Super D-helix

Jin-Ho Cho* and Phillial Oh†
BK21 Physics Research Division and Institute of Basic Science, Sungkyunkwan University, Suwon 440-746, Korea

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We study the "Myers effect" for a bunch of D1-branes with type-IIB superstrings moving in one direction along the branes. We show that the "blown-up" configuration is a helical D1-brane, which is self-supported from collapse by the axial momentum flow. The tilting angle of the helix is determined by the number of D1-branes. The radius of the helix is stabilized to a certain value depending on the number of D1-branes and the momentum carried by type-IIB superstrings. This helix is actually a T-dual version of the supertube recently found as the "blown-up" configuration of a bunch of type-IIA superstrings carrying a D0-brane charge. It is shown that the helical D1 configuration preserves one-quarter of the supersymmetry of the type-IIB Minkowski vacuum.

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I. INTRODUCTION

Dp-branes interacting with higher form Ramond-Ramond (RR) fields [$(p+2n+1)$-form, for example] become D$(p+2n)$-branes, which was first suggested by Emparan [1] and nontrivial interactions were explicitly obtained by Myers [2]. In the presence of magnetic RR fields, these interactions are caused by the motion of the branes. With the motion in compact space (thus carrying angular momentum) the higher-dimensional brane increases its size. Since the size is bounded in the compact space, the angular momentum is also bounded. This fact possibly explains the stringy exclusion principle in the dual gravity setup [3].

Recently Mateos and Townsend showed that this angular momentum can be given in a different guise [4]. In some special setup of a tubular D2-brane, the Poynting vector of the electromagnetic field on the D2 world volume provides the angular momentum. This configuration can be thought of as the "blown-up" configuration of a bunch of type-IIA superstrings with D0-branes evenly distributed on it. It is self-supported from collapse by the angular momentum supplied by the electric and magnetic field associated with the number of type-IIA superstrings and D0-branes, respectively. The important fact is that the tube solution preserves one-quarter of the supersymmetry of the type-IIA Minkowski vacuum. This was generically possible for the intersecting D-branes with relative codimension four [5].

In this paper we pursue the issue further to see how this "blowing-up" effect can be understood in the T-dual setup. Several string duality transformations will yield straightforward generalizations of the supertube. In Ref. [4], S-dual configurations (therefore of M theory) of the supertube were also discussed. These configurations will generate lots of lower-dimensional descendants upon different compactifications, which are to be related with one another via U-dualities. Although these are the naive expectations, recent interests [6–9] on this subject warrant to produce more explicit results.

*E-mail address: jhcho@taegeug.skku.ac.kr
†E-mail address: ploh@dirac.skku.ac.kr

We first show that an array of Dp-branes ($p\leqslant 6$) along some axis, say the X axis [see Eq. (1)] when threaded vertically by superstrings over the entire volume of Dp-branes, is "blown-up" to a D$(p+2)$-brane of topology $\boldsymbol{R}^{p+1}\times S^1$ acquiring extra tubular two dimensions. This result is obtained by taking T duality along various directions transverse to type-IIA superstrings carrying D0-brane charges. The radius of the circle S^1 is invariant under these T dualities. An astonishing result is obtained when we take T duality along the axial direction of the configuration. The bound state of D0-branes and type-IIA superstrings becomes that of D1-branes with type-IIB superstrings moving in one direction on them. We show that its corresponding blown-up configuration is a single helical D1-brane (D-helix) traveling with the speed of light along its axis. This result is peculiar in that the dimensionality is not changed upon the blowing-up. This D-helix should be related with the helical IIA string discussed in Ref. [4] via a sequence of S and T duality.

To begin with, we recapitulate briefly the results of Ref. [4], where the configuration of D0-branes evenly arrayed along X direction and threaded by a bunch of type-IIA superstrings was considered.

The configuration is embedded in a flat geometry parametrized as

$$ds^2=-dT^2+dX^2+R^2d\phi^2+dR^2+ds^2(\mathbb{E}^{(6)}). \tag{1}$$

Therefore it is free from any background gravitational effect and there is no background field of any kind. It was shown in Ref. [4] that this configuration can be considered as zero radius limit of a tubular D2-brane. D0-brane charge is "dissolved" as the magnetic flux on D2-brane while type-IIA superstrings are dissolved as the electric field along the X direction.

With the static gauge for the world-volume coordinates ($t=T$, $x=X$, $\varphi=\phi$) on D2-brane, Born-Infeld (BI) two-form field strength is given by

$$F=E\,dt\wedge dx+B\,dx\wedge d\varphi. \tag{2}$$

The Lagrangian for the tubular D2-brane is that of Dirac-Born-Infeld (DBI) which can be simplified as

$$\mathcal{L}=-\sqrt{R^2(1-E^2)+B^2}. \tag{3}$$

For fixed momentum $\Pi\equiv\partial\mathcal{L}/\partial E$ and magnetic field B, the Hamiltonian $\mathcal{H}=R^{-1}\sqrt{(\Pi^2+R^2)(B^2+R^2)}$ is minimized at $R=\sqrt{|\Pi B|}$. The same physics can be viewed from D0-brane side. In this case, the system is described by the matrix model. Some extended solutions, including multisupertube configurations, were found in Ref. [6].

In the next section, we consider the cases of planar Dp-brane array threaded vertically by superstrings. These are obtained by T-dual transformation along various directions transverse to the type-IIA superstrings carrying D0-brane charge. In Sec. III, we study the case obtained by a T-dual transformation along the longitudinal direction of the original configuration. We show that the blown-up configuration is the helical D1-brane moving with light velocity along its axis. In Sec. IV, we explicitly show that 1/4 of the supersymmetry of the type-IIB Minkowski vacuum is preserved in the D-helix configuration. In the last section we conclude with some discussions and remarks on further works.

II. D*P*-BRANES THREADED BY SUPERSTRINGS

The first question that arises from the supertube physics is whether similar blowing-up happens in the type-IIB setup. One simple way to see this is to take T duality along some directions for the original tpe-IIA superstrings carrying D0-brane charge and check whether the same T duality for the supertube gives sensible blown-up configuration. This scheme is based on the fact that supersymmetry is preserved under T duality [10]. Since the supertube configuration encodes those charges of D0-brane array and type-IIA superstrings, so must its T-dual counterpart.

We first take T duality along some directions transverse to the type-IIA superstring. The D0-brane array threaded by type-IIA superstrings are dualized to be an array of D1-branes crossed by a bunch of type-IIB superstrings. We expect a D3-brane of topology $\boldsymbol{R}^2\times S^1$ as the blown-up configuration. This D3-brane is nothing but T-dual version of the type-IIA supertube. To be more specific, we take the X_4 direction of $\mathbb{E}^{(6)}$ as the T-dual direction.

Taking T duality directly on DBI action is not simple. It is very obscure in the DBI action to start from the $U(\infty)$ matrix and constrain its components (using the orbifold technique used in Ref. [11]) to describe D-brane array along some compact direction. Instead we take an indirect way. With the knowledge about BI fields on the D3-brane, which encodes the dissolved D1-branes and type-IIB superstrings, we construct DBI action for this type-IIB setup. On the resulting D3-brane, the array of D1-branes is dissolved as magnetic flux and the number of type-IIB superstrings is encoded as the electric field along the X direction. In the static gauge for the additional world-volume coordinate $x_4=X_4$, the induced geometry and the BI fields on the world volume become

$$ds^2=-dt^2+dx^2+R^2d\varphi^2+dx_4^2,$$

$$F=E\,dt\wedge dx+B\,dx\wedge d\varphi. \tag{4}$$

One can see that a D3-brane of topology $\boldsymbol{R}^2\times S^1$ is blown-up to have nonvanishing size of the circle direction because a DBI Lagrangian constructed from the above configuration is the same as that of the supertube. The basic reason why this case gives the same results as the type-IIA case is that T duality along X_4 preserves the relative codimension of the D2-brane and D0-branes dissolved in it. Hence the same results will be obtained for further T dualities along the directions along $X_{5,6,7,8,9}$.

Summing up the result for an array of Dp-branes ($p\leqslant 6$) along the X axis, we can say as follows: when threaded vertically by superstrings over the entire volume of the Dp-brane, it is blown-up to the D$(p+2)$-brane. The extra two dimensions obtained are tubular, which are extended along the X direction and embedded in the residual dimensions. In all cases the stabilized radius is the same and governed by the Neveu-Schwarz (NS) field E of the superstrings, and the magnetic field B which is produced effectively by Dp-branes dissolved in the D$(p+2)$-brane.

III. D1-BRANE WITH TRAVELING TYPE-IIB SUPERSTRINGS

In this section we deal with another type-IIB setup, by taking T duality along the X direction. The basic question here is about the type-IIB counterpart of the supertube. The D0-brane array threaded by type-IIA superstrings is T dual to a bunch of D1-branes along which type-IIB superstrings are traveling in one direction. Since the former configuration is not stable in the type-IIA setup, neither should be the latter in the type-IIB setup. At first sight, one might think this latter system will be blown up to a D3-brane acquiring extra spherical two dimensions because there is no two-dimensional object in the type-IIB setup. We show below that this is not the case. Actually the stabilized configuration remains one dimensional. It turns out to be a D-helix with axial momentum flow, whose radius is the same as that of the supertube.

The basic tool is again T duality acting on the supertube. In order to see the resulting configuration, we study the boundary conditions of type-IIB superstrings. These can be obtained by T dualizing the boundary conditions of type-IIA superstrings living on the supertube;

$$\begin{aligned}\text{type IIA:}\quad &(\partial_\sigma X^0+E\partial_\tau X^1)|_{\sigma=0,\pi}=0,\\ &(\partial_\sigma X^1+E\partial_\tau X^0-B\partial_\tau\phi)|_{\sigma=0,\pi}=0,\\ &(R^2\partial_\sigma\phi+B\partial_\tau X^1)|_{\sigma=0,\pi}=0.\end{aligned} \tag{5}$$

T duality along the X^1 direction interchanges $\partial_\tau X^1$ with $\partial_\sigma X^1$. With $\tilde{X}^\mu$ denoting T-dualized coordinates, the above boundary conditions are T dualized as

$$\begin{aligned}\text{type IIB:}\quad &\partial_\sigma(\tilde{X}^0+E\tilde{X}^1)|_{\sigma=0,\pi}=0,\\ &\partial_\tau(\tilde{X}^1+E\tilde{X}^0-B\tilde{\phi})|_{\sigma=0,\pi}=0,\\ &\partial_\sigma(R^2\tilde{\phi}+B\tilde{X}^1)|_{\sigma=0,\pi}=0.\end{aligned} \tag{6}$$

From the second condition, we note that the hypersurface $\tilde{X}^1 + E\tilde{X}^0 - B\tilde{\phi} = c$ (with the omitted conditions for other transverse directions) defines D1 world sheet. We take the constant c to be zero for simplicity.

The other two conditions define the longitudinal directions of the D1-brane. Since both of them are Neumann conditions, one can take arbitrary two independent combinations of the coordinates $\tilde{X}^0 + E\tilde{X}^1$ and $R\tilde{\phi} + B\tilde{X}^1/R$ to make one temporal coordinate and one spatial coordinate. The simplest choice will be the "orthonormal" pair $(\tilde{X}^0,\ R\tilde{\phi})$. This choice is transparent if we see dual background geometry obtained by Buscher's duality [12];

$$\begin{aligned}\tilde{d}s^2 &= -(d\tilde{X}^0)^2 + (d\tilde{X}^1)^2 + R^2(d\tilde{\phi})^2 \\ &= -(d\overline{X^0})^2 + (d\overline{X^1} - E\,d\overline{X^0} + B\,d\overline{\phi})^2 \\ &\quad + R^2(d\overline{\phi})^2, \end{aligned} \tag{7}$$

where one can easily see the relation between the orthonormal coordinates $(\tilde{X}^0,\ R\tilde{\phi},\ \tilde{X}^1)$ and the tilted coordinates $(\overline{X^0},\ R\overline{\phi},\ \overline{X^1})$;

$$\tilde{X}^0 = \overline{X^0}, \quad R\tilde{\phi} = R\overline{\phi}, \quad \tilde{X}^1 = \overline{X^1} - E\overline{X^0} + B\overline{\phi}. \tag{8}$$

In the tilted coordinates, the D1-brane is defined by the hypersurface $\overline{X^1} = 0$. The other coordinates $(\tilde{X}^0,\ R\tilde{\phi})$ are orthonormal longitudinal coordinates. The metric induced on the hypersurface $(\overline{X^1} = 0)$ becomes

$$\tilde{d}s^2 = -(1-E^2)(d\tilde{X}^0)^2 - 2EB\,d\tilde{X}^0 d\tilde{\phi} + (R^2 + B^2)d\tilde{\phi}^2, \tag{9}$$

from which one gets a DBI Lagrangian (in static gauge as $\hat{\tau} = \tilde{X}^0, \hat{\sigma} = \tilde{\phi}$);

$$\mathcal{L} = -\sqrt{(1-E^2)R^2 + B^2}. \tag{10}$$

The DBI Lagrangian is of the same form as that of the type-IIA case. This result is rather expected because T duality in general leaves the D-brane action invariant. The only difference is the change of some field strength components into the derivatives of transverse scalar $\tilde{X}^1$ denoting transverse fluctuations of the D1-brane with respect to the world-sheet coordinates. In the case at hand, we can see this change explicitly in the relation $\tilde{X}^1 = -E\hat{\tau} + B\hat{\sigma}$;

$$\frac{\partial \tilde{X}^1}{\partial \hat{\tau}} = -E, \quad \frac{\partial \tilde{X}^1}{\partial \hat{\sigma}} = B. \tag{11}$$

Therefore E is now the axial velocity of the D-helix. Figure 1 summarizes the configuration.

As in the case of the supertube, the equations of motion just tells us that the momentum Π is conserved. The momentum Π stabilizes the radius of the D-helix as $R = \sqrt{|\Pi B|}$, at which the velocity becomes the speed of light, $E = \pm 1$, and the Hamiltonian saturates its bound as $\mathcal{H} = |\Pi| + |B|$. One could consider the type-IIB strings traveling along the D-helix. However, uniform motion of those strings are not physical because of the world-sheet reparametrization symmetry.

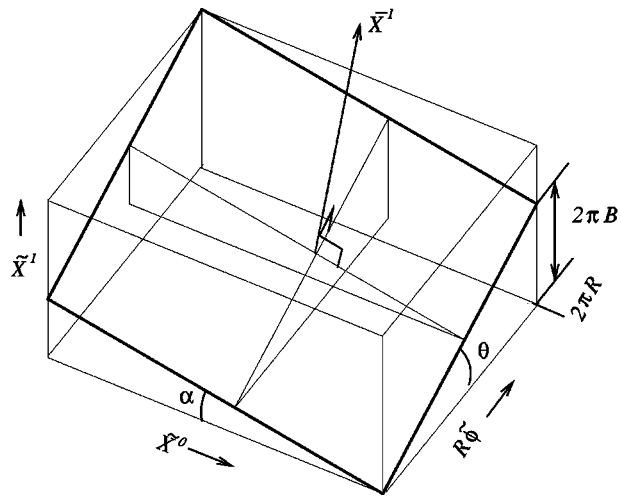

FIG. 1. The solid square represents D1-brane world sheet. In the presence of $E = \tan\alpha$, D1-brane is not static. It is tilted with an angle of $\tan\theta = B/R$, and is thus helical. The helix pitch is $2\pi B$.

IV. SUPERSYMMETRY OF THE D-HELIX

In this section we show that the above D-helix configuration preserves one-quarter of the supersymmetry of the Minkowski vacuum, as in the case of the supertube. This is, in fact, easy to understand because T duality in general preserves supersymmetry [10]. However, it looks not so transparent to see supersymmetry directly in the D-helix configuration. Here we give a rigorous proof of that. We closely follow the procedure of Ref. [4]. Supersymmetry is determined by the independent Killing spinors ϵ satisfying

$$\Gamma \epsilon = \epsilon, \tag{12}$$

where Γ is the matrix defining κ transformation on the world volume of D-branes [13]. In the type-IIB case at hand, it is

$$\begin{aligned}\Delta\Gamma &= \sigma_1 \otimes \overline{\Gamma}, \\ \overline{\Gamma} &= (B\Gamma_{\tilde{0}\tilde{1}} - E\Gamma_{\tilde{1}\tilde{\phi}} + \Gamma_{\tilde{0}\tilde{\phi}}), \end{aligned} \tag{13}$$

where $\Delta \equiv \sqrt{-|\tilde{g}|}$ and the static gauge is chosen. Therefore the Killing spinor relation (12) becomes

$$\begin{pmatrix} 0 & \overline{\Gamma} \\ \overline{\Gamma} & 0 \end{pmatrix} \begin{pmatrix} \epsilon^{1,\alpha} \\ \epsilon^{2,\beta} \end{pmatrix} = \Delta \begin{pmatrix} \epsilon^{1,\alpha} \\ \epsilon^{2,\beta} \end{pmatrix}. \tag{14}$$

In type-IIB case, chiral projection should be understood for the above spinors, so for 32-component spinors ϵ^1 and ϵ^2. (See Ref. [13] for details.) Taking into account positive chirality components, we can put

$$\epsilon^{1,\alpha}=\begin{pmatrix}\epsilon^{1,\alpha_1}\\ 0\end{pmatrix},\quad \epsilon^{2,\alpha}=\begin{pmatrix}\epsilon^{2,\alpha_1}\\ 0\end{pmatrix}. \tag{15}$$

Therefore, they are effectively 16-component spinors.

In order to delimitate the coordinate dependent part of the spinors, we make use of the fact that Killing vectors can be written as bilinears of Killing spinors; $\xi^\mu=\bar{\epsilon}\Gamma^\mu\epsilon$ [14]. In flat geometry, Killing spinors can be written as

$$\epsilon^{1,2}=e^{\tilde{\phi}/2\Gamma_{\tilde{R}\tilde{\phi}}}\epsilon_0^{1,2}, \tag{16}$$

where ϵ_0's are constant spinors and $M_\pm=e^{\pm\tilde{\phi}\Gamma_{\tilde{R}\tilde{\phi}}/2}$ are Lorentz transformations acting on the chiral spinors. (Note that $\epsilon^{1,2}$ are of the same chirality.) With this in mind, one sees that the Killing spinor relation (14), i.e., $\bar{\Gamma}\epsilon^1=\Delta\epsilon^2$ becomes

$$M_+(B\Gamma_{\tilde{0}\tilde{1}}\epsilon_0^1-\Delta\epsilon_0^2)+M_-(\Gamma_{\tilde{0}\tilde{\phi}}-E\Gamma_{\tilde{1}\tilde{\phi}})\epsilon_0^1=0. \tag{17}$$

In order to satisfy the above relation for an arbitrary value of $\tilde{\phi}$, the first two terms and the last term should vanish separately;

$$B\Gamma_{\tilde{0}\tilde{1}}\epsilon_0^1-\Delta\epsilon_0^2=0,$$

$$(\Gamma_{\tilde{0}\tilde{\phi}}-E\Gamma_{\tilde{1}\tilde{\phi}})\epsilon_0^1=0. \tag{18}$$

From the second relation, we see that $\Gamma_{\tilde{0}\tilde{1}}\epsilon_0^1=-E\epsilon_0^1$, from which it is clear that $E=\pm 1$. (This result was also obtained when we insert $R=\sqrt{|\Pi B|}$ into the expression of E.) Therefore the D-helix should travel with the speed of light on its axis to make the configuration supersymmetric. With this value of E inserted, the first relation of Eq. (18) tells us that

$$\epsilon_0^1=-\mathrm{sgn}(B)\epsilon_0^2. \tag{19}$$

Hence ϵ_0^2 can be written in terms of ϵ_0^1. Since ϵ_0^1 is constrained by the projection operator $\Gamma_{\tilde{0}\tilde{1}}$, only eight components of Killing spinors remain independent; thereby one quarter supersymmetry of the type-IIB vacuum are preserved for the configuration.

V. DISCUSSION

In this paper, we studied the supertube physics in the *T*-dual setup. In the transversely *T*-dual case, the result looks somewhat plain; the blowing-up effect happens for an array of D-branes when they are threaded by superstrings. However, we have to mention one interesting point. In the case of the D1 array crossed by the superstring array, we can see why the stabilized radius is expressed by the product of Π and B. If we take the *S* duality, the role of D1-branes and that of superstrings are interchanged. In order for the radius to be invariant under *S* duality, its square (by dimensional analysis) should be proportional to type-ΠB.

In the longitudinally *T*-dual case, we showed that a bunch of D1-branes with type-IIB superstrings moving in one direction along the branes is blown-up to a D helix traveling with the speed of light along its axis. It carries 1/4 of the supersymmetry of the type-IIB vacuum.

One could consider a D3-brane of topology $\boldsymbol{R}\times S^2$ as the blown-up configuration of the aforementioned type-IIB case. However, that configuration is not consistent with the type-IIA result because it cannot be obtained by *T* duality from the supertube configuration. In fact, when one construct DBI action by assuming flat geometry in the spherical polar coordinates and NS *B* field over the spherical part of the topology, the stabilized radius is vanishing. Spherically blown-up configurations can be possible only with appropriate background fields. Some examples are given in Ref. [3] and other examples include D-branes embedded in the flux branes [7,15].

The helical configuration of D-branes appeared in a different context. In Ref. [16], a double helix with fundamental string rungs is obtained by *T* dualizing the D-brane setup of the quantum Hall soliton [17]. With some additional input of D-branes (such as an array of D6-branes along the *X* axis, which is transverse to their world volume, and *T* duality as before) the super D-helix considered in this paper could be a starting point to study such a system further.

Finally, we mention two related problems to be solved. The first one concerns the well-known bound state of D1/D5-branes. As for an array of D3-branes threaded vertically by type-IIB superstrings, one can apply the *S* duality to change these superstrings into the same number of D1-branes. Subsequent *T* dualities along the direction of D1-branes and another direction transverse to both branes result in the familiar D1/D5 bound state. The same sequence of dualities applied to the blown-up version of the initial configuration result in the Kaluza-Klein monopolelike configuration [18]. This is very puzzling. If the blowing-up effect is preserved under *U* duality, the result of supertube predicts that the D1/D5 bound state is indeed unstable (even though it is known as a typical one-quarter supersymmetric state) and should be understood as a Kaluza-Klein monopolelike configuration. The same argument applies to the D0/D4 bound state, which is to be blown-up to NS5-brane wrapping a trivial homology cycle. This nontrivial observation cannot be easily understood by direct consideration of D1/D5 without recourse to *S* and *T* duality. Further study on these bound states should be pursued.

The second one is another *T* duality, which is not considered in this paper. It is the *T* duality along the angular direction of the supertube. This duality is quite intriguing as it involves a fixed point. Since the resulting geometry is singular, it should be taken with care. This will be studied elsewhere.

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