{\mu\nu}(x)+(\tilde{U}_{\mu\nu}(x))^{\dagger}]\right). \quad (14)$$

The disorder parameter introduced in Refs. [4,5] is given by

$$\langle\mu(\vec{y}_0,0)\rangle=\frac{\int(\mathcal{D}U)e^{-S_M(\vec{y}_0,0)}}{\int(\mathcal{D}U)e^{-S}}, \quad (15)$$

where the functional integral of e^{-S} is taken with periodic boundary conditions and the integral of e^{-S_M} with $C^{\star}$-periodic boundary conditions [14,15]

$$U_i(\vec{n},t=N_t)=U_i^{*}(\vec{n},t=0), \quad (16)$$

N_t being the temporal extension of the lattice and U_i^{*} being the complex conjugated of U_i. To study more in detail the dependence on the projecting operator, we will modify the definition of Φ as follows.

(1) We choose a projecting operator and we diagonalize it but without fixing the order of the eigenvalues, or better with the order of the eigenvalues randomly chosen,

$$\Phi_i(\vec{n},\vec{y})=GPe^{iTb_i(\vec{n}-\hat{i},\vec{y})}PG^{\dagger}, \quad (17)$$

where P is a random $N\times N$ permutation matrix. This corresponds to a sort of average of μ over the class of operators differing from O on each point by the order of the eigenvalues. We refer to this case as Abelian projection with random ordering (APRO).

(2) We do not perform the Abelian projection, i.e., we take

$$\Phi_i(\vec{n},\vec{y})=e^{iTb_i(\vec{n}-\hat{i},\vec{y})}, \quad (18)$$

with T a generator in the Cartan subalgebra. In a given configuration this amounts to making the Abelian projection with any operator Φ which is diagonal in that representation. For instance in SU(3) the Cartan subalgebra is generated by $[\lambda_3,\lambda_3-(\lambda_8/\sqrt{3})]$. Since, however, what is defined as λ_3,λ_8 in another configuration is independent of λ_3 and λ_8 in the previous one, definition (18) is equivalent to a sort of average of μ over all the possible Abelian projections. We refer to this case as random Abelian projection (RAP).

As discussed in Refs. [4,5], a direct computation of $\langle\mu\rangle$ with Monte Carlo techniques is problematic, because this quantity has large fluctuations, being the exponential of a sum over the physical volume. A more convenient quantity to study in numerical simulations is [8,4,5]

$$\rho=\frac{\partial}{\partial\beta}\log\langle\mu\rangle=\langle S\rangle_S-\langle S_M\rangle_{S_M}. \quad (19)$$

ρ is the difference of two average actions, the Wilson action and the modified action S_M (the latter being averaged with the modified measure $((\mathcal{D}U)e^{-S_M})/(\int(\mathcal{D}U)e^{-S_M})$). ρ has smaller fluctuations and contains all relevant information. The value of $\langle\mu\rangle$ is related to ρ by the relationship

$$\langle\mu\rangle=\exp\left(\int_0^{\beta}\rho(\beta')\mathrm{d}\beta'\right). \quad (20)$$

III. GAUGE FIXING AND SIMULATION ALGORITHMS

We have determined the temperature dependence of ρ for SU(2) and SU(3) pure Yang-Mills theories, for both definitions (17) and (18) of Φ on an asymmetric lattice $N_s^3\times N_t$ with $N_t\ll N_s$. For both definitions of Φ, the simulation of the Wilson term $\langle S\rangle_S$ has been performed on a lattice with periodic boundary conditions by using a standard mixture of heatbath and overrelaxed algorithms.

As for the APRO case, we have chosen the Polyakov line as the operator to identify the Abelian projection, following the definition in Eq. (31) of Ref. [4] and Eq. (19) of Ref. [5], with the only difference that at each spatial point the ordering of the eigenvalues is selected randomly among the possible different permutations n_p [$n_p=2$ (6) for SU(2) (SU(3)) pure gauge theory]. This effectively corresponds to averaging over $n_p^{N_s^3}$ different definitions of the Abelian projection. The Abelian generator F^8 ($F^8=\lambda^8/2$, with λ^i the Gell-Mann matrices), has been chosen to define the monopole field for the SU(3) case. We use $C^{\star}$ boundary conditions in time to compute $\langle S_M\rangle_{S_M}$ in Eq. (19). In this case, as explained in Ref. [4], it is not possible to use a standard heatbath or overrelaxed algorithm to simulate the modified action, since, e.g., in the case of Polyakov projection, the change of any temporal link induces a nonlinear change in the modified action. So we have performed simulations by using a mixed (heatbath plus overrelaxed) algorithm for the update of spatial links, and a Metropolis algorithm for the update of the temporal links.

In the case of the random Abelian projection, as it appears in Eq. (18), one does not need any gauge fixing to define the monopole field. As a consequence, the change of any link always induces a linear change in the modified action. There-

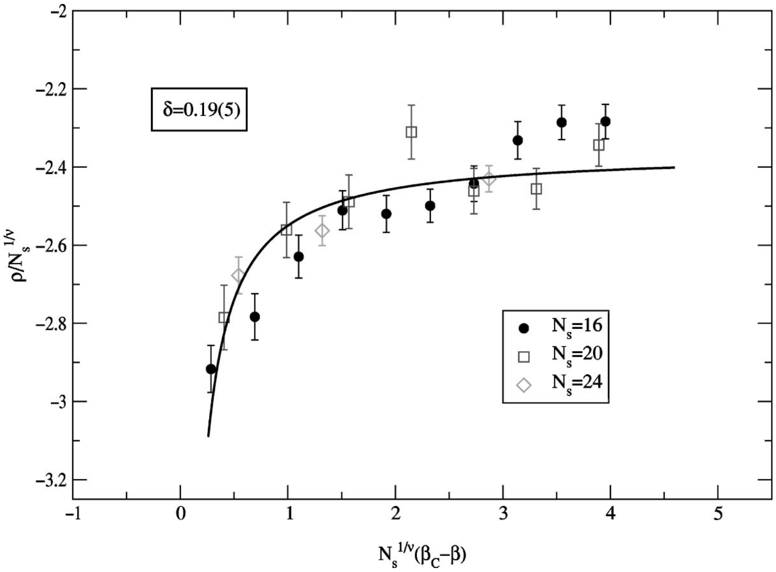

FIG. 1. Quality of scaling in the RAP case for SU(2).

fore we have used a standard (heatbath plus overrelaxed) algorithm in this case. In the SU(3) case, the Abelian generator F^8 has been used again to define the monopole field.

IV. NUMERICAL RESULTS

The phase transition is known to be second order for SU(2), and weak first order for SU(3) [17]. As usual we shall speak of critical indices in both cases, meaning, for SU(3), effective critical indices at small values of $(1-T/T_c)$, but not too small.

In Refs. [4,5] it was shown that the critical indices of the confinement transitions for SU(2) and SU(3) did not depend on the particular type of Abelian projection used to define the monopole condensation. Here we will show numerically that the critical exponents, and also the value of the critical coupling, are the same even for the random Abelian projection and the Abelian projection with random ordering, both for SU(2) and SU(3).

The critical behavior of the disorder parameter $\langle\mu\rangle$ is governed by an exponent δ. For finite lattice sizes $(N_s^3\times N_t)$, finite size scaling states that

$$\langle\mu\rangle = N_s^{-\delta/\nu} F\left(\frac{\xi}{N_s}, \frac{a}{\xi}, \frac{N_t}{N_s}\right), \tag{21}$$

where a and ξ are the lattice spacing and the correlation length of the system, respectively.

Near the critical point, for $\beta<\beta_C$,

$$\xi \propto (\beta_C-\beta)^{-\nu}, \tag{22}$$

where ν is the corresponding critical exponent. In the limit $N_s \gg N_t$ and for $a/\xi \ll 1$, i.e., sufficiently close to the critical point,

$$\langle\mu\rangle = N_s^{-\delta/\nu}\tilde{F}[N_s^{1/\nu}(\beta_C-\beta)], \tag{23}$$

or equivalently

$$\frac{\rho}{N_s^{1/\nu}} = f[N_s^{1/\nu}(\beta_C-\beta)]. \tag{24}$$

The ratio $\rho/N_s^{1/\nu}$ is a universal function of the scaling variable:

$$x = N_s^{1/\nu}(\beta_C-\beta). \tag{25}$$

We will use the known values of β_C and ν of SU(2) and SU(3) pure gauge theories to see that scaling holds with the present data. In order to obtain the critical exponent δ, we use an expression equivalent to Eq. (23):

$$\langle\mu\rangle = (\beta_c-\beta)^{\delta}\mathcal{F}(x). \tag{26}$$

From this we obtain

$$\frac{\rho}{N_s^{1/\nu}} = -\frac{\delta}{x} - \frac{\mathcal{F}'(x)}{\mathcal{F}(x)}. \tag{27}$$

To obtain δ we need additional assumptions about the unknown scaling function $\mathcal{F}(x)$. We will see that fits of good quality are obtained with the simple parametrization

$$\frac{\rho}{N_s^{1/\nu}} = -\frac{\delta}{x} - C, \tag{28}$$

where C is a constant term. This form is suggested by the fact that when $x\to 0$, both $\mathcal{F}(x)$ and its derivative should go to a constant.

A. SU(2) gauge theory

1. Random Abelian projection

The quality of the scaling [Eq. (24)] for SU(2) in the RAP case can be seen in Fig. 1. Here we used the known values of $\beta_C=2.2986$ and $\nu=0.63$ [16]. The curve in the figure corresponds to the best fit to Eq. (28). We obtain δ

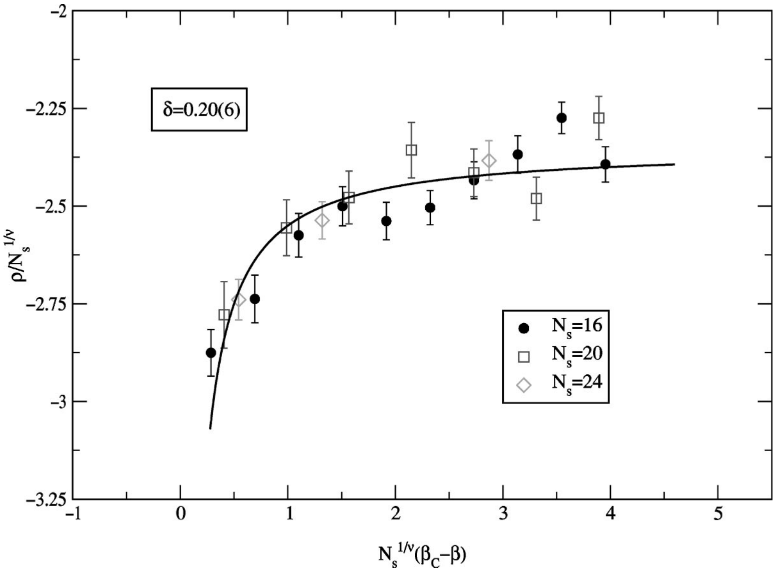

FIG. 2. Quality of scaling in the APRO case for SU(2).

$=0.19(5)$, with a χ^2/d.o.f.~ 1.5, in good agreement with the value obtained in the plaquette and Polyakov gauges in Ref. [4]: $\delta = 0.20(8)$.

2. Abelian projection with random ordering

Results obtained in the APRO case are shown in Fig. 2, where again known values of β_C and ν have been used. The curve in the figure corresponds to the best fit to Eq. (28), which gives $\delta = 0.20(6)$, with a χ^2/d.o.f.~ 1.1. The agreement with the results obtained in the RAP case and in the plaquette and Polyakov gauges in Ref. [4] is very good.

B. SU(3) gauge theory

The confinement transition in pure SU(3) gauge theory is a first order transition [17]. One therefore expects a pseudocritical behavior, with $\nu = 1/3$, that is, the inverse of the number of spatial dimensions. As remarked upon in Ref. [5], the scaling relation [Eq. (24)] has to be modified in this case to include finite size violations to scaling,

$$\frac{\rho}{N_s^{1/\nu}} = f[N_s^{1/\nu}(\beta_C - \beta)] + \Psi(N_s), \tag{29}$$

where $\Psi(N_s)$ parametrizes these effects. A simple assumption is

$$\Psi(N_s) = \frac{a}{N_s^3}, \tag{30}$$

valid up to $O(1/N_s^6)$.

1. Random Abelian projection

Figure 3 shows the scaling behavior expressed by Eq. (29), where for $\Psi(N_s)$ we have taken the form of Eq. (30). As an input we used the values $\beta_C(N_t = 4) = 5.6925$ and $\nu = 1/3$ [18]. The curve in the figure corresponds to a best fit to Eq. (28), modified by including the term $\Psi(N_s)$, which gives the value $\delta = 0.50(3)$, with a χ^2/d.o.f.$= 3.2$.

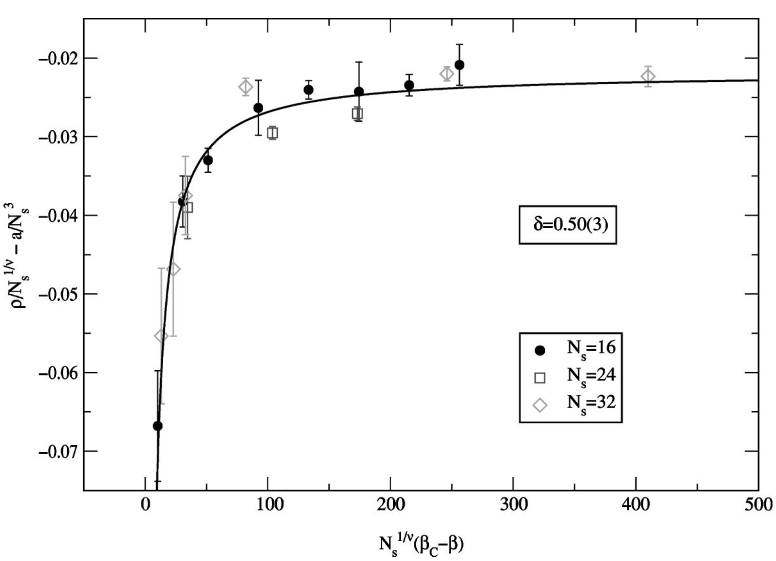

FIG. 3. Quality of scaling in the RAP case for SU(3).

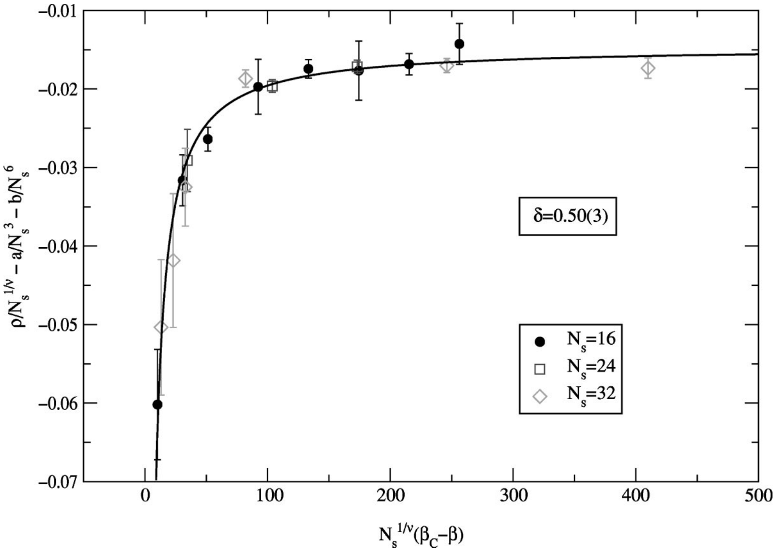

FIG. 4. The same as in Fig. 3, using Eq. (31) for $\Psi(N_s)$.

The quality of the fit improves considerably if one includes a second term in the expression for $\Psi(N_s)$:

$$\Psi(N_s) = \frac{a}{N_s^3} + \frac{b}{N_s^6}. \tag{31}$$

Then the value of δ remains the same, while χ^2/d.o.f. = 0.7. This demonstrates the importance of the finite size effects in SU(3) gauge theory. The fit is shown in Fig. 4.

2. Abelian projection with random ordering

In the APRO case we have obtained a good scaling behavior as well, as shown in Fig. 5. However in this case a fit to Eq. (29) with function (30) has a very bad χ^2/d.o.f. (of order 16).

Also in this case the use of expression (31) is essential: the best fit, shown in the figure, gives $\delta = 0.43(3)$ with χ^2/d.o.f. ~ 1.4. The value of δ is nearly compatible with the one obtained in the RAP case: $\delta = 0.50(3)$. The result obtained in Ref. [5] is $\delta = 0.54(4)$.

V. CONCLUSIONS

We have produced further and compelling evidence that monopole condensation is independent of the Abelian projection used to define the monopoles. If the idea of duality is correct, the nonlocal excitations which are expected to be the fields of the dual description of QCD and weakly interacting in the confined phase, should have nonzero magnetic charge in all the Abelian projections. This is a very important symmetry property, which can help in identifying them.

There have been a number of papers in the literature of the past years, claiming that the fundamental fields of the dual description are the monopoles defined by the maximal Abelian projection. The claim that monopoles defined by the maximal Abelian projection could be dual excitations does not appear to be in good shape after the quantitative attempts to construct the dual theory, which go beyond the initial em-

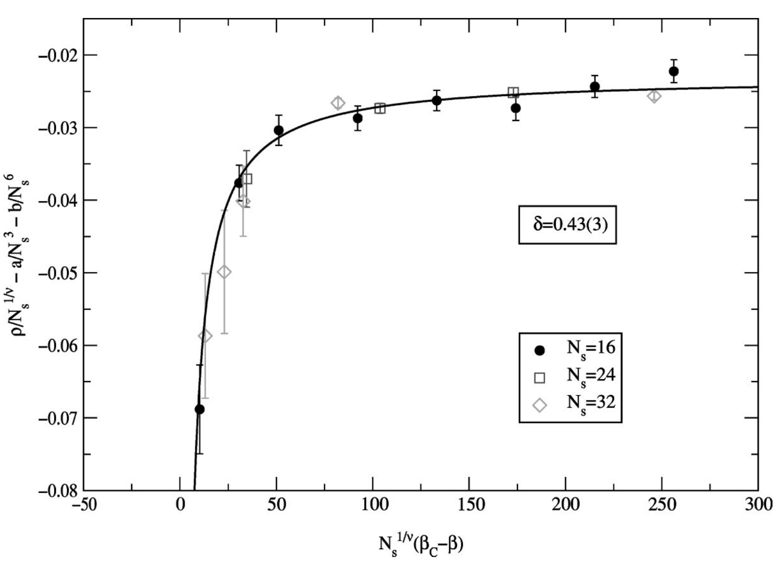

FIG. 5. Quality of scaling in the APRO case for SU(3), using Eq. (31) for $\Psi(N_s)$.

pirical observation of Abelian dominance [19]. If this were true, maximal Abelian monopoles should be magnetically charged in all Abelian projections. This does not seem to be plausible, since one single Abelian projection does not confine the U(1) neutral particles belonging to the adjoint representation, which would instead be confined in other Abelian projections.

An analysis of the Z_N vortices could give some hints, and investigation has been started in this direction [20]. We think that the problem is still open.

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[1] G. 't Hooft, in *High Energy Physics*, edited by A. Zichichi (EPS International Conference, Palermo, 1975).
[2] G.'t Hooft, Nucl. Phys. **B190**, 455 (1981).
[3] S. Mandelstam, Phys. Rep. **23C**, 245 (1976).
[4] A. Di Giacomo, B. Lucini, L. Montesi, and G. Paffuti, Phys. Rev. D **61**, 034503 (2000).
[5] A. Di Giacomo, B. Lucini, L. Montesi, and G. Paffuti, Phys. Rev. D **61**, 034504 (2000).
[6] G. 't Hooft, Nucl. Phys. **B79**, 276 (1974).
[7] T.A. De Grand and D. Toussaint, Phys. Rev. D **22**, 2478 (1980).
[8] L. Del Debbio, A. Di Giacomo, and G. Paffuti, Phys. Lett. B **349**, 513 (1995).
[9] A. Di Giacomo and G. Paffuti, Phys. Rev. D **56**, 6816 (1997).
[10] L. Del Debbio, A. Di Giacomo, G. Paffuti, and P. Pieri, Phys. Lett. B **355**, 255 (1995).
[11] P. Cea and D. Cosmai, Phys. Rev. D **62**, 094510 (2000).
[12] T. Suzuki and I. Yotsuyanagi, Phys. Rev. D **42**, 4257 (1990).
[13] J. Stack, S. Neiman, and R. Wensley, Phys. Rev. D **50**, 3399 (1994).
[14] U.J. Wiese, Nucl. Phys. **B375**, 45 (1992).
[15] A.S. Kronfeld and U.J. Wiese, Nucl. Phys. **B401**, 190 (1992).
[16] J. Fingberg, U.M. Heller, and F. Karsch, Nucl. Phys. **B392**, 493 (1993).
[17] M. Fukugita, M. Okawa, and A. Ukawa, Phys. Rev. Lett. **63**, 1768 (1989).
[18] G. Boyd, G. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lütgemeier, and B. Petersson, Phys. Rev. Lett. **75**, 4169 (1995).
[19] See, e.g., talk by T. Suzuki, in *Confinement, Duality and Non-Perturbative Aspects of QCD*, Vol. 368 of NATO Advanced Study Institute, Series B, edited P. Van Baal (Plenum, New York, 1998), pp. 439–452.
[20] T.G. Kovacs and E.T. Tomboulis, Phys. Rev. Lett. **85**, 704 (2000); Ch. Hoelbling, C. Rebbi, and V.A. Rubakov, Phys. Rev. D **63**, 034506 (2001); L. Del Debbio, A. Di Giacomo, and B. Lucini, Phys. Lett. B **500**, 326 (2001); Nucl. Phys. **B594**, 287 (2001); Ph. de Forcrand, M. D'Elia, and M. Pepe, Phys. Rev. Lett. **86**, 1438 (2001).