Softly broken A_4 symmetry for nearly degenerate neutrino masses

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The leptonic Higgs doublet model of neutrino masses is implemented with an A_4 discrete symmetry (the even permutation of four objects or equivalently the symmetry of the tetrahedron) which has four irreducible representations: $\underline{1}, \underline{1}', \underline{1}''$, and $\underline{3}$. The resulting spontaneous and soft breaking of A_4 provides a realistic model of charged-lepton masses as well as a nearly degenerate neutrino mass matrix. The phenomenological consequences at and below the TeV scale are discussed.

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I. INTRODUCTION

Since the experimental evidence of neutrino oscillations [1–3] requires only neutrino mass differences, the possibility of nearly degenerate neutrino masses is often considered [4]. However, the charged lepton masses are certainly not degenerate, so whatever symmetry we use to maintain the neutrino mass degeneracy must be broken. To implement this idea in a renormalizable field theory, the symmetry in question should be broken only spontaneously and by explicit soft terms (if it is not a gauge symmetry).

Recently, a simple model of neutrino masses was proposed [5] using a leptonic Higgs doublet $\eta=(\eta^+,\eta^0)$ and three right-handed singlet fermions N_{iR}, all of which are at or below the TeV energy scale. It was further shown [6] that this model is able to account for the recent measurement [7] of the muon anomalous magnetic moment, provided that neutrino masses are nearly degenerate [8].

In this paper, the specific choice of a discrete symmetry, i.e., A_4, which is the symmetry group of the even permutation of four objects or equivalently that of the tetrahedron, is used to sustain this degeneracy, which is then broken both spontaneously to generate the different charged-lepton masses, and softly to account for the mass splitting and mixing of the neutrinos. In Sec. II, the group A_4 and its irreducible representations are discussed. In Sec. III, the structure of the leptonic model, which has altogether four Higgs doublets, is presented. In Sec. IV, phenomenological consequences of this model are explored. In Sec. V, the quark sector is discussed. In Sec. VI, there are some concluding remarks.

II. DISCRETE SYMMETRY A_4

The finite group of the even permutation of four objects, i.e., A_4, has 12 elements, which are divided into 4 classes, with the number of elements 1,4,4,3, respectively. This means that there are 4 irreducible representations, with dimensions n_i, such that $\Sigma_i n_i^2=12$. There is only one solution: $n_1=n_2=n_3=1$ and $n_4=3$, and the character table of the 4 representations is shown in Table I.

The complex number ω is the cube root of unity, i.e., $e^{2\pi i/3}$. Hence $1+\omega+\omega^2=0$. Calling the 4 irreducible representations $\underline{1}, \underline{1}', \underline{1}''$, and $\underline{3}$ respectively, we have the decomposition

$$\underline{3}\times\underline{3}=\underline{1}+\underline{1}'+\underline{1}''+\underline{3}+\underline{3}. \tag{1}$$

In particular, denoting $\underline{3}$ as (a,b,c), we have

$$\underline{1}=a_1a_2+b_1b_2+c_1c_2, \tag{2}$$

$$\underline{1}'=a_1a_2+\omega^2 b_1b_2+\omega c_1c_2, \tag{3}$$

$$\underline{1}''=a_1a_2+\omega b_1b_2+\omega^2 c_1c_2. \tag{4}$$

For completeness, the 3×3 representation matrices of the 12 group elements are given below.

$$C_1:\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}, \tag{5}$$

$$C_2:\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix},\begin{bmatrix}0&0&1\\-1&0&0\\0&-1&0\end{bmatrix},\begin{bmatrix}0&0&-1\\1&0&0\\0&-1&0\end{bmatrix},\begin{bmatrix}0&0&-1\\-1&0&0\\0&1&0\end{bmatrix}, \tag{6}$$

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$$C_3:\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix},\begin{bmatrix}0&1&0\\0&0&-1\\-1&0&0\end{bmatrix},\begin{bmatrix}0&-1&0\\0&0&1\\-1&0&0\end{bmatrix},\begin{bmatrix}0&-1&0\\0&0&-1\\1&0&0\end{bmatrix},\tag{7}$$

$$C_4:\begin{bmatrix}1&0&0\\0&-1&0\\0&0&-1\end{bmatrix},\begin{bmatrix}-1&0&0\\0&1&0\\0&0&-1\end{bmatrix},\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&1\end{bmatrix}.\tag{8}$$

The reason that we choose A_4 for discussing degenerate neutrino masses is that it is the simplest discrete symmetry, which admits one three-dimensional representation as well as three inequivalent one-dimensional representations. As shown in the next section, this is ideal for having degenerate Dirac neutrino masses while allowing arbitrary charged-lepton masses. In contrast, the S_3 discrete symmetry [9] has one two-dimensional and two one-dimensional representations, whereas S_4 [10] has two three-dimensional, one two-dimensional, and two one-dimensional representations. If continuous groups are considered, then $SO(3)$ has a three-dimensional representation and may be used as well.

III. MODEL OF NEARLY DEGENERATE NEUTRINO MASSES

Under A_4 and L (lepton number), the color-singlet fermions and scalars of this model transform as follows.

$$(\nu_i,l_i)_L\sim(\underline{3},1),\tag{9}$$

$$l_{1R}\sim(\underline{1},1),\tag{10}$$

$$l_{2R}\sim(\underline{1}',1),\tag{11}$$

$$l_{3R}\sim(\underline{1}'',1),\tag{12}$$

$$N_{iR}\sim(\underline{3},0),\tag{13}$$

$$\Phi_i=(\phi_i^+,\phi_i^0)\sim(\underline{3},0),\tag{14}$$

$$\eta=(\eta^+,\eta^0)\sim(\underline{1},-1).\tag{15}$$

Hence its Lagrangian has the invariant terms

$$\frac{1}{2}MN_{iR}^2+f\overline{N}_{iR}(\nu_{iL}\eta^0-l_{iL}\eta^+)+h_{ijk}\overline{(\nu_i,l_i)}_L l_{jR}\Phi_k+\text{H.c.},\tag{16}$$

TABLE I. Character table of A_4.

Class	$\chi^{(1)}$	$\chi^{(2)}$	$\chi^{(3)}$	$\chi^{(4)}$
C_1	1	1	1	3
C_2	1	ω	ω^2	0
C_3	1	ω^2	ω	0
C_4	1	1	1	-1

where

$$h_{i1k}=h_1\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\quad h_{i2k}=h_2\begin{bmatrix}1&0&0\\0&\omega&0\\0&0&\omega^2\end{bmatrix},$$

$$h_{i3k}=h_3\begin{bmatrix}1&0&0\\0&\omega^2&0\\0&0&\omega\end{bmatrix}.\tag{17}$$

Thus the neutrino mass matrix (in this basis) is proportional to the unit matrix with magnitude f^2u^2/M, where $u=\langle\eta^0\rangle$, whereas the charged-lepton mass matrix is given by

$$\mathcal{M}_l=\begin{bmatrix}h_1v_1&h_2v_1&h_3v_1\\h_1v_2&h_2\omega v_2&h_3\omega^2v_2\\h_1v_3&h_2\omega^2v_3&h_3\omega v_3\end{bmatrix}.\tag{18}$$

If $v_1=v_2=v_3=v$, then $\mathcal{M}_l$ is easily diagonalized:

$$U_L^\dagger\mathcal{M}_lU_R=\begin{bmatrix}\sqrt{3}h_1v&0&0\\0&\sqrt{3}h_2v&0\\0&0&\sqrt{3}h_3v\end{bmatrix}=\begin{bmatrix}m_e&0&0\\0&m_\mu&0\\0&0&m_\tau\end{bmatrix},\tag{19}$$

where

$$U_L=\frac{1}{\sqrt{3}}\begin{bmatrix}1&1&1\\1&\omega&\omega^2\\1&\omega^2&\omega\end{bmatrix},\quad U_R=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.\tag{20}$$

The 6×6 Majorana mass matrix spanning $(\overline{\nu}_e,\overline{\nu}_\mu,\overline{\nu}_\tau,N_1,N_2,N_3)$ is then given by

$$\mathcal{M}_{(\nu,N)}=\begin{bmatrix}0&U_L^\dagger fu\\U_L^*fu&M\end{bmatrix}.\tag{21}$$

Hence the 3×3 see-saw mass matrix for (ν_e,ν_μ,ν_τ) becomes

$$\mathcal{M}_\nu = \frac{f^2u^2}{M} U_L^T U_L = \frac{f^2u^2}{M} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{22}$$

This shows that ν_μ mixes maximally with ν_τ, but since all physical neutrino masses are degenerate, there are no neutrino oscillations. To break the degeneracy, arbitrary soft terms of the form $m_{ij}N_{iR}N_{jR}$ may be added to Eq. (16). As a result, it is possible to have, for example, a bimaximal mixing pattern with the appropriate small neutrino mass-squared differences for atmospheric [1] and solar [2] neutrino oscillations used in Ref. [6].

IV. PHENOMENOLOGICAL CONSEQUENCES

Whereas the minimal standard model has only one Higgs scalar doublet, our A_4 model has four, $\Phi_{1,2,3}$ and η. The interplay between Φ_i and η is the same as in Ref. [5], which allows $u = \langle \eta^0 \rangle$ to be small. The new feature here is the structure of the Higgs sector containing Φ_i. The corresponding A_4-invariant Higgs potential is given by

$$\begin{aligned} V = {} & m^2 \sum_i \Phi_i^\dagger \Phi_i + \frac{1}{2}\lambda_1 \left(\sum_i \Phi_i^\dagger \Phi_i \right)^2 + \lambda_2 (\Phi_1^\dagger \Phi_1 \\ & + \omega^2 \Phi_2^\dagger \Phi_2 + \omega \Phi_3^\dagger \Phi_3)(\Phi_1^\dagger \Phi_1 + \omega \Phi_2^\dagger \Phi_2 + \omega^2 \Phi_3^\dagger \Phi_3) \\ & + \lambda_3 [(\Phi_2^\dagger \Phi_3)(\Phi_3^\dagger \Phi_2) + (\Phi_3^\dagger \Phi_1)(\Phi_1^\dagger \Phi_3) + (\Phi_1^\dagger \Phi_2) \\ & \times (\Phi_2^\dagger \Phi_1)] + \left\{ \frac{1}{2}\lambda_4 [(\Phi_2^\dagger \Phi_3)^2 + (\Phi_3^\dagger \Phi_1)^2 + (\Phi_1^\dagger \Phi_2)^2] \right. \\ & \left. + \text{H.c.} \right\}. \end{aligned} \tag{23}$$

Let $\langle \phi_i^0 \rangle = v_i$, then the minimum of V is

$$\begin{aligned} V_{min} = {} & m^2 (|v_1|^2 + |v_2|^2 + |v_3|^2) + \frac{1}{2}\lambda_1 (|v_1|^2 + |v_2|^2 + |v_3|^2)^2 \\ & + \lambda_2 (|v_1|^2 + \omega^2 |v_2|^2 + \omega |v_3|^2)(|v_1|^2 + \omega |v_2|^2 \\ & + \omega^2 |v_3|^2) + \lambda_3 (|v_2|^2 |v_3|^2 + |v_3|^2 |v_1|^2 + |v_1|^2 |v_2|^2) \\ & + \left\{ \frac{1}{2}\lambda_4 [(v_2^*)^2 v_3^2 + (v_3^*)^2 v_1^2 + (v_1^*)^2 v_2^2] + \text{c.c.} \right\} \end{aligned} \tag{24}$$

The minimization conditions on v_i are given by

$$\begin{aligned} 0 = \frac{\partial V_{min}}{\partial v_1^*} = {} & m^2 v_1 + \lambda_1 v_1 (|v_1|^2 + |v_2|^2 + |v_3|^2) \\ & + \lambda_2 v_1 (2|v_1|^2 - |v_2|^2 - |v_3|^2) + \lambda_3 v_1 (|v_2|^2 + |v_3|^2) \\ & + \lambda_4 v_1^* (v_2^2 + v_3^2), \end{aligned} \tag{25}$$

and other similar equations. Hence the solution

$$v_1 = v_2 = v_3 = v = \sqrt{\frac{-m^2}{3\lambda_1 + 2\lambda_3 + 2\lambda_4}} \tag{26}$$

is allowed if λ_4 is real.

The mass-squared matrices in the Re ϕ_i^0, Im ϕ_i^0, and $\phi_i^\pm$ bases are all of the form

$$\mathcal{M}^2 = \begin{pmatrix} a & b & b \\ b & a & b \\ b & b & a \end{pmatrix}, \tag{27}$$

where

$$\text{Re}\,\phi_i^0: \quad a = 2(\lambda_1 + 2\lambda_2)v^2, \quad b = 2(\lambda_1 - \lambda_2 + \lambda_3 + \lambda_4)v^2, \tag{28}$$

$$\text{Im}\,\phi_i^0: \quad a = -4\lambda_4 v^2, \quad b = 2\lambda_4 v^2, \tag{29}$$

$$\phi_i^\pm: \quad a = -2(\lambda_3 + \lambda_4)v^2, \quad b = (\lambda_3 + \lambda_4)v^2. \tag{30}$$

The eigenvalues of $\mathcal{M}^2$ are $a+2b$, $a-b$, and $a-b$. Hence $(\Phi_1+\Phi_2+\Phi_3)/\sqrt{3}$ has the properties of the standard-model Higgs doublet with mass-squared eigenvalues $2(3\lambda_1+2\lambda_3+2\lambda_4)v^2$, 0, and 0 for $\text{Re}(\phi_1^0+\phi_2^0+\phi_3^0)/\sqrt{3}$, $\text{Im}(\phi_1^0+\phi_2^0+\phi_3^0)/\sqrt{3}$, and $(\phi_1^\pm+\phi_2^\pm+\phi_3^\pm)/\sqrt{3}$, respectively. The two other linear combinations are mass degenerate in each sector with mass-squared eigenvalues given by $M_R^2 = 2(3\lambda_2-\lambda_3-\lambda_4)v^2$, $M_I^2 = -6\lambda_4 v^2$, and $M_\pm^2 = -3(\lambda_3+\lambda_4)v^2$, respectively.

The distinct phenomenological signatures of our A_4 model are thus given by the two new Higgs doublets. They are predicted to be pairwise degenerate in mass and their Yukawa interactions are given by

$$\begin{aligned} \mathcal{L}_{int} = {} & \left(\frac{m_\tau}{v} \overline{(\nu_e, e)}_L \tau_R + \frac{m_\mu}{v} \overline{(\nu_\tau, \tau)}_L \mu_R + \frac{m_e}{v} \overline{(\nu_\mu, \mu)}_L e_R \right) \Phi' \\ & + \left(\frac{m_\tau}{v} \overline{(\nu_\mu, \mu)}_L \tau_R + \frac{m_\mu}{v} \overline{(\nu_e, e)}_L \mu_R \right. \\ & \left. + \frac{m_e}{v} \overline{(\nu_\tau, \tau)}_L e_R \right) \Phi'' + \text{H.c.}, \end{aligned} \tag{31}$$

where

$$\Phi' = \frac{1}{\sqrt{3}} (\Phi_1 + \omega \Phi_2 + \omega^2 \Phi_3),$$

$$\Phi'' = \frac{1}{\sqrt{3}} (\Phi_1 + \omega^2 \Phi_2 + \omega \Phi_3). \tag{32}$$

This means that lepton flavor is necessarily violated and serves as an unmistakable prediction of this model.

Using Eq. (31), we find that the most prominent (with strength $m_\tau m_\mu / v^2$) exotic decays of this model are

$$\tau_R^- \to \mu_L^- \mu_R^- e_R^+, \quad \tau_R^- \to \mu_L^- \mu_L^+ e_L^-, \tag{33}$$

through $(\phi'')^0$ exchange. The former amplitude is proportional to $M_0^{-2}=M_R^{-2}+M_I^{-2}$ and the latter to $M_1^{-2}=|M_R^{-2}-M_I^{-2}|$. Hence

$$B(\tau^-\to\mu^-\mu^-e^+)=\left(\frac{9m_\tau^2m_\mu^2}{M_0^4}\right)\left(\frac{v_0^2}{3v^2}\right)^2 B(\tau\to\mu\nu\nu), \tag{34}$$

where $v_0=(2\sqrt{2}G_F)^{-1/2}$ and $3v^2<v_0^2$. Using $B(\tau\to\mu\nu\nu)=0.174$, we find

$$B(\tau^-\to\mu^-\mu^-e^+)=5.5\times10^{-10}\left(\frac{v_0^2}{3v^2}\right)^2\left(\frac{100\ \text{GeV}}{M_0}\right)^4, \tag{35}$$

as compared to the experimental upper bound of 1.5×10^{-6}. Similarly, $B(\tau^-\to\mu^-\mu^+e^-)$ is also given by Eq. (35) with M_0 replaced by M_1 (which is always greater than M_0) as compared to the experimental upper bound of 1.8×10^{-6}. Other τ decays are further suppressed because they are proportional to $m_\tau m_e$ or $m_\mu m_e$. Note the important fact that there is no tree-level $\mu\to eee$ decay in this model.

From Eq. (31), there are also tree-level contributions to τ and μ decays through charged-scalar exchange. For example,

$$\mu_R^-\to e_R^-\nu_\tau\overline{\nu}_\mu, \quad \mu_R^-\to e_R^-\nu_e\overline{\nu}_\tau, \tag{36}$$

through $(\phi')^\pm$ and $(\phi'')^\pm$ exchange, respectively. However, these amplitudes are proportional to $m_\mu m_e$ and only add incoherently to the dominant $\mu_L^-\to e_L^-\nu_\mu\overline{\nu}_e$ amplitude. Hence they are completely negligible. The same holds true for τ decays, but to a lesser extent.

Consider next the muon anomalous magnetic moment, which receives a contribution proportional to m_τ^2 from $(\phi'')^0$. A straightforward calculation yields

$$\Delta a_\mu=\frac{G_Fm_\tau^2}{4\sqrt{2}\pi^2}\left(\frac{m_\mu^2}{M_0^2}\right)\left(\frac{v_0^2}{3v^2}\right)$$
$$=7.4\times10^{-13}\left(\frac{v_0^2}{3v^2}\right)\left(\frac{100\ \text{GeV}}{M_0}\right)^2, \tag{37}$$

as compared to the possible discrepancy [11] of $(426\pm165)\times10^{-11}$, based on the recent experimental measurement [7]. Hence the contribution to Δa_μ from Eq. (31) is negligible, and the latter's theoretical explanation remains that of η and N exchange as proposed in Ref. [6].

Radiative lepton-flavor-changing decays (i.e., $\tau\to\mu\gamma,\tau\to e\gamma,\mu\to e\gamma$) through η and N exchange are suppressed by the near degeneracy of the neutrino mass matrix, as explained in Ref. [6]. However, they also receive contributions from Eq. (31). The most prominent process is actually $\mu\to e\gamma$ from $(\phi')^0$ exchange, with an amplitude given by

$$\mathcal{A}=\frac{e}{32\pi^2}\frac{m_\tau^2}{M_{eff}^2}\frac{m_\mu}{v^2}\epsilon^\alpha q^\beta\overline{e}\sigma_{\alpha\beta}\left(\frac{1+\gamma_5}{2}\right)\mu, \tag{38}$$

where

$$\frac{1}{M_{eff}^2}=\frac{1}{M_R^2}\left(\ln\frac{M_R^2}{m_\tau^2}-\frac{3}{2}\right)-\frac{1}{M_I^2}\left(\ln\frac{M_I^2}{m_\tau^2}-\frac{3}{2}\right). \tag{39}$$

Hence

$$B(\mu\to e\gamma)=\frac{27\alpha}{8\pi}\frac{m_\tau^4}{M_{eff}^4}\left(\frac{v_0^2}{3v^2}\right)^2. \tag{40}$$

Using the experimental upper bound [12] of 1.2×10^{-11}, we find $M_{eff}>284\ \text{GeV}(v_0/\sqrt{3}v)$.

V. QUARK SECTOR

In the quark sector, we could also try having the three left-handed quark doublets transform as $\underline{3}$ under A_4, and the right-handed quark singlets as $\underline{1},\underline{1}'$, and $\underline{1}''$. In that case, the quark mass matrices corresponding to Eq. (19) are diagonal like those of the charged leptons. Since the soft breaking of A_4 is not possible in the quark sector, the only way that a charged-current mixing matrix may arise is from the violation of $v_1=v_2=v_3$. However, because the mixing is further suppressed by the ratio of quark masses, the final effect is negligible.

Suppose we assign both quark doublets and singlets to be $\underline{3}$ under A_4. Then there are 2 invariant couplings to Φ_i as shown by Eq. (1). However, the mass eigenvalues in this case are those of Eq. (27), which do not match the observed quark masses.

To accommodate realistic quark mass matrices with the correct charged-current mixing matrix, we can just go back to the standard model, i.e., all quarks are trivial under A_4 as well as another Higgs doublet Φ_4. Thus

$$v_0^2=v_1^2+v_2^2+v_3^2+v_4^2+u^2=3v^2+v_4^2+u^2. \tag{41}$$

VI. CONCLUDING REMARKS

In conclusion, we have shown how nearly degenerate neutrino masses can be obtained in the context of a softly and spontaneously broken discrete A_4 (tetrahedral) symmetry while allowing realistic charged-lepton and quark masses [13]. In addition to the standard-model particles, we have three heavy neutral right-handed singlet fermions N_i at the TeV scale or below, whose decay into charged leptons would map out the neutrino mass matrix as discussed in Ref. [5]. The nearly mass-degenerate N_i can explain the possible discrepancy of the muon anomalous magnetic moment as discussed in Ref. [6]. The three new Higgs scalar doublets Φ_i of this model have distinct experimental signatures. One combination, i.e., $(\Phi_1+\Phi_2+\Phi_3)/\sqrt{3}$ behaves like the standard-model Higgs doublet, except that it couples only to leptons.

The other two, i.e., Φ' and Φ'' of Eq. (32), are predicted to be pairwise mass degenerate and have precisely determined flavor-changing couplings as given by Eq. (31). They are consistent with all present experimental bounds and amenable to experimental discovery below a TeV.

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