Baryons in O(4) and the vibron model

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The structure of the reported excitation spectra of light unflavored baryons is described in terms of multispin valued Lorentz group representations of the so called Rarita-Schwinger (RS) type $(K/2,K/2)\otimes[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$ with $K=1,3$, and 5. We first motivate the legitimacy of such a pattern as fundamental fields as they emerge in the decomposition of triple fermion constructs into Lorentz representations. We then study the baryon realization of RS fields as composite systems by means of the quark version of the U(4) symmetric diatomic rovibron model. In using the $U(4)\supset O(4)\supset O(3)\supset O(2)$ reduction chain, we are able to reproduce the quantum numbers and mass splittings of the above resonance assemblies. We present essentials of the four dimensional angular momentum algebra, and construct electromagnetic tensor operators. The predictive power of the model is illustrated by ratios of reduced probabilities concerning electric de-excitations of various resonances to the nucleon.

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I. O(4) DEGENERACY MOTIF IN BARYON SPECTRA: AN INTRODUCTION

One of the basic quality tests for any model of composite baryons is the level of accuracy reached in describing the nucleon and Δ excitation spectra. In this respect, a knowledge of the degeneracy group of baryon spectra appears as a key tool in constructing the underlying Hamiltonian of the strong-interaction dynamics as a function of the Casimir operators of the symmetry group. To uncover the latter, one can analyze isospin by isospin how the masses of the resonances from the full baryon listing in Ref. [1] spread with spin and parity. Such an analysis was performed in prior work [2], where it was found that Breit-Wigner masses reveal on the mass/spin (M/J) plane a well pronounced spin and parity clustering. There it was further shown that the quantum numbers of the resonances belonging to a particular cluster fit into O(1,3) Lorentz group representations of the so called Rarita-Schwinger (RS) type [3]

$$\Psi_{\mu_1\mu_2\ldots\mu_K}:=\left(\frac{K}{2},\frac{K}{2}\right)\otimes\left[\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\right]. \tag{1}$$

To be specific, one finds the three RS clusters with $K=1,3$, and 5 in both the nucleon (N) and Δ spectra. As long as the *Lorentz group is locally isomorphic to* O(4), multiplets with the quantum numbers of the RS representations also appear in typical O(4) problems such as the levels of an electron with spin in the hydrogen atom. There the principal quantum number of the Coulomb problem is associated with $K+1$ while the role of the boost generators is taken by the Runge-Lenz vector. The Rarita-Schwinger fields are the so-called "diagonal case" (i.e., $a=b=K/2$) of the more general representations $(a,b)\otimes[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$.

A. Rarita-Schwinger fields as multispin-parity states

RS fields are described in terms of totally symmetric traceless rank-K Lorentz tensors with Dirac spinor components that satisfy the Dirac equation for each Lorentz index μ_i, associated with a four-vector $(\frac{1}{2},\frac{1}{2})$ space

$$(i\partial_\lambda\gamma^\lambda-M)\Psi_{\mu_1\mu_2\cdots\mu_K}=0. \tag{2}$$

Fields of the type in Eq. (1) were considered six decades ago by Rarita and Schwinger [3], the most popular being the $K=1$ field frequently applied to the description of spin-3/2 particles. Around the mid 1960's, Weinberg [4] continued the tradition of the original Rarita-Schwinger work [3] and considered $\Psi_{\mu_1\mu_2\ldots\mu_K}$ as fields suited for the description of *pure spin-*$J=K+\frac{1}{2}$ states of *fixed parity*. The conjecture that $\Psi_{\mu_1\mu_2\ldots\mu_K}$ can be reduced to a single-spin state was based upon the belief that its lower-spin components are redundant, unphysical states which can be removed by means of the two auxiliary conditions $\partial^{\mu_1}\Psi_{\mu_1\ldots\mu_k}=0$ and $\gamma^{\mu_1}\Psi_{\mu_1\ldots\mu_k}=0$. That these conditions do not serve the above purpose was demonstrated in Ref. [5]. There the first auxiliary condition was shown to solely test consistency with the mass-shell relation $E^2-\vec{p}^{\,2}=m^2$, while the second condition amounted to the acausal energy-momentum dispersion relation $E=-m\pm\sqrt{\vec{p}^{\,2}}$. It is that type of acausality that must be at the heart of the Velo-Zwanziger problem [6]. The RS fields in O(4) are in fact compilations of fermions of different spins and parities. To illustrate this statement, and for the sake of

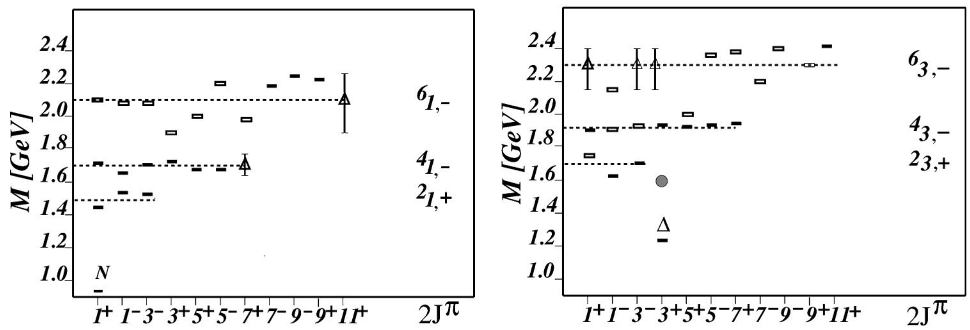

FIG. 1. Rarita-Schwinger clustering of light unflavored baryon resonances. The full bricks stand for three- to four-star resonances, the empty bricks are one- to two-star states, while the triangles represent states that are "missing" for the completeness of the three RS clusters. Note that "missing" F_{17} and $H_{1,11}$ nucleon excitations (left figure) appear as four-star resonances in the Δ spectrum (right figure). The "missing" Δ excitations P_{31}, P_{33}, and D_{33} from $6_{3,-}$ are one- to two-star resonances in the nucleon counterpart $6_{1,-}$. The $\Delta(1600)$ resonance (shadowed oval) drops out of our RS cluster systematics and we view it as an independent hybrid state.

concreteness, here we consider the coupling of, say, a positive parity Dirac fermion to the $(K/2,K/2)$ hyperboson, the latter being composed of O(3) states of either natural $(\eta=+)$, or, unnatural $(\eta=-)$ parities. These (mass degenerate) O(3) states carry all integer internal angular momenta l, with $l=0,\ldots,K$, and transform (for the odd K's of interest) with respect to the space inversion operation $\mathcal{P}$ according to

$$\mathcal{P}|K;\eta;lm\rangle=\eta e^{i\pi l}|K;\eta;l-m\rangle,$$
$$l^P=0^{\eta},1^{-\eta},\ldots,K^{-\eta},\quad m=-l,\ldots,l. \tag{3}$$

In coupling now the Dirac spinor to $(K/2,K/2)$ from above, the following spin (J) and parity (P) quantum numbers are created:

$$J^P=\frac{1}{2}^{\eta},\frac{1}{2}^{-\eta},\frac{3}{2}^{-\eta},\ldots,\left(K+\frac{1}{2}\right)^{-\eta}. \tag{4}$$

In the following, for the spin sequence in Eq. (4) we will use the short hand notation $\sigma_{2I,\eta}$, with $\sigma=K+1$, or, equivalently

$$\sigma_{2I,\eta}=\left(\frac{\sigma-1}{2},\frac{\sigma-1}{2}\right)\otimes\left[\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\right]\chi^I. \tag{5}$$

Here χ^I stands for the isospin spinor attributed to the states under consideration.

A glance at the baryon spectra teaches us that actually Nature strongly favors excitations of multispin-valued resonance clusters over that of pure higher-spin states. This circumstance suggests a new data supported interpretation of the RS fields as complete resonances packages.

B. Clustering principle for baryon resonances

In terms of the notations introduced above, all reported light-quark baryons with masses below 2500 MeV [up to the $\Delta(1600)$ resonance that is most probably an independent quark-gluon hybrid state [7]] were shown in Ref. [2] to be completely accommodated by the RS clusters $2_{2I,+}$, $4_{2I,-}$, and $6_{2I,-}$, having states of highest spin-$3/2^-$, $7/2^+$, and $11/2^+$, respectively (see Fig. 1). In each of the Δ nucleon and Λ hyperon spectra, the natural parity cluster $2_{2I,+}$ is always of lowest mass. We consider it to reside in a Fock space $\mathcal{F}_+$ built on top of a scalar vacuum. Equations (3) and (4) illustrate how $2_{2I,+}$ clusters (with $I=1/2,3/2$, and 0) always unite the first spin-$\frac{1}{2}^+$, $\frac{1}{2}^-$, and $\frac{3}{2}^-$ resonances. For nonstrange baryons, $2_{2I,+}$ is followed by the unnatural parity clusters $4_{2I,-}$, and $6_{2I,-}$, which we view to reside in a different Fock space $\mathcal{F}_-$, built on top of a pseudoscalar vacuum that is orthogonal [for an ideal O(4) symmetry] to the previous scalar vacuum. To be specific, one finds all seven Δ-baryon resonances S_{31}, P_{31}, P_{33}, D_{33}, D_{35}, F_{35} and F_{37} from $4_{3,-}$ to be squeezed within the narrow mass region from 1900 to 1950 MeV, while the $I=1/2$ resonances paralleling them, of which only the F_{17} state is still "missing" from the data, are located around 1700^{+20}_{-50} MeV (see Fig. 1 left). Therefore, the F_{17} resonance is the only nonstrange state with a mass below 2000 MeV which is "missing" for the completeness of the present RS classification scheme. In further paralleling baryons from the third nucleon and Δ clusters with $K+1=6$, one finds, in addition, the four states $H_{1,11}$, P_{31}, P_{33}, and D_{33} with masses above 2000 MeV to be "missing" for the completeness of the new classification scheme. The $H_{1,11}$ state is needed to parallel the well established $H_{3,11}$ baryon, while the Δ states P_{31}, P_{33}, and D_{33} are required as partners to the (less established) $P_{11}(2100)$, $P_{13}(1900)$, and $D_{13}(2080)$ nucleon resonances. For Λ hyperons, incomplete data prevent a conclusive analyses. Even so, Fig. 2 (left) indicates that the RS motif may already show up in the reported spectrum. The (approximate) degeneracy group of baryon spectra, as already suggested in Ref. [2], is, therefore, confirmed to be

$$SU(2)_I\otimes O(1,3)\simeq SU(2)_I\otimes O(4), \tag{6}$$

i.e., isospin$\otimes$space-time symmetry. To summarize, here we state the principle that light unflavored baryon excitations are patterned after Lorentz multiplets. For example, the Rarita-Schwinger spinors $\Psi_{\mu_1\ldots\mu_K}$, with $K=1,3$, and 5, accommodate all the πN resonances according to

$$\mathcal{F}_+:2_{2I,+}:\Psi_{\mu_1}:P_{2I,1};S_{2I,1},D_{2I,3}\quad\text{for}\quad I=0,\frac{1}{2},\frac{3}{2}$$

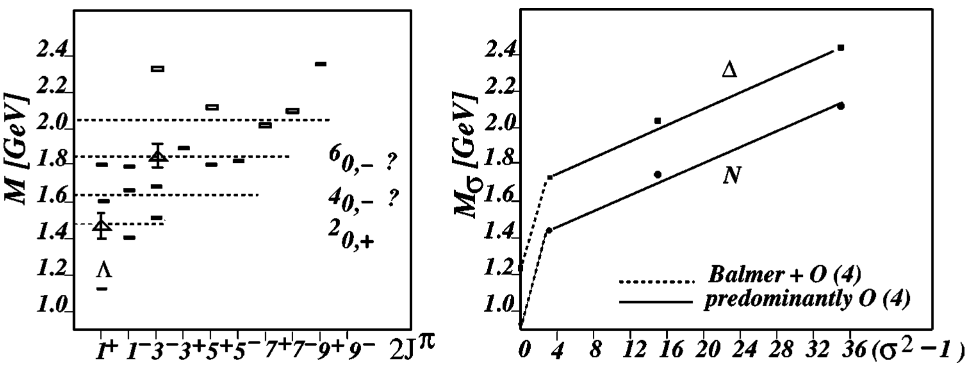

FIG. 2. Clustering traces in the Λ hyperon spectrum (left). O(4) rotational bands of nucleon (N) and (Δ) excitations (right). Notations are as in Fig. 1.

and

$$\mathcal{F}_-:4_{2I,-}:\Psi_{\mu_1\mu_2\mu_3}:S_{2I,1};P_{2I,1}P_{2I,3};D_{2I,3},D_{2I,5};F_{2I,5},F_{2I,7},$$

$$\mathcal{F}_-:6_{2I,-}:\Psi_{\mu_1\mu_2\ldots\mu_5}:S_{2I,1};P_{2I,1}P_{2I,3};D_{2I,3},D_{2I,5};F_{2I,5},$$

$$F_{2I,7};G_{2I,7},G_{2I,9};H_{2I,9},H_{2I,11} \quad \text{for} \quad I=\frac{1}{2},\frac{3}{2}, \tag{7}$$

with the five "missing" states $F_{17}, H_{1,11}, P_{31}, P_{33}$, and D_{33}.

Occasionally, the above structures will be referred to as LAMPF clusters to emphasize their close relationship to LAMPF physics. The scalar vacuum in the first Fock space reflects the Nambu-Goldstone mode of chiral symmetry near the ground state. As argued in Ref. [8], its change to a pseudoscalar between the first and second clusters may be related to a change of the mode of chiral symmetry realization in baryonic spectra. Within our scheme, the intercluster spacing of 200–300 MeV is larger by a factor of 3–6 as compared to the mass spread within the clusters. For example, the $2_{1,+}$, $2_{3,+}$, $4_{1,-}$, and $4_{3,-}$ clusters carry the maximal internal mass splitting of 50–70 MeV.

Finally, the reported mass averages of the resonances from RS multiplets with $K=1,3$, and 5 are well described by means of the following simple *empirical* relation:

$$M_{\sigma;I}=M_I-m_1\frac{1}{\sigma^2}+m_2\frac{\sigma^2-1}{4}, \quad I=\frac{1}{2},\frac{3}{2}, \tag{8}$$

where, again, $\sigma=K+1$. The two mass parameters take for the nucleon ($I=\frac{1}{2}$) the values $m_1=600$ MeV and $m_2=70$ MeV, respectively. The Δ spectrum ($I=\frac{3}{2}$) is best fitted by the smaller m_2 value of $m_2=40$ MeV (Fig. 2, right).

It is the goal of this paper to develop a constituent model for baryons that explains the observed clustering in the spectra of the light unflavored baryons. The paper is organized as follows. In Sec. II we motivate the legitimacy of fundamental fields of specified mass and unspecified spin as they emerge in the decomposition of a triple-Dirac-fermion system into Lorentz group representations. In Sec. III we present the quark version of the diatomic rovibron model [9] and study its excitation modes. There we also establish a correspondence between excited rovibron states and baryonic RS clusters. We further make all the observed resonances and some of the "missing" resonances distinguishable in organizing them into different rovibron modes. We construct the relevant quark Hamiltonian and recover Eq. (8). We finally outline the construction of electric transition operators and calculate selected electric transitions of cluster inhabitants to the nucleon. The paper is completed with a brief summary and outlook.

II. MULTISPIN STATES AS LORENTZ COVARIANT REPRESENTATIONS

The relativistic description of three-Dirac-spinor systems was studied in detail in Ref. [10]. Starting with the well known Lorentz invariance of the ordinary Dirac equation

$$(\gamma^\mu p_\mu - m)u(\vec{p})=0, \tag{9}$$

the authors showed that the direct product of three Dirac spinors gives rise to a 64-dimensional linear equation of the type

$$(\Gamma^\mu p_\mu - m)\mathcal{U}(\vec{p})=0$$

with

$$\Gamma^\mu=\sum_{r=1}^{3}\gamma_r^\mu, \quad \gamma_1^\mu=\gamma^\mu\otimes I\otimes I, \quad \gamma_2^\mu=I\otimes\gamma^\mu\otimes I,$$

$$\gamma_3^\mu=I\otimes I\otimes\gamma^\mu. \tag{10}$$

Here I stays for the four dimensional unit matrix, while the index r indicates the position of the Dirac matrix γ^μ in γ_r^μ. Under Lorentz transformations (a_ν^μ) of the γ matrices, the matrices Γ^μ from Eq. (10) change according to $\Gamma^{\mu\prime}=U\Gamma^\mu U^{-1}$ with $U=U_1\otimes U_2\otimes U_3$, and U_r defined as the matrix that covers the Lorentz transformation $\gamma_{r}^{\mu\prime}=a_\nu^\mu\gamma_r^\nu=U_r\gamma_r^\mu U_r^{-1}$ of γ_r. Equation (10) is therefore Lorentz invariant. Moreover, it was demonstrated that Eq. (10) has U(4) as an additional dynamical symmetry.

The 64 states from above are distributed over different irreducible representations (irreps) of U(4) and the permutational group group $\mathcal{S}_3$ as well. To be specific, one finds two 20plets in turn associated with the Young schemes [3000] and [2100]. They are completed by the quartet [1110]. The three-Dirac spinor state (denoted by s^3) can be characterized by the set of quantum numbers

$$|s^3[f]X,\{f\}R\rangle. \tag{11}$$

Here X stands for a set of quantum numbers characterizing the U(4) basis vectors of the $[f]$ irrep, while R denotes the Yamanouchi symbol labeling the basis vectors of the $\mathcal{S}_3$ representation $\{f\}$ [11]. The Yamanouchi symbols for the [3000], [2100], and [1110] are 1, 2, 1, and 1, respectively. The complete number (N_{s^3}) of 64 states of the three-quark (q^3) system is then encoded by the relation

$$N_{q^3}=\sum_{[f]} \dim\{f\}\dim\{f\}, \tag{12}$$

where $\dim[f]$ and $\dim\{f\}$ are in turn the dimensionalities of the U(4) irrep $[f]$, and the $\mathcal{S}_3$ irrep $\{f\}$, respectively. Considering, now the reduction chain $\mathrm{U}(4)\supset\mathrm{O}(5)$ allows for a more detailed specification of the spin content of the U(4) multiplets from above (see Ref. [10] for details).

The quantum numbers of the irreducible representations (irreps) of O(5) are labeled by the two numbers $(\lambda_1\lambda_2)$ which can be either an integer or a half-integer. The states participating a given O(5) irrep can be further specified by the quantum numbers of the irreps of the O(5) subgroups appearing in the reduction chain $\mathrm{O}(5)\supset\mathrm{O}(4)\supset\mathrm{O}(3)\supset\mathrm{O}(2)$. To specify the O(4) irreps in the context of the O(5) reduction down to O(2) it is more convenient to use, instead of the pair (a,b) from above, a pair $(m_1\, m_2)$ with the mapping

$$m_1=a+b, \quad m_2=a-b. \tag{13}$$

Finally, the O(3) irreps in the $\mathrm{O}(3)\supset\mathrm{O}(2)$ reduction scheme are labeled by the well known spin number (J) and the magnetic quantum number (M). The complete set of quantum numbers specifying a member of a O(5) multiplet $|(\lambda_1\,\lambda_2);(m_1\,m_2);JM\rangle$ satisfy the inequalities

$$\lambda_1\geqslant m_1\geqslant\lambda_2\geqslant|m_2|,$$
$$m_1\geqslant J\geqslant|m_2|, \quad J\geqslant M\geqslant -J. \tag{14}$$

The U(4) irrep [2100] is of particular interest for the present work. In the $\mathrm{U}(4)\supset\mathrm{O}(5)$ reduction chain it splits into O(5) irreps according to

$$[2100]\rightarrow\left(\frac{3}{2}\,\frac{1}{2}\right)\oplus\left(\frac{1}{2}\,\frac{1}{2}\right). \tag{15}$$

The first irrep on the right-hand side of the last equation is 16 dimensional, while the second is four dimensional and associated with a Dirac spinor. As we shall see below, the O(5) 16plet $(\frac{3}{2}\frac{1}{2})$ is nothing but the RS field with $K=1$. Indeed, from Eq. (14) follows that

$$\frac{3}{2}\geqslant m_1, \quad m_1\geqslant\frac{1}{2} \quad \text{and} \quad \frac{1}{2}\geqslant|m_2|. \tag{16}$$

The inequalities in the latter equation are satisfied for $m_1=3/2$ and 1/2, and for $m_2=1/2$ and $-1/2$. In accordance with the second equation in Eq. (14), J can take the three values $J=3/2,1/2$, and 1/2. Thus the $(\frac{3}{2}\frac{1}{2})$ irrep of O(5) describes a spin-3/2 state and two spin-1/2 states, and coincides with the lowest 16-dimensional Rarita-Schwinger field.

The above consideration gives an idea of how Lorentz representations of the RS type can emerge as *fundamental* free particles of definite mass and indefinite spin within the context of a relativistic space-time treatment. Though such pointlike particles have not been detected so far, the N and Δ spectra strongly indicate existence of *composite* RS fields. In the following, we shall focus onto that very realization of multispin Lorentz representations and explore their internal structure by means of constituent models. For a more profound textbook presentation on the various aspects of higher-dimensional relativistic supermultiplets, the interested reader is referred to Ref. [12].

III. QUARK VERSION OF THE DIATOMIC ROVIBRON MODEL AND THE RS CLUSTERING IN BARYON SPECTRA

Baryons in the quark model are considered to be constituted of three quarks in a color singlet state. It appears natural, therefore, to undertake an attempt of describing the baryonic system by means of algebraic models developed for the purposes of triatomic molecules, a path already pursued in Ref. [13]. There, the three body system was described in terms of two vectorial ($\vec{p}^{\,+}$) boson degrees of freedom and one scalar (s^+) boson degree of freedam that transform as the fundamental U(7) septet. In the dynamical symmetry limit

$$\mathrm{U}(7)\rightarrow\mathrm{U}(3)\times\mathrm{U}(4) \tag{17}$$

the degrees of freedom associated with the one vectorial boson factorize from those associated with the scalar boson and the remaining vectorial boson. Because of that the physical states constructed within the U(7) IBM model are often labeled by means of $\mathrm{U}(3)\times\mathrm{U}(4)$ quantum numbers. Below we will focus on that very sub-model of the IBM, and show that it perfectly accommodates the RS clusters from above, and thereby the LAMPF data on the nonstrange baryon resonances.

The dynamical limit $\mathrm{U}(7)\rightarrow\mathrm{U}(3)\times\mathrm{U}(4)$ corresponds to the quark-diquark approximation of the three quark system, when two of the quarks reveal a stronger pair correlation to a diquark (Dq) [14], while the third quark (q) acts as a spectator. The diquark approximation turned out to be rather convenient in particular in describing various properties of the ground state baryons [15,16]. Within the context of the quark-diquark (q-Dq) model, the ideas of the rovibron model, known from the spectroscopy of diatomic molecules [9], can be applied to the description of the rotational-vibrational (rovibron) excitations of the q-Dq system.

A. Rovibron model for the quark-diquark system

In the rovibron model (RVM) the relative q-Dq motion (see Fig. 3) is described by means of four types of boson creation operators s^+,p_1^+,p_0^+, and p_{-1}^+ (compare Ref. [9]).

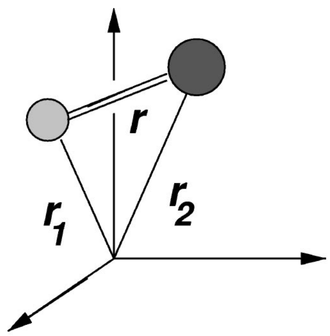

FIG. 3. Schematic presentation of a q-Dq two-body system.

The operators s^+ and p_m^+ in turn transform as rank-0 and rank-1 spherical tensors, i.e., the magnetic quantum number m takes in turn the values $m=1,\ 0$, and -1. In order to construct boson-annihilation operators that also transform as spherical tensors, one introduces the four operators $\tilde{s}=s$ and $\tilde{p}_m=(-1)^m p_{-m}$. Constructing rank-$k$ tensor product of any rank-k_1 and rank-k_2 tensors, say, $A^{k_1}_{m_1}$ and $A^{k_2}_{m_2}$, is standard and given by

$$[A^{k_1}\otimes A^{k_2}]^k_m=\sum_{m1,m_2}(k_1m_1k_2m_2|km)A^{k_1}_{m_1}A^{k_2}_{m_2}. \quad (18)$$

Here, $(k_1m_1k_2m_2|km)$ are the well known O(3) Clebsch-Gordan coefficients.

Now the lowest states of the two-body system are identified with N boson states, and are characterized by the ket vectors $|n_s n_p l\ m\rangle$ (or, a linear combination of them) within a properly defined Fock space. The constant $N=n_s+n_p$ stands for the total number of s and p bosons, and plays the role of a parameter of the theory. In molecular physics, the parameter N is usually associated with the number of molecular bound states. The group symmetry of the rovibron model is well known to be U(4). The 15 generators of the associated su(4) algebra are determined as the following set of bilinears:

$$A_{00}=s^+\tilde{s}, \quad A_{0m}=s^\dagger\tilde{p}_m,$$

$$A_{m0}=p_m^+\tilde{s}, \quad A_{mm'}=p_m^\dagger\tilde{p}_{m'}. \quad (19)$$

The u(4) algebra is then recovered by the commutation relations

$$[A_{\alpha\beta},A_{\gamma\delta}]_-=\delta_{\beta\gamma}A_{\alpha\delta}-\delta_{\alpha\delta}A_{\gamma\beta}. \quad (20)$$

The operators associated with physical observables can then be expressed as combinations of the u(4) generators. To be specific, the three-dimensional angular momentum takes the form

$$L_m=\sqrt{2}[p^+\otimes\tilde{p}]^1_m. \quad (21)$$

Further operators are (D_m)– and (D'_m), defined as

$$D_m=[p^+\otimes\tilde{s}+s^+\otimes\tilde{p}]^1_m, \quad (22)$$

$$D'_m=i[p^+\otimes\tilde{s}-s^+\otimes\tilde{p}]^1_m, \quad (23)$$

respectively. Here $\vec{D}$ plays the role of the electric dipole operator.

Finally, a quadrupole operator Q_m can be constructed as

$$Q_m=[p^+\otimes\tilde{p}]^2_m, \quad \text{with} \quad m=-2,\ldots,+2. \quad (24)$$

The u(4) algebra has the two algebras su(3), and so(4), as respective subalgebras. The su(3) algebra is constituted by the three generators L_m, and the five components of the quadrupole operator Q_m. Its so(4) subalgebra is constituted by the three components of the angular momentum operator L_m, on the one hand, and the three components of the operator D'_m, on the other hand. Thus there are two exactly soluble RVM limits that correspond to the two different chains of reducing U(4) down to O(3). These are

$$\mathrm{U}(4)\supset\mathrm{U}(3)\supset\mathrm{O}(3) \quad \text{and} \quad \mathrm{U}(4)\supset\mathrm{O}(4)\supset\mathrm{O}(3), \quad (25)$$

respectively. The Hamiltonian of the RVM in these exactly soluble limits is then constructed as a properly chosen function of the Casimir operators of the algebras of either the first or the second chain. For example, in case one approaches O(3) via U(3), the Hamiltonian of a dynamical SU(3) symmetry can be cast into the form

$$H_{\mathrm{SU}(3)}=H_0+\alpha\mathcal{C}_2(\mathrm{SU}(3))+\beta\mathcal{C}_2(\mathrm{SO}(3)). \quad (26)$$

Here H_0 is a constant, and $\mathcal{C}_2(\mathrm{SU}(3))$ and $\mathcal{C}_2(\mathrm{SO}(3))$ are in turn the quadratic (in terms of the generators) Casimirs of the groups SU(3) and SO(3), respectively, while α and β are constants, to be determined from data fits.

A similar expression (in obvious notations) can be written for the RVM Hamiltonian in the $\mathrm{U}(4)\supset\mathrm{O}(4)\supset\mathrm{O}(3)$ exactly solvable limit:

$$H_{\mathrm{SO}(4)}=H_0+\tilde{\alpha}\mathcal{C}_2(\mathrm{SO}(4))+\tilde{\beta}\mathcal{C}_2(\mathrm{SO}(3)). \quad (27)$$

The Casimir operator $\mathcal{C}_2(\mathrm{SO}(4))$ is defined accordingly as

$$\mathcal{C}_2(\mathrm{SO}(4))=\frac{1}{4}(\vec{L}^2+\vec{D}'^2) \quad (28)$$

and has an eigenvalue of $K/2[(K/2)+1]$. In molecular physics, only linear combinations of the Casimir operators are used, as a rule. However, as known from the hydrogen atom [17], the Hamiltonian is determined by the inverse power of $\mathcal{C}_2(\mathrm{SO}(4))$ according to

$$H_{Coul}=f[-4C_2(\mathrm{SO}(4))-1]^{-1}, \quad (29)$$

where f is a parameter with the dimensionality of mass. This Hamiltonian predicts the energy of the states as $E_K=-f/(K+1)^2$ and does not follow the simple linear pattern [also see Eq. (27)].

In order to demonstrate how the RVM applies to baryon spectroscopy, let us consider the case of q-Dq states associated with $N=5$ and for the case of a SO(4) dynamical symmetry. From now on we shall refer to the quark rovibron model as QRVM. It is of common knowledge that the totally symmetric irreps of the u(4) algebra with the Young scheme $[N]$ contain the SO(4) irreps $(K/2,K/2)$ with

$$K=N,N-2,\ldots,1 \quad \text{or} \quad 0. \tag{30}$$

Each one of these SO(4) irreps contains SO(3) multiplets with three dimensional angular momenta

$$l=K,K-1,\ K-2,\ldots,1,0. \tag{31}$$

In applying the branching rules in Eqs. (30) and (31) to the case $N=5$, one encounters the series of levels

$$\begin{aligned} K=1: &\quad l=0,1,\\ K=3: &\quad l=0,1,2,3,\\ K=5: &\quad l=0,1,2,3,4,5. \end{aligned} \tag{32}$$

The parity carried by these levels is $\eta(-1)^l$ where η is the parity of the relevant vacuum. In coupling now the angular momenta in Eq. (32) to the spin-1/2 of the three quarks in the nucleon, the following sequence of states is obtained:

$$\begin{aligned} K=1: &\quad \eta J^\pi=\frac{1}{2}^+,\frac{1}{2}^-,\frac{3}{2}^-,\\ K=3: &\quad \eta J^\pi=\frac{1}{2}^+,\frac{1}{2}^-,\frac{3}{2}^-,\frac{3}{2}^+,\frac{5}{2}^+,\frac{5}{2}^-,\frac{7}{2}^-,\\ K=5: &\quad \eta J^\pi=\frac{1}{2}^+,\frac{1}{2}^-,\frac{3}{2}^-,\frac{3}{2}^+,\frac{5}{2}^+,\frac{5}{2}^-,\frac{7}{2}^-,\frac{7}{2}^+,\\ &\quad \frac{9}{2}^-,\frac{11}{2}^-. \end{aligned} \tag{33}$$

Thus rovibron states of half-integer spin will transform according to $(K/2,K/2)\otimes[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$ representations of SO(4). The isospin structure is accounted for pragmatically through attaching an isospin spinor χ^I to the RS clusters, with I taking the values $I=\frac{1}{2}$ and $\frac{3}{2}$ for the nucleon and the Δ states, respectively. As illustrated in Fig. 1, the above quantum numbers cover both the nucleon and the Δ excitations.

Note that in the present simple version of the rovibron model, the spin of the quark-diquark system is $S=\frac{1}{2}$, and the total spin J takes the values $J=l\pm\frac{1}{2}$ in accordance with Eqs. (32) and (33). The strong relevance of *same* picture for both the nucleon and the $\Delta(1232)$ spectra (where the diquark is in a vector-isovector state) hints onto the dominance of a scalar diquark for both the excited nucleon and $\Delta(1232)$ states. This situation is reminiscent of the $^2 10$ configuration of the $70(1^-)$plet of the canonical $SU(6)_{SF}\otimes O(3)_L$ symmetry where the mixed symmetric-antisymmetric character of the $S=1/2$ wave function in spin space is compensated for by a mixed symmetric-antisymmetric wave function in coordinate space, while the isotriplet $I=3/2$ part is totally symmetric. Here we will leave aside the discussion of the generic problem of the various incarnations of the IBM model regarding the symmetry properties of the resonance wave functions to a later date, and concentrate in Sec. III B on the "missing" resonance problem.

B. Observed and "missing" resonance clusters within the rovibron model

The comparison of states in Eq. (33) with the reported states in Eq. (7) shows that the predicted sets are in agreement with the characteristics of the nonstrange baryon excitations with masses below ~ 2500 MeV, provided the parity η of the vacuum changes from scalar ($\eta=1$) for the $K=1$ cluster to pseudoscalar ($\eta=-1$) for the $K=3$ and 5 clusters. A pseudoscalar "vacuum" can be modeled in terms of an excited composite diquark carrying an internal angular momentum $L=1^-$ and maximal spin $S=1$. In one of the possibilities the total spin of such a system can be $|L-S|=0^-$. To explain the properties of the ground state, one has to consider even N values separately, such as, say, $N'=4$. In this case another branch of excitations, with $K=4$, 2, and 0 will emerge. The $K=0$ value characterizes the ground state, $K=2$ corresponds to $(1,1)\otimes[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$, while $K=4$ corresponds to $(2,2)\otimes[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$. These are the multiplets that we will associate with the "missing" resonances predicted by the rovibron model. In this manner, reported and "missing" resonances fall apart and populate distinct U(4)- and SO(4) representations. In making observed and "missing" resonances distinguishable, reasons for their absence or presence in the spectra are easier to search for. As to the parity of resonances with even K's, there is some ambiguity. As a guidance one may consider the decomposition of the three-quark (q^3) Hilbert space into Lorentz group representations as performed in Ref. [8]. There, two states of the type $(1,1)\otimes[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$ were found. The first one arose from the decomposition of the q^3 Hilbert space spanned by the $1s-1p-2s$ single-particle states. It was close to $(\frac{1}{2},\frac{1}{2})\otimes[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$ and carried opposite parity to the latter. Therefore, it accommodated, unnatural parity resonances. The second $K=2$ state was part of the $(1s-3s-2p-1d)$ single-particle configuration space and was closer to $(\frac{3}{2},\frac{3}{2})\otimes[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$. It also carried a parity opposite to the latter and accommodated natural parity resonances. Finally, the $K=4$ cluster $(2,2)\otimes[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$ emerged in the decomposition of the one-particle–one-hole states within the $(1s-4s-3p-2d-1f-1g)$ configuration space and also carried natural parity, that is, a parity opposite to $(\frac{5}{2},\frac{5}{2})\otimes[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$. In accordance with the above results, here we will treat the $N=4$ states to be all of natural parities, and identify them with the nucleon ($K=0$), the natural parity $K=2$, and the natural parity $K=4$ RS clusters.

The unnatural parity $K=2$ cluster from Ref. [8] could be generated through an unnatural parity $N=2$ excitation mode. However, this mode would require a manifest chiral symmetry up to ≈ 1550 MeV which contradicts at least present data. With this observation in mind, we here will restrict ourselves to the consideration of the natural parity $N=4$

TABLE I. Predicted mass distribution of observed (obs), and missing (miss) rovibron clusters (in MeV) according to Eqs. (34) and (35). The sign of η in Eq. (3) determines natural ($\eta=+1$) or unnatural ($\eta=-1$) parity states. All Δ excitations have been calculated with $m_2=40$ MeV rather than with the nucleon value of $m_2=70$ MeV. The experimental mass averages of the resonances from a given RS cluster have been labeled "exp." The nucleon and Δ ground state masses M_N and M_Δ were taken to be equal to their experimental values.

K	sign η	N^{obs}	N^{expt}	Δ^{obs}	Δ^{expt}	N^{miss}	Δ^{miss}
0	+	939	939	1232	1232		
1	+	1441	1498	1712	1690		
2	+					1612	1846
3	−	1764	1689	1944	1922		
4	+					1935	2048
5	−	2135	2102	2165	2276		

clusters. In this manner the unnatural parity $K=2$ state from Ref. [8] will be dropped from the current version of the rovibron model. From now on we will refer to the excited $N=4$ states as to "missing" rovibron clusters.

Now the QRVM Hamiltonian that reproduces the mass formula from Eq. (8) is given by the following function of $\mathcal{C}_2(\mathrm{SO}(4))$:

$$H_{QRVM}=H_0-f_1[4\mathcal{C}_2(\mathrm{SO}(4))+1]^{-1}+f_2(\mathcal{C}_2(\mathrm{SO}(4))). \tag{34}$$

The states in Eq. (33) are degenerate and the dynamical symmetry is SO(4). The parameter set

$$H_0=M_{N/\Delta}+f_1, \quad +f_1=m_1, \quad f_2=m_2, \tag{35}$$

with $I=\frac{1}{2}$ and $\frac{3}{2}$, recovers the empirical mass formula in Eq. (8). Thus the SO(4) dynamical symmetry limit of the QRVM picture of baryon structure motivates existence of quasidegenerate clusters of resonances in the nucleon- and Δ baryon spectra. In Table I we list the masses of the RS clusters concluded from Eqs. (34) and (35).

The data on the Λ, Σ, and Ω^- hyperon spectra are still far from being as complete as those of the nucleon and the Δ baryons and do not allow, at least at the present stage, a conclusive statement on relevance or irrelevance of the rovibron picture (Fig. 1). The presence of the heavier strange quark can significantly influence the excitation modes of the q^3 system. If the presence of an s quark in the hyperon structure is essential, the $U(4)\supset U(3)\supset O(3)$ chain can be favored over the $U(4)\supset O(4)\supset O(3)$ chain, and a different clustering motif can appear here. For the time being, this issue will be dropped from further consideration. In Sec. III C, we shall outline the calculational scheme for branching ratios of reduced probabilities for electromagnetic transitions.

C. O(4) angular momentum algebra and multipole operators

In the following, resonance states from a RS cluster will be denoted as

$$|N;0^\eta;(a,b);l^\pi;S;J^\pi M_J\rangle. \tag{36}$$

Here $\eta=\pm$ denotes the parity of the vacuum of the Fock space accommodating the RS cluster, $(a,b)=(K/2,K/2)$, l^π is the underlying three-dimensional angular momentum, S is the quark spin, and J^π and M_J are in turn total spin and magnetic quantum numbers of the resonance under consideration. In fact, K is nothing but the four-dimensional angular momentum. Within the framework of the rovibron model one can describe three different types of transitions.

(i) Transitions without change of the quantum numbers N and K, i.e., transitions between resonances from same cluster. In such a case, the transition operator is the D'_m generator of the $so(4)$ algebra. One can calculate the reduced probabilites $B(\alpha_1,J_1\rightarrow\alpha_1 J_2;T^{\alpha;1})$ for electric dipole transitions. Note that the reduced transition probability, for an electric transition of the multipolarity λ between states of initial and final spins J_1 and J_2, respectively, is defined as [19]

$$B(\alpha_1,J_1\rightarrow\alpha_2,J_2;T^{\alpha,\lambda})=\frac{1}{2J_1+1}|(\alpha_2 J_2||T^{\alpha,\lambda}||\alpha_1 J_1)|^2. \tag{37}$$

Unfortunately, such transitions are difficult and perhaps even beyond any possibility of being observed.

(ii) Transitions between states of same number of bosons N but of different four dimensional angular momenta, $\Delta K\neq 0$, i.e., transitions between resonances belonging to different RS clusters. Operators that can realize transitions between different O(4) multiplets are U(4) generators (or, tensor products of them) lying outside of the so(4) sub algebra. The latter operators constitute the set

$$Q_m=[p^+\otimes\tilde{p}]^2_m, \quad E_m=\frac{1}{\sqrt{2}}D_m,$$

$$E0=\frac{1}{2\sqrt{3}}(3n_s-n_p). \tag{38}$$

It is not difficult to prove that the nine operators in Eq. (38) behave with respect to SO(4) transformation as the components of the totally symmetric rank-2 tensor, $T^{(1,1)lm}$, where

$$T^{(1,1)2m}:=Q_m, \quad T^{(1,1)1m}:=E_m,$$

$$T^{(1,1)00}:=E0. \tag{39}$$

The tensor $T^{(1,1)lm}$ is the one of lowest rank that can realize transitions between SO(4) multiplets having same number of bosons N and differing by two units in K.

(iii) Transitions between U(4) multiplets whose number of bosons differ by one unit ($\Delta N=1$), the most interesting being resonance de-excitation modes into the nucleon:

$$|N_1=5;0^\eta;K_1;l_1;S_1=\tfrac{1}{2};J_1M_1\rangle \rightarrow|N_2=4;0^+;K_2=0;l_2=0;S_2=\tfrac{1}{2};\tfrac{1}{2}m_{1/2}\rangle. \tag{40}$$

In the following we will be mainly interested in transitions of the third type. At the present stage, however, it is convenient to first outline the general scheme of the SO(4) Racah algebra.

Tensor products $[T^{(a_1,b_2)}\otimes T^{(a_2,b_2)}]^{(a,b)lm}$ in SO(4) are defined as (see Refs. [11,18] for details)

$$[T^{(a_2,b_2)}\otimes T^{(a_1,b_1)}]^{(a,b)lm}=\sum_{l_1m_1l_2m_2}((a_1b_1)l_1m_1(a_2b_2)\times l_2m_2|(a_1b_1)(a_2b_2);(ab)lm)\times T^{(a_1,b_1)l_1m_1}T^{(a_2,b_2)l_2m_2}. \quad (41)$$

The matrix elements of any tensor operator $T^{(a,b)lm}$ between O(4) states are expressed as

$$\langle(a_1,b_1)l_1m_1|T^{(a,b)lm}|(a_2,b_2)l_2m_2\rangle =((a_2b_2)l_2m_2(ab)lm|(a_2b_2)(ab);(a_1b_1)\times l_1m_1)((a_1,b_1)|||T^{(a,b)}|||(a_2,b_2)). \quad (42)$$

The SO(4) Clebsch-Gordan coefficients entering the last equation are determined by

$$((a_2b_2)l_2m_2(ab)lm|(a_1b_1)(a_2b_2);(a_1b_1)l_1m_1) =\sqrt{(2l_1+1)(2l_2+1)(2l+1)(2a+1)(2b+1)}\times(-1)^{(l-m)}\begin{pmatrix} l_1 & l_2 & l\\ -m_1 & -m_2 & m\end{pmatrix}\times\begin{Bmatrix} a_1 & a_2 & a\\ b_1 & b_2 & b\\ l_1 & l_2 & l\end{Bmatrix}. \quad (43)$$

The last equation shows that the ratios of the reduced probabilities of electromagnetic transitions between resonances with different K quantum numbers are determined as ratios of the squared SO(4) Clebsch-Gordan coeffecients, as the triple barred transition matrix elements cancel out. As an example of that type of transitions let us consider the electromagnetic de-excitations of the natural parity resonances with spins $3/2^-$ and $1/2^-$ from the first cluster to the nucleon. Obviously, the relevant tensor operator in SO(4) space is $T^{(1/2,1/2)lm}$. The latter should connect U(4) states with different numbers of bosons, i.e., $\Delta N=1$. Therefore, it can be taken in the forms

$$T^{(1/2,1/2)1m}=p_m^+,\quad T^{(1/2,1/2)00}=s^+. \quad (44)$$

Transitions of the above type can then be calculated by means of ordinary Racah algebra in considering $\alpha_i :=N(N')(a_i,b_i)=N(N')(K_i/2,K_i/2)$ (with $i=1,2$) as an intrinsic quantum number according to

$$\left\langle \alpha_1,l_1;\frac{1}{2};J^\pi M_J\middle|T^{\alpha,lm}\middle|\alpha_2,0;\frac{1}{2};\frac{1}{2}^+ m_{1/2}\right\rangle =(-1)^{(J-M_J)}\begin{pmatrix} J & l & \frac{1}{2}\\ -M_J & m & m_{1/2}\end{pmatrix}\times\left(\alpha_1,l_1;\frac{1}{2};J^\pi\middle|\middle|T^{\alpha,l}\middle|\middle|\alpha_2;0;\frac{1}{2}^+\right). \quad (45)$$

In order to express double barred matrix element in terms of triple barred matrix elements, the following relations should be taken into account:

$$\left(\alpha_1,l_1;\frac{1}{2};J^\pi\middle|\middle|T^{\alpha,l}\middle|\middle|\alpha_2;0^+;\frac{1}{2};\frac{1}{2}^+\right) =\delta_{l_1l}\sqrt{2(2J+1)}(\alpha_1,l||T^{\alpha,l}||\alpha_2,0),$$

$$(N(a_1,b_1);l_1||T^{(a,b)l}||N'(a_2,b_2);l_2) =\sqrt{(2l_1+1)(2l_2+1)(2l+1)(2a_1+1)(2b_1+1)}\times\begin{Bmatrix} a_2 & b_2 & l_2\\ a & b & l\\ a_1 & b_1 & l_1\end{Bmatrix}\times(N(a_1,b_1)|||T^{(a,b)}|||N'(a_2,b_2)),$$

$$\begin{Bmatrix} 0 & 0 & 0\\ a & b & l\\ a_1 & b_1 & l_1\end{Bmatrix} =\delta_{a_1a}\delta_{b_1b}\delta_{l_1l}\frac{1}{\sqrt{(2l+1)(2a+1)(2b+1)}}. \quad (46)$$

Combining Eqs. (45) and (46) results in

$$\left|\left(N(a_1,b_1);l_1;\frac{1}{2};J^\pi\middle|\middle|T^{(a,b)l}\middle|\middle|N'(0,0);0^+;\frac{1}{2};\frac{1}{2}^+\right)\right|^2 =(2J+1)|(N(a,a)|||T^{(a,a)}|||N'(0,0))|^2. \quad (47)$$

D. Electric de-excitations of resonances to the nucleon

Equations (45)–(47) can be applied to calculate the ratio of, say, the electric dipole de-excitations $D_{13}(1520)\rightarrow p+\gamma$, and $S_{11}(1535)\rightarrow p+\gamma$. In this case $l_1^\pi=l^\pi=1^-$, $a_1=a=\frac{1}{2}$, $b_1=b=\frac{1}{2}$, and J^π takes the two values $J^\pi=\frac{3}{2}^-$ and $\frac{1}{2}^-$, respectively.

Substitution of the relevant quantum numbers into Eqs. (45) and (46), followed by a calculation of the ratio of the squared values of the $J^\pi=\frac{3}{2}^-$ and $J=\frac{1}{2}^-$ matrix elements, yields the theoretical ratio of the electric dipole widths of interest $\Gamma_\gamma^{D_{13}}$, and $\Gamma_\gamma^{S_{11}}$ of the respective $D_{13}(1520)$ and $S_{11}(1535)$ states as

$$\mathcal{R}^{\rm th}=\left(\frac{\Gamma_\gamma^{D_{13}}}{\Gamma_\gamma^{S_{11}}}\right)^{\rm th}=1. \quad (48)$$

In order to compare to data, one may approximate the dipole widths with the total γ widths and obtain their experimental values from the full widths and the branching ratios listed in Ref. [1]. The full widths of the $D_{13}(1520)$ and $S_{11}(1535)$ resonances are reported as 120 and 150 MeV, respectively. The $D_{13}(1520)\to p+\gamma$ branching ratio is reported as 0.46–0.56 %, while the $S_{11}(1535)$ takes values within the broader range from 0.15% to 0.35%. The theoretical prediction corresponds to a $S_{11}(1535)\to p+\gamma$ ratio of 0.35%, and thereby lies at the upper bound of the data range. This ratio is in fact J independent. This shows that a purely algebraic description is insufficient to reproduce the electromagnetic properties of the resonances in great detail. In that regard, further development of the model is needed with the aim to account for the internal diquark structure.

Remarkably, the internal structure of the diquark does not show up in the spectra, and seems to be irrelevant for the gross feature of the excitation modes. At the vertex level, however, it will gain more importance. The merit of the rovibron model is that there it can be treated as a correction rather than as a leading mechanism from the very beginning.

One can further compare gamma-widths of resonances carrying different internal O(3) quantum numbers l. This effect is easiest to study on the example of the natural parity resonances from the "missing" rovibron clusters. To be specific, we will compare the reduced probabilities for the following two transitions:

$$\left|4;0^+;(2,2);1^-;\frac{1}{2};\frac{3}{2}^-\ m_{3/2}\right\rangle \overset{T^{(2,2)1m}}{\rightarrow} \left|4;0^+;(0,0);0^+;\frac{1}{2};\frac{1}{2}^+\ m_{1/2}\right\rangle,$$

$$\left|4;0^+;(2,2);3^-;\frac{1}{2};\frac{5}{2}^-\ m_{5/2}\right\rangle \overset{T^{(2,2)3m}}{\rightarrow} \left|4;0^+;(0,0);0^+;\frac{1}{2};\frac{1}{2}^+\ m_{1/2}\right\rangle. \tag{49}$$

The relevant transition operator is

$$T^{(2,2)lm}=[T^{(1,1)}\otimes T^{(1,1)}]^{(2,2)lm}. \tag{50}$$

Here l can take the values $l=0,1,2,\ 3$, and 4. The first of the transitions in Eq. (49) is governed by the electric dipole operator $T^{(2,2)1m}$, while the second is controlled by the electric octupole $T^{(2,2)3m}$. We are going to calculate the ratio $\mathcal{R}_2$ of the quantities

$$\mathcal{R}_2=\frac{B\left(\alpha_1,\frac{5}{2}^-\to\alpha_2,\frac{1}{2}^+;T^{(2,2)1}\right)}{B\left(\alpha_1,\frac{3}{2}^-\to\alpha_2,\frac{1}{2}^+;T^{(2,2)3}\right)}. \tag{51}$$

Here

$$B\left(\alpha_1,\frac{3}{2}^-\to\alpha_2,\frac{1}{2}^+;T^{(2,2)1}\right)=\frac{1}{4}\left|\left(4;0^+;(1,1);1^-;\frac{1}{2};\frac{3}{2}^-\right|\left|[T^{(2,2)1}\otimes 11]^{(2,2)1}\right|\left|4;0^+;(0,0);0;\frac{1}{2};\frac{1}{2}^+\right)\right|^2,$$

$$B\left(\alpha_1,\frac{5}{2}^-\to\alpha_2,\frac{1}{2}^+;T^{(2,2)3}\right)=\frac{1}{6}\left|\left(4;0^+;(1,1);3^-;\frac{1}{2};\frac{5}{2}^-\right|\left|[T^{(2,2)3}\otimes 11]^{(2,2)3}\right|\left|4;0^+;(0,0);0;\frac{1}{2};\frac{1}{2}^-\right)\right|^2. \tag{52}$$

Usage of Eq. (47) yields equal reduced probabilities for both the dipole and octupole deexcitations and thereby the unit value for $\mathcal{R}_2$. Thus, within this early version of the rovibron model, a given RS cluster will have a common partial $(\gamma+p)$ decay width, that is insensitive to its O(3) spin content.

A more interesting situation occurs in the case of LAMPF clusters, such like $|5;0^-;(\frac{3}{2},\frac{3}{2});2^-;\frac{1}{2};\frac{3}{2}^-m\frac{3}{2}\rangle$. There one encounters a *suppression of electromagnetic transitions to the nucleon*. Indeed, in the rigorous case of an ideal O(4) symmetry, due to the unnatural parities of the nucleon resonances with masses above 1535 MeV (and the Δ excitations with masses above 1700 MeV), transitions of the type

$$\left|5;0^-;\left(\frac{3}{2},\frac{3}{2}\right);2^-;\frac{1}{2};\frac{3}{2}^-\ m_{3/2}\right\rangle \to\left|4;0^+;(0,0);0^+;\frac{1}{2};\frac{1}{2}^+\ m_{1/2}\right\rangle \tag{53}$$

cannot proceed either via electric $E\lambda$ or magnetic $M\lambda$ multipoles (to be presented elsewhere). In the less rigid scenario of a violated O(4) symmetry, mixing between states of same parity and total spins but different K's may occur. For example, the above unnatural parity spin-$\frac{3}{2}^-$ resonance from the $K=3$ multiplet may mix up with the spin $\frac{3}{2}^-$ of natural $(l=1^-)$ from the $K'=2$ multiplet

$$\left|J^\pi=\frac{3}{2}^- m_{3/2}\right\rangle$$
$$=\sqrt{1-\alpha^2}\left|5;0^-;\left(\frac{3}{2},\frac{3}{2}\right);2^-;\frac{1}{2};\frac{3}{2}^- m_{3/2}\right\rangle$$
$$+\alpha\left|4;0^+;(1,1),1^-;\frac{1}{2};\frac{3}{2}^- m_{3/2}\right\rangle. \quad (54)$$

For similar reasons, a mixing with $K'=4$ states can also take place. Within this mixing scheme, unnatural parity resonances can be excited electrically via their natural parity component. As long as the relevant transition operator for such transitions is $T^{((K'/2)(K'/2))lm}$, its matrix element between the nucleon and the resonance of interest will be proportional to the mixing parameter α. To be specific,

$$\left\langle J^\pi=\frac{3}{2}^- m_{3/2}\right|T^{(1,1)1m}\left|4;0^+;(0,0);0^+,\frac{1}{2};\frac{1}{2}^+ m_{1/2}\right\rangle$$
$$=\alpha\left\langle 4;0^+;(1,1);1^-;\frac{1}{2};\frac{3}{2}^- m_{3/2}\right|$$
$$\times T^{(1,1)1m}\left|4;(0,0);0^+;\frac{1}{2};\frac{1}{2}^+ m_{1/2}\right\rangle. \quad (55)$$

It is obvious from the last equation that electric excitations of the nucleon into the unnatural parity resonances will be suppressed by the factor of α^2. At the present early stage of development of the quark rovibron model, the mixing parameter α cannot be calculated but has to be considered as free and determined from data. A theoretical prediction for α would require more fundamental approach to the internal diquark dynamics. In case the O(4) symmetry is slightly violated, one may assume α to be same for all cluster inhabitants and perform some calculations as to what extent such states can be linked via electromagnetic transitions to the nucleon.

IV. SUMMARY AND OUTLOOK

The results of the present study can be summarized as follows.

(1) The present investigation communicated an idea of how Lorentz representations of the RS type can emerge as *fundamental* as well as *composite* free particles of definite mass and indefinite spin within the context of a relativistic space-time treatment of the three Dirac-fermion system. Though structureless RS particles have not been detected so far, the N and Δ spectra strongly indicate existence of *composite* RS fields.

(2) Excited light unflavored baryons preferably exist as multiresonance clusters that are described in terms of RS multiplets such as the (predominantly) observed LAMPF clusters $2_{2I,+}$, $4_{2I,-}$ and $6_{2I,-}$, and the "missing" clusters $3_{2I,+}$ and $5_{2I,+}$.

(3) The above RS clusters accommodate all the resonances observed so far in the πN decay channel [up to the $\Delta(1600)$ state]. The LAMPF data constitute, therefore, an almost accomplished excitation mode in its own rights, as only five resonances are "missing" for the completeness of this structure.

(4) We modeled composite RS fields within the framework of the quark rovibron model, and constructed a Hamiltonian that fits the masses of the LAMPF clusters.

(5) In using the Hamiltonian we predicted, from a different but the $SU(6)_{SF}\otimes O(3)_L$ perspective, the masses of two "missing" clusters of natural parity resonances, in support of the TJNAF "missing" resonance search program [20]. "Missing" resonances under debate in the literature, such as $P_{11}(1880)$ [21] and $P_{13}(1910)$ [22] could neatly fit into the $(2,2)\otimes[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$ RS cluster at 1935 MeV in Table I.

(6) We constructed electric transition operators, outlined the essentials of the O(4) Racah algebra, and calculated ratios of reduced probabilities of various resonance deexcitations to the nucleon. We found the internal structure of the diquark to be of minor importance for the gross features of the excitation modes. At the vertex level, however, a pointlike diquark was shown to be insufficient to account for differences in the branching ratios of resonances from same cluster. It is that place where the present early version of the QRVM model of baryon structure needs further improvements. Treating the internal structure of the diquark as a correction rather than as a leading mechanism from the very beginning is a major merit of the quark rovibron model.

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