Perturbative QCD analysis of $B\to\phi K$ decays and power counting

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(Received 17 July 2001; published 5 November 2001)

We investigate the exclusive nonleptonic B meson decays $B\to\phi K$ in the perturbative QCD formalism. It is shown that the end-point (logarithmic and linear) singularities in the decay amplitudes do not exist, after k_T and threshold resummations are included. Power counting for emission and annihilation topologies of diagrams, including both factorizable and nonfactorizable ones, is discussed with Sudakov effects taken into account. Our predictions for the branching ratios $B(B\to\phi K)\sim 10\times 10^{-6}$ are larger than those ($\leqslant 4\times 10^{-6}$) from the factorization approach because of dynamical enhancement of penguin contributions. Whether this enhancement is essential for penguin-dominated modes can be justified by experimental data.

DOI: 10.1103/PhysRevD.64.112002 PACS number(s): 12.38.Bx, 11.10.Hi, 12.38.Qk, 13.25.Hw

I. INTRODUCTION

The perturbative QCD (PQCD) factorization theorem for the semileptonic decay $B\to\pi l\bar{\nu}$ has been proved in [1], which states that the soft divergences in the $B\to\pi$ form factor can be factorized into a light-cone B meson distribution amplitude and the collinear divergences can be factorized into a pion distribution amplitude order by order. The remaining finite contribution is assigned to a hard amplitude, which is calculable in perturbation theory. A meson distribution amplitude, though not calculable, is universal, since it absorbs long-distance dynamics, which is insensitive to specific decays of the b quark into light quarks with large energy release. The universality of nonperturbative distribution amplitudes is one of the important ingredients of the PQCD factorization theorem. Because of this universality, one extracts distribution amplitudes from experimental data and then employ's them to make model-independent predictions for other processes. In this paper we shall assume that the PQCD factorization theorem holds for two-body nonleptonic B meson decays, to which there is no difficulty in generalizing the proof [1]. The one-loop proof for the PQCD factorization of two-body decays has been given in [2].

The PQCD formalism for the charmed decays $B\to D^{(*)}\pi(\rho)$ [2,3] is restricted to twist-2 (leading-twist) distribution amplitudes. For charmless decays such as $B\to K\pi$, $\pi\pi$, and KK [4–8], contributions from two-parton twist-3 (next-to-leading-twist) distribution amplitudes are introduced via the penguin operators O_{5-8} in the effective Hamiltonian for weak decays. It has been argued [9] that two-parton twist-3 contributions are in fact not suppressed by a power of $1/M_B$, M_B being the B meson mass. A complete leading-power PQCD analysis of the heavy-to-light $B\to\pi$, ρ form factors, including both twist-2 and twist-3 contributions, has been performed in [10]. There exist many other higher-twist sources in B meson decays, whose contributions are indeed down by a power of $1/M_B$. These sources include the B meson and b quark mass difference $\bar{\Lambda}=M_B-m_b$, the light quark masses m_u, m_d, and m_s, and the light pseudo-scalar meson masses M_π and M_K. Those from three-parton distribution amplitudes are further suppressed by the coupling constant α_s. All these sub-leading contributions will be neglected in the current formalism.

In this work we shall perform a PQCD analysis of the $B\to\phi K$ decays up to corrections of $O(\bar{\Lambda}/M_B)$. These modes involve different topologies of diagrams, such as factorizable (also nonfactorizable) emission and annihilaton. We predict the branching ratios and CP asymmetries of the modes

$$B_d^0\to\phi K^0,\quad B^\pm\to\phi K^\pm. \tag{1}$$

It will be found that two-parton twist-3 contributions are comparable to twist-2 ones as expected. Our predictions for the braching ratios

$$B(B^\pm\to\phi K^\pm)=(10.2^{+3.9}_{-2.1})\times 10^{-6}$$

$$B(B_d^0\to\phi K^0)=(9.6^{+3.7}_{-2.0})\times 10^{-6}, \tag{2}$$

are larger than those from the factorization approach [11] and from the QCD factorization approach [12–14], which are located within $4.3^{+3.0}_{-1.4}\times 10^{-6}$ with the uncertainty arising mainly from the inclusion of annihilation contributions [14]. The mechanism responsible for the larger branching ratios in

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the PQCD formalism is dynamical enhancement of penguin contributions [4,5]. Note that the current experimental data of $B(B^{\pm}\to\phi K^{\pm})$,

$$\text{CLEO [15]:}\quad (5.5^{+2.1}_{-1.8}\pm 0.6)\times 10^{-6},$$

$$\text{Belle [16]:}\quad (10.6^{+2.1}_{-1.9}\pm 2.2)\times 10^{-6},$$

$$\text{BaBar [17]:}\quad (7.7^{+1.6}_{-1.4}\pm 0.8)\times 10^{-6}, \tag{3}$$

and those of $B(B^{0}\to\phi K^{0})$,

$$\text{CLEO [15]:}\quad <12.3\times 10^{-6},$$

$$\text{Belle [16]:}\quad (8.7^{+3.8}_{-3.0}\pm 1.5)\times 10^{-6},$$

$$\text{BaBar [17]:}\quad (8.1^{+3.1}_{-2.5}\pm 0.8)\times 10^{-6}, \tag{4}$$

are still not very consistent with each other.

The PQCD factorization formulas for $B\to\phi K$ decay amplitudes have been derived independently in [18]. Here we shall further discuss the power behavior of the factorizable emission and annihilation amplitudes, and the nonfactorizable amplitudes in $1/M_B$, whose relative importance is given by

$$\text{emission:annihilation:nonfactorizable}=1:\frac{2m_0}{M_B}:\frac{\bar{\Lambda}}{M_B}, \tag{5}$$

with m_0 being the chiral symmetry breaking scale. In the heavy quark limit the annihilation and nonfactorizable amplitudes are indeed power-suppressed compared to the factorizable emission ones. Therefore, the PQCD formalism for two-body charmless nonleptonic B meson decays coincides with the factorization approach as $M_B\to\infty$. We shall also explain why the annihilation amplitudes are mainly imaginary and investigate theoretical uncertainty in the PQCD approach.

Dynamical enhancement is a unique feature of the PQCD approach, which does not exist in the factorization or QCD factorization approach. We argue that the $B\to\phi K$ modes are more appropriate for testing this mechanism in penguin-dominated nonleptonic B meson decays compared to the $B\to K\pi$ decays [9]. The large $B\to K\pi$ branching ratios may not be regarded as evidence of dynamical enhancement: they can also be achieved by chiral enhancement (the kaon is a pseudo-scalar meson) and by choosing a large unitarity angle $\phi_3\sim 120°$ [19], which leads to constructive interference between penguin and emission contributions. The $B\to\phi K$ modes are not chirally enhanced, because ϕ is a vector meson, and insensitive to the variation of the angle ϕ_3, because they are pure penguin processes. If the data of $B(B\to\phi K)$ are settled down at values around 10×10^{-6} in the future, dynamical enhancement will gain a strong support.

On the other hand, precise measurement of the CP asymmetry in the $B\to\phi K$ decays is important for new physics search and for the determination of the unitarity angle ϕ_1 with high degree of accuracy [20,21]. This measurement is experimentally accessible at the early stage of the asymmetric B factories. The $B\to\phi K$ decays arise from penguin (loop) effects, while the $B\to J/\psi K$ decays involve only tree amplitudes. The search for different CP asymmetries in the $B\to J/\psi K_s$ and ϕK_s decays, with the common source from B^0-$\bar{B}^0$ mixing, provides a promising way to discover new physics [22,23]: a difference of $|A_{CP}(J/\psi K_s)-A_{CP}(\phi K_s)|>5\%$ would be an indication of new physics. This subject will be addressed elsewhere [24]. Furthermore, the ϕ and J/ψ mesons are bound states of the $s\bar{s}$ and $c\bar{c}$ quarks. It is also interesting to compare their branching ratios, which reflect the mass effect from charm quarks.

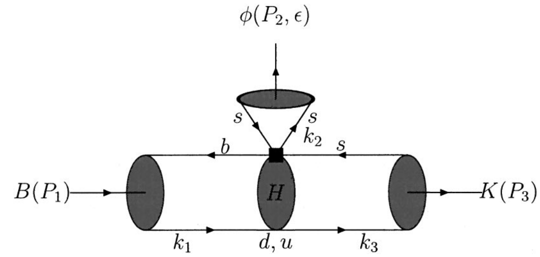

FIG. 1. Leading contribution to the $B\to\phi K$ decays, where H denotes the hard amplitude and k_i, $i=1$, 2, and 3 are the parton momenta.

We demonstrate the importance of k_T and threshold resummations by studying the $B\to\pi,K$ transition form factors in Sec. II. The power counting and the factorization formulas for various topologies of amplitudes are given in Sec. III. The numerical analysis is performed in Sec. IV. Section V is the conclusion. Twist-2 and two-parton twist-3 distribution amplitudes for the kaon and for the ϕ meson are defined in the Appendix.

II. SUDAKOV SUPPRESSION

In this section we briefly review the importance of k_T and threshold resummations for an infrared finite PQCD calculation of heavy-to-light transition form factors [10]. Consider the leading diagram shown in Fig. 1 for the $B\to\phi K$ decays in the kinematic region with a fast-recoil kaon. The B meson momentum P_1, the ϕ meson momentum P_2, the longitudinal polarization vector ϵ, and the kaon momentum P_3 are chosen, in light-cone coordinates, as

$$P_1=\frac{M_B}{\sqrt{2}}(1,1,\mathbf{0}_T),\quad P_2=\frac{M_B}{\sqrt{2}}(1,r_\phi^2,\mathbf{0}_T),$$

$$P_3=\frac{M_B}{\sqrt{2}}(0,1-r_\phi^2,\mathbf{0}_T),\quad \epsilon=\frac{1}{\sqrt{2}r_\phi}(1,-r_\phi^2,\mathbf{0}_T), \tag{6}$$

with the ratio $r_\phi=M_\phi/M_B$, M_ϕ being the ϕ meson mass. At the end of the derivation of the factorization formulas, the terms $r_\phi^2\sim 0.04$ in the above kinematic variables will be neglected. We treat the kaon as a massless particle, and define the ratio $r_K=m_0/M_B$ for the kaon, which will appear in the

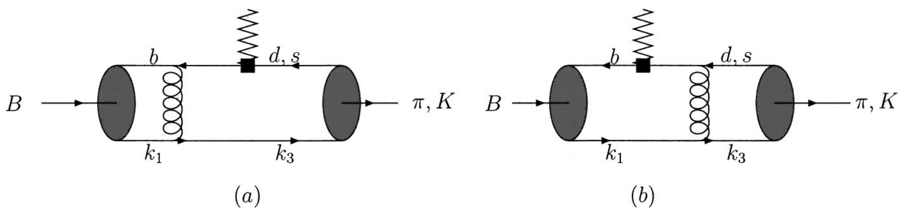

FIG. 2. Lowest-order diagrams for the $B\to\pi,K$ transition form factors.

normalization of the twist-3 kaon distribution amplitudes. The B meson is at rest under the above parametrization of momenta.

A. k_T and threshold resummations

It has been known that the lowest-order diagram in Fig. 2(a) for the $B\to\pi$ form factor $F^{B\pi}$ is proportional to $1/(x_1x_3^2)$ without including parton transverse momenta k_T, where $x_1=k_1^+/P_1^+$ $(x_3=k_3^-/P_3^-)$ is the momentum fraction associated with the spectator quark on the B meson (pion) side. If the pion distribution amplitude vanishes like x_3 as $x_3\to 0$ (in the twist-2 case), $F^{B\pi}$ is logarithmically divergent. If the pion distribution amplitude is a constant as $x_3\to 0$ (in the twist-3 case), $F^{B\pi}$ even becomes linearly divergent. These end-point singularities have caused critiques on the perturbative evaluation of the $B\to\pi$ form factor. Several methods have been proposed to regulate the above singularities. An on-shell b quark propagator has been subtracted from the hard amplitude as $x_3\to 0$ in [25]. However, this subtraction renders the lepton energy spectrum of the semileptonic decay $B\to\pi l\bar{\nu}$ as vanishing as the lepton energy is equal to half of its maximal value. Obviously, this vanishing is unphysical, indicating that the subtraction may not be an appropriate way to regulate the singularity. The subtraction also leads to a value of $F^{B\pi}$ at maximal recoil, which is much smaller than the expected one 0.3. A lower bound of x_3 of $O(\bar{\Lambda}/M_B)$ has been introduced in [26] to make the convolution integral finite. However, the outcomes depend on the cutoff sensitively, and PQCD loses its predictive power.

A self-consistent prescription has been proposed in [27], where parton transverse momenta k_T are retained in internal particle propagators involved in a hard amplitude. In the end-point region the invariant mass of the exchanged gluon is only $O(\bar{\Lambda}^2)$ without including k_T. The inclusion of k_T brings in large double logarithms $\alpha_s \ln^2(k_T/M_B)$ through radiative corrections, which should be resummed in order to improve perturbative expansion. k_T resummation [28,29] then gives a distribution of k_T with the average $\langle k_T^2\rangle \sim O(\bar{\Lambda}M_B)$ for $M_B\sim 5$ GeV. The off-shellness of internal particles then remains $O(\bar{\Lambda}M_B)$ even at the end point, and the singularities are removed. Hence, it cannot be self-consistent to treat k_T as a higher-twist effect as the end-point region is important. The expansion parameter $\alpha_s(\bar{\Lambda}M_B)/\pi \sim 0.13$ is also small enough to justify PQCD evaluation of heavy-to-light form factors [4,5]. This result is the so-called Sudakov suppression on the end-point singularities in exclusive processes [30].

The above discussion applies to the $B\to K$ form factor and to the $B\to\phi K$ decays. k_T resummation of large logarithmic corrections to the B, ϕ, and K meson distribution amplitudes lead to the exponentials S_B, S_ϕ, and S_K, respectively,

$$S_B(t)=\exp\left[-s(x_1P_1^+,b_1)-2\int_{1/b_1}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma(\alpha_s(\bar{\mu}^2))\right],$$

$$S_\phi(t)=\exp\left[-s(x_2P_2^+,b_2)-s((1-x_2)P_2^+,b_2)\right.$$
$$\left.-2\int_{1/b_2}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma(\alpha_s(\bar{\mu}^2))\right],$$

$$S_K(t)=\exp\left[-s(x_3P_3^-,b_3)-s((1-x_3)P_3^-,b_3)\right.$$
$$\left.-2\int_{1/b_3}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma(\alpha_s(\bar{\mu}^2))\right], \tag{7}$$

with the quark anomalous dimension $\gamma=-\alpha_s/\pi$. The variables b_1, b_2, and b_3, conjugate to the parton transverse momenta k_{1T}, k_{2T}, and k_{3T}, represent the transverse extents of the B, ϕ, and K mesons, respectively. The expression for the exponent s is referred to [28–30]. The above Sudakov exponentials decrease fast in the large b region, such that the $B\to\phi K$ hard amplitudes remain sufficiently perturbative in the end-point region.

Recently, the importance of threshold resummation [31–33] has been observed in exclusive B meson decays [34]. As $x_3\to 0$ [to be precise, $x_3\sim O(\bar{\Lambda}/M_B)$] in Fig. 2(a), the internal b quark, carrying the momentum P_1-k_3, becomes almost on shell, indicating that the end-point singularity is associated with the b quark. Additional soft divergences then appear at higher orders, and the double logarithm $\alpha_s\ln^2 x_3$ is produced from the loop correction to the weak decay vertex. This double logarithm can be factored out of the hard amplitude systematically, and its resummation introduces a Sudakov factor $S_t(x_3)$ into PQCD factorization formulas [34]. Similarly, another lowest-order diagram [Fig. 2(b)] with a hard gluon exchange between the $d(s)$ quark and the spectator quark gives an amplitude proportional to $1/(x_1^2x_3)$. In the threshold region with $x_1\to 0$ [to be precise, $x_1\sim O(\bar{\Lambda}^2/M_B^2)$], additional collinear divergences are associated with the internal $d(s)$ quark carrying the momentum P_3-k_1. The double logarithm $\alpha_s\ln^2 x_1$ is then produced from the loop correction to the weak decay vertex. Resummation of this type of double logarithm leads to the Sudakov factor $S_t(x_1)$.

The above formalism can be generalized to factorizable annihilation diagrams easily. For the lowest-order diagram with the internal quark carrying the momentum P_2+k_3, the end-point region corresponds to $x_3\to 0$. Hence, threshold resummation of the double logarithm gives the Sudakov factor $S_t(x_3)$. For the lowest-order diagram with the internal quark carrying the momentum P_3+k_2, the end-point region corresponds to $x_2\to 0$. Threshold resummation of the double logarithm then gives the Sudakov factor $S_t(x_2)$. The Sudakov factor from threshold resummation is universal, independent of flavors of internal quarks, twists and topologies of hard amplitudes, and decay modes. To simplify the analysis, we have proposed the parametrization [10],

$$S_t(x)=\frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \tag{8}$$

with the parameter $c=0.3$. This parametrization, symmetric under the interchange of x and $1-x$, is convenient for evaluation of factorizable annihilation amplitudes. It is obvious that threshold resummation modifies the end-point behavior of the meson distribution amplitudes, rendering them vanishing faster at $x\to 0$.

Threshold resummation for nonfactorizable diagrams is weaker and negligible. The reason is understood as follows. Consider the diagram with a hard gluon exchange between the spectator quark and the s quark in the ϕ meson, in which the internal s quark carries the momentum $k_2-k_1+k_3$. To obtain additional infrared divergences, i.e., the double logarithms, the s quark must be close to mass shell. We then have the threshold region defined, for example, by $k_1\sim O(\bar{\Lambda}^2/M_B)$, $k_2\sim O(\bar{\Lambda}^2/M_B)$, and $k_3\sim O(M_B)$ simultaneously. That is, this region has more limited phase space compared to that for factorizable amplitudes. Furthermore, soft contribution to a pair of nonfactorizable diagrams cancels, such that the end-point region is not important. Based on the above observations, we shall not include threshold resummation for nonfactorizable amplitudes.

k_T and threshold resummations arise from different subprocesses in PQCD factorization. They can be derived in perturbation theory, and are not free parameters. Their combined effect suppresses the end-point contributions, making PQCD evaluation of exclusive B meson decays reliable. If excluding the resummation effects, the PQCD predictions for the $B\to K$ form factor are infrared divergent. If including only k_T resummation, the PQCD predictions are finite. However, the two-parton twist-3 contributions are still huge, so that the $B\to K$ form factor has an unreasonably large value $F^{BK}\sim 0.57$ at maximal recoil. The reason is that the double logarithms $\alpha_s \ln^2 x$ have not yet been organized. If including both resummations, we obtain the reasonable result $F^{BK}\sim 0.35$. This study indicates the importance of resummations in PQCD analyses of B meson decays. In conclusion, if the PQCD evaluation of the heavy-to-light form factors is performed self-consistently, there exist no end-point singularities, and both twist-2 and twist-3 contributions are well-behaved.

The mechanism of Sudakov suppression can be easily understood by regarding a meson as a color dipole. In the region with vanishing k_T and x, the meson possesses a huge extent in the transverse and longitudinal directions, respectively. That is, the meson carries a large color dipole. At fast recoil, this large color dipole, strongly scattered during B meson decays, tends to emit infinitely many real gluons. However, these emissions are forbidden in an exclusive process with final-state particles specified. As a consequence, contributions to the $B\to\pi,K$ form factors from the kinematic region with vanishing k_T and x must be highly suppressed.

B. Form factors

We demonstrate that k_T and threshold resummations must be taken into account in order to obtain reasonable results for the $B\to K,\pi$ form factors. In the PQCD approach these form factors are derived from the diagrams with one hard gluon exchange as shown in Fig. 2. Soft contribution from the diagram without any hard gluon is Sudakov suppressed [10]. For a rigorous justification of this statement in QCD sum rules, refer to [35]. The two form factors $F^{BK}_{+,0}(q^2)$ are defined by

$$\begin{aligned}\langle K(P_3)|\bar{b}(0)\gamma_\mu s(0)|B(P_1)\rangle &= F^{BK}_+(q^2)\left[(P_1+P_3)_\mu-\frac{M_B^2-M_K^2}{q^2}q_\mu\right]\\ &\quad +F^{BK}_0(q^2)\frac{M_B^2-M_K^2}{q^2}q_\mu ,\end{aligned} \tag{9}$$

where $q=P_1-P_3$ is the outgoing lepton-pair momentum. F^{BK}_+ and F^{BK}_0 at maximal recoil are, quoted from Eq. (39) below, written as

$$\begin{aligned}F^{BK}_{+,0}(0)&=8\pi C_F M_B^2\int_0^1 dx_1\, dx_3\int_0^\infty b_1\, db_1 b_3\, db_3 \Phi_B(x_1,b_1)\{[(1+x_3)\Phi_K(x_3)+(1-2x_3)r_K(\Phi^p_K(x_3)+\Phi^\sigma_K(x_3))]\\ &\quad\times\alpha_s(t_e^{(1)})S_B(t_e^{(1)})S_K(t_e^{(1)})h_e(x_1,x_3,b_1,b_3)+2r_K\Phi^p_K(x_3)\alpha_s(t_e^{(2)})S_B(t_e^{(2)})S_K(t_e^{(2)})h_e(x_3,x_1,b_3,b_1)\},\end{aligned} \tag{10}$$

with $C_F=4/3$ being a color factor. The hard function $h_e(x_1,x_3,b_1,b_3)$, referred to in Eq. (44), contains the threshold resummation factor $S_t(x_3)$. The hard scales t_e are defined in Eq. (46). The expression for the $B\to\pi$ form factor $F^{B\pi}(q^2)$ is similar to Eq. (10).

In Eq. (10) we have included the complete two-parton twist-3 distribution amplitudes associated with the pseudoscalar and pseudo-tensor structures of the kaon. We adopt the mass $m_0=1.7$ GeV [4,5], the B meson distribution amplitude Φ_B proposed in [4,5] [see Eq. (61)], and the kaon distribution amplitudes Φ_K, Φ_K^p, and Φ_K^σ derived from QCD sum rules [36] [see Eqs. (65)–(67)]. Since Φ_B is still not well determined, we consider the variation of its shape parameter within 0.36 GeV$<\omega_B<$0.44 GeV [37] around its central value $\omega_B=0.4$ GeV. Turning off k_T and threshold resummations and fixing α_s, the $B\to\pi,K$ form factors are divergent and not calculable. With k_T resummation, the results become finite as shown in Table I, but still much larger than the expected ones 0.3–0.4. Further including the threshold resummation effect, we obtain the reasonable values $F^{BK}(0)\sim 0.35\pm 0.06$ and $F^{B\pi}(0)=0.30\pm 0.04$. These ranges of the form factors have been usually adopted as model inputs in the literature, and are consistent with the results from lattice calculations [38–40] extrapolated to the small q^2 region and from light-cone QCD sum rules [41,42].

We also present our results of the time-like form factors $F^{\phi K}$, which govern the factorizable annihilation amplitudes. The factorization formulas are, quoted from Eqs. (41) and (42), given by

$$\begin{aligned}F^{\phi K}_{(V-A)}=&8\pi C_F M_B^2\int_0^1 dx_2\, dx_3\int_0^\infty b_2\, db_2 b_3\, db_3\{[(1-x_3)\Phi_\phi(x_2)\Phi_K(x_3)+2r_K r_\phi\Phi_\phi^s(x_2)((2-x_3)\Phi_K^p(x_3)+x_3\Phi_K^\sigma(x_3))]\\&\times\alpha_s(t_a^{(1)})S_\phi(t_a^{(1)})S_K(t_a^{(1)})h_a(x_2,1-x_3,b_2,b_3)\\&-[x_2\Phi_\phi(x_2)\Phi_K(x_3)+2r_K r_\phi((1+x_2)\Phi_\phi^s(x_2)-(1-x_2)\Phi_\phi^t(x_2))\Phi_K^p(x_3)]\\&\times\alpha_s(t_a^{(2)})S_\phi(t_a^{(2)})S_K(t_a^{(2)})h_a(1-x_3,x_2,b_3,b_2)\}. \end{aligned}\tag{11}$$

$$\begin{aligned}F^{\phi K}_{(V+A)}=&-8\pi C_F M_B^2\int_0^1 dx_2\, dx_3\int_0^\infty b_2\, db_2 b_3\, db_3\{[2r_K(1-x_3)\Phi_\phi(x_2)(\Phi_K^p(x_3)+\Phi_K^\sigma(x_3))+4r_\phi\Phi_\phi^s(x_2)\Phi_K(x_3)]\\&\times\alpha_s(t_a^{(1)})S_\phi(t_a^{(1)})S_K(t_a^{(1)})h_a(x_2,1-x_3,b_2,b_3)+[4r_K\Phi_\phi(x_2)\Phi_K^p(x_3)+2x_2 r_\phi(\Phi_\phi^s(x_2)-\Phi_\phi^t(x_2))\Phi_K(x_3)]\\&\times\alpha_s(t_a^{(2)})S_\phi(t_a^{(2)})S_K(t_a^{(2)})h_a(1-x_3,x_2,b_3,b_2)\}. \end{aligned}\tag{12}$$

The hard function $h_a(x_2,x_3,b_2,b_3)$, referred to in Eq. (45), contains the threshold resummation factor $S_t(x_3)$. The hard scales t_a and the ϕ meson distribution amplitudes Φ_ϕ, Φ_ϕ^t, and Φ_ϕ^s are defined in Eq. (47) and in Eqs. (62)–(64), respectively. It is obvious that $F^{\phi K}_{(V-A)}$ contains both logarithmic and linear divergences, and $F^{\phi K}_{(V+A)}$ contains logarithmic divergences [14,43], if the Sudakov factors are excluded. To include annihilation contributions in the QCD factorization approach, several arbitrary complex infrared cutoffs must be introduced to regulate the above end-point singularities [44]. These cutoffs are process dependent. Hence, the PQCD approach with the Sudakov effects has a better control on annihilation contributions.

We obtain $F^{\phi K}_{(V-A)}=(1.78+0.63i)\times 10^{-2}$ and $F^{\phi K}_{(V+A)}=(-3.48+13.52i)\times 10^{-2}$ for $\omega_B=0.4$ GeV and $m_0=1.7$ GeV. The smaller value of $F^{\phi K}_{(V-A)}$ is due to a mechanism similar to helicity suppression: note the partial cancellation between the two terms in Eq. (11). $F^{\phi K}_{(V+A)}$ is mainly imaginary, whose reason will become clear after we explain the power behavior of the various decay amplitudes in the next section. The time-like form factors are independent of the shape parameter ω_B. The change of the $B\to\pi,K$ form factors and of the time-like form factors with the chiral enhancement factor m_0 is displayed in Table II.

III. POWER COUNTING AND DECAY AMPLITUDES

We discuss the power counting rules in the presence of the Sudakov effects and present the factorization formulas for the $B\to\phi K$ decays. The effective Hamiltonian for the flavor-changing $b\to s$ transition is given by

$$H_{\rm eff}=\frac{G_F}{\sqrt{2}}\sum_{q=u,c}V_q\left[C_1(\mu)O_1^{(q)}(\mu)+C_2(\mu)O_2^{(q)}(\mu)+\sum_{i=3}^{10}C_i(\mu)O_i(\mu)\right],\tag{13}$$

with the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $V_q=V_{qs}^*V_{qb}$ and the operators

TABLE I. $B\to\pi,K$ transition form factors without threshold resummation (column A) and with threshold resummation (column B).

Form factors	$F^{B\pi}(0)$		$F^{BK}(0)$	
ω_B (GeV)	A	B	A	B
0.35	0.603	0.356	0.703	0.430
0.36	0.579	0.343	0.675	0.413
0.37	0.557	0.330	0.647	0.398
0.38	0.535	0.318	0.621	0.382
0.39	0.515	0.306	0.597	0.368
0.40	0.496	0.295	0.574	0.354
0.41	0.478	0.285	0.552	0.342
0.42	0.461	0.275	0.532	0.329
0.43	0.445	0.266	0.512	0.318
0.44	0.429	0.257	0.494	0.307
0.45	0.414	0.248	0.476	0.296

$$O_1^{(q)}=(\bar{s}_i q_j)_{V-A}(\bar{q}_j b_i)_{V-A}, \quad O_2^{(q)}=(\bar{s}_i q_i)_{V-A}(\bar{q}_j b_j)_{V-A},$$

$$O_3=(\bar{s}_i b_i)_{V-A}\sum_q (\bar{q}_j q_j)_{V-A},$$

$$O_4=(\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V-A},$$

$$O_5=(\bar{s}_i b_i)_{V-A}\sum_q (\bar{q}_j q_j)_{V+A},$$

$$O_6=(\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V+A}, \tag{14}$$

$$O_7=\frac{3}{2}(\bar{s}_i b_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V+A},$$

$$O_8=\frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V+A},$$

$$O_9=\frac{3}{2}(\bar{s}_i b_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V-A},$$

$$O_{10}=\frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V-A},$$

i and j being the color indices. Using the unitarity condition, the CKM matrix elements for the penguin operators O_3-O_{10} can also be expressed as $V_u+V_c=-V_t$. The unitarity angle ϕ_3 is defined via

$$V_{ub}=|V_{ub}|\exp(-i\phi_3). \tag{15}$$

Here we adopt the Wolfenstein parametrization for the CKM matrix up to $O(\lambda^3)$,

TABLE II. Time-like form factors $F^{\phi K}$.

Form factors	$F^{B\pi}(0)$	$F^{BK}(0)$	$F^{\phi K}_{(V-A)}\times 10^2$		$F^{\phi K}_{(V+A)}\times 10^2$	
m_0 (GeV)	Re	Re	Re	Im	Re	Im
1.4	0.295	0.312	1.49	0.56	−2.58	12.72
1.5	0.308	0.326	1.61	0.57	−3.06	13.13
1.6	0.320	0.340	1.71	0.62	−3.20	13.41
1.7	0.333	0.354	1.78	0.63	−3.48	13.52
1.8	0.345	0.368	1.92	0.68	−3.79	13.88
1.9	0.357	0.383	2.02	0.73	−3.99	14.67
2.0	0.370	0.396	2.10	0.74	−4.40	14.82

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{16}$$

A recent analysis of quark-mixing matrix yields [45]

$$\lambda=0.2196\pm0.0023,$$

$$A=0.819\pm0.035,$$

$$R_b\equiv\sqrt{\rho^2+\eta^2}=0.41\pm0.07. \tag{17}$$

The hard amplitudes contain factorizable diagrams, where hard gluons attach the valence quarks in the same meson, and nonfactorizable diagrams, where hard gluons attach the valence quarks in different mesons. The annihilation topology is also included, and classified into factorizable and nonfactorizable ones according to the above definitions. The $B\to\phi K$ decay rates have the expressions

$$\Gamma=\frac{G_F^2 M_B^3}{32\pi}|A|^2. \tag{18}$$

The amplitudes for $B_d\to\phi\bar{K}^0$, $\bar{B}_d\to\phi K^0$, $B^+\to\phi K^+$, and $B^-\to\phi K^-$ are written, respectively, as

$$A=f_\phi V_t^* F_e^{P(s)}+V_t^*\mathcal{M}_e^{P(s)}+f_B V_t^* F_a^{P(d)}+V_t^*\mathcal{M}_a^{P(d)}, \tag{19}$$

$$\bar{A}=f_\phi V_t F_e^{P(s)}+V_t\mathcal{M}_e^{P(s)}+f_B V_t F_a^{P(d)}+V_t\mathcal{M}_a^{P(d)}, \tag{20}$$

$$A^+=f_\phi V_t^* F_e^{P(s)}+V_t^*\mathcal{M}_e^{P(s)}+f_B V_t^* F_a^{P(u)}+V_t^*\mathcal{M}_a^{P(u)} -f_B V_u^* F_a-V_u^*\mathcal{M}_a, \tag{21}$$

$$A^-=f_\phi V_t F_e^{P(s)}+V_t\mathcal{M}_e^{P(s)}+f_B V_t F_a^{P(u)}+V_t\mathcal{M}_a^{P(u)} -f_B V_u F_a-V_u\mathcal{M}_a. \tag{22}$$

In the above expressions F ($\mathcal{M}$) denote factorizable (nonfactorizable) amplitudes, the subscripts e (a) denote the emission (annihilation) diagrams, and the superscripts $P(q)$ denote amplitudes from the penguin operators involving the q-$\bar{q}$ quark pair, and f_B (f_ϕ) is the B (ϕ) meson decay constant.

A. Power counting

Before presenting the factorization formulas of the above amplitudes, we discuss their power behavior in $1/M_B$. The spectator quark in the B meson, forming a soft cloud around the heavy b quark, carries momentum $O(\bar{\Lambda})$. The spectator quark on the kaon side carries momentum $O(M_B)$ in order to form the fast-moving kaon with the s quark produced in the b quark decay. These dramatic different orders of magnitude in momenta explain why a hard gluon is necessary. Based on this argument, the hard gluon is off shell by order of $\bar{\Lambda}M_B$. This special scale, characterizing heavy-to-light decays, is essential for developing the PQCD formalism of exclusive B meson decays. Below we shall explicitly show how to construct this power behavior, and argue that all the topologies of diagrams should be taken into account in the leading-power PQCD analysis.

We start with the twist-2 contribution to the factorizable emission amplitude, which contains the integrand [see Eq. (10)],

$$I^{e2}=\frac{x_3M_B^2}{[x_1x_3M_B^2+O(\bar{\Lambda}M_B)][x_3M_B^2+O(\bar{\Lambda}M_B)]}, \quad (23)$$

where the factor x_3 in the numerator comes from the twist-2 kaon distribution amplitude $\Phi_K(x_3)$ and the first and second factors in the denominator come from the virtual gluon and quark propagators, respectively. The terms $O(\bar{\Lambda}M_B)$ represent the order of magnitude of parton transverse momenta k_T^2 under Sudakov suppression. The momentum fraction x_1 is assumed to be of $O(\bar{\Lambda}/M_B)$. Here we concentrate on the important end-point region with $x_3\sim O(\bar{\Lambda}/M_B)$. It is trivial to find that in the end-point region I^{e2} behaves like

$$I^{e2}\sim\frac{1}{\bar{\Lambda}M_B}, \quad (24)$$

as argued above.

We then consider the first twist-3 term in Eq. (10),

$$I^{e3}=\frac{r_KM_B^2}{[x_1x_3M_B^2+O(\bar{\Lambda}M_B)][x_3M_B^2+O(\bar{\Lambda}M_B)]}. \quad (25)$$

For small $x_3\sim O(\bar{\Lambda}/M_B)$, we have the power law,

$$I^{e3}\sim\frac{r_K}{\bar{\Lambda}^2}=\frac{m_0}{\bar{\Lambda}}\frac{1}{\bar{\Lambda}M_B}. \quad (26)$$

Hence, the twist-3 contribution is not power-suppressed compared to the twist-2 one in the $M_B\to\infty$ limit [9,10], though the two-parton twist-3 distribution amplitudes are proportional to the ratio $r_K=m_0/M_B$. The presence of the potential linear divergence in the twist-3 contribution modifies the naive power counting rules from twist expansion of meson distribution amplitudes. The power behavior of the other twist-3 term in Eq. (10) is the same.

A folklore for annihilation contributions is that they are suppressed by a power of the small ratio f_B/M_B, and negligible compared to emission contributions. The annihilation amplitudes from the operators $O_{1,2,3,4}$ with the structure $(V-A)(V-A)$ are small because of the mechanism of helicity suppression. This argument applies exactly to the $B\to\pi\pi$ decays, and partially to the $B\to K\pi$ and ϕK decays, since the kaon distribution amplitudes are not symmetric in the momentum fraction x, and the two final-state mesons are not identical. Those from the operators $O_{5,6}$ with the structure $(S-P)(S+P)$ survive under helicity suppression [4,5]. Below we shall argue that the annihilation amplitudes from $O_{5,6}$ are proportional to $2r_K$, which is in fact $O(1)$ for $M_B\sim5$ GeV. That is, annihilation contributions vanish in the heavy quark limit $m_b\to\infty$, but are important for the physical mass m_b.

Referring to Eq. (11), the integrand for the twist-2 annihilation amplitudes from $O_{1,2,3,4}$ is written as

$$I^{a2}_{(V-A)}=\frac{x_2x_3^2M_B^2}{[x_2x_3M_B^2-O(\bar{\Lambda}M_B)+i\epsilon][x_3M_B^2-O(\bar{\Lambda}M_B)+i\epsilon]}, \quad (27)$$

where the factor x_2 in the numerator comes from $\Phi_\phi(x_2)$ and x_3^2 comes from $x_3\Phi_K(x_3)$. Here we have interchanged x_3 and $1-x_3$ for the convenience of discussion. In the region with $x_3\sim O(\bar{\Lambda}/M_B)$ and with arbitrary x_2, we obtain the power laws of the real and imaginary parts of Eq. (27),

$$\mathrm{Re}(I^{a2}_{(V-A)})\sim\frac{1}{M_B^2}, \quad \mathrm{Im}(I^{a2}_{(V-A)})\sim\frac{1}{M_B^2}. \quad (28)$$

Hence, the twist-2 annihilation contributions from $O_{1,2,3,4}$ are negligible.

The integrand for the $O(r^2)$ terms in Eq. (11) is written as

$$I^{a4}_{(V-A)}=\frac{2r_Kr_\phi M_B^2}{[x_2x_3M_B^2-O(\bar{\Lambda}M_B)+i\epsilon][x_3M_B^2-O(\bar{\Lambda}M_B)+i\epsilon]}. \quad (29)$$

We express the quark propagator as

$$\frac{1}{x_3M_B^2-O(\bar{\Lambda}M_B)+i\epsilon}=\frac{P}{x_3M_B^2-O(\bar{\Lambda}M_B)}-\frac{i\pi}{M_B^2}\delta\left(x_3-O\left(\frac{\bar{\Lambda}}{M_B}\right)\right), \quad (30)$$

where P denotes the principle-value prescription. The gluon propagator is expressed in a similar way. The real part then behaves like

$$\mathrm{Re}(I^{a4}_{(V-A)})=2r_Kr_\phi M_B^2\left[\frac{P}{[x_2x_3M_B^2-O(\bar\Lambda M_B)]}\frac{P}{[x_3M_B^2-O(\bar\Lambda M_B)]}-\frac{\pi^2}{M_B^4}\delta\left(x_2x_3-O\left(\frac{\bar\Lambda}{M_B}\right)\right)\delta\left(x_3-O\left(\frac{\bar\Lambda}{M_B}\right)\right)\right],$$
$$\sim 2r_Kr_\phi\left(\frac{1}{\bar\Lambda^2}-\frac{\pi^2}{\bar\Lambda M_B}\right)=2r_K\frac{M_\phi}{\bar\Lambda}\frac{1}{\bar\Lambda M_B}\left(1-\frac{\pi^2\bar\Lambda}{M_B}\right). \tag{31}$$

The imaginary part of Eq. (29) behaves like

$$\mathrm{Im}(I^{a4}_{(V-A)})=-2\pi r_Kr_\phi\left[\frac{P}{[x_2x_3M_B^2-O(\bar\Lambda M_B)]}\times\delta\left(x_3-O\left(\frac{\bar\Lambda}{M_B}\right)\right)+\frac{P}{[x_3M_B^2-O(\bar\Lambda M_B)]}\delta\left(x_2x_3-O\left(\frac{\bar\Lambda}{M_B}\right)\right)\right],$$
$$\sim -2r_Kr_\phi\frac{2\pi}{\bar\Lambda M_B}=-2r_K\frac{M_\phi}{\bar\Lambda}\frac{2\pi}{M_B^2}. \tag{32}$$

For the above estimates the end point $x_3\sim O(\bar\Lambda/M_B)$ with arbitrary x_2 corresponds to the important region. Note that the ratio $M_\phi/\bar\Lambda$ is of $O(1)$. Compared to Eq. (24), the first term in Eq. (31), multiplied by a chiral factor $2r_K\sim O(1)$, is not down by the small ratio f_B/M_B. The second term in Eq. (31) and the imaginary part, though scaling like $1/M_B^2$, are enhanced by the factors π^2 and 2π, respectively. However, the mechanism of helicity suppression renders that these annihilation contributions turn out to be small ($\sim 1/M_B^2$) as shown in Table II.

Referring to Eq. (12), the general integrand for the twist-3 annihilation amplitudes from $O_{5,6}$ is given by,

$$I^{a3}_{(V+A)}=\frac{2r_Kx_2x_3M_B^2}{[x_2x_3M_B^2-O(\bar\Lambda M_B)+i\epsilon][x_3M_B^2-O(\bar\Lambda M_B)+i\epsilon]}. \tag{33}$$

By means of a similar argument, the real and imaginary parts behave, respectively, like

$$\mathrm{Re}(I^{a3}_{(V+A)})\sim 2r_K\frac{1}{\bar\Lambda M_B}\left(1-\frac{\pi^2\bar\Lambda}{M_B}\right), \tag{34}$$

$$\mathrm{Im}(I^{a3}_{(V+A)})\sim -2r_K\frac{2\pi}{M_B^2}. \tag{35}$$

It is observed that the twist-3 amplitudes in Eq. (12) possess the same power law as the r^2 terms in Eq. (11). The difference is only the $O(1)$ ratio $M_\phi/\bar\Lambda$. As stated before, the potential linear divergence in the r^2 terms alters the naive power counting rules from twist expansion.

Because $O(M_B/\bar\Lambda)\sim\pi^2$ for $M_B\sim 5$ GeV, the two terms in Eq. (34) almost cancel each other. The imaginary part in Eq. (35) is enhanced by the factor 2π. There is no helicity suppression in this case. It is then understood that the annihilation amplitudes from $O_{5,6}$ are mainly imaginary, and their magnitude is a few times smaller than the factorizable emission one as exhibited in Table II. If the mass M_B changes, the relative importance of the real and imaginary parts will change. For example, as M_B increases, the cancellation in Eq. (34) is not exact, such that the real part becomes larger. On the other hand, the imaginary part in Eq. (35) decreases. It has been checked that for $M_B=10$ GeV, $\mathrm{Re}(F^{\phi K}_{(V+A)})$ is of the same order as $\mathrm{Im}(F^{\phi K}_{(V+A)})$. We conclude that the smallness of $\mathrm{Re}(F^{\phi K}_{(V+A)})$ in Table II is due to the special value of the B meson mass $M_B\sim 5$ GeV. For general M_B, it is more appropriate to state that annihilation amplitudes from $O_{5,6}$, without distinguishing their real and imaginary parts, scale like $2r_K/(\bar\Lambda M_B)$ according to Eq. (34).

The above reasoning is applicable to nonfactorizable amplitudes. It can be found, referring to Eqs. (49)–(51) and to the asymptotic behavior of the meson distribution amplitudes, that the twist-2 term of each nonfactorizable diagram scales like $1/(\bar\Lambda M_B)$. However, because of the soft cancellation between a pair of nonfactorizable diagrams in the end-point region of x_3, the sum of twist-2 terms turns out to scale like $1/M_B^2$. The twist-3 and $O(r^2)$ terms in each nonfactorizable diagram scale like

$$\frac{r_K}{\bar\Lambda M_B},\quad \frac{r_Kr_\phi}{\bar\Lambda^2}=r_K\frac{M_\phi}{\bar\Lambda}\frac{1}{\bar\Lambda M_B}, \tag{36}$$

respectively. The cancellation between a pair of nonfactorizable diagrams modifies the above power behaviors into

$$\frac{r_K}{M_B^2},\quad \frac{r_Kr_\phi}{\bar\Lambda M_B}=r_K\frac{M_\phi}{\bar\Lambda}\frac{1}{M_B^2}, \tag{37}$$

respectively. For the nonfactorizable annihilation amplitudes [see Eqs. (53) and (54)], the soft cancellation does not exist, since the B meson is a heavy-light system. However, it can be shown that the twist-2, twist-3 and $O(r^2)$ terms in each nonfactorizable annihilation diagram possess the power behaviors,

$$\frac{1}{M_B^2},\quad \frac{r_K}{M_B^2},\quad r_K\frac{M_\phi}{\bar\Lambda}\frac{1}{M_B^2}, \tag{38}$$

respectively.

We emphasize that it is more appropriate to count the power of each individual diagram, instead of the power of sum of diagrams. In some cases, factorizable contributions are suppressed by a vanishing Wilson coefficient, so that nonfactorizable contributions become dominant. For example, factorizable internal-W emisson contributions are strongly suppressed by the Wilson coefficient a_2 in the $B\to J/\psi K^{(*)}$ decays [3]. In some cases, such as the $B\to D\pi$ decays, there is no soft cancellation between a pair of nonfactorizable diagrams, and nonfactorizable contributions are significant [3]. In summary, we derive the relative importance of the various topologies of amplitudes given in Eq. (5). The annihilation and nonfactorizable amplitudes are indeed negligible compared to the factorizable emission ones in the heavy quark limit. However, for the physical mass $M_B\sim 5$ GeV, the annihilation contributions should be included.

B. Factorization formulas

Below we calculate the hard amplitudes for the emission and annihilation topologies, whcih have been obtained independently in [18]. The factorizable penguin contribution is written as

$$F_e^{P(s)}=-8\pi C_F M_B^2\int_0^1 dx_1\,dx_3\int_0^\infty b_1\,db_1 b_3\,db_3\Phi_B(x_1,b_1)\times\{[(1+x_3)\Phi_K(x_3)+r_K(1-2x_3)\times(\Phi_K^p(x_3)+\Phi_K^\sigma(x_3))]E_e^{(s)}(t_e^{(1)})h_e(x_1,x_3,b_1,b_3)+2r_K\Phi_K^p(x_3)E_e^{(s)}(t_e^{(2)})h_e(x_3,x_1,b_3,b_1)\}. \tag{39}$$

The factorizable annihilation contribution is given by

$$F_a^{P(q)}=F_{a4}^{P(q)}+F_{a6}^{P(q)}, \tag{40}$$

with

$$F_{a4}^{P(q)}=\pi C_F M_B^2\int_0^1 dx_2\,dx_3\int_0^\infty b_2\,db_2 b_3\,db_3\{[(1-x_3)\Phi_\phi(x_2)\Phi_K(x_3)+2r_K r_\phi\Phi_\phi^s(x_2)((2-x_3)\Phi_K^p(x_3)+x_3\Phi_K^\sigma(x_3))]\times E_{a4}^{(q)}(t_a^{(1)})h_a(x_2,1-x_3,b_2,b_3)-[x_2\Phi_\phi(x_2)\Phi_K(x_3)+2r_K r_\phi((1+x_2)\Phi_\phi^s(x_2)-(1-x_2)\Phi_\phi^t(x_2))\Phi_K^p(x_3)]\times E_{a4}^{(q)}(t_a^{(2)})h_a(1-x_3,x_2,b_3,b_2)\}, \tag{41}$$

$$F_{a6}^{P(q)}=-8\pi C_F M_B^2\int_0^1 dx_2\,dx_3\int_0^\infty b_2\,db_2 b_3\,db_3\{[2r_K(1-x_3)\Phi_\phi(x_2)(\Phi_K^p(x_3)+\Phi_K^\sigma(x_3))+4r_\phi\Phi_\phi^s(x_2)\Phi_K(x_3)]\times E_{a6}^{(q)}(t_a^{(1)})h_a(x_2,1-x_3,b_2,b_3)+[4r_K\Phi_\phi(x_2)\Phi_K^p(x_3)+2x_2 r_\phi(\Phi_\phi^s(x_2)-\Phi_\phi^t(x_2))\Phi_K(x_3)]\times E_{a6}^{(q)}(t_a^{(2)})h_a(1-x_3,x_2,b_3,b_2)\}, \tag{42}$$

for the light quarks $q=u$ and d. F_a in Eqs. (21) and (22) are the same as $F_{a4}^{P(u)}$ but with the Wilson coefficient $a_4^{(u)}$ replaced by a_2.

The factors $E(t)$ contain the evolution from the W boson mass to the hard scales t in the Wilson coefficients $a(t)$, and from t to the factorization scale $1/b$ in the Sudakov factors $S(t)$,

$$E_e^{(q)}(t)=\alpha_s(t)a_e^{(q)}(t)S_B(t)S_K(t),$$

$$E_{ai}^{(q)}(t)=\alpha_s(t)a_i^{(q)}(t)S_\Phi(t)S_K(t). \tag{43}$$

The hard functions h are

$$h_e(x_1,x_3,b_1,b_3)=K_0(\sqrt{x_1x_3}M_Bb_1)S_t(x_3)\times[\theta(b_1-b_3)K_0(\sqrt{x_3}M_Bb_1)\times I_0(\sqrt{x_3}M_Bb_3)+\theta(b_3-b_1)\times K_0(\sqrt{x_3}M_Bb_3)I_0(\sqrt{x_3}M_Bb_1)], \tag{44}$$

$$h_a(x_2,x_3,b_2,b_3)=\left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(\sqrt{x_2x_3}M_Bb_2)S_t(x_3)\times[\theta(b_2-b_3)H_0^{(1)}(\sqrt{x_3}M_Bb_2)\times J_0(\sqrt{x_3}M_Bb_3)+\theta(b_3-b_2)\times H_0^{(1)}(\sqrt{x_3}M_Bb_3)\times J_0(\sqrt{x_3}M_Bb_2)], \tag{45}$$

where $S_t(x)$ is the evolution function from threshold resummation discussed in Sec. II, and K_0, I_0, H_0, and J_0 are the Bessel functions.

The hard scales t are chosen as the maxima of the virtualities of internal particles involved in the hard amplitudes, including $1/b_i$,

$$t_e^{(1)}=\max(\sqrt{x_3}M_B,1/b_1,1/b_3),$$

$$t_e^{(2)}=\max(\sqrt{x_1}M_B,1/b_1,1/b_3), \tag{46}$$

$$t_a^{(1)}=\max(\sqrt{1-x_3}M_B,1/b_2,1/b_3),$$
$$t_a^{(2)}=\max(\sqrt{x_2}M_B,1/b_2,1/b_3), \tag{47}$$

which decrease higher-order corrections [46]. The Sudakov factor in Eq. (7) suppresses long-distance contributions from the large b [i.e., large $\alpha_s(t)$] region, and improves the applicability of PQCD to B meson decays. We emphasize that the special intermediate scales $t\sim O(\sqrt{\bar{\Lambda}M_B})$ lead to predictions for penguin-dominated decay modes, such as $B\to\phi K$, which are larger than those from the factorization and QCD factorization approaches. When PQCD analyses are extended to $O(\alpha_s^2)$ [46], the hard scales can be determined more precisely and the scale independence of our predictions will be improved. The $O(\alpha_s^2)$ corrections to two-body nonleptonic B meson decays have been computed in the generalized factorization approach [47,48], which indeed improve the scale independence of the predictions.

For the nonfactorizable amplitudes, the factorization formulas involve the kinematic variables of all the three mesons [49], and the Sudakov factor is given by $S=S_B S_\Phi S_K$. Their expressions are

$$\mathcal{M}_e^{P(q)}=\mathcal{M}_{e3}^{P(q)}+\mathcal{M}_{e4}^{P(q)}+\mathcal{M}_{e5}^{P(q)}+\mathcal{M}_{e6}^{P(q)}, \tag{48}$$

with

$$\begin{aligned}\mathcal{M}_{e4}^{P(q)}=&16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1\,db_1 b_2\,db_2\Phi_B(x_1,b_1)\Phi_\phi(x_2)\{[(x_2+x_3)\Phi_K(x_3)-r_K x_3(\Phi_K^p(x_3)+\Phi_K^\sigma(x_3))]\\&\times E_{e4}^{(q)\prime}(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)-[(1-x_2)\Phi_K(x_3)-r_K x_3(\Phi_K^p(x_3)-\Phi_K^\sigma(x_3))]\\&\times E_{e4}^{(q)\prime}(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1,b_2)\},\end{aligned} \tag{49}$$

$$\begin{aligned}\mathcal{M}_{e5}^{P(q)}=&16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1\,db_1 b_2\,db_2\Phi_B(x_1,b_1)\Phi_\phi(x_2)\{[x_2\Phi_K(x_3)-r_K x_3(\Phi_K^p(x_3)-\Phi_K^\sigma(x_3))]\\&\times E_{e5}^{(q)\prime}(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)-[(1-x_2+x_3)\Phi_K(x_3)-r_K x_3(\Phi_K^p(x_3)+\Phi_K^\sigma(x_3))]\\&\times E_{e5}^{(q)\prime}(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1,b_2)\},\end{aligned} \tag{50}$$

$$\begin{aligned}\mathcal{M}_{e6}^{P(q)}=&-16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1\,db_1 b_2\,db_2\Phi_B(x_1,b_1)\{[r_\phi x_2(\Phi_\phi^t(x_2)-\Phi_\phi^s(x_2))\Phi_K(x_3)+r_K r_\phi(x_2-x_3)\\&\times(\Phi_\phi^t(x_2)\Phi_K^p(x_3)+\Phi_\phi^s(x_2)\Phi_K^\sigma(x_3))+r_K r_\phi(x_2+x_3)(\Phi_\phi^t(x_2)\Phi_K^\sigma(x_3)-\Phi_\phi^s(x_2)\Phi_K^p(x_3))]\\&\times E_{e6}^{(q)\prime}(t_d^{(1t)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)+[r_\phi(1-x_2)(\Phi_\phi^t(x_2)+\Phi_\phi^s(x_2))\Phi_K(x_3)+r_K r_\phi(1-x_2-x_3)\\&\times(\Phi_\phi^t(x_2)\Phi_K^p(x_3)-\Phi_\phi^s(x_2)\Phi_K^\sigma(x_3))-r_K r_\phi(1-x_2+x_3)(\Phi_\phi^t(x_2)\Phi_K^\sigma(x_3)-\Phi_\phi^s(x_2)\Phi_K^p(x_3))]\\&\times E_{e6}^{(q)\prime}(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1,b_2)\},\end{aligned} \tag{51}$$

where N_c is the number of colors and $d[x]$ denotes $dx_1dx_2dx_3$. The amplitude $\mathcal{M}_{e3}^{P(q)}$ is the same as $\mathcal{M}_{e4}^{P(q)}$ but with the Wilson coefficient $a_3^{(q)\prime}$. The nonfactorizable annihilation amplitudes are given by

$$\mathcal{M}_a^{P(q)}=\mathcal{M}_{a3}^{P(q)}+\mathcal{M}_{a5}^{P(q)}, \tag{52}$$

with

$$\begin{aligned}\mathcal{M}_{a3}^{P(q)}=&-16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1\,db_1 b_2\,db_2\Phi_B(x_1,b_1)\{[(1-x_3)\Phi_\phi(x_2)\Phi_K(x_3)+r_K r_\phi(1-x_2-x_3)\\&\times(\Phi_\phi^t(x_2)\Phi_K^p(x_3)-\Phi_\phi^s(x_2)\Phi_K^\sigma(x_3))-r_K r_\phi(1+x_2-x_3)(\Phi_\phi^t(x_2)\Phi_K^\sigma(x_3)-\Phi_\phi^s(x_2)\Phi_K^p(x_3))]\\&\times E_{a3}^{(q)\prime}(t_f^{(1)})h_f^{(1)}(x_1,x_2,x_3,b_1,b_2)-[x_2\Phi_\phi(x_2)\Phi_K(x_3)-r_K r_\phi(1-x_2-x_3)(\Phi_\phi^t(x_2)\Phi_K^p(x_3)+\Phi_\phi^s(x_2)\Phi_K^\sigma(x_3))\\&+r_K r_\phi(1-x_2+x_3)(\Phi_\phi^t(x_2)\Phi_K^\sigma(x_3)+(3+x_2-x_3)\Phi_\phi^s(x_2)\Phi_K^p(x_3))]E_{a3}^{(q)\prime}(t_f^{(2)})h_f^{(2)}(x_1,x_2,x_3,b_1,b_2)\},\end{aligned} \tag{53}$$

$$\mathcal{M}_{a5}^{P(q)}=-16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1\,db_1 b_2\,db_2\Phi_B(x_1,b_1)\{[r_\phi x_2(\Phi_\phi^t(x_2)+\Phi_\phi^s(x_2))\Phi_K(x_3)$$
$$-r_K(1-x_3)\Phi_\phi(x_2)(\Phi_K^p(x_3)-\Phi_K^\sigma(x_3))]E_{a5}^{(q)\prime}(t_f^{(1)})h_f^{(1)}(x_1,x_2,x_3,b_1,b_2)+[r_\phi(2-x_2)(\Phi_\phi^t(x_2)+\Phi_\phi^s(x_2))$$
$$\times\Phi_K(x_3)-r_K(1+x_3)\Phi_\phi(x_2)(\Phi_K^p(x_3)-\Phi_K^\sigma(x_3))]E_{a5}^{(q)\prime}(t_f^{(2)})h_f^{(2)}(x_1,x_2,x_3,b_1,b_2)\}. \tag{54}$$

The amplitude $\mathcal{M}_a$ is the same as $\mathcal{M}_{a3}^{P(u)}$ but with Wilson coefficient a_1'.

The evolution factors are given by

$$E_{ei}^{(q)\prime}(t)=\alpha_s(t)a_i^{(q)\prime}(t)S(t)|_{b_3=b_1},$$
$$E_{ai}^{(q)\prime}(t)=\alpha_s(t)a_i^{(q)\prime}(t)S(t)|_{b_3=b_2}. \tag{55}$$

The hard functions $h^{(j)}$, $j=1$ and 2, are written as

$$h_d^{(j)}=[\theta(b_1-b_2)K_0(DM_Bb_1)I_0(DM_Bb_2)+\theta(b_2-b_1)K_0(DM_Bb_2)I_0(DM_Bb_1)]$$
$$\times K_0(D_jM_Bb_2),\quad \text{for } D_j^2\geqslant 0$$
$$\times\frac{i\pi}{2}H_0^{(1)}(\sqrt{|D_j^2|}M_Bb_2),\quad \text{for } D_j^2\leqslant 0, \tag{56}$$

$$h_f^{(j)}=\frac{i\pi}{2}[\theta(b_1-b_2)H_0^{(1)}(FM_Bb_1)J_0(FM_Bb_2)+\theta(b_2-b_1)H_0^{(1)}(FM_Bb_2)J_0(FM_Bb_1)]$$
$$\times K_0(F_jM_Bb_1),\quad \text{for } F_j^2\geqslant 0$$
$$\times\frac{i\pi}{2}H_0^{(1)}(\sqrt{|F_j^2|}M_Bb_1),\quad \text{for } F_j^2\leqslant 0, \tag{57}$$

with the variables

$$D^2=x_1x_3,$$
$$D_1^2=(x_1-x_2)x_3,$$
$$D_2^2=-(1-x_1-x_2)x_3, \tag{58}$$
$$F^2=x_2(1-x_3),$$
$$F_1^2=(x_1-x_2)(1-x_3),$$
$$F_2^2=x_1+x_2+(1-x_1-x_2)(1-x_3). \tag{59}$$

The hard scales $t^{(j)}$ are chosen as

$$t_d^{(1)}=\max(DM_B,\sqrt{|D_1^2|}M_B,1/b_1,1/b_2),$$
$$t_d^{(2)}=\max(DM_B,\sqrt{|D_2^2|}M_B,1/b_1,1/b_2),$$
$$t_f^{(1)}=\max(FM_B,\sqrt{|F_1^2|}M_B,1/b_1,1/b_2),$$
$$t_f^{(2)}=\max(FM_B,\sqrt{|F_2^2|}M_B,1/b_1,1/b_2). \tag{60}$$

In the above factorization formulas the Wilson coefficients are defined by

$$a_1=C_1+\frac{C_2}{N_c},\quad a_1'=\frac{C_1}{N_c},$$
$$a_2=C_2+\frac{C_1}{N_c},\quad a_2'=\frac{C_2}{N_c},$$

$$a_3^{(q)}=C_3+\frac{C_4}{N_c}+\frac{3}{2}e_q\left(C_9+\frac{C_{10}}{N_c}\right),$$

$$a_3^{(q)\prime}=\frac{1}{N_c}\left(C_3+\frac{3}{2}e_qC_9\right),$$

$$a_4^{(q)}=C_4+\frac{C_3}{N_c}+\frac{3}{2}e_q\left(C_{10}+\frac{C_9}{N_c}\right),$$

$$a_4^{(q)\prime}=\frac{1}{N_c}\left(C_4+\frac{3}{2}e_qC_{10}\right),$$

$$a_5^{(q)}=C_5+\frac{C_6}{N_c}+\frac{3}{2}e_q\left(C_7+\frac{C_8}{N_c}\right),$$

$$a_5^{(q)\prime}=\frac{1}{N_c}\left(C_5+\frac{3}{2}e_qC_7\right),$$

$$a_6^{(q)}=C_6+\frac{C_5}{N_c}+\frac{3}{2}e_q\left(C_8+\frac{C_7}{N_c}\right),$$

$$a_6^{(q)\prime}=\frac{1}{N_c}\left(C_6+\frac{3}{2}e_qC_8\right),$$

$$a_e^{(q)}=a_3^{(q)}+a_4^{(q)}+a_5^{(q)}.$$

IV. NUMERICAL ANALYSIS

For the B meson distribution amplitude, we adopt the model [4,5]:

$$\Phi_B(x,b)=N_Bx^2(1-x)^2\exp\left[-\frac{1}{2}\left(\frac{xM_B}{\omega_B}\right)^2-\frac{\omega_B^2b^2}{2}\right], \quad (61)$$

with the shape parameter $\omega_B=0.4$ GeV. The normalization constant $N_B=91.784$ GeV is related to the decay constant $f_B=190$ MeV (in the convention $f_\pi=130$ MeV). As stated before, Φ_B has a sharp peak at $x\sim\bar{\Lambda}/M_B\sim0.1$. The ϕ and K meson distribution amplitudes are derived from QCD sum rules [36,50]:

$$\Phi_\phi(x)=\frac{3f_\phi}{\sqrt{2N_c}}x(1-x), \quad (62)$$

$$\Phi_\phi^t(x)=\frac{f_\phi^T}{2\sqrt{2N_c}}\Big\{3(1-2x)^2+0.21[3-30(1-2x)^2+35(1-2x)^4]+0.69\left(1+(1-2x)\ln\frac{x}{1-x}\right)\Big\}, \quad (63)$$

$$\Phi_\phi^s(x)=\frac{f_\phi^T}{4\sqrt{2N_c}}\left[3(1-2x)(4.5-11.2x+11.2x^2)+1.38\ln\frac{x}{1-x}\right], \quad (64)$$

$$\Phi_K(x)=\frac{3f_K}{\sqrt{2N_c}}x(1-x)\{1+0.51(1-2x)+0.3[5(1-2x)^2-1]\}, \quad (65)$$

$$\Phi_K^p(x)=\frac{f_K}{2\sqrt{2N_c}}[1+0.24C_2^{1/2}(1-2x)-0.11C_4^{1/2}(1-2x)], \quad (66)$$

$$\Phi_K^\sigma(x)=\frac{f_K}{2\sqrt{2N_c}}(1-2x)[1+0.35(10x^2-10x+1)], \quad (67)$$

with the Gegenbauer polynomials

$$C_2^{1/2}(\xi)=\tfrac{1}{2}[3\xi^2-1], \quad C_4^{1/2}(\xi)=\tfrac{1}{8}[35\xi^4-30\xi^2+3]. \quad (68)$$

To derive the coefficients of the Gegenbauer polynomials, we have assumed $M_K=0.49$ GeV and $m_0=1.7$ GeV. The terms $1-2x$, rendering the kaon distribution amplitudes a bit

TABLE III. Twist-2 and higher-twist contributions to the $B\to\phi K$ decay amplitudes.

Decay	$B^{\pm}\to\phi K^{\pm}$					
Components	Twist-2 (10^{-4} GeV)		Higher-twist (10^{-4} GeV)		Total (10^{-4} GeV)	
Amplitudes	Re	Im	Re	Im	Re	Im
$f_\phi F_e^P$	9.48	—	27.71	—	37.19	—
M_e^P	−3.30	2.44	1.60	−1.11	−1.70	1.33
$f_B F_a^P$	0.07	−0.01	−2.57	−15.47	−2.50	−15.48
M_a^P	0.06	0.41	0.30	−0.25	0.36	0.16
$f_B F_a$	−1.21	0.47	41.00	13.52	39.79	13.99
M_a	0.88	−12.74	−7.05	2.61	-6.17	−10.13
Decay	$B^0\to\phi K^0$					
Components	Twist-2 (10^{-4} GeV)		Higher-twist (10^{-4} GeV)		Total (10^{-4} GeV)	
Amplitudes	Re	Im	Re	Im	Re	Im
$f_\phi F_e^P$	9.48	—	27.71	—	37.19	—
M_e^P	−3.30	2.44	1.60	−1.11	−1.70	1.33
$f_B F_a^P$	0.07	−0.01	−2.68	−15.76	−2.61	−15.77
M_a^P	−0.03	0.77	0.48	−0.28	0.45	0.49

asymmetric, corresponds to the $SU(3)$ symmetry breaking effect. We employ $G_F=1.16639\times10^{-5}$ GeV^{-2}, the Wolfenstein parameters $\lambda=0.2196$, $A=0.819$, and $R_b=0.38$, the unitarity angle $\phi_3=90°$, the masses $M_B=5.28$ GeV and $M_\Phi=1.02$ GeV, the decay constants $f_\phi=237$ MeV, $f_\phi^T=220$ MeV, and $f_K=160$ MeV, and the $\overline{B}_d^0$ (B^-) meson lifetime $\tau_{B^0}=1.55$ ps ($\tau_{B^-}=1.65$ ps) [51]. Note that the $B\to\phi K$ branching ratios are insensitive to the variation of ϕ_3.

We present values of the factorizable and nonfactorizable amplitudes from the emission and annihilation topologies in Table III. Contributions from twist-2 and two-parton twist-3 distribution amplitudes are displayed separately. It is found that the latter, not power suppressed, are in fact more important for $f_\phi F_e^P$. According to the power counting in Sec. III, the twist-2 contributions to the annihilation amplitudes $f_B F_a^P$ are negligible. This has been explicitly confirmed in Table III. As expected, the factorizable amplitudes $f_\phi F_e^P$ dominate, and the annihilation amplitudes $f_B F_a^P$ are almost imaginary and their magnitudes are only a few times smaller than $f_\phi F_e^P$. The nonfactorizable amplitudes M_e^P and M_a^P are down by a power of $\overline{\Lambda}/M_B\sim0.1$ compared to the factorizable ones $f_\phi F_e^P$ and $f_B F_a^P$, respectively. The cancellation between the twist-2 and twist-3 contributions makes them even smaller. M_a and $f_B F_a$ from the operators $O_{1,2}$ are of the same order because of the partial cancellation between the two terms in the factorization formula for F_a (helicity suppression).

We demonstrate the importance of penguin enhancement in Table IV. It has been known that the RG evolution of the Wilson coefficients $C_{4,6}(t)$ dramatically increases as t

TABLE IV. Enhancement effects in the $B^{\pm}\to\phi K^{\pm}$ decay amplitudes.

Scales	$\mu=t$		$\mu=2.5$ GeV	
Amplitudes	Re (10^{-4} GeV)	Im (10^{-4} GeV)	Re (10^{-4} GeV)	Im (10^{-4} GeV)
$f_\phi F_e^P$	37.19	—	23.14	—
M_e^P	−1.70	1.33	−1.05	0.62
$f_B F_a^P$	−2.50	−15.48	1.92	−12.83
M_a^P	0.36	0.16	−0.05	0.19
$f_B F_a$	39.79	13.99	37.73	13.14
M_a	−6.17	−10.13	−0.56	−9.05
Branching ratio (without Ann.)	9.8×10^{-6}		3.8×10^{-6}	
Branching ratio (with Ann.)	10.2×10^{-6}		5.6×10^{-6}	

$<m_b/2$, while that of $C_{1,2}(t)$ almost remains constant [52]. With this penguin enhancement of about 40%, the branching ratios of the $B\to K\pi$ decays, dominated by penguin contributions, are about four times larger than those of the $B\to\pi\pi$ decays, which are dominated by tree contributions. This is the reason we can explain the observed $B\to K\pi$ and $\pi\pi$ branching ratios using a smaller unitarity angle $\phi_3<90°$ [4,5]. In the factorization approach [11] and in the QCD factorization approach [12], it is assumed that factorizable contributions are not calculable. The leading contribution to a nonleptonic decay amplitude is then expressed as a convolution of a hard part with a form factor and a meson distribution amplitude. In both approaches the hard scale is m_b and the intermediate scale $\bar{\Lambda}M_B$ cannot appear, so that the dynamical enhancement of penguin contributions does not exist. To accommodate the $B\to K\pi$ data in the factorization and QCD factorization approaches, one relies on the chiral enhancement by increasing the mass m_0 to a large value $m_0\sim 3$ GeV, or on a large unitarity angle $\phi_3\sim 120°$ [19], which leads to constructive (destructive) interference between penguin and emission amplitudes for the $B\to K\pi$ ($B\to\pi\pi$) decays.

Whether dynamical enhancement or chiral enhancement is essential for the penguin-dominated decay modes can be tested by measuring the $B\to\phi K$ modes. In these modes penguin contributions dominate, and their branching ratios are almost independent of the angle ϕ_3. Since ϕ is a vector meson, the mass m_0 is replaced by the ϕ meson mass $M_\phi\sim 1$ GeV, and chiral enhancement does not exist. Annihilation contributions cannot enhance the $B\to\phi K$ branching ratios too much, because they are assumed to be a $1/m_b$ effect in the QCD factorization approach [12]. In the PQCD approach annihilation amplitudes reach 40%, which is reasonable according to our power counting. However, they, being mainly imaginary, are not responsible for the large $B\to\phi K$ branching ratios as shown in Table IV. If the $B\to\phi K$ branching ratios are around 4×10^{-6} [13,14], the chiral enhancement may be essential. If the branching ratios are around 10×10^{-6}, the dynamical enhancement may be essential. Therefore, the $B\to\phi K$ decays are the appropriate modes to distinguish the QCD and PQCD factorization approaches. The branching ratios of $B_d^0\to\phi K^0$ and of $B^\pm\to\phi K^\pm$ are almost euqal. We have also evaluated the CP asymmetries of the $B\to\phi K$ decays, and found that they are not significant: their maxima, appearing at $\phi_3\sim 90°$, are less than 2%.

We emphasize that $m_0(\mu)$, appearing along with the twist-3 kaon distribution amplitudes, is defined at the factorization scale $1/b$ as low as 1 GeV in the PQCD formalism [2,3]. Hence, its value should be located within 1.6 ± 0.2 GeV [36]. Between the hard scale and the factorization scale, there is the Sudakov evolution. In the QCD factorization approach, $m_0(\mu)$ defined at m_b is as large as 3 GeV, leading to chiral enhancement. It has been argued that $m_0(\mu)$ and $a_6(\mu)$ form a scale-independent product (that is, $m_0(\mu)$ increases, while $a_6(\mu)$ decreases with μ), such that chiral and dynamical enhancements cannot be distinguished [53]. However, dynamical enhancement exists in both twist-2 and twist-3 contributions, but chiral enhancement exists only in twist-3 ones [see Eq. (39)]. Therefore, they are indeed a different mechanism.

At last, we examine the uncertainty from the variation of the hard scales t, which provides the informaiton of higher-order corrections to the hard amplitudes. We notice that this is the major source of the theoretical uncertainty. The values of ω_B and m_0 have been fixed at around 0.4 GeV and 1.7 GeV, respectively, which are preferred by the $B\to K\pi$, $\pi\pi$ data. The light meson distribution amplitudes have been fixed more or less in QCD sum rules. The possible 30% variation of the coefficients of the Gegenbauer polynomials lead to minor changes of our predictions. Since the analyses of the $B\to\pi$ and $B\to K$ form factors are the same, we constraint the ranges of the hard scales t, such that our predictions for the $B\to K\pi$ branching ratios are within the data uncertainties. The resultant approximate range of the hard scales t_e is given by

$$\max(0.75\sqrt{x_3}M_B,1/b_1,1/b_3) < t_e^{(1)} < \max(1.25\sqrt{x_3}M_B,1/b_1,1/b_3),$$
$$\max(0.75\sqrt{x_1}M_B,1/b_1,1/b_3) < t_e^{(2)} < \max(1.25\sqrt{x_1}M_B,1/b_1,1/b_3). \quad (69)$$

Note that the coefficients of the factorization scales $1/b$, associated with the definition of the meson distribution amplitudes, do not change. The variation of the other hard scales t is similar, but does not affect the results very much. The theoretical uncertainty for the $B\to\phi K$ branching ratios in Eq. (2) is then obtained.

V. CONCLUSION

In this paper we have shown that a leading-power PQCD formalism should contain contributions from both twist-2 and two-parton twist-3 distribution amplitudes. Threshold and k_T resummations are essential for infrared finite PQCD analyses of B meson decays. Without Sudakov suppression from these resummations, all topologies of decay amplitudes possess infrared (logarithmic or linear) divergences. We have explained the power counting rules for the factorizable (also nonfactorizable) emission and annihilation amplitudes under the sufficiently strong Sudakov effects. The annihilation and nonfactorizable amplitudes are suppressed by $2m_0/M_B$ and by $\bar{\Lambda}/M_B$ in the heavy quark limit, respectively, compared to the factorizable emission ones. For the physical mass $M_B\sim 5$ GeV, the former should be taken into account. In the PQCD formalism the annihilation amplitudes can be calculated in the same way as the emission ones without introducing any new free parameters. Hence, our formalism has more precise control on the annihilation effects than the QCD factorization approach. Annihilation contributions of 40% at the amplitude level are reasonable according to our power counting. However, these amplitudes are not responsible for the large $B\to\phi K$ branching ratios, since they are mainly imaginary.

We have emphasized that exclusive heavy meson decays are characterized by a lower scale $\bar{\Lambda}M_B$, for which penguin contributions are dynamically enhanced. This enhancement renders that penguin-dominated decay modes acquire branching ratios larger than those from the factorization and QCD factorization approaches, even when the final-state particle is a vector meson. We have proposed the $B\to\phi K$ decays as the ideal modes to test the importance of this mechanism. If their branching ratios are as large as 10×10^{-6} (independent of the unitarity angle ϕ_3), dynamical enhancement will gain a convincing support. The answer will become clear, when the consistency among the BaBar, Belle, and CLEO data is achieved. We have also found that the CP asymmetries in the $B\to\phi K$ modes are vanishingly small (less than 2%).

ACKNOWLEDGMENTS

We thank S. Brodsky, H. Y. Cheng, L. Dixon, G. Hiller, H. Quinn, A. I. Sanda, and K. C. Yang for helpful discussions. The work was supported in part by Grant-in Aid for Special Project Research (Physics of CP Violation), by Grant-in Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan. The work of H.N.L. was supported in part by the National Science Council of R.O.C. under Grant No. NSC-89-2112-M-006-033 and by the Theory Group SLAC. The work of Y.Y.K. was supported in part by National Science Council of R.O.C. under Grant No. NSC-90-2811-M-002.

APPENDIX: TWO-PARTON DISTRIBUTION AMPLITUDES

1. *B* meson distribution amplitudes

The B meson distribution amplitudes are written as [10,54]

$$\frac{(\not{P}_1+M_B)\gamma_5}{\sqrt{2N_c}}\Phi_B(x,b),\quad \frac{(\not{P}_1+M_B)\gamma_5}{\sqrt{2N_c}}\frac{\not{n}_+-\not{n}_-}{\sqrt{2}}\bar{\Phi}_B(x,b), \tag{A1}$$

with the dimensioless vectors $n_+=(1,0,0_T)$ and $n_-=(0,1,0_T)$. As shown in [10], the contribution from $\bar{\Phi}_B$ is negligible, after taking into account the equation of motion between Φ_B and $\bar{\Phi}_B$. Hence, we consider only a single B meson distribution amplitude in the heavy quark limit in this work. As the transverse extent b approaches zero, the B meson distribution amplitude $\Phi_B(x,b)$ reduces to the standard parton model $\Phi_B(x)=\Phi_B(x,b=0)$, which satisfies the normalization

$$\int_0^1\Phi_B(x)dx=\frac{f_B}{2\sqrt{2N_c}}. \tag{A2}$$

2. ϕ meson distribution amplitudes

To define the ϕ meson distribution amplitudes, we consider the following nonlocal matrix elements [50]:

$$\langle 0|\bar{s}(0)\gamma_\mu s(z)|\phi(P_2)\rangle=f_\phi M_\phi\Bigg[P_{2\mu}\frac{\epsilon\cdot z}{P_2\cdot z}\int_0^1 dx_2\,e^{-ix_2P_2\cdot z}\phi_\parallel(x_2)+\epsilon_{T\mu}\int_0^1 dx_2\,e^{-ix_2P_2\cdot z}g_T^{(v)}(x_2)$$
$$-\frac{1}{2}z_\mu\frac{\epsilon\cdot z}{(P_2\cdot z)^2}M_\phi^2\int_0^1 dx_2\,e^{-ix_2P_2\cdot z}g_3(x_2)\Bigg], \tag{A3}$$

$$\langle 0|\bar{s}(0)\sigma_{\mu\nu}s(z)|\phi(P_2)\rangle=if_\phi^T\Bigg[(\epsilon_{T\mu}P_{2\nu}-\epsilon_{T\nu}P_{2\mu})\int_0^1 dx_2\,e^{-ix_2P_2\cdot z}\phi_T(x_2)+(P_{2\mu}z_\nu-P_{2\nu}z_\mu)\frac{\epsilon\cdot z}{(P_2\cdot z)^2}M_\phi^2$$
$$\times\int_0^1 dx_2\,e^{-ix_2P_2\cdot z}h_\parallel^{(t)}(x_2)+\frac{1}{2}(\epsilon_{T\mu}z_\nu-\epsilon_{T\nu}z_\mu)\frac{M_\phi^2}{P_2\cdot z}\int_0^1 dx_2\,e^{-ix_2P_2\cdot z}h_3(x_2)\Bigg], \tag{A4}$$

$$\langle 0|\bar{s}(0)Is(z)|\phi(P_2)\rangle=\frac{i}{2}\left(f_\phi^T-f_\phi\frac{2m_s}{M_\phi}\right)\epsilon\cdot zM_\phi^2\int_0^1 dx_2\,e^{-ix_2P_2\cdot z}h_\parallel^{(s)}(x_2), \tag{A5}$$

where f_ϕ and f_ϕ^T are the decay constants of the ϕ meson with longitudinal and transverse polarizations, respectively, ϵ_T the transverse polarization vector, x_2 the momentum associated with the s quark at the coordinate $z\propto(0,1,0_T)$, and m_s the s quark mass. The explicit expressions of the distribution amplitudes ϕ, g, and h with unity normalization are referred to in [50].

The contributions from the distribution amplitudes $g_T^{(v)}$, ϕ_T, and h_3 vanish for two-body B meson decays, in which only longitudinally polarized ϕ mesons are produced. The contributions from $\phi_\parallel$, $h_\parallel^{(t)}$, $h_\parallel^{(s)}$, and g_3 are twist-2, twist-3, twist-3, and twist-4, respectively. It is easy to confirm that g_3 does not contribute to the factorizable emission and annihilation amplitudes. The term proportional to the small ratio $2m_s/M_\phi$ in Eq. (A5) is negligible. Therefore, up to twist 3, we consider the following three ϕ meson final-state distribution amplitudes:

$$\frac{M_\phi \not\epsilon}{\sqrt{2N_c}}\Phi_\phi(x_2), \quad \frac{\not\epsilon \not P_2}{\sqrt{2N_c}}\Phi^t_\phi(x_2), \quad \frac{M_\phi}{\sqrt{2N_c}}\Phi^s_\phi(x_2), \tag{A6}$$

with

$$\Phi_\phi=\frac{f_\phi}{2\sqrt{2N_c}}\phi_\parallel, \quad \Phi^t_\phi=\frac{f^T_\phi}{2\sqrt{2N_c}}h^{(t)}_\parallel,$$

$$\Phi^s_\phi=\frac{f^T_\phi}{4\sqrt{2N_c}}\frac{d}{dx}h^{(s)}_\parallel. \tag{A7}$$

The spin structures associated with the ϕ meson distribution amplitudes can be derived from Eqs. (A3)–(A5).

3. Kaon distribution amplitudes

The general expressions of the relevant nonlocal matrix elements for a kaon are given by [36],

$$\langle 0|\overline{s}(0)\gamma_5\gamma_\mu u(z)|K(P_3)\rangle=-if_K P_{3\mu}\int_0^1 dx_3\, e^{-ix_3P_3\cdot z}\phi_v(x_3)-\frac{i}{2}f_K M_K^2\frac{z_\mu}{P_3\cdot z}\int_0^1 dx_3\, e^{-ix_3P_3\cdot z}g_K(x_3), \tag{A8}$$

$$\langle 0|\overline{s}(0)\gamma_5 u(z)|K(P_3)\rangle=-if_K m_0\int_0^1 dx_3\, e^{-ix_3P_3\cdot z}\phi_p(x_3), \tag{A9}$$

$$\langle 0|\overline{s}(0)\gamma_5\sigma_{\mu\nu}u(z)|K(P_3)\rangle=\frac{i}{6}f_K m_0\left(1-\frac{M_K^2}{m_0^2}\right)(P_{3\mu}z_\nu-P_{3\nu}z_\mu)\int_0^1 dx_3\, e^{-ix_3P_3\cdot z}\phi_\sigma(x_3), \tag{A10}$$

with the mass

$$m_0=\frac{M_K^2}{m_s+m_d}. \tag{A11}$$

f_K is the kaon decay constant and x_3 the momentum fraction associated with the u quark at the coordinate $z\propto(1,0,0_T)$. The explicit expression of the wave fucntions ϕ and g_K with unit normalization are referred to in [36].

The contributions from the distribution amplitudes ϕ_v, ϕ_p, ϕ_σ, and g_K are twist-2, twist-3, twist-3, and twist-4, respectively. Note that g_K does not contribute to factorizable emission and annihilation amplitudes. Hence, the factorizable annihilation amplitude in Eq. (41) is complete in the r^2 terms. We consider the following three kaon final-state distribution amplitudes

$$\frac{\gamma_5 \not P_3}{\sqrt{2N_c}}\Phi_K, \quad \frac{m_0\gamma_5}{\sqrt{2N_c}}\Phi^p_K, \quad \frac{m_0\gamma_5(\not n_- \not n_+ -1)}{\sqrt{2N_c}}\Phi^\sigma_K, \tag{A12}$$

with

$$\Phi_K(x)=\frac{f_K}{2\sqrt{2N_c}}\phi_v(x), \quad \Phi^p_K(x)=\frac{f_K}{2\sqrt{2N_c}}\phi_p(x),$$

$$\Phi^\sigma_K(x)=\frac{f_K}{12\sqrt{2N_c}}\frac{d}{dx}\phi_\sigma(x), \tag{A13}$$

where the term M_K^2/m_0^2 in Eq. (A10) has been neglected. The spin structures associated with the kaon distribution amplitudes can be derived from Eqs. (A8)–(A10).

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