Bimaximal neutrino mixing and small U_{e3} from Abelian flavor symmetry

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Atmospheric neutrino data strongly suggest a near-maximal ν_μ-ν_τ mixing and also solar neutrino data can be nicely explained by another near-maximal ν_e-ν_μ or ν_e-ν_τ mixing. We examine the possibility that this bimaximal mixing of atmospheric and solar neutrinos arises naturally, while keeping U_{e3} and $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ small enough, as a consequence of Abelian flavor symmetry. Two simple scenarios of Abelian flavor symmetry within the supersymmetric framework are considered to obtain the desired form of the neutrino mass matrix and the charged lepton mass matrix parametrized by the Cabibbo angle $\lambda\approx 0.2$. Future experiments at a neutrino factory measuring the size of U_{e3} and the sign of Δm^2_{32} could discriminate between those scenarios as they predict distinctive values of U_{e3} in connection with $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ and also with the order of the neutrino mass eigenvalues.

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I. INTRODUCTION

Atmospheric and solar neutrino experiments have suggested for a long time that neutrinos oscillate into different flavors. In particular, the super-Kamiokande data strongly indicate that the observed deficit of atmospheric muon neutrinos is due to the near-maximal $\nu_\mu \to \nu_\tau$ oscillation [1]. Solar neutrino data from the recent SNO experiment combined with those of Homestake, SAGE, GALLEX, and super-Kamiokande [2] provide also a strong observational basis for $\nu_e \to \nu_\mu$ or ν_τ oscillation [3]. Thus, the "standard" framework to accommodate the atmospheric and solar neutrino anomalies is to introduce small but nonzero masses of the three known neutrino species.

The low-energy effective Lagrangian relevant to the neutrino masses and mixing can be written as

$$\Delta\mathcal{L} = \overline{e_L} M^e e_R + g W^{-\mu} \overline{e_L}\gamma_\mu \nu_L + \frac{1}{2}(\overline{\nu_L})^c M^\nu \nu_L + \text{H.c.}, \tag{1}$$

where the charged lepton mass matrix M^e and the neutrino mass matrix M^ν are not diagonal in general in the weak interaction eigenbasis. Diagonalizing M^e and M^ν as

$$(U^e)^\dagger M^e V^e = D^e = \text{diag}(m_e, m_\mu, m_\tau),$$
$$(U^\nu)^T M^\nu U^\nu = D^\nu = \text{diag}(m_1, m_2, m_3), \tag{2}$$

one finds the effective Lagrangian written in terms of the mass eigenstate fermion fields:

$$\Delta\mathcal{L} = \overline{e_L} D^e e_R + g W^{-\mu} \overline{e_L}\gamma_\mu U^{\rm MNS} \nu_L + \frac{1}{2}(\overline{\nu_L})^c D^\nu \nu_L + \text{H.c.}, \tag{3}$$

where the Maki-Nakagawa-Sakata (MNS) lepton mixing matrix [4] is given by

$$U^{\rm MNS} = (U^e)^\dagger U^\nu. \tag{4}$$

The MNS mixing matrix can be parametrized as

$$U^{\rm MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13} \end{pmatrix}, \tag{5}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Within this parametrization, the mass-square differences for atmospheric and solar neutrino oscillation can be chosen to be

$$\Delta m^2_{\rm atm} = |\Delta m^2_{32}| = |m_3^2 - m_2^2|,$$

$$\Delta m^2_{\rm sol} = |\Delta m^2_{21}| = |m_2^2 - m_1^2|.$$

Then the corresponding mixing angles are given by

$$\theta_{\rm atm} = \theta_{23}, \quad \theta_{\rm sol} = \theta_{12}, \quad \theta_{\rm rea} = \theta_{13}, \tag{6}$$

where $\theta_{\rm rea}$ describes the neutrino oscillation $\nu_\mu \to \nu_e$ in reactor experiments such as the CHOOZ experiment.

The atmospheric neutrino data strongly suggest near-maximal $\nu_\mu \to \nu_\tau$ oscillation with

$$\Delta m_{32}^2 \sim 3\times 10^{-3}\ \ {\rm eV}^2,\ \ \sin^2 2\theta_{23} \sim 1. \tag{7}$$

As for the solar neutrino anomaly, the following four solutions are possible:

$$\begin{aligned}
&[\text{small mixing angle (SMA)}]:\ \Delta m_{21}^2 \sim 5.0\times 10^{-6}\ {\rm eV}^2,\\
&\qquad \sin^2 2\theta_{12} \sim 2.4\times 10^{-3},\\
&[\text{large mixing angle (LMA)}]:\ \Delta m_{21}^2 \sim 3.2\times 10^{-5}\ {\rm eV}^2,\\
&\qquad \sin^2 2\theta_{12} \sim 0.75,\\
&[\text{low mass (LOW)}]:\ \Delta m_{21}^2 \sim 1.0\times 10^{-7} {\rm eV}^2,\\
&\qquad \sin^2 2\theta_{12} \sim 0.96,\\
&[\text{vaccum oscillation (VAC)}]:\ \Delta m_{21}^2 \sim 8.6\times 10^{-10}\ {\rm eV}^2,\\
&\qquad \sin^2 2\theta_{12} \sim 0.96.
\end{aligned} \tag{8}$$

These values represent the *best-fit* points for each region and the LMA region extends to larger $\Delta m_{21}^2 \sim 2\times 10^{-4}$ [5]. Recent reports by Super-Kamiokande [6] and SNO [3] favor the solutions with large θ_{12}. On the other hand, the third mixing angle θ_{13} is constrained by the CHOOZ reactor experiment [7] as

$$U_{e3}^{\rm MNS} = \sin\theta_{13} \lesssim 0.2. \tag{9}$$

The above neutrino oscillation parameters indicate that the neutrino mass matrix has the same nontrivial flavor structure as the quark and charged lepton mass matrices. (It has been noted that the near-maximal atmospheric neutrino oscillation and the LMA solar neutrino oscillation can be achieved from an anarchical neutrino mass matrix if one accepts a certain degree of accidental cancellation [8].) One of the most popular schemes to explain the hierarchical quark masses and mixing angles is the Frogatt-Nielsen mechanism with a spontaneously broken Abelian flavor symmetry [9–13]. In this scheme, flavor symmetry is assumed to be broken by $\langle\phi\rangle/M_* \simeq \lambda$ ($\equiv$ Cabibbo angle $\simeq 0.2$) where ϕ is a symmetry-breaking scalar field and M_* denotes the fundamental scale of the model, e.g., the Planck scale or the string scale. Then all Yukawa couplings are suppressed by an appropriate power of λ as determined by the flavor charge of the corresponding operator, thereby leading to hierarchical fermion masses and mixing angles. It is then quite natural to expect that the nontrivial flavor structure of neutrino mass matrix can be understood also by the Abelian flavor symmetry explaining the hierarchical quark and charged lepton masses.

In cases of large solar neutrino mixing, i.e. in the LMA, LOW, and VAC solutions, we have two *near-maximal* mixing angles θ_{12}, θ_{23} and one *small* mixing angle θ_{13}, as well as the *small* mass-square ratio $\Delta m_{21}^2/\Delta m_{32}^2$. It may turn out in future neutrino experiments that θ_{13} is significantly smaller than the current bound (9), and then the hierarchy between θ_{23} and θ_{13} will become more severe. In this paper, we wish to examine the possibility that small θ_{13} and $\Delta m_{21}^2/\Delta m_{32}^2$ naturally arise together with near bimaximal θ_{23} and θ_{12} as a consequence of Abelian flavor symmetry. Our basic assumption is that the flavor symmetry is broken by order parameters which have the Cabibbo angle size λ. Since the simplest scheme with single anomalous $U(1)$ flavor symmetry and single symmetry-breaking parameter cannot produce the desired form of M^e and M^ν, we need to extend the scheme. In this regard, we consider two simple extensions, scenarios A and B, which are assumed to be realized in supersymmetric models. Flavor symmetry of scenario A is a nonanomalous $U(1)_X$, so is broken by two scalar fields with opposite $U(1)_X$ charges $x=\pm 1$. In scenario B, flavor symmetry is extended to $U(1)_X \times U(1)_{X'}$, where $U(1)_X$ is anomalous while $U(1)_{X'}$ is nonanomalous. It is then assumed to be broken by two scalar fields with the flavor charges $(x,x')=(-1,-1)$ and (0,1), for which the symmetry-breaking parameters naturally have the Cabibbo angle size.

Depending upon the way that it is generated, M^ν can be determined either by the weak scale selection rule involving only the flavor charges of the weak scale fields, or by a more involved selection rule. For instance, in seesaw models with heavy singlet neutrinos N_i [14], the selection rule for M^ν involves the flavor charges of N_i as well as those of the weak scale fields. Sometimes this feature enables us to build a greater variety of models, although in most cases it is possible to find the flavor charges of N_i for which M^ν is determined simply by the weak scale selection rule.

Measuring the mixing angle θ_{13} is one of the main targets of the proposed neutrino factory, which can achieve precision down to $\theta_{13} \sim 10^{-2}$ [15]. This would allow us to distinguish several different $\theta_{13} \simeq \lambda^n$ by future experiments. A nonzero $\theta_{13} \simeq \lambda$ or λ^2 would give a detectable $\nu_e \leftrightarrow \nu_\mu$ transition. On the other hand, $\theta_{13} \simeq \lambda^3$ may or may not be detectable, and $\theta_{13} \lesssim \lambda^4$ would give an undetectably small $\nu_e \leftrightarrow \nu_\mu$ transition. In this sense, it is meaningful to explore the possibility that θ_{13} is as small as λ^3 or even less. CP-violating effects could also be probed if the rephasing invariant

$$J_{CP} = \frac{1}{4} c_{13}^2 s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin\delta$$

is sizable and the LMA solution of the solar neutrino problem is realized [15]. Note that the CP-violating phase is *not* controlled by Abelian flavor symmetry, so $\sin\delta$ is generically of order 1 in our scheme. Another important result expected in the future neutrino experiments is the determination of the sign of Δm_{32}^2. Once $\nu_e \to \nu_\mu$ oscillations are established, matter effects can be measured to discriminate the sign of Δm_{32}^2 [15]. That is, one would be able to determine whether neutrino masses follow the normal ($\Delta m_{32}^2 > 0$) or inverted ($\Delta m_{32}^2 < 0$) mass hierarchy. As we will see, the information

TABLE I. Possible ranges of θ_{13} for each of the scenarios A and B, neutrino mass matrix of classes (ii) and (iii), and the LMA, LOW, and VAC solar neutrino oscillations. Note that class (i) cannot be obtained within our framework. classes (ii) and (iii) are a pseudo-Dirac-type neutrino mass matrix with $\Delta m_{32}^2>0$ and $\Delta m_{32}^2<0$, respectively.

	Solar ν oscillation	A-ii	A-iii	B-ii	B-iii
	LMA	$\lambda^2-\lambda$	λ	λ	λ^2
WSSR	LOW	$\lambda^3-\lambda$	$\lambda^3-\lambda$	λ^2	λ^6
	VAC	$\lambda^4-\lambda^2$	$\lambda^4-\lambda^2$	λ^3	λ^9
	LMA	$\lambda^2-\lambda$	$\lambda^6-\lambda^2$	λ	λ^2
Seesaw	LOW	$\lambda^3-\lambda$	$\lambda^7-\lambda$	λ^2	λ^6
	VAC	$\lambda^4-\lambda$	$\lambda^4-\lambda^2$	λ^3	λ^9

on θ_{13} and/or Δm_{32}^2 together with the solar neutrino solution will provide meaningful constraints on models of Abelian flavor symmetry.

The organization of this paper is as follows. In the next section, we discuss some aspects of Abelian flavor symmetry and the associated selection rule which are relevant to our subsequent discussions. In Sec. III, we discuss the textures of M^e and M^ν which would give small θ_{13} and $\Delta m_{21}^2/\Delta m_{32}^2$ while keeping the θ_{23} and θ_{12} near-bimaximal. We focus on three types of M^ν: class (i) with $M_{33}^\nu \gg M_{11}^\nu \simeq M_{12}^\nu \simeq M_{22}^\nu$ so $m_1\simeq m_2 \ll m_3$, pseudo-Dirac type class (ii) with $M_{33}^\nu \gtrsim M_{12}^\nu \gg M_{11}^\nu, M_{22}^\nu$ so the normal mass hierarchy $m_1\simeq m_2 \lesssim m_3$, and pseudo-Dirac type class (iii) with $M_{12}^\nu \gg M_{11}^\nu, M_{22}^\nu, M_{33}^\nu$ so the inverted mass hierarchy $m_1 \simeq m_2 \gg m_3$. In Sec. IV, we discuss examples of Abelian flavor symmetry for scenarios A and B, leading to the mass textures discussed in Sec. III under the assumption that M^ν is determined by the weak scale selection rule. We first list examples with the largest possible θ_{13} for each of the three types of mass textures, i.e., classes (i)–(iii), and the three types of solar neutrino oscillations with large θ_{12}, i.e., LMA, LOW, and VAC. We then explore the possibility of having a smaller θ_{13}. Under the condition that the lepton doublets L_i have integer-valued flavor charges $|l_i|<10$ when the flavor charges of symmetry-breaking fields are normalized to be ± 1, we find the possible range of θ_{13} for each type of mass textures and solar neutrino oscillations and the results are summarized in Table I. In Sec. V, we discuss seesaw models containing singlet neutrinos N_i with integer-valued flavor charges $|n_i|<10$ and also with $|l_i|<10$ to find the possible range of θ_{13}. Some seesaw models are explicitly presented as examples producing M^ν, which cannot be obtained under the weak selection rule. The results on the range of θ_{13} in seesaw models are summarized also in Table I. Section VI is devoted to the conclusion.

II. FROGATT-NIELSEN MECHANISM FOR ABELIAN FLAVOR SYMMETRY

The simplest framework to implement the Frogatt-Nielsen mechanism with Abelian flavor symmetry is to introduce single anomalous $U(1)_X$ symmetry, which is assumed to be broken by the single symmetry-breaking scalar field $\langle\phi\rangle/M_* \simeq \lambda$. This framework is best motivated from compactified heterotic string theory with anomalous $U(1)$. In such theory, the scalar potential includes the contribution from the string-loop–induced Fayet-Illiopoulos D term, so

$$V=\frac{g_X^2}{2}(\xi^2-|\phi|^2)^2,$$

where $\xi^2=\mathrm{tr}(X)M_*^2/96\pi^2$ for the string scale M_* and all other $U(1)_X$-charged scalar fields are set to zero for simplicity. This framework is particularly attractive since the symmetry-breaking parameter naturally has the Cabibbo angle size:

$$\frac{\langle\phi\rangle}{M_*}=\left(\frac{\mathrm{tr}(X)}{96\pi^2}\right)^{1/2}\simeq\lambda.$$

Then the generic $U(1)_X$-invariant superpotential is given by

$$W=\sum_i \left(\frac{\phi}{M_*}\right)^{x_i} O_i=\sum_i \lambda^{x_i} O_i \quad (x_i \geqslant 0), \tag{10}$$

where the $U(1)_X$ charges of ϕ and O_i are -1 and x_i, respectively. With this selection rule, we can control the size of Yukawa couplings by assigning $U(1)_X$ charge appropriately to the low-energy fields. One important consequence of this selection rule is that the operator O_i with negative $U(1)_X$ charge is forbidden due to the holomorphicity. This point is very useful and enables us to build the nontrivial Yukawa matrix.

It is well known that realistic quark and charged lepton mass matrices can be easily obtained within the framework of single anomalous $U(1)_X$ and single symmetry-breaking parameter [12]. However, this framework cannot provide the textures of M^e and M^ν, which will be discussed in the next section as producing bimaximal θ_{23}, θ_{12} together with small $\theta_{13}, \Delta m_{21}^2/\Delta m_{32}^2$. One simple modification of the model which would provide the desired forms of M^e and M^ν is to assume that $U(1)_X$ is *nonanomalous*, thus it is broken by two symmetry-breaking scalar fields ϕ_1, ϕ_2 with opposite $U(1)_X$ charges ± 1. The D-term scalar potential is then given by

$$V=\frac{g_X^2}{2}(|\phi_1|^2-|\phi_2|^2)^2$$

which ensures

$$\langle\phi_1\rangle/M_*=\langle\phi_2\rangle/M_*.$$

However there is no good reason in this framework that $\langle\phi_1\rangle/M_*$ has the Cabibbo angle size. A simple way to avoid this difficulty is to have one anomalous $U(1)_X$ and another nonanomalous $U(1)_{X'}$ which are broken by two scalar fields ϕ_1 and ϕ_2 having the flavor charges $(-1,-1)$ and (0,1). In this case, the D-term potential of ϕ_1 and ϕ_2 is given by

$$V=\frac{g_X^2}{2}(\xi^2-|\phi_1|^2)^2+\frac{g_{X'}^2}{2}(|\phi_2|^2-|\phi_1|^2)^2, \tag{11}$$

which guarantees that

$$\frac{\langle\phi_1\rangle}{M_*}=\frac{\langle\phi_2\rangle}{M_*}=\frac{\xi}{M_*}\simeq\lambda. \tag{12}$$

In this paper, we will explore the possibility of obtaining the desired textures of M^e and M^ν within the following two scenarios of Abelian flavor symmetry.

Scenario A. Single nonanomalous $U(1)_X$ with two symmetry-breaking parameters $\langle\phi_1\rangle/M_*=\langle\phi_2\rangle/M_*\simeq\lambda$ with $U(1)_X$ charges $x=\pm1$. The selection rule in this scenario is given by

$$W=\sum_i \lambda^{|x_i|}O_i, \tag{13}$$

where x_i denotes the $U(1)_X$ charge of O_i.

Scenario B. $U(1)_X\times U(1)_{X'}$ with two symmetry-breaking parameters $\langle\phi_1\rangle/M_*=\langle\phi_2\rangle/M_*\simeq\lambda$ with flavor charges $(x,x')=(-1,-1)$ and $(0,1)$. The resulting selection rule is given by

$$W=\sum_i \left(\frac{\phi_2}{M_*}\right)^{x_i-x_i'}\left(\frac{\phi_1}{M_*}\right)^{x_i}O_i=\sum_i c_iO_i, \tag{14}$$

where

$$c_i=\begin{cases}0 & \text{if } x_i<0 \text{ or } x_i<x_i',\\ \lambda^{2x_i-x_i'} & \text{otherwise,}\end{cases} \tag{15}$$

for (x_i,x_i') denoting the $U(1)_X\times U(1)_{X'}$ charge of O_i.

The above selection rules are derived at energy scales just below the flavor symmetry-breaking scale M_X. If some heavy fields have masses depending upon the symmetry-breaking order parameter, the low-energy effective couplings of light fields induced by the exchange of such heavy fields may not obey the selection rule as determined by the flavor charges of light fields alone. This can happen for instance in singlet seesaw models containing heavy singlet neutrinos with flavor-dependent masses.

Usually, the smallness of neutrino masses is explained by assuming that neutrino masses are induced by the exchange of superheavy particles. At the weak scale, neutrino masses are described by $d=5$ operators in the effective superpotential:

$$\Delta W_{\rm eff}=\frac{M_{ij}^\nu}{\langle H_2\rangle^2}L_iH_2L_jH_2 \tag{16}$$

where L_i $(i=1,2,3)$ and H_2 denote the lepton and Higgs superfields, respectively. In singlet seesaw models, exchanged heavy particles are the singlet neutrinos N_i having the superpotential couplings

$$\Delta W=\frac{M_{ij}^D}{\langle H_2\rangle}H_2L_iN_j+M_{ij}^MN_iN_j+\text{H.c.}, \tag{17}$$

which lead to the well-known seesaw formula

$$M^\nu=M^D(M^M)^{-1}(M^D)^T. \tag{18}$$

Although M^M and M^D obey the selection rule as determined by the flavor charges of the corresponding operators, the resulting M^ν may *not* obey the selection rule as determined by the flavor charges of the effective operator $L_iH_2L_jH_2$. In most cases, there exist some sets of the flavor charges of N_i for which M^ν can be determined simply by applying the selection rule to the weak scale effective operator $L_iH_2L_jH_2$, which we call the weak scale selection rule (WSSR). However it is also possible that M^ν does not obey the WSSR, thus it can be determined only through the seesaw formula (18).

This complication does not occur in triplet seesaw models in which M^ν is generated by the exchange of superheavy $SU(2)_L$ triplet Higgs fields T_1,T_2 [16]. Such models include the superpotential couplings

$$\Delta W=h_{ij}T_1L_iL_j+h_0T_2H_2H_2+M_TT_1T_2,$$

which give

$$M_{ij}^\nu=h_0h_{ij}\frac{\langle H_2\rangle^2}{M_T}. \tag{19}$$

In this case, M^ν can be determined always by the WSSR, which is applied to the effective superpotential (16) at the weak scale.

Before closing this section, we note that physical Yukawa couplings can be affected by nonholomorphic flavor-mixing terms in the Kähler potential, e.g., $\Phi_i^*\Phi_j(\phi/M_*)^{k_{ij}}$ [13]. However, it turns out that such Kähler mixing terms give negligible corrections in all models discussed in this paper.

III. TEXTURES FOR BIMAXIMAL MIXING WITH SMALL U_{e3}

There have been many discussions in the literature about the possibility of bimaximal θ_{23} and θ_{12} [17]. Most of them rely on the assumption that M^e is (approximately) diagonal so that U^e is an identity matrix. However, comparing Eq. (4) and Eq. (5) gives another interesting possibility. If U^e and U^ν are given by

$$U^e\simeq\begin{pmatrix}1&0&0\\0&\frac{1}{\sqrt2}&\frac{1}{\sqrt2}\\0&-\frac{1}{\sqrt2}&\frac{1}{\sqrt2}\end{pmatrix},\quad U^\nu\simeq\begin{pmatrix}\frac{1}{\sqrt2}&\frac{1}{\sqrt2}&0\\-\frac{1}{\sqrt2}&\frac{1}{\sqrt2}&0\\0&0&1\end{pmatrix}, \tag{20}$$

the resulting $U^{\rm MNS}$ *naturally* has a small θ_{13} together with bimaximal $\theta_{23}\sim\theta_{12}\sim\pi/4$. In this section, we categorize what textures of M^e and M^ν can realize this idea while giving the correct (small) value of $\Delta m_{21}^2/\Delta m_{32}^2$. Recall that our goal is to realize these textures within the framework of Abelian flavor symmetry in which all mass matrix elements are expressed in powers of the Cabibbo angle $\lambda\simeq0.2$. Any matrix element not shown explicitly should be understood to be small enough not to disturb the basic feature of the texture.

The charged lepton mass matrix that gives U^e of Eq. (20) is given by

$$M^e_{i3}=m_\tau\begin{pmatrix}\lambda^n\\ 1\\ 1\end{pmatrix}, \tag{21}$$

where $n\geqslant 1$ and the first and second column should be smaller than the third one. Within the framework of Abelian flavor symmetry, there is no way to get U^e of Eq. (20) other than this form of M^e. However, for the neutrino mass matrix, there are several different ways to get U^ν of Eq. (20). Among them, the following texture with a pseudo-Dirac 2×2 block is of particular interest:

$$M^\nu=m_{\max}\begin{pmatrix}\lambda^n & \lambda^l & \\ \lambda^l & \lambda^m & \\ & & \lambda^k\end{pmatrix}, \tag{22}$$

where $m_{\max}$ denotes the largest mass eigenvalue, $l\geqslant 0$, $k\geqslant 0$ and $n,m>l$. For $k=0$, this M^ν gives the normal mass hierarchy $m_3\gtrsim m_2,m_1$, while $k>l=0$ gives the inverted hierarchy $m_2\simeq m_1\gg m_3$. The mass eigenvalues of the above pseudo-Dirac M^ν give

$$\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}\sim\lambda^{q+l}, \tag{23}$$

where $q\equiv\min(n,m)$. The size of this ratio can be read off from the oscillation data of Eqs. (7) and (8), implying

$$\text{LMA}: q+l=2-4,$$
$$\text{LOW}: q+l=6-7, \tag{24}$$
$$\text{VAC}: q+1=9-10.$$

Including the case of plain large mixing, textures of M^ν which would give U^ν of Eq. (20) together with the right value $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ can be categorized as follows.

Class (i). Plain large mixing with $n\geqslant 1$, which gives $m_1\simeq m_2\ll m_3$,

$$M^\nu\simeq m_3\begin{pmatrix}\lambda^n & \lambda^n & \\ \lambda^n & \lambda^n & \\ & & 1\end{pmatrix}\ \text{or}\ M^\nu\simeq m_3\begin{pmatrix} & & \lambda^n\\ & & \lambda^n\\ \lambda^n & \lambda^n & 1\end{pmatrix}. \tag{25}$$

Class (ii). Pseudo-Dirac type with $n,m>l\geqslant 0$, which gives the normal mass hierarchy $m_1\simeq m_2\lesssim m_3$,

$$M^\nu\simeq m_3\begin{pmatrix}\lambda^n & \lambda^l & \\ \lambda^l & \lambda^m & \\ & & 1\end{pmatrix}. \tag{26}$$

Class (iii). Pseudo-Dirac type with the inverted mass hierarchy $m_1\simeq m_2\gg m_3$,

$$M^\nu\simeq m_2\begin{pmatrix}\lambda^n & 1 & \\ 1 & \lambda^m & \\ & & \lambda^l\end{pmatrix}. \tag{27}$$

In all the cases, we will scan the possible charge assignments to find the allowed ranges of θ_{13}, which may turn out to be within the reach of future neutrino experiments and can give a large CP-violating quantity J_{CP}. Note that classes (i) and (ii) give $\Delta m^2_{32}>0$ and class (iii) gives $\Delta m^2_{32}<0$.

IV. MODELS OBEYING THE WEAK SCALE SELECTION RULE

In this section, we discuss the models in which the selection rule can be applied to the *weak scale* effective superpotential:

$$W_{\rm eff}=\frac{M^e_{ij}}{\langle H_1\rangle}H_1L_iE^c_j+\frac{M^\nu_{ij}}{\langle H_2\rangle^2}L_iH_2L_jH_2,$$

where L_i,E^c_i and H_1,H_2 denote the lepton doublets, antilepton singlets, and the two Higgs doublets, respectively. As was noted in Sec. II, this weak scale selection rule may not be valid in some singlet seesaw models, which will be discussed in the next section. Here we consider only the models with integer-valued flavor charges when the flavor charges of the symmetry-breaking fields are normalized to be ± 1. We further limit ourselves to the cases that L_i have the flavor charges $|l_i|<10$. On the other hand, E^c_i are allowed to have larger flavor charges, otherwise most of the LOW and VAC models presented below cannot be obtained.

Scenario A. Let us first show that the neutrino mass matrix of class (i) *cannot* be obtained under the weak scale selection rule in scenario A. To proceed, let l_i,e_i,h_1,h_2 denote the $U(1)_X$ charges of the superfields L_i,E^c_i,H_1,H_2. Then the charged lepton mass matrix (21) requires

$$|l_1+e_3+h_1|\neq|l_2+e_3+h_1|=|l_3+e_3+h_1|$$

while the neutrino mass matrix (25) requires

$$|l_1+a|=|l_2+a|\neq|l_3+a|,$$

where a is a certain combination of $U(1)_X$ charges. These conditions inevitably lead to M^ν which *cannot* give either a correct value of $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ or a small θ_{13}. It appears also difficult to find a desirable class (i) model even in the framework of singlet seesaw models.

On the other hand, it is rather easy to get a pseudo-Dirac M^ν of class (ii) under the weak scale selection rule. Let us first list examples with the largest possible θ_{13} for each of the LMA, LOW, and VAC solutions. Considering the charge assignments

$$\text{LMA:}\ \ l_i=(1,-2,0),\ e_i=(5,5,1),\ h_1=h_2=0, \tag{28}$$

$$\text{LOW:}\ \ l_i=(4,-7,-1),\ e_i=(-12,12,4),$$

$$h_1=h_2=0,$$

$$\text{VAC:}\quad l_i=(8,-5,1),\ e_i=(-16,10,2),\ h_1=h_2=0,$$

we get the following mass textures:

$$\text{LMA:}\ \frac{M^\nu}{m_3}\simeq\begin{pmatrix}\lambda^2 & \lambda & \lambda\\ \lambda & \lambda^4 & \lambda^2\\ \lambda & \lambda^2 & 1\end{pmatrix},\quad \frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^5 & \lambda^5 & \lambda\\ \lambda^2 & \lambda^2 & 1\\ \lambda^4 & \lambda^4 & 1\end{pmatrix},$$

$$\text{LOW:}\ \frac{M^\nu}{m_3}\simeq\begin{pmatrix}\lambda^6 & \lambda & \lambda\\ \lambda & \lambda^{12} & \lambda^6\\ \lambda & \lambda^6 & 1\end{pmatrix},$$

$$\frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^5 & \lambda^{13} & \lambda^5\\ \lambda^{16} & \lambda^2 & 1\\ \lambda^{10} & \lambda^8 & 1\end{pmatrix}, \tag{29}$$

$$\text{VAC:}\ \frac{M^\nu}{m_3}\simeq\begin{pmatrix}\lambda^{14} & \lambda & \lambda^7\\ \lambda & \lambda^8 & \lambda^2\\ \lambda^7 & \lambda^2 & 1\end{pmatrix},$$

$$\frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^5 & \lambda^{15} & \lambda^7\\ \lambda^{18} & \lambda^2 & 1\\ \lambda^{12} & \lambda^8 & 1\end{pmatrix},$$

for which

$$(\theta_{13},\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm})\simeq(\lambda,\lambda^3)_{\rm LMA},(\lambda,\lambda^7)_{\rm LOW},(\lambda^2,\lambda^9)_{\rm VAC}.$$

For class (iii), the following charge assignments are possible:

$$\text{LMA:}\quad l_i=(2,-3,1),\ e_i=(-9,7,1),\ h_1=h_2=0,$$

$$\text{LOW:}\quad l_i=(5,-4,2),\ e_i=(-13,9,1),\ h_1=h_2=0, \tag{30}$$

$$\text{VAC:}\quad l_i=(5,-5,-3),\ e_i=(-11,8,4),\ h_1=h_2=0,$$

to produce the mass textures

$$\text{LMA:}\ \frac{M^\nu}{m_2}\simeq\begin{pmatrix}\lambda^3 & 1 & \lambda^2\\ 1 & \lambda^5 & \lambda\\ \lambda^2 & \lambda & \lambda\end{pmatrix},\quad \frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^5 & \lambda^7 & \lambda\\ \lambda^{10} & \lambda^2 & 1\\ \lambda^6 & \lambda^6 & 1\end{pmatrix},$$

$$\text{LOW:}\ \frac{M^\nu}{m_2}\simeq\begin{pmatrix}\lambda^9 & 1 & \lambda^6\\ 1 & \lambda^7 & \lambda\\ \lambda^6 & \lambda & \lambda^3\end{pmatrix}, \tag{31}$$

$$\frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^5 & \lambda^{11} & \lambda^3\\ \lambda^{14} & \lambda^2 & 1\\ \lambda^8 & \lambda^8 & 1\end{pmatrix},$$

$$\text{VAC:}\ \frac{M^\nu}{m_2}\simeq\begin{pmatrix}\lambda^{10} & 1 & \lambda^2\\ 1 & \lambda^{10} & \lambda^8\\ \lambda^2 & \lambda^8 & \lambda^6\end{pmatrix},$$

$$\frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^5 & \lambda^{12} & \lambda^8\\ \lambda^{15} & \lambda^2 & 1\\ \lambda^{13} & \lambda^4 & 1\end{pmatrix},$$

which give

$$(\theta_{13},\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm})\simeq(\lambda,\lambda^3)_{\rm LMA},(\lambda,\lambda^7)_{\rm LOW},(\lambda^2,\lambda^{10})_{\rm VAC}.$$

The value of $\theta_{13}\simeq\lambda$ is perhaps the most interesting possibility since it is just below the current bound (9). For the LMA and LOW, we could easily get $\theta_{13}\simeq\lambda$ under the WSSR for both classes of models. However, for the VAC solution θ_{13} can be *only* as large as λ^2 under the WSSR. As we will see in the next section, $\theta_{13}\simeq\lambda$ can be obtained for the VAC in the framework of singlet seesaw models for class (ii).

Since it may be possible to determine θ_{13} with a precision of order 10^{-2}, it is worthwhile to explore a smaller θ_{13} including $\theta_{13}\lesssim\lambda^3$. In this regard, the LMA in scenario A has a special property. Class (ii) LMA models can have only $\theta_{13}\simeq\lambda$ or λ^2, while class (iii) LMA models can have only $\theta_{13}\simeq\lambda$. Actually the LMA model shown in Eq. (30) is the unique one which gives the LMA solution with inverted mass hierarchy. A class (ii) LMA example with $\theta_{13}\simeq\lambda^2$ is given by

$$l_i=(2,-2,0),\ e_i=(5,5,1),\ h_1=h_2=0, \tag{32}$$

which lead to

$$\frac{M^\nu}{m_3}\simeq\begin{pmatrix}\lambda^4 & 1 & \lambda^2\\ 1 & \lambda^4 & \lambda^2\\ \lambda^2 & \lambda^2 & 1\end{pmatrix},\quad \frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^6 & \lambda^6 & \lambda^2\\ \lambda^2 & \lambda^2 & 1\\ \lambda^4 & \lambda^4 & 1\end{pmatrix}. \tag{33}$$

Note that this form of mass matrix can give both the normal mass hierarchy ($m_1\simeq m_2\lesssim m_3$) or the inverted mass hierarchy ($m_1\simeq m_2\gtrsim m_3$) depending on the precise values of M^ν_{12} and M^ν_{33}, both of which are of order unity.

The LOW and VAC solutions in scenario A can have smaller $\theta_{13}\lesssim\lambda^3$. Here are such examples:

$$\text{LOW, (ii):}\quad l_i=(3,-4,0),\ e_i=(-10,8,2),\ h_1=h_2=0,$$

$$\text{VAC, (ii):}\quad l_i=(-6,4,0),\ e_i=(13,-8,-2),$$

$$h_1=h_2=0, \tag{34}$$

$$\text{LOW, (iii):}\quad l_i=(4,-5,1),\ e_i=(-12,10,2)\ h_1=h_2=0,$$

$$\text{VAC, (iii):}\quad l_i=(5,-5,-1),\ e_i=(-12,9,3)\ h_1=h_2=0,$$

which produce the mass textures

$$\text{LOW, (ii):}\quad \frac{M^\nu}{m_3} \simeq \begin{pmatrix} \lambda^6 & \lambda^1 & \lambda^3 \\ \lambda^1 & \lambda^8 & \lambda^4 \\ \lambda^3 & \lambda^4 & 1 \end{pmatrix},$$

$$\frac{M^e}{m_\tau} \propto \begin{pmatrix} \lambda^5 & \lambda^9 & \lambda^3 \\ \lambda^{12} & \lambda^2 & 1 \\ \lambda^8 & \lambda^6 & 1 \end{pmatrix},$$

$$\text{VAC, (ii):}\quad \frac{M^\nu}{m_3} \simeq \begin{pmatrix} \lambda^{12} & \lambda^2 & \lambda^6 \\ \lambda^2 & \lambda^8 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix},$$

$$\frac{M^e}{m_\tau} \simeq \begin{pmatrix} \lambda^5 & \lambda^{12} & \lambda^6 \\ \lambda^{15} & \lambda^2 & 1 \\ \lambda^{11} & \lambda^6 & 1 \end{pmatrix},$$

$$\text{LOW, (iii):}\quad \frac{M^\nu}{m_3} \simeq \begin{pmatrix} \lambda^7 & 1 & \lambda^4 \\ 1 & \lambda^9 & \lambda^3 \\ \lambda^4 & \lambda^3 & \lambda \end{pmatrix},$$

$$\frac{M^e}{m_\tau} \simeq \begin{pmatrix} \lambda^5 & \lambda^{11} & \lambda^3 \\ \lambda^{14} & \lambda^2 & 1 \\ \lambda^8 & \lambda^8 & 1 \end{pmatrix},$$

$$\text{VAC, (iii):}\quad \frac{M^\nu}{m_3} \simeq \begin{pmatrix} \lambda^{10} & 1 & \lambda^4 \\ 1 & \lambda^{10} & \lambda^6 \\ \lambda^4 & \lambda^6 & \lambda^2 \end{pmatrix},$$

$$\frac{M^e}{m_\tau} \simeq \begin{pmatrix} \lambda^5 & \lambda^{12} & \lambda^6 \\ \lambda^{15} & \lambda^2 & 1 \\ \lambda^{11} & \lambda^6 & 1 \end{pmatrix},$$

for which

$$(\theta_{13}, \Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}) = (\lambda^3, \lambda^7)_{\rm LOW,II}, (\lambda^4, \lambda^{10})_{\rm VAC,II}, \quad (\lambda^3, \lambda^7)_{\rm LOW,III}, (\lambda^4, \lambda^{10})_{\rm VAC,III}.$$

The examples shown in this section are the models giving either the largest or the smallest value of θ_{13} under the limitation $|l_i| < 10$. The reason for the occurrence of these bounds on θ_{13} is that M^ν_{13}, M^ν_{23} and M^ν_{11}, M^ν_{22} are closely related by the $U_X(1)$ charge of L_2, L_3 fields. It is thus difficult to suppress (enhance) M^ν_{13}, M^ν_{23} arbitrarily to get smaller (larger) θ_{13} while keeping the right size of M^ν_{11}, M^ν_{22} to obtain the right size of $m^2_{\rm sol}/m^2_{\rm atm}$ for each of the solar neutrino oscillations. This explains also that the VAC allows smaller θ_{13} ($\lambda^2 \sim \lambda^4$) than the LMA or LOW ($\lambda \sim \lambda^3$). The allowed ranges of θ_{13} are summarized in Table I for the class (ii) and (iii) mass textures and the LMA, LOW, and VAC solar neutrino oscillations.

Scenario B. For this scenario, we use the notation that $\Phi_i(x,x')$ denotes the superfield Φ with $U(1)_X \times U(1)_{X'}$ charge (x,x'). Let us first note that we need

$$l_1^{\rm eff} \neq l_2^{\rm eff} = l_3^{\rm eff}, \tag{35}$$

where $l_i^{\rm eff} = 2l_i - l_i'$ to get the desired form of M^e. Then, it is easy to see that class (I) *cannot* be realized as it requires $l_1^{\rm eff} = l_2^{\rm eff} \neq l_3^{\rm eff}$.

On the other hand, the condition (35) can be reconciled with the pseudo-Dirac structure of the class (ii) neutrino mass matrix by imposing holomorphic zeros. This leads us to get the following texture:

$$\frac{M^\nu}{m_3} \simeq \begin{pmatrix} \lambda^{2x} & \lambda^x & \lambda^x \\ \lambda^x & 0 & 0 \\ \lambda^x & 0 & 1 \end{pmatrix}, \tag{36}$$

where $x = l_1^{\rm eff} - l_2^{\rm eff} = l_1^{\rm eff} - l_3^{\rm eff}$. In this texture, M^ν_{22} and M^ν_{23} are forbidden due to the holomorphicity, and the sizes of nonzero elements are entirely determined by the condition Eq. (35). This texture exhibits an interesting correlation of θ_{13} with the mass-squared difference ratio as follows:

$$\theta_{13} \sim \lambda^x, \quad \Delta m^2_{\rm sol}/\Delta m^2_{\rm atm} \sim \lambda^{3x}. \tag{37}$$

Hence $x = 1$, 2, or 3 is required for the LMA, LOW or VAC, respectively, in order to give correct square mass difference (24). We then have the following specific predictions:

$$(\theta_{13}, \Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}) \simeq (\lambda, \lambda^3)_{\rm LMA}, (\lambda^2, \lambda^6)_{\rm LOW}, (\lambda^3, \lambda^9)_{\rm VAC}.$$

Explicit charge assignments realizing the texture (36) are given by

$$\begin{aligned} \text{LMA:}\ & L_1(0,-1),\ L_2(1,2),\ L_3(0,0), \\ & E_1(3,0), E_2(2,0), E_3(1,0), \\ \text{LOW:}\ & L_1(0,-2),\ L_2(1,2),\ L_3(0,0), \\ & E_1(2,-1), E_2(2,0), E_3(1,0), \\ \text{VAC:}\ & L_1(0,-3),\ L_2(1,2),\ L_3(0,0), \\ & E_1(2,0),\ E_2(2,0),\ E_3(1,0), \end{aligned} \tag{38}$$

producing

$$\text{LMA:}\ \frac{M^\nu}{m_3} \simeq \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 0 & 0 \\ \lambda & 0 & 1 \end{pmatrix},\ \frac{M^e}{m_\tau} \simeq \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda^1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \tag{39}$$

$$\text{LOW:}\ \frac{M^\nu}{m_3} \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 0 & 0 \\ \lambda^2 & 0 & 1 \end{pmatrix},\ \frac{M^e}{m_\tau} \simeq \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix},$$

$$\text{VAC:}\ \frac{M^\nu}{m_3}\simeq\begin{pmatrix}\lambda^6 & \lambda^3 & \lambda^3\\ \lambda^3 & 0 & 0\\ \lambda^3 & 0 & 1\end{pmatrix},\ \frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^5 & \lambda^5 & \lambda^3\\ \lambda^2 & \lambda^2 & 1\\ \lambda^2 & \lambda^2 & 1\end{pmatrix},$$

where H_1 and H_2 are assumed to be neutral under $U(1)_X\times U(1)_{X'}$.

Following the same argument as above, we find that class (iii) requires the following texture:

$$\frac{M^\nu}{m_3}\simeq\begin{pmatrix}\lambda^x & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\tag{40}$$

where $x=l_1^{\rm eff}-l_2^{\rm eff}=l_1^{\rm eff}-l_3^{\rm eff}$. Here, all zero elements are again forbidden due to the holomorphicity. This texture gives θ_{13} and the square mass difference ratio as

$$\theta_{13}\sim\lambda^x,\quad \Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}\sim\lambda^x.\tag{41}$$

Here we should take $x=q+l$ in Eq. (24) in order to produce the right value of $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$. Then the largest possible values of θ_{13} and $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ are predicted to be

$$(\theta_{13},\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm})\simeq(\lambda^2,\lambda^2)_{\rm LMA},(\lambda^6,\lambda^6)_{\rm LOW},(\lambda^9,\lambda^9)_{\rm VAC}.$$

Equations (37) and (41) show that the LOW and VAC solutions have smaller θ_{13} than the LMA solution, and also the inverted mass hierarchy gives smaller θ_{13} than the normal hierarchy. In particular, the LOW and VAC models with inverted mass hierarchy predict very small θ_{13}, which cannot give any observable $\nu_e\leftrightarrow\nu_\mu$ transition in the future long-baseline experiments and neutrino factory.

Explicit examples of class (iii) can be obtained by assuming the charge assignments:

$$\begin{aligned}
\text{LMA:}\quad & L_1(0,-1),L_2(0,1),L_3(-1,-1),\\
& E_1(3,1),E_2(2,0),E_3(1,0),\\
\text{LOW:}\quad & L_1(1,-2),L_2(0,2),L_3(-2,-2),\\
& E_1(4,5),E_2(3,0),E_3(2,0),\\
\text{VAC:}\quad & L_1(1,-5),L_2(0,2),L_3(-2,-2),\\
& E_1(-1,-2),E_2(3,0),E_3(2,0),
\end{aligned}\tag{42}$$

which give

$$\text{LMA:}\ \frac{M^\nu}{m_2}\simeq\begin{pmatrix}\lambda^2 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\ \frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^5 & \lambda^4 & \lambda^3\\ \lambda^3 & \lambda^2 & 1\\ \lambda^3 & \lambda^2 & 1\end{pmatrix},$$

$$\text{LOW:}\ \frac{M^\nu}{m_2}\simeq\begin{pmatrix}\lambda^6 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\ \frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^5 & \lambda^8 & \lambda^6\\ 0 & \lambda^2 & 1\\ 0 & \lambda^2 & 1\end{pmatrix},\tag{43}$$

$$\text{VAC:}\ \frac{M^\nu}{m_2}\simeq\begin{pmatrix}\lambda^9 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\ \frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^5 & \lambda^{11} & \lambda^9\\ 0 & \lambda^2 & 1\\ 0 & \lambda^2 & 1\end{pmatrix},$$

and so

$$(\theta_{13},\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm})\simeq(\lambda^2,\lambda^2)_{\rm LMA},(\lambda^6,\lambda^6)_{\rm LOW},(\lambda^9,\lambda^9)_{\rm VAC}.$$

It should be noted that all the models discussed so far can be easily extended to the quark sector. For instance, one can assume the following charge assignment in scenario A:

$$(q_{13},q_{23})=(3,2),\ (u_{13},u_{23})=(5,2),\ (d_{13},d_{23})=(1,0)\tag{44}$$

to obtain the quark mass matrices

$$\frac{M^u}{m_t}\simeq\begin{pmatrix}\lambda^8 & \lambda^5 & \lambda^3\\ \lambda^7 & \lambda^4 & \lambda^2\\ \lambda^5 & \lambda^2 & 1\end{pmatrix},\quad \frac{M^d}{m_b}\simeq\begin{pmatrix}\lambda^4 & \lambda^3 & \lambda^3\\ \lambda^3 & \lambda^2 & \lambda^2\\ \lambda^1 & 1 & 1\end{pmatrix},\tag{45}$$

where $q_{ij}=q_i-q_j$, $u_{ij}=u_i-u_j$, $d_{ij}=d_i-d_j$ for q_i,u_i,d_i, which are the $U(1)_X$ charges of the quark superfields Q_i,U_i^c,D_i^c. The same form of the quark mass matrices can be obtained in scenario B also from the $U(1)_X\times U(1)_{X'}$ charge assignment:

$$\begin{aligned}
& Q_1(3,3),Q_2(2,2),\quad Q_3(0,0),\\
& U_1^c(5,5),U_2^c(2,2),U_3^c(0,0),\\
& D_1^c(1,1),D_2^c(0,0),D_3^c(0,0).
\end{aligned}\tag{46}$$

V. SEESAW MODELS

In singlet seesaw models, the light neutrino mass matrix is given by

$$M^\nu_{ij}=\sum_{k,l}(M^M)^{-1}_{kl}M^D_{ik}M^D_{jl},\tag{47}$$

where M^D and M^M denote the Dirac and heavy-Majorana mass matrices, respectively. This formula can be understood as a summation of nine singular matrices $M^D_{ik}M^D_{jl}$ weighted by $(M^M)^{-1}_{kl}$. This feature offers a greater variety of ways to get nontrivial neutrino mixing together with hierarchical mass eigenvalues. For example, if one contribution among the nine contributions in Eq. (47) domimates over the others, we can obtain some interesting models [18]. However, here we do not pursue this possibility, but look for the models without such special dominance.

Scenario A. Since the seesaw framework involves more degrees of freedom, i.e., the flavor charges of N_i, one might expect that it can reproduce all the models found under the weak scale selection rule. However, it is not true. For instance, the LMA model of class (iii) in Eq. (30) has no realization in the seesaw framework. Furthermore, it turns out that $\theta_{13}\sim\lambda$ *cannot* be realized in class (iii) LMA models in the seesaw framework. On the other hand, the seesaw framework allows a wider range of θ_{13} than the weak scale selection rule (see Table I) since it provides generically a greater variety of models. For instance, some VAC models of class (ii) with $\theta_{13}\simeq\lambda$ can be obtained in the seesaw framework, which was not possible under the weak scale selection rule. One such model has the flavor charges

$$\text{VAC: } l_i=(7,-6,-2),\ e_i=(-14,10,4),\ n_i=(-4,4,0), \tag{48}$$

for which the resulting M^ν and M^e are given by

$$\text{VAC: } \frac{M^\nu}{m_3}\simeq\begin{pmatrix}\lambda^{10} & \lambda & \lambda\\ \lambda & \lambda^8 & \lambda^4\\ \lambda & \lambda^4 & 1\end{pmatrix},\quad \frac{M^e}{m_\tau}\simeq\begin{pmatrix}\lambda^5 & \lambda^{15} & \lambda^9\\ \lambda^{18} & \lambda^2 & 1\\ \lambda^{14} & \lambda^6 & 1\end{pmatrix}. \tag{49}$$

Note that one obtains a completely different neutrino mass texture if one applies the weak scale selection rule to the above model.

We have explored the possible range of θ_{13} under the restriction $|l_i|<10$ and $|n_i|<10$. Even in the seesaw framework, it appears to be difficult to find a desirable form of class (i) model in scenario A. However there is a potentially interesting example of class (i), yielding $\theta_{13}\simeq\lambda^2$:

$$l_i=(2,-2,0),\ e_i=(5,5,1),\ n_i=(0,0,0), \tag{50}$$

which gives

$$\frac{M^\nu}{m_3}\simeq\begin{pmatrix}\lambda^4 & \lambda^4 & \lambda^2\\ \lambda^4 & \lambda^4 & \lambda^2\\ \lambda^2 & \lambda^2 & 1\end{pmatrix}. \tag{51}$$

The resulting $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}\simeq\lambda^8$ is close to either the LOW value $\lambda^6-\lambda^7$ or the VAC value $\lambda^9-\lambda^{10}$, so it may fit to the LOW or VAC if a somewhat large or small coefficient of order 1 is involved. For the LMA and LOW model of class (ii), we found that the range of θ_{13} is the same as the case of the weak scale selection rule. For the VAC of class (ii), $\theta_{13}\simeq\lambda$ is added as we have noted above. For class (iii) models, we find θ_{13} can be as small as λ^6 and λ^7 for the LMA and LOW cases, respectively. The maximal value of θ_{13} for the LMA model of class (iii) turns out to be of order λ^2, not of order λ, which is noted also in the above discussion. For the VAC model of class (iii), the range of θ_{13} is the same as the case of the weak scale selection rule. All of these results on θ_{13} are summarized in Table I.

Scenario B. Similar to scenario, the neutrino mass of class (i) *cannot* be obtained even in the seesaw framework. For classes (ii) and (iii), we need a pseudo-Dirac form of M^M to get a pseudo-Dirac M^ν. We find that all models found under the weak scale selection rule can be realized in the seesaw framework. For purposes of illustration, we show only the seesaw realization of the LMA solution of class (ii) in Eq. (38). For this, we introduce the singlet neutrinos with the following $U(1)$ charges:

$$N_1(0,-1),N_2(0,1),N_3(0,0), \tag{52}$$

giving

$$M^M\propto\begin{pmatrix}\lambda^2 & 1 & \lambda\\ 1 & 0 & 0\\ \lambda & 0 & 1\end{pmatrix},\quad M^D\propto\begin{pmatrix}\lambda^2 & 1 & \lambda\\ \lambda & 0 & 0\\ \lambda & 0 & 1\end{pmatrix}. \tag{53}$$

The resulting M^ν is given by

$$\frac{M^\nu}{m_3}\simeq\begin{pmatrix}\lambda^2 & \lambda & \lambda\\ \lambda & 0 & 0\\ \lambda & 0 & 1\end{pmatrix}, \tag{54}$$

which has the same form as determined by the weak scale selection rule.

We remark that the selection rule (15) of scenario B is very restrictive so that the seesaw framework does not provide more freedom than the case of the weak scale selection rule. Basically, the positivity of the exponents for the nonvanishing mass matrix elements forbids us to modify the structure of holomorphic zeros in the textures (37) and (41) even in the presence of singlet neutrinos. Therefore, no new model can be found by considering the seesaw mechanism.

VI. CONCLUSION

In conclusion, we have examined the possibility that the near-bimaximal mixing of atmospheric and solar neutrinos naturally arises together with small $U_{e3}=\sin\theta_{13}$ and $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ as a consequence of Abelian flavor symmetry. We have considered two simple scenarios where the mass textures are expressed in terms of the Cabibbo angle λ within the supersymmetric framework. Scenario A has a single nonanomalous $U(1)$ broken by two scalar fields with opposite $U(1)$ charge and scenario B involves one anomalous $U(1)_X$ and another nonanomalous $U(1)_{X'}$ which are broken by two scalar fields with the $U(1)_X\times U(1)_{X'}$ charges $(-1,-1)$ and $(0,1)$. In the latter scenario, all symmetry-breaking order parameters naturally have the Cabbibo angle size $\lambda\simeq 0.2$. Concentrating on the scheme where the large atmospheric neutrino mixing comes from the charged lepton mass matrix, we found that the neutrino mass textures of pseudo-Dirac type (with normal or inverted hierarchy) can produce nicely a large solar neutrino mixing angle while keeping θ_{13} appropriately small. The current bound on θ_{13} is of order λ, however it may be measured down to order λ^3 in future neutrino experiments. Table I summarizes the possible ranges of θ_{13} predicted by the models under consideration.

While the models of scenario A produce relatively broad ranges of θ_{13}, those of scenario B give more specific predictions which are strongly correlated with $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ and also with the sign of Δm^2_{32}. Generically, larger $\Delta m^2_{\rm sol}$ come with larger θ_{13} and the normal hierarchy ($\Delta m^2_{32}>0$) has larger θ_{13} than the inverted hierarchy ($\Delta m^2_{32}<0$). Table I shows that various models of neutrino mass textures could be discriminated by future solar and terrestrial neutrino experiments which would pin down the specific solution of the solar neutrino problem and give information about θ_{13} and the sign of Δm^2_{32}.

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[1] Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).
[2] B.T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998); W. Hampel, *et al.*, Phys. Lett. B **447**, 127 (1999); J.N. Abdurashitov *et al.*, Phys. Rev. C **60**, 055801 (1999); Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996); S. Fukuda *et al.*, *ibid.* **86**, 5651 (2001); Q.R. Ahmad *et al.*, *ibid.* **87**, 071301 (2001).
[3] V. Barger *et al.*, hep-ph/0106207; G.L. Fogli *et al.*, Phys. Rev. D **64**, 093007 (2001); J.N. Bahcall *et al.*, J. High Energy Phys. **08**, 014 (2001).
[4] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 247 (1962);
[5] M.C. Gonzalez-Garcia, M. Maltoni, C. Peña-Garay, and J.W.F. Valle, Phys. Rev. D **63**, 033005 (2001).
[6] T. Takeuchi, talk at *XXX International Conference on HEP*, ICHEP 2000, Osaka, Japan, 2000.
[7] CHOOZ Collaboration, Phys. Lett. B **420**, 397 (1998).
[8] L. Hall, H. Murayama, and N. Weiner, Phys. Rev. Lett. **84**, 2572 (2000).
[9] C.D. Frogatt and H.B. Nielsen, Nucl. Phys. **B147**, 277 (1979).
[10] M. Leurer, Y. Nir, and N. Seiberg, Nucl. Phys. **B398**, 319 (1993); **B420**, 468 (1994); L. Ib'anez and G.G. Ross, Phys. Lett. B **332**, 100 (1993); P. Binetruy and P. Ramond, *ibid.* **350**, 49 (1995); V. Jain and R. Shrock, *ibid.* **352**, 83 (1995); E. Dudas, S. Pokorski, and C.A. Savoy, *ibid.* **356**, 45 (1995); P. Binetruy, S. Lavignac, and P. Ramond, Nucl. Phys. **B477**, 353 (1996); Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B **396**, 150 (1997).
[11] N. Irges, S. Lavignac, and P. Ramond, Phys. Rev. D **58**, 035003 (1998); P. Binetruy, E. Dudas, S. Lavignac, and S.A Savoy, Phys. Lett. B **422**, 171 (1998); J. Ellis, S. Lola, and G.G. Ross, Nucl. Phys. **B526**, 115 (1998); C.D. Frogatt, M. Gibson, and H.B. Nielson, Phys. Lett. B **446**, 256 (1999); Y. Nir and Y. Shadmi, J. High Energy Phys. **05**, 023 (1999); J. Feng, Y. Nir, and Y. Shadmi, Phys. Rev. D **61**, 113005 (2000); Q. Shafi and Z. Tavartkiladze, Phys. Lett. B **482**, 145 (2000); M. Tanimoto, *ibid.* **501**, 231 (2001); M. Berger and K. Siyeon, Phys. Rev. D **63**, 057302 (2001).
[12] E.J. Chun and A. Lukas, Phys. Lett. B **387**, 99 (1996).
[13] K. Choi, E.J. Chun, and K. Hwang, Phys. Rev. D **60**, 031301 (1999); K. Choi, E.J. Chun, and H. Kim, Phys. Lett. B **394**, 89 (1997).
[14] M. Gell-Mann, P. Ramond, and R. Slansky, in Sanibel Talk, CALT-68-709, 1979, and in *Supergravity* (North-Holland, Amsterdam, 1979); T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe, edited by O. Sawada and A. Sugamoto, KEK Report no. 79-18, Tsukuba, 1979; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
[15] C. Albright *et al.*, "Physics at a Neutrino Factory," hep-ex/0008064.
[16] E. Ma and Utpal Sarkar, Phys. Rev. Lett. **80**, 5716 (1998); E. Ma, *ibid.* **81**, 1171 (1998); K. Choi and A. Santamaria, Phys. Lett. B **267**, 504 (1991).
[17] V. Barger *et al.*, Phys. Lett. B **437**, 107 (1998); H. Georgi and S.L. Glashow, Phys. Rev. D **61**, 097301 (2000); A.J. Baltz *et al.*, Phys. Rev. Lett. **81**, 5730 (1998); S. Davidson and S.F King, Phys. Lett. B **445**, 191 (1998); C. Giunti, Phys. Rev. D **59**, 077301 (1999); Y. Nomura and T. Yanagida, *ibid.* **59**, 017303 (1999); R.N. Mohapatra and S. Nussinov, Phys. Lett. B **441**, 299 (1998); M. Tanimoto, hep-ph/9807517; H. Fritzsch and Z. Xing, Phys. Lett. B **440**, 313 (1998); S.K. Kang and C.S. Kim, Phys. Rev. D **59**, 091302 (1999); C. Jarlskog *et al.*, Phys. Lett. B **449**, 240 (1999); R. Barieri *et al.*, hep-ph/9901241; Q. Shafi and A Tavartkiladze, Phys. Lett. B **451**, 129 (1999); A.H. Guth *et al.*, J. High Energy Phys. **08**, 018 (1999); J. Ellis and S. Lola, Phys. Lett. B **458**, 310 (1999); M. Jezabek and Y. Sumino, *ibid.* **457**, 139 (1999); J.A. Casas *et al.*, Nucl. Phys. **B556**, 3 (1999).
[18] S. F. King, Phys. Lett. B **439**, 350 (1998); Nucl. Phys. **B562**, 57 (1999); **B576**, 85 (2000).