Model independent radiative corrections in processes of polarized electron-nucleon elastic scattering

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Explicit formulas for radiative correction (RC) calculations for elastic *ep* scattering are presented. Two typical measurements of polarization observables, such as beam-target asymmetry or recoil proton polarization, are considered. The possibilities of taking into account realistic experimental acceptances are discussed. The FORTRAN code MASCARAD for providing the RC procedure is presented. A numerical analysis is done for the kinematical conditions of CEBAF.

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I. INTRODUCTION

Precise polarization measurements of nucleon form factors in electron scattering are an essential component of the research program at new-generation electron accelerators such as the Continuous Electron Beam Accelerator Facility (CEBAF) [1]. This unprecedented precision requires knowledge of higher-order electromagnetic effects at a percent level. The purpose of our work is to analyze radiative corrections in elastic electron-proton scattering and develop proper computational techniques that could be used in experiments at Jefferson Lab (CEBAF) and other electron accelerator laboratories.

The modern approach to radiative correction (RC) calculations assumes exact calculations of the lowest-order model independent correction. This correction includes the QED processes of radiation of an unobserved real photon, vacuum polarization, and lepton-photon vertex corrections. These processes give the largest contributions that can be calculated exactly. Uncertainties of the model independent RC can come only from fits and data used for structure functions. The calculation of model dependent corrections (box-type diagrams, emission by hadrons) requires additional assumptions about hadron interactions, so it has additional purely theoretical uncertainties, which are hard to control. The model dependent correction is much smaller compared to leptonic radiation because it does not include a large logarithmic term $\ln(Q^2/m^2)$. In this paper we concentrate on the calculation of the model independent correction as the main contribution to the total RC. Treatment of the model dependent correction requires different methods and will be the subject of a separate investigation.

There are two basic methods of calculation of model independent QED radiative corrections. The first one is connected with the introduction of an artificial parameter (Δ) separating the momentum phase space into soft and hard parts. One can find a classical review introducing this formalism and further developments for elastic *ep* scattering in Refs. [2–4]. For the soft-photon part, the calculation is performed in the soft-photon approximation, in which the photon energy is considered to be small with respect to all momenta and masses in the problem. So this parameter (Δ) should be chosen as small as possible to reduce the region evaluated approximately. However, it cannot be chosen too small because of possible numerical instabilities in calculating hard-photon emission. An exact calculation within the approach of Mo and Tsai has been performed only for the case of unpolarized deep inelastic scattering. At the end of the 1970s Bardin and Shumeiko developed an approach [5] involving extraction and cancellation of infrared divergences without introducing this artificial parameter.[1] Later on many calculations were performed within this approach and a few FORTRAN codes were created to deal with numerical calculations. The best known of them are TERAD and POLRAD. A detailed review of the approach is presented in Ref. [7]. In this paper we use this approach to calculate the RC of lowest order to the transferred polarization and asymmetry in elastic electron-proton scattering. The method allows us to calculate the model independent correction exactly. By "exact" we mean the calculation of the lowest-order correction for which extraction and cancellation of the infrared divergence are performed without introducing the artificial parameter that separates the soft and hard parts in phase space, and for which integration over photon phase space is performed without approximations like peaking or leading logarithm. Instead, this integration is performed numerically within the given accuracy. This accuracy does not usually exceed 0.1%, so contributions of the order of the electron mass squared can be dropped. In general, the result for the RC can be presented in the form of a series in powers of m^2:

$$\sigma_{RC}=\alpha\left[A\ln\frac{Q^2}{m^2}+B+O\left(\frac{m^2}{Q^2}\right)\right]. \tag{1}$$

The coefficients A and B are responsible for the first-order leading and next-to-leading contributions, respectively. They

[1] A detailed comparison of explicit formulas obtained within the two approaches considered is given in Ref. [6] for the case of deep inelastic scattering.

are independent of the electron mass, and our approach allows us to calculate them explicitly. The term of order m^2/Q^2 is always negligible for values Q^2 in the GeV^2 region and above.

We should mention here that there exists another approach that satisfies the properties listed above. It is a modified method of electron structure functions. Usually this approach provides a direct calculation of leading terms in all orders. However, in Ref. [8] it was shown that it can be improved to obtain exactly the first-order next-to-leading contribution and even to reconstruct the main part of the second-order one. In [9] this approach was used to calculate the radiative correction in the case of recoil polarization measurement within so-called leptonic variables.

The observed cross section of the process

$$e(k_1)+N(p_1)\to e'(k_2)+N(p_2) \tag{2}$$

is described by one independent variable, which is usually chosen to be the square of the four-momentum transfer. There are two ways to reconstruct the variable when both lepton and nucleon final momenta are measured. In the first case it will be denoted as $Q_l^2=-(k_1-k_2)^2$, and in the second case it is $Q_h^2=-(p_2-p_1)^2$. It is clear that there is no difference between these definitions at the Born level. However, emission of an additional photon in the final state of reaction (2) makes the definitions of Q^2 nonidentical. We consider both cases in this paper. In the first case the structure of the bremsstrahlung cross section looks like

$$\frac{d\sigma}{dQ_l^2}\sim\alpha^3\int\frac{d^3k}{k_0}\sum\mathcal{K}\mathcal{F}^2(Q_h^2)\mathcal{A} \tag{3}$$

where $\mathcal{K}$ is a kinematical coefficient calculable exactly in the lowest order. It depends on photon variables. $\mathcal{F}^2$ is a bilinear combination of nucleon form factors dependent on Q_h^2 only, which is a function of photon momentum. Usually only final momenta are measured in a certain range controlled by a function of the acceptance $\mathcal{A}$, which is 1 or 0, depending on whether the final particles make it to the detectors or not. The integral (3) should not be analytically calculated for two reasons. The first one is the dependence of the form factors on Q_h^2. We avoid using specific models for them. The second one is that the acceptance is usually a very complicated function of the kinematical variables, dependent on the photon momentum.

For the second method of reconstruction of the transferred momentum squared, the structure of the cross section is

$$\frac{d\sigma}{dQ_h^2}\sim\alpha^3\sum\mathcal{F}^2(Q_h^2)\int\frac{d^3k}{k_0}\mathcal{K}\mathcal{A}. \tag{4}$$

In this case the squared form factor does not depend on the photon momentum and for 4π kinematics ($\mathcal{A}=1$) this integral can be calculated analytically. In the experimental conditions at the Jefferson Lab the (JLab) [1], both the final electron and the proton were detected in order to reduce background. However, elastic scattering kinematics was restored by the final proton kinematics, while electron momentum was integrated over. Therefore, the formalism of Eq. (4) applies for this case.

In this paper we calculate the model independent RC to two experimental situations which are currently dealt with in Collaborations at CEBAF, Mainz accelerater (MAMI), and MIT Bates: measurement of polarization asymmetry in terms of leptonic variables; measurement of asymmetry in recoil proton in terms of hadronic variables. Our approach allows one to take into account the lowest-order RC exactly and to calculate the RC within experimental cuts.

II. KINEMATICS AND BORN PROCESS

The Born[2] cross section of the process (2) can be written in the form

$$d\sigma_0=\frac{M_0^2}{4p_1k_1}d\Gamma_0=M_0^2\frac{dQ^2}{16\pi S^2}, \tag{5}$$

where $S=2k_1p$. Kinematical limits for Q^2 are defined as

$$0\leqslant Q^2\leqslant\frac{\lambda_s}{S+m^2+M^2},\quad \lambda_s=S^2-4m^2M^2, \tag{6}$$

where m,M are the electron and proton masses. Because of axial symmetry the integration over the azimuthal angle ϕ can be performed analytically. However, in our case the kinematical cuts are dependent on this angle so we will consider the two-dimensional Born cross section

$$d\sigma_0=\frac{M_0^2}{4p_1k_1}d\Gamma_0=M_0^2\frac{dQ^2d\phi}{32\pi^2S^2}. \tag{7}$$

The born matrix element is

$$M^2=\frac{e^4}{Q^4}L^0_{\mu\nu}W_{\mu\nu}. \tag{8}$$

We use standard definitions for the (unpolarized) leptonic tensor and for the hadronic tensor

$$W^u_{\mu\nu}=\sum_{i=1}^{2}w^i_{\mu\nu}\mathcal{F}_i \tag{9}$$

with

$$w^1_{\mu\nu}=-g_{\mu\nu},\quad w^2_{\mu\nu}=\frac{p_{1\mu}p_{1\nu}}{M^2}, \tag{10}$$

and $\tau_p=Q^2/4M^2$,

$$\mathcal{F}_1=4\tau_pM^2G_M^2,\quad \mathcal{F}_2=4M^2\frac{G_E^2+\tau_pG_M^2}{1+\tau_p}. \tag{11}$$

[2]Throughout the paper, by "Born" we mean "one photon exchange."

It is convenient to define the contractions

$$2\theta_B^1 = L^0_{\mu\nu} w^1_{\mu\nu} = 2Q^2, \tag{12}$$

$$2\theta_B^2 = L^0_{\mu\nu} w^2_{\mu\nu} = \frac{1}{M^2}[S(S-Q^2) - M^2 Q^2]. \tag{13}$$

As a result, for the Born cross section we obtain the well known formula

$$\frac{d\sigma_0}{dQ^2} = \frac{2\pi\alpha^2}{S^2 Q^4} \sum_i \theta_B^i \mathcal{F}_i, \tag{14}$$

which can be reduced to

$$\frac{d\sigma_0}{dQ^2} \approx \frac{4\pi\alpha^2}{Q^4} \frac{G_E^2 + \tau_p G_M^2}{1+\tau_p} \tag{15}$$

in the ultrarelativistic approximation $M^2 \ll S^2$.

A. Polarized part of cross section

We consider two possible polarization measurements.

(1) The initial proton is polarized and the final electron is detected to reconstruct Q^2. In this case there are four experimental situations for asymmetry definition: the target is polarized along (perpendicular) to the beam or $\vec{q}$ ($q = p_2 - p_1$). Corresponding polarization four-vectors are denoted as η_L (η_T) or η_L^q (η_T^q).

(2) Polarization and momentum of the final proton are measured. Two polarization states should be considered: the final proton is polarized along (η_L') and perpendicular (η_T') to $\vec{q}$.

If the polarization vector is kept in a general form the polarization part of the hadronic tensor can be written as

$$W^p_{\mu\nu} = \sum_{i=3}^{4} w^i_{\mu\nu} \mathcal{F}_i \tag{16}$$

with

$$w^3_{\mu\nu} = -i\epsilon_{\mu\nu\lambda\sigma} \frac{q_\lambda \eta_\sigma}{M}, \quad w^4_{\mu\nu} = i\epsilon_{\mu\nu\lambda\sigma} \frac{q_\lambda p_{1\sigma} \eta q}{M^3}. \tag{17}$$

For the case of initially polarized particles, we have to choose the corresponding representation for the polarization vector and structure functions in the forms

$$\mathcal{F}_3 = -2M^2 G_E G_M, \quad \mathcal{F}_4 = -M^2 G_M \frac{G_E - G_M}{1+\tau_p}, \tag{18}$$

when the total hadronic tensor $W_{\mu\nu} = W^u_{\mu\nu} + W^p_{\mu\nu}$. In the case of final polarization states ($\eta \to \eta'$) the same formulas are used for structure functions (18) up to the different sign for the last term ($\mathcal{F}_4 \to -\mathcal{F}_4$).

The polarization four-vector η can be expressed in terms of the four-momentum of the particles in the reaction. The four considered cases correspond to four representations of polarization vectors:

$$\eta_L = \frac{1}{\sqrt{\lambda_s}} \left(k_1 - \frac{S}{M} p_1 \right), \tag{19}$$

$$\eta_T = \frac{1}{\sqrt{\lambda_s \lambda}} [(-SX + 2M^2 Q^2 + 4m^2 M^2) k_1 + \lambda_s k_2 - (SQ^2 + 2m^2 S_x) p_1],$$

$$\eta_L^q = \frac{1}{\sqrt{\lambda_q}} \left(2M(k_1 - k_2) - \frac{(S-X)}{M} p_1 \right),$$

$$\eta_T^q = \frac{1}{\sqrt{\lambda_q \lambda}} [(2M^2 Q^2 - S_x X) k_1 + (2M^2 Q^2 + S_x S) k_2 - Q^2 (S+X) p_1],$$

where $\lambda = SXQ^2 - m^2 \lambda_q - M^2 Q^4$ and $\lambda_q = S_x^2 + 4M^2 Q^2$, $S_x = S - X$. We note that the task of the calculation is reduced to contraction of the leptonic tensors at the Born and RC levels with $w^{3,4}_{\mu\nu}$, using the corresponding polarization vector representation of the general form

$$\eta = 2(a_\eta k_1 + b_\eta k_2 + c_\eta p_1) \tag{20}$$

and subsequent integration. The variable X is calculated in different ways for the Born and radiative processes. In the first case it is defined as $S - Q^2$ and it depends on the inelasticity $S - Q^2 - v$ for RC. The definition of inelasticity v is given in the next section. It should be noted that we do not consider effects of normal polarization, because the polarization parts of both the Born and model independent RC cross sections are exactly zero in this case. It allows us to keep only three basis vectors in Eq. (20).

The polarization part of the Born cross section is given by Eq. (14) for two additional terms in the sum over $i = 3,4$. The functions θ_i^B have the forms

$$\theta_3^B = \frac{2m}{M}(q\eta\, k_2 \xi - \eta\xi Q^2),$$

$$\theta_4^B = \frac{mQ^2 q\eta}{M^3}(2p_1\xi - k_2\xi), \tag{21}$$

where the lepton polarization vector can be defined as

$$\xi = \frac{2}{\sqrt{\lambda_s}} \left(\frac{S}{m} k_1 - m\, p_1 \right). \tag{22}$$

If Q^2 is calculated in terms of hadronic variables, the polarization vector expansion reads

$$\eta'_{L,T} = 2[a'_{L,T} k_1 + b'_{L,T}(p_1 - p_2) + c'_{L,T} p_1] \tag{23}$$

with

$$a'_L=0,\quad b'_L=-\frac{Q^2+2M^2}{2M\sqrt{\lambda_M}},\quad c'_L=\frac{Q^2}{2M\sqrt{\lambda_M}},\tag{24}$$

and

$$a'_T=\frac{Q^2(Q^2+4M^2)}{2\sqrt{\lambda_h}\sqrt{\lambda_M}},\tag{25}$$

$$b'_T=\frac{Q^2S+2M^2Q_u^2}{2\sqrt{\lambda_h}\sqrt{\lambda_M}},\tag{26}$$

$$c'_T=-\frac{Q^2(2S-Q_u^2)}{2\sqrt{\lambda_h}\sqrt{\lambda_M}}.\tag{27}$$

Here $\lambda_h=SQ^2(S-Q_u^2)-M^2Q_u^4-m^2\lambda_M$ and $\lambda_M=Q^2(Q^2+4M^2)$. As in the case above, we keep the variable Q_u^2, which is Q^2 and Q^2+u for the Born and RC cases, respectively. The quantity u is related to the invariant mass of the unobserved state. It is also called inelasticity and is defined below [for the case of hadronic variables see Eq. (54)].

It is easy to verify that the four-vectors η'_L and η'_T satisfy the necessary conditions of normalization and orthogonality:

$$\eta'_L p_2=\eta'_T p_2=0,\quad \eta'_L\eta'_T=0,\quad \eta_L'^2=\eta_T'^2=-1.$$

In the rest frame of the final proton $p_2=(M,\vec{0})$ the vector of longitudinal polarization reads

$$\eta'_L=(0,\vec{n}),\quad \vec{n}^2=1.\tag{28}$$

The direction of the three-vector $\vec{n}$ coincides with the direction of the three-vector $\vec{p}_2$ in the laboratory system. Therefore, η'_L indeed describes the longitudinal polarization of the scattered proton. The four-vector η'_T has the form $(0,\vec{m})$, $\vec{n}\cdot\vec{m}=0$, in both the laboratory and rest frame systems of the scattered proton. Thus, it describes the transverse polarization in the scattered plane. It can be defined up to a sign only.

In the case of longitudinal polarization, only the term with G_M^2 contributes to the spin dependent part of the cross section. The reason is that the part proportional to G_EG_M goes to zero for $\eta=\eta^{\parallel}$. The situation is just contrary in the case of transverse polarization. A simple calculation gives

$$\frac{\eta'_L}{\eta'_T}=\frac{G_M}{G_E}\sqrt{\frac{-q^2}{M^2}}\frac{k_1p_1+k_1p_2}{\sqrt{4(k_1p_1+k_1p_2)^2+q^2(4M^2-q^2)}}.\tag{29}$$

It is easy to verify that in the Breit system, where $p_1=(E,-\vec{q}/2)$, $p_2=(E,\vec{q}/2)$, $q=(0,\vec{q})$, the right side of Eq. (29) coincides (up to a sign) with the expression given in [10]. Indeed, in this system $\varepsilon_1=\varepsilon_2=\sqrt{-q^2}/2\sin\theta_B/2$ (θ_B is the electron scattering angle in the Breit system) and

$$\frac{(k_1p_1+k_1p_2)}{\sqrt{4(k_1p_1+k_1p_2)^2+q^2(4M^2-q^2)}}=\frac{1}{2\cos\theta_B/2}.$$

III. RADIATIVE EFFECTS

A. Leptonic variables

For the cross section of the radiative process

$$e(k_1)+N(p_1)\rightarrow e'(k_2)+\gamma(k)+N(p_2),\tag{30}$$

we have the expression

$$d\sigma_r=\frac{M_r^2}{4p_1k_1}d\Gamma_r.\tag{31}$$

The cross section of the process depends on Q_l^2 which for simplicity is referred to as Q^2 in this section. The phase space

$$d\Gamma_r=\frac{1}{(2\pi)^5}\frac{d^3p_2}{2p_{20}}\frac{d^3k_2}{2k_{20}}\frac{d^3k}{2k_0}\delta(p_1+k_1-k_2-k-p_2)\tag{32}$$

can be parametrized in terms of three variables: inelasticity $v=\Lambda^2-M^2$ ($\Lambda=p_1+k_1-k_2$), $\tau=kq/kp_1$, and the angle ϕ_k between planes $(\mathbf{q},\mathbf{k})$ and $(\mathbf{k}_1,\mathbf{k}_2)$. Using the result (V.7.7) of [11] we have

$$d\Gamma_r=\frac{dQ^2}{4(2\pi)^4S}\int_0^{v_m}\frac{dv}{4\sqrt{\lambda_q}}\int_{\tau_{min}}^{\tau_{max}}d\tau\frac{v}{(1+\tau)^2}\int_0^{2\pi}d\phi_k,\tag{33}$$

where $\lambda_q=(v+Q^2)^2+4M^2Q^2$ and we use the variable τ instead of the standard t,

$$t=Q^2+v-R,\quad R=\frac{v}{1+\tau}.\tag{34}$$

It allows us to present the final result in a form close to that in Refs. [12–16]. The limits of integration are defined as

$$v_m=\frac{1}{2m^2}(\sqrt{\lambda_s}\sqrt{\lambda_m}-2m^2Q^2-Q^2S)\tag{35}$$

$$=\frac{2Q^2[\lambda_s-Q^2(S+m^2+M^2)]}{Q^2S+2m^2Q^2+\sqrt{\lambda_s}\sqrt{\lambda_m}}\tag{36}$$

$$\approx S-Q^2-\frac{M^2Q^2}{S}\tag{37}$$

and

$$2M^2\tau_{max,min}=v+Q^2\pm\sqrt{\lambda_q}.\tag{38}$$

The matrix element squared of the radiative process is

$$M_r^2 = \frac{e^6}{Q_h^4} L^r_{\mu\nu} W_{\mu\nu}. \tag{39}$$

The leptonic tensor of the radiative process is standard and can be found, for example, in [14]. We note that here we use a more standard definition of the tensors (with an additional factor of 2 compared to those from Ref. [14]). For contractions we have

$$L^r_{\mu\nu} w^i_{\mu\nu} = 4\pi\sqrt{\lambda_q} \sum_{j=1}^{3} R^{j-3}\theta_{ij}. \tag{40}$$

The functions θ_{ij} are similar to the ones given in Appendix B of Ref. [13]. However, there they are integrated over ϕ_k. We refrain from this integration because of possible dependence of the acceptance function on this angle. The explicit form of the functions in our general case is discussed in the Appendix.

We note that the well-known formula for the soft-photon approximation is immediately obtained on keeping the term with $j=1$ and restricting integration over v as $v_1 < v < v_2 \ll S$ (small photon energy):

$$\frac{d\sigma_r}{dQ^2} = \frac{2\alpha}{\pi}(l_m - 1)\ln\frac{v_2}{v_1}\frac{d\sigma_0}{dQ^2}, \tag{41}$$

where $l_m = \ln(Q^2/m^2)$. For angular integration the formula (27) of Ref. [15] was used.

Straightforward integration over photon phase space is not possible because of infrared divergence. The first step of the solution is the identity transformation of the integrand

$$\sigma_R = \sigma_R - \sigma_{IR} + \sigma_{IR} = \sigma_F + \sigma_{IR}, \tag{42}$$

where σ_F is finite for $k \to 0$ (here and below we use the shortened notation for differential cross sections $\sigma_R \equiv d\sigma_R/dQ^2$, and so on). There is some ambiguity in the definition of σ_{IR}. Only the asymptotic expression in the limit $k \to 0$ is unambiguous.[3] In our case we construct σ_{IR} using the term with $j=1$ in Eq. (40) and form factors estimated at the Born point. This term is factorized in front of the Born cross section as

$$\sigma_0 \frac{2}{\pi}\int \frac{d^3k}{2k_0} F_{IR}, \quad F_{IR} = \left(\frac{k_1}{2k_1k} - \frac{k_2}{2k_2k}\right)^2. \tag{43}$$

As a result, the infrared part can be written in the factorized form

$$\sigma_{IR} = \frac{\alpha}{\pi}\delta_R^{IR}\sigma_0 = \frac{\alpha}{\pi}(\delta_S + \delta_H)\sigma_0. \tag{44}$$

[3]There is one more limitation. We must provide the conditions of applicability of the theorem about changing the order of integration and the limit. For example, uniform convergence is required. In practice, this means that we may subtract the quantity with the same denominator.

The quantities δ_S and δ_H appear after additional splitting of the integration region over the inelasticity v by the infinitesimal parameter $\bar{v}$:

$$\delta_S = \frac{-1}{\pi}\int_0^{\bar{v}} dv \int \frac{d^{n-1}k}{(2\pi\mu)^{n-4}k_0} F_{IR}\,\delta((\Lambda - k)^2 - M^2),$$

$$\delta_H = \frac{-1}{\pi}\int_{\bar{v}}^{v_m} dv \int \frac{d^3k}{k_0} F_{IR}\,\delta((\Lambda - k)^2 - M^2). \tag{45}$$

We note that, in contrast to the Mo and Tsai formalism, here the artificial parameter $\bar{v}$ completely cancels in the final expressions. The way to calculate these integrals was suggested in the [5] (see also [15] and the review [7]). In our case we have

$$\delta_S = 2\left(P_{IR} + \ln\frac{\bar{v}}{\mu M}\right)(l_m - 1) + \ln\frac{S(S-Q^2)}{m^2M^2} + S_\phi,$$

$$\delta_H = 2(l_m - 1)\ln\frac{v_m}{\bar{v}}. \tag{46}$$

These contributions have to be added to the vertex correction, which is standard:

$$\delta_V = -2\left(P_{IR} + \ln\frac{m}{\mu}\right)(l_m - 1) - \frac{1}{2}l_m^2 + \frac{3}{2}l_m - 2 + \frac{\pi^2}{6}. \tag{47}$$

For this sum we have the following expression where the infrared divergent term P_{IR} and the quadratic term l_m^2 are explicitly canceled out:

$$\frac{\alpha}{\pi}(\delta_S + \delta_H + \delta_V) = \delta_{inf} + \delta_{VR}, \tag{48}$$

where

$$\delta_{inf} = \frac{\alpha}{\pi}(l_{m-1})\ln\frac{v_m^2}{S(S-Q^2)},$$

$$\delta_{VR} = \frac{\alpha}{\pi}\left[\frac{3}{2}l_m - 2 - \frac{1}{2}\ln^2\frac{S}{S-Q^2} + \mathrm{Li}_2\left(1 - \frac{M^2Q^2}{S(S-Q^2)}\right) - \frac{\pi^2}{6}\right]. \tag{49}$$

Here we used the ultrarelativistic expression for the function S_ϕ from [17].

Finally, the cross section that takes into account radiative effects can be written as

$$\sigma_{obs} = \sigma_0 e^{\delta_{inf}}(1 + \delta_{VR} + \delta_{vac}) + \sigma_F. \tag{50}$$

Here the corrections δ_{inf} and δ_{vac} come from radiation of soft photons and effects of vacuum polarization. The correction δ_{VR} is an infrared-free sum of factorized parts of the real and virtual photon radiation, and σ_F is the infrared-free contribution of the bremsstrahlung process:

$$\sigma_F=-\frac{\alpha^3}{2S^2}\int_0^{v_m}dv\int_{\tau_{min}}^{\tau_{max}}\frac{d\tau}{1+\tau}\int_0^{2\pi}d\phi_k \times\sum_i\left[\sum_{j=1}^{3}\mathcal{A}R^{j-2}\theta_{ij}\frac{\mathcal{F}_i}{Q_h^4}-4F_{IR}^0\theta_i^B\frac{\mathcal{F}_i^0}{RQ_l^4}\right]. \quad (51)$$

Here $\mathcal{A}$ is integrated over the ϕ acceptance function.

B. Hadronic variables

For the cross section of the radiative process

$$e(k_1)+N(p_1)\to e'(k_2)+\gamma(k)+N(p_2), \quad (52)$$

we have an expression similar to Eq. (31):

$$d\sigma_r=\frac{M_r^2}{4p_1k_1}d\Gamma_r. \quad (53)$$

The parametrization of photonic phase space and integration over it developed for so-called hadronic emission within the Bardin and Shumeiko approach [5] can be directly applied to this case. Thus the phase space (32) can be parametrized in terms of three invariant variables [18,19]; namely, inelasticity $u=(k_2+k)^2-m^2=2k_2k$, $w=2k_1k$, and $z=2p_2k$:

$$d\Gamma_r=\frac{dQ^2d\phi}{(4\pi)^4S}\int_0^{u_m}du\int_{w_{min}}^{w_{max}}dw\int_{z_{min}}^{z_{max}}\frac{dz}{\pi\sqrt{-R_z}}, \quad (54)$$

where R_z comes from the Gramm determinant [16$R_z=\Delta(k_1,p_1,p_2,k)$] and coincides with the standard R_z function appearing in the Bardin-Shumeiko approach [5,18]. Explicitly it reads

$$R_z=A_zz^2-2B_zz+C_z. \quad (55)$$

For completeness we give the coefficients in our notation:[4]

$$A_z=\lambda_q=(u+Q^2)^2+4Q^2m^2, \quad (56)$$

$$B_z=u(u+Q^2)s_q-(u-Q^2)Sw-2m^2Q^2(u-w),$$

$$C_z=(us_q-Sw)^2+4M^2uw(Q^2+u-w)-4M^2m^2(u-w)^2,$$

where $s_q=S-Q^2+w-u$.

We note that we introduced the invariant variable z which corresponds to the azimuthal angle in Eq. (33). This variable is more convenient for introducing explicit expressions for experimental cuts.

The limits of integration in Eq. (54) are defined as

$$u_m=\frac{1}{2M^2}(\sqrt{\lambda_s}\sqrt{\lambda_M}-2M^2Q^2-Q^2S) \quad (57)$$

$$=\frac{2Q^2[\lambda_s-Q^2(S+m^2+M^2)]}{Q^2S+2M^2Q^2+\sqrt{\lambda_s}\sqrt{\lambda_M}} \quad (58)$$

and

$$w_{max,min}=\frac{u}{2(u+m^2)}(Q^2+u+2m^2\pm\sqrt{\lambda_q}). \quad (59)$$

The limits $z_{min,max}$ are defined as solutions of the equation $R_z=0$:

$$\lambda_qz_{max,min}=B_z\pm\sqrt{D}, \quad (60)$$

where

$$D=4[M^2\lambda_q+m^2Q^4-Q^2S(S-Q^2-u)] \times[m^2(w-u)^2+uw(w-u-Q^2)]. \quad (61)$$

The two solutions of the equation $D=0$ give limits on w [Eq. (59)].

The matrix element squared of the radiative process is calculated as

$$M_r^2=-\frac{e^6}{Q^4}L^r_{\mu\nu}W_{\mu\nu}=-\frac{e^6}{Q^4}(T_{IR}+T_3+T_4+T_{34}) \quad (62)$$

where

$$T_3=\frac{4a'_{L,T}\mathcal{F}_3}{MS}\left(2\frac{m^2}{w^2}(Q^4z-2Q^2Su+Q^2uz-u^2S) +\frac{SQ^2}{u}(w-2Q^2)+\frac{S}{w}(4Q^4+3Q^2u+u^2)-Sw\right),$$

$$T_4=\frac{-2Q^2a'_{L,T}\mathcal{F}_4}{M^3S}\left(\frac{2m^2}{w^2}(Q^2Su-Q^2uz-2S^2u+Su^2 +2Suz-2uz^2)+\frac{S}{w}(-2Q^4+4Q^2S-2Q^2u-2Q^2z +2Su-u^2)-2S^2+Sw+2Sz\right),$$

$$T_{34}=\frac{2Q^2}{M^3S}(a'_{L,T}\mathcal{F}_4Q^2-2M^2c'_{L,T}\mathcal{F}_3-2b'_{L,T}\mathcal{F}_4Q^2 -c'_{L,T}\mathcal{F}_4Q^2)\left(2\frac{m^2}{w^2}(Q^2z-Su-2Sz+2z^2) +\frac{2SzQ^2}{uw}+\frac{S}{u}(2S-w-2z)+\frac{S}{w}(2Q^2-2S+u)\right).$$

The contribution T_{IR} is

$$T_{IR}=4\left(\frac{m^2}{w^2}+\frac{m^2}{u^2}-\frac{Q^2}{uw}\right)T_0, \quad (63)$$

[4] Q^2 in subsections B, C is Q_h^2.

$$T_0=-\frac{Q^2}{M^3}[2M^2\mathcal{F}_3(a'_{L,T}Q^2-c'_{L,T}Q^2+2c'_{L,T}S)+Q^2(Q^2-2S)\mathcal{F}_4(a'_{L,T}-2b'_{L,T}-c'_{L,T})].$$

The structure functions $\mathcal{F}_3$ and $\mathcal{F}_4$ are defined in Eq. (18) and are functions of Q_h^2. We note that the Born cross section can be written in terms of T_0 for a_η,b_η,c_η taken for $u\to 0$:

$$\sigma_0=\frac{\pi\alpha^2}{Q^4S^2}T_0^B. \tag{64}$$

The radiative cross section can be presented as

$$\sigma_R=\frac{-\alpha^3}{4S^2Q^4}\int\frac{du\,dw\,dz}{\pi\sqrt{-R_z}}\mathcal{A}(T_{IR}+T_3+T_4+T_{34}). \tag{65}$$

Performing the explicit integration over the region of small energies $(0<u_1<u_2\ll m^2,M^2,Q^2,S)$, we obtain the well-known result in the soft-photon limit:

$$\sigma_R=-\frac{\alpha}{\pi}\sigma_0\int_{u_1}^{u_2}du\left[\frac{1}{u}+\frac{m^2}{u(u+m^2)}-\frac{Q^2}{u\sqrt{\lambda_q}}\ln\frac{w_{max}}{w_{min}}\right]$$

$$=\frac{2\alpha}{\pi}\ln\frac{u_2}{u_1}(l_m-1)\sigma_0. \tag{66}$$

The radiative cross section has an infrared divergence, so for this case an identity transformation like (42) has to be performed also. However, the form factor is not dependent on photon variables, so only the acceptance function should be subtracted. The relevant integrals look similar. As a result, the infrared part can be written in the factorized form

$$\sigma_{IR}=\frac{\alpha}{\pi}\delta_R^{h,IR}\sigma_0=\frac{\alpha}{\pi}(\delta_S^h+\delta_H^h)\sigma_0. \tag{67}$$

The quantities δ_S and δ_H appear after additional splitting of the integration region over the inelasticity v by the infinitesimal parameter $\bar{v}$:

$$\delta_S^h=\frac{-1}{\pi}\int_0^{\bar{u}}du\int\frac{d^{n-1}k}{(2\pi\mu)^{n-4}k_0}F_{IR}\delta((\Lambda_h-k)^2-m^2),$$

$$\delta_H^h=\frac{-1}{\pi}\int_{\bar{u}}^{u_m}du\int\frac{d^3k}{k_0}F_{IR}\delta((\Lambda_h-k)^2-m^2), \tag{68}$$

where $\Lambda_h=k_1+p_1-p_2$ and F_{IR} is defined in Eq. (43). The integration gives the following explicit results:

$$\delta_S^h=2\left(P^{IR}+\ln\frac{\bar{u}}{m\mu}\right)(l_m-1)+1+l_m-l_m^2-\frac{\pi^2}{6},$$

$$\delta_H^h=2(l_m-1)\ln\frac{u_m}{\bar{u}}-\frac{1}{2}\ln^2\frac{u_m}{m^2}+\ln\frac{u_m}{m^2}-l_w(l_m+l_w-l_v)-\frac{\pi^2}{6}-\mathrm{Li}_2\left(-\frac{u_m}{Q^2}\right),$$

$$l_v=\ln\frac{u_m}{Q^2},\quad l_w=\ln\left(1+\frac{u_m}{Q^2}\right). \tag{69}$$

Their sum and the contribution of the vertex function yield again the result free of infrared divergence:

$$\delta_{VR}^h=l_m\left(l_v-l_w+\frac{3}{2}\right)-l_v-1-\frac{3}{2}l_w^2-\frac{1}{2}l_v^2+2l_wl_v-\mathrm{Li}_2\left(\frac{Q^2}{Q^2+u_m}\right). \tag{70}$$

The cross section that takes into account radiative effects can be written as

$$\sigma_{obs}=\sigma_0(1+\delta_{VR}^h+\delta_{vac})+\sigma_F^h. \tag{71}$$

The explicit expression for σ_F^h is

$$\sigma_F^h=-\frac{\alpha^3}{4Q^4S^2}\int_0^{u_m}du\int_{w_{min}}^{w_{max}}dw\int_{z_{min}}^{z_{max}}\frac{dz}{\pi\sqrt{-R_z}}\times[\mathcal{A}(T_{IR}+T_3+T_4+T_{34})-T_{IR}^B] \tag{72}$$

with $T_{IR}^B=T_{IR}^B(T_0\to T_0^B)$.

C. Kinematical cuts

In this section we show how experimental cuts can be introduced in this approach. As an example we consider the conditions of the experiment [1] at CEBAF.

The following restrictions on the momentum of the final electron and proton have to be made. We consider the high-resolution spectrometer (HRS) [1] as a rectangular area with some energy acceptance. We describe this rectangle by two angles between corresponding planes: $\theta_x^{e,p}$ and $\theta_y^{e,p}$. For one of the scattered particles these angular definitions are given in Fig. 1. The upper index corresponds to the detected particle, namely, the electron or the proton. The angles and momenta of the final particles being measured in the laboratory frame have to be expressed in terms of kinematical invariants. The simplest way to do it is to apply the formalism of Gramm determinants, for which a detailed description can be found in Ref. [11]. We will give starting expressions in terms of four-momenta and Gramm determinants as well as explicit results for the invariant variables used in Eq. (54). In terms of Gramm determinants the momenta (in the rest frame of p_1) of the final proton and electron are given by the formulas

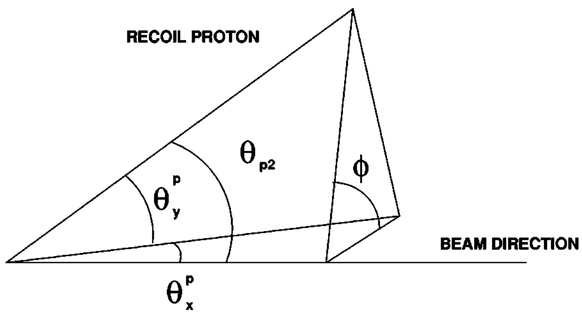

FIG. 1. Recoil proton angle definitions. The beam and the hadron-arm detector of the HRS define the horizontal plane.

$$|\mathbf{p}_2|^2 = -\frac{\Delta(p_1,p_2)}{p_1^2}, \quad |\mathbf{k}_2|^2 = -\frac{\Delta(p_1,k_2)}{p_1^2}. \tag{73}$$

This gives immediately for the Born process

$$|\mathbf{p}_2| = \frac{\sqrt{\lambda_q}}{2M}, \quad |\mathbf{k}_2| = \frac{\sqrt{\lambda_z}}{2M} \tag{74}$$

for both Born and radiative processes. Here $\lambda_z = (S-Q^2-z_1)^2 - 4M^2m^2$, $S_{qz} = S - Q^2 - z_1$.

The cosine of the polar angle of the direction of the final proton (with respect to the beam direction) is defined as

$$\cos\theta_{p2} = \frac{k_1p_1\, p_2p_1 - p_1^2\, p_2k_1}{\Delta(p_1,k_1)\Delta(p_1,p_2)}. \tag{75}$$

It gives

$$\sin^2\theta_{p2} = \frac{4\lambda_{sx}M^2}{\lambda_M S^2} \tag{76}$$

where $\lambda_{sx} = SXQ^2 - S_x^2M^2$. We have to use $X = S - Q^2$ and $S_x = Q^2$ for the Born process and $X = S - Q^2 - u$ and $S_x = Q^2 + u$ for the radiative one. In terms of the angle the horizontal and vertical angles of the proton momentum are

$$\sin\theta_y^p = \sin\phi\sin\theta_{p2},$$

$$\tan\theta_x^p = \cos\phi\tan\theta_{p2}, \tag{77}$$

where ϕ was introduced in Eq. (7).

At the Born level all vectors ($\mathbf{p}_2$, $\mathbf{k}_1$, and $\mathbf{k}_2$) are in the same plane. However, for the unobserved photon there is a nonzero angle $\Delta\phi$ between the planes ($\mathbf{k}_1,p_2$) and ($\mathbf{k}_1,k_2$). In terms of Gramm determinants it is defined as

$$\cos\Delta\phi = \frac{G\begin{pmatrix} p_1 & k_1 & p_2 \\ p_1 & k_1 & k_2 \end{pmatrix}}{\Delta(p_1,k_1,k_2)\Delta(p_1,k_1,p_2)}. \tag{78}$$

Explicitly we have

$$\sin^2\Delta\phi = -\frac{S^2}{4Q_e^2\lambda_{sx}\lambda_{qz}}\{[S(u-w) - z_1u + Q^2w]^2 + 4Q_e^2M^2uw + Q^2(2w+z_1)z_1(Q^2+u) + z_1^2uQ^2 + 2z_1Q^2uw - 2Sz_1(u+w)Q^2\} \tag{79}$$

where $\lambda_{qz} = SS_{qz} - 2M^2Q_e^2$, $Q_e^2 = Q^2 + u - w$, and $z_1 = z + u - w$. Now we can define the angles of the final electron for the radiative process:

$$\sin\theta_y^e = \sin(\phi+\Delta\phi)\sin\theta_{k2},$$

$$\tan\theta_x^e = \cos(\phi+\Delta\phi)\tan\theta_{k2}, \tag{80}$$

where θ_{k2} is the polar angle of the final electron. For the latter we have

$$\sin^2\theta_{k2} = \frac{4Q_e^2\lambda_{qz}M^2}{S_{qz}^2S^2}. \tag{81}$$

IV. NUMERICAL RESULTS

In this section we present the FORTRAN code MASCARAD (Sec. IV A) developed on the basis of the formalism presented in the last sections. This code uses Monte Carlo methods to calculate the radiative corrections to the observable quantities in polarized ep scattering measurements. The numerical results of applying this code for the kinematical conditions of Jlab are given in Secs. IV B and IV C with leptonic and hadronic variables, respectively. In the last subsection we discuss the influence of experimental cuts on observable quantities in polarized scattering.

A. FORTRAN code MASCARAD

There are two variants of the code: MASCARAD_L.F and MASCARAD_H.F dealing with leptonic and hadronic variables, respectively. The first code does not require any external libraries. However, the histogramming by HBOOK can be optionally included in the second variant. In this case MASCARAD_H.F requires CERNLIB installed. In the external file one can choose the kinematical variables, the accuracy of the calculation, and the value of the cut on inelasticity. An option to include kinematical cuts described in Sec. III C is available for MASCARAD_H.F. As output one has the value of the Born cross section and the radiative correction factor (with estimation of the statistical error) at chosen kinematical points. The source code for MASCARAD can be obtained at http://www.jlab.org/~aku/RC/

B. Numerical results: Leptonic variables

Both the spin averaged and spin dependent parts of the cross section (σ^u and σ^p) can be presented as

$$\sigma_{obs}^{u,p} = (1+\delta)\sigma_0^{u,p} + \sigma_R^{u,p}. \tag{82}$$

Both the factorized correction δ and the unfactorized cross section coming from the bremsstrahlung process contribute

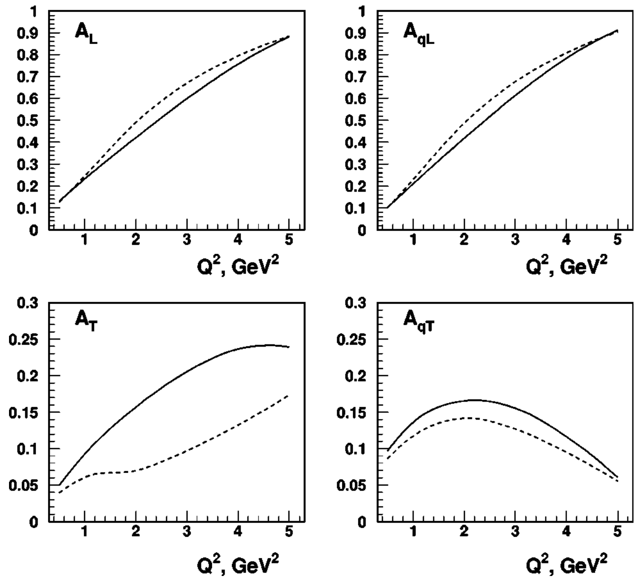

FIG. 2. Born (solid line) and observed (dashed line) asymmetries vs Q^2. No kinematical cuts on inelasticity were used. Electron beam energy $E=4$ GeV.

to the cross section. When polarization asymmetries of the elastic processes are considered, the factorized part of the total RC tends to cancel but the unfactorized part can give an important contribution.

Absolute and relative corrections to the asymmetry can be defined as [see Eq. (82)]

$$\Delta A_i = A_i - A_{i0} = \frac{(1+\delta)\sigma_0^p + \sigma_R^p}{(1+\delta)\sigma_0^u + \sigma_R^u} - \frac{\sigma_0^p}{\sigma_0^u}, \tag{83}$$

$$\Delta_i = \frac{A_i - A_{i0}}{A_{i0}} = \frac{\delta_p - \delta_u}{1+\delta+\delta_u}, \tag{84}$$

where the index i runs over all considered cases: $i = L,T,qL,qT$; $\delta_{u,p} = \sigma_R^{u,p}/\sigma_0^{u,p}$. Here the correction δ is usually large because of contributions of leading logarithms. However, it exactly cancels in the numerator of the expression for the correction to the asymmetry. This is the reason why the correction to the cross section can be large, while the correction to the asymmetry is relatively small.

The Born and observed asymmetries are presented in Fig. 2. The four lines correspond to the four considered cases defined in Sec. II A. No cuts were used for the missing mass. As a result, hard-photon emission gives different contributions to the spin averaged and spin dependent parts of the cross section due to its unfactorizing properties. For longitudinal asymmetries $\delta_p > \delta_u$ and there are positive contributions to the RC. The situation with transverse asymmetries is opposite.

One can see that the transverse asymmetry A_T with respect to the beam direction has a large correction. This is not in contradiction with the other plots in this figure. The polarized parts of the cross sections in the cases (L,T) and (qL,qT) are related to each other by some unitary transformation

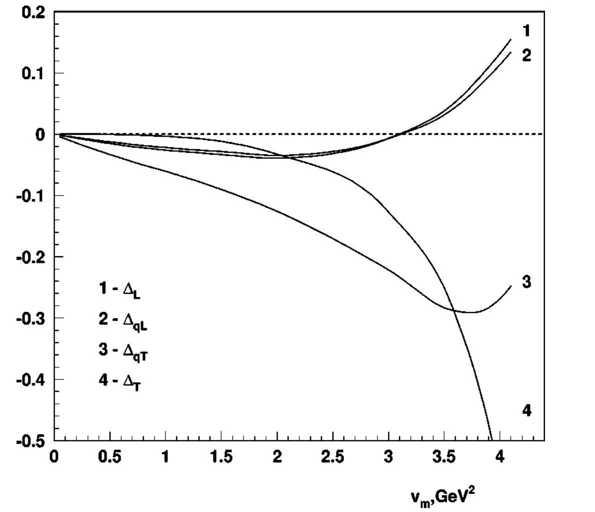

FIG. 3. Relative RC to asymmetries defined in Eq. (84) vs value of inelasticity cut for $Q^2=3$ GeV^2; $E=4$ GeV.

$$\frac{d\sigma_t^p}{dQ^2 dv} = \cos\gamma \frac{d\sigma_{qt}^p}{dQ^2 dv} - \sin\gamma \frac{d\sigma_{ql}^p}{dQ^2 dv}. \tag{85}$$

Because of the dependence of the polarization vectors on inelasticity [see Eq. (19) and definitions there] this angle γ is a function of v:

$$\sin^2\gamma = \frac{4M^2\lambda}{\lambda_s\lambda_q}, \quad \cos\gamma = \frac{SS_x + 2M^2Q^2}{\sqrt{\lambda_s\lambda_q}}, \tag{86}$$

and only unintegrated cross sections are related as Eq. (85). This sine strongly suppresses the cross section of the hard photon emission. Weighted with the sine and cosine, the cross sections in the right-hand side of Eq. (85) have the same signs and similar magnitude and therefore compensate each other. As a result, $\delta_p \ll \delta_u$ and the asymmetry A_T has a large negative contribution.

In practice, the RC to the asymmetries can be essentially reduced by applying a cut on the missing mass or inelasticity which is also a measured quantity in elastic electron-proton scattering. In Fig. 3 we show how these relative corrections depend on the value of the cut on missing mass or inelasticity.

C. Numerical results: Hadronic variables

As in the case of leptonic variables, let us define the relative RC to the ratio of recoil nucleon polarizations P_T/P_L as

$$\Delta = \frac{P_T/P_L - P_T^0/P_L^0}{P_T^0/P_L^0}. \tag{87}$$

In Fig. 4 this correction is given for several values of the cut on missing mass. It can be seen that the radiative correction in this case is smaller than in the case of leptonic variables.

In practice, however, the experimental situation can be more complicated than simply applying a single kinematical cut on the missing mass. In past and future experiments

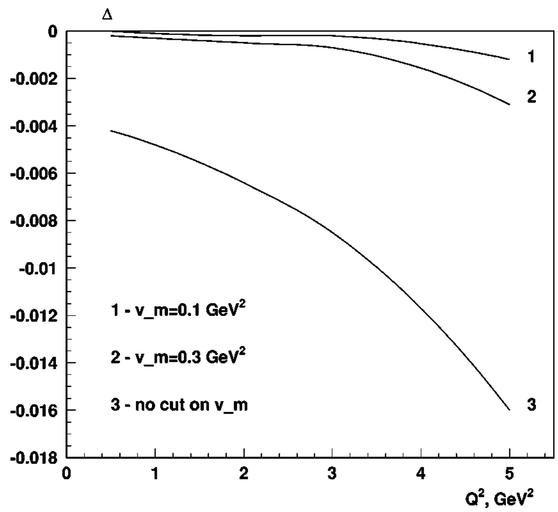

FIG. 4. Relative RC to the ratio of recoil proton polarizations vs Q^2 for three values of inelasticity cut.

TABLE I. The results for the RC to the asymmetry P_T/P_L.

E (GeV)	Q^2 (GeV^2)	Δ (%)
0.934	0.45–0.53	−1.01
0.934	0.77–0.81	−1.54
1.821	1.11–1.25	−0.95
3.395	1.37–1.59	−0.59
3.395	1.65–1.89	−0.64
4.087	1.75–2.01	−0.62
4.090	2.30–2.64	−0.71
4.087	2.77–3.17	−0.80
4.090	3.27–3.67	−0.95
4.845	3.5	−0.80
4.845	4.2	−0.96
5.545	4.9	−0.95
6.045	5.6	−0.97

[1,20] at JLab on measuring the ratio of elastic form factors of the proton, all the events appearing in detectors[5] were accepted for analysis. To calculate the RC in this situation we have to apply all cuts discussed in the previous section. Table I gives the results for the past [1] and future [20] experiments (above and below the line). As we can see the RC does not exceed 1%.

V. DISCUSSION AND CONCLUSION

In this paper we applied the approach of Bardin and Shumeiko [5] for calculation of the model independent radiative correction of lowest order in processes of elastic electron-proton scattering. Current experiments on the process measure different polarization observables such as spin asymmetries in different combinations of polarizations of initial particles and the ratio of recoil proton polarizations, allowing one to access the ratio of electromagnetic form factors of the proton. That is why special attention was paid to radiative corrections to the polarized parts of the cross section.

The chosen method of calculation allowed us to obtain explicit formulas in the cases of so-called electron and hadron variables. They correspond to the cases when the kinematics of the measured process is reconstructed from the momentum of the final electron and proton, respectively. It was shown that, although the formulas for the Born case are exactly the same for both cases, all ingredients of the RC calculation are different in these cases. The physical (or kinematical) reason for this is the fact that in the first case the radiating particle (electron) is measured, but in the second case it escapes unmeasured or is integrated over.

The explicit formulas for the lowest-order model independent radiative correction are exact up to the ultrarelativistic approximation which neglects the terms $\sim m^2/Q^2$. However, this is not a limitation of the approach. These terms can easily be restored if needed (for example, if the approach is applied to muon scattering and accuracy of $\sim m_\mu^2/Q^2$ is required).

In contrast to the case of inclusive deep inelastic scattering, the integration here is left for numerical analysis. All integrals are finite after using the procedure of covariant cancellation of infrared divergences. This form of solution allows one to include acceptance effects in the integrand. The function of acceptance usually depends on the final angles and momenta, which can be expressed in terms of integration variables. A proper way to do this is discussed in Sec. III C.

On the basis of the exact formulas, the FORTRAN package MASCARAD was developed. It includes codes for both the electron and hadron variable measurements. Applying the package to the radiative correction procedure allows one to include the model independent correction in the data analysis of current experiments (including polarization) of elastic electron-nucleon scattering. Our numerical analysis shows that radiative effects can be important especially in the cases of transversely polarized targets. However, using kinematical cuts such as a single cut on inelasticity or a cut on kinematical variables of the second (undetected) particle allows one to reduce the effect considerably.

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[5]Here we do not consider possible lost events in the detector. In other words, we take into account only geometrical acceptance. However, the influence of apparatus acceptance also can be taken into account in our approach by introducing some map of device acceptance to MASCARAD. There is a certain place in the code to do it.

APPENDIX

The functions θ_{ij} are defined the same way as in Appendix B of Ref. [12] or with more details in Appendix B of Ref. [13]. However, there the formulas are integrated over the photon azimuthal angle [or equivalently over z; see Eq. (B7) of Ref. [12]] Below we define the procedure for writing the explicit form of these functions in our case.

Formulas (B1),(B2) of Ref. [12] or (B.1)–(B.11) of Ref. [13] can be applied unchanged for our case. Instead of the functions F from (B5) [12] or (B.12) [13] we use the following expressions:

$$F_d = \frac{F}{z_1 z_2}, \quad F_{1+} = \frac{F}{z_1} + \frac{F}{z_2}, \quad F_{2\pm} = F\left(\frac{m^2}{z_2^2} \pm \frac{m^2}{z_1^2}\right), \tag{A1}$$

where

$$F = 1/(2\pi\sqrt{\lambda_Q}), \quad F_{IR} = F_{2+} - Q^2 F_d,$$

and

$$z_1 = \frac{1}{\sqrt{\lambda_q}}[Q^2 S_p + \tau(SS_x + 2M^2Q^2) - 2M\sqrt{\lambda_z}\cos\phi_k],$$

$$z_2 = \frac{1}{\sqrt{\lambda_q}}[Q^2 S_p + \tau(XS_x - 2M^2Q^2) - 2M\sqrt{\lambda_z}\cos\phi_k],$$

with

$$\lambda_z = (\tau - \tau_{min})(\tau_{max} - \tau)(SXQ^2 - M^2Q^4 - m^2\lambda_q). \tag{A2}$$

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