Fermionic zero modes and spontaneous symmetry breaking on the light front

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Spontaneous symmetry breaking is studied in a simple version of the light-front $O(2)$ sigma model with fermions. Its vacuum structure is derived by an implementation of global symmetries in terms of unitary operators in a finite volume with a periodic Fermi field. Because of the dynamical fermion zero mode, the vector and axial $U(1)$ charges do not annihilate the light-front vacuum. The latter is transformed into a continuous set of degenerate vacuum states, leading to the spontaneous breakdown of the axial symmetry. The existence of associated massless Goldstone boson is demonstrated.

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The phenomenon of spontaneous symmetry breaking represents a challenge in the light-front (LF) formulation of quantum field theory. In contrast with the usual quantization on spacelike (SL) surfaces, the vacuum of the theory quantized at constant LF time x^+ can be defined kinematically as a state with minimum (zero) longitudinal LF momentum p^+, since the operator P^+ has a positive spectrum [1]. Thus neglecting modes of quantum fields with $p^+=0$ [zero modes (ZM's)], the vacuum of even the interacting theory does not contain dynamical quanta. This ''triviality'' of the ground state is very advantageous for a Fock-state description of the bound states [2], but it seems to forbid important nonperturbative aspects such as vacuum degeneracy and the formation of condensates. Since Dirac's front form of relativistic dynamics [3,4] defines a consistent quantum theory [1,5], it should be a sensible strategy to look for a genuine LF description of spontaneous symmetry breaking (SSB) [6] and related aspects of the vacuum structure, which would complement the complicated SL picture of a dynamical vacuum state. Various approaches to the LF vacuum problem were previously developed in Refs. [7–9], for example. Note in this context that, in the continuum LF theory, unlike the SL quantization [10,11], even those charges which correspond to nonconserved currents do annihilate the vacuum [12,13]. Thus we may expect similar ''surprises'' in other aspects of the LF field theory.

A convenient regularized framework for studying these and related problems of nonperturbative nature is quantization in a finite volume with fields obeying periodic boundary conditions. This allows one to separate infrared aspects (ZM operators relevant for vacuum properties) from the remainder of the dynamics [14]. Note that to have a well-defined theory, one also has to specify boundary conditions (BC's) in the continuum formulation [15].

For self-interacting LF scalar theories a bosonic ZM is not a dynamical degree of freedom [14] but a constrained variable. Thus the vacuum remains indeed ''empty'' and one expects that physics of SSB is contained in solutions of a complicated operator ZM constraint [16,17]. If a continuous symmetry is spontaneously broken, a massless Nambu-Goldstone (NG) boson should be present in the spectrum of states. However, as emphasized by Yamawaki and coworkers [6], the Goldstone theorem cannot exist on the light front as long as all charges annihilate the LF vacuum. Instead, charge nonconservation has been suggested as the manifestation of the NG phase in the LF scalar theories.

The situation is different for $(3+1)$-dimensional theories with fermions. For fermionic LF fields, one usually imposes antiperiodic BC's in x^- to avoid subtleties with zero modes. However, periodic BC's, which imply a dynamical zero mode in the expansion of the independent component ψ_+, are perfectly valid as well, for both massless and *massive* fermions (see below). Recently, it was demonstrated within the massive LF Schwinger model with antiperiodic fermion field that the residual symmetry under large gauge transformations, when realized quantum mechanically, gives rise to a nontrivial vacuum structure in terms of gauge-field zero mode as well as of fermion excitations [18]. It is the purpose of the present work to demonstrate that a dynamical fermion ZM provides a similar mechanism for a simple non-gauge field theory with fermions. Charges, which are generators of global symmetries of the given system, contain a ZM part and consequently transform the trivial vacuum into a continuous set of degenerate vacuum states. This leads to a SSB in the usual sense [19–26] with nonzero vacuum expectation values of certain operators and a massless NG state in the spectrum of states. Much of what we demonstrate is of a rather general nature.

In string theory, periodic Fermi fields are known as Ramond fermions [27]. The corresponding zero modes lead to degenerate vacua [27,28]. In our approach, vacuum degeneracy is related to symmetries of the Hamiltonian, and the operators generating multiple vacua are bilinear in fermion Fock operators.

In the LF field theory, dynamical symmetry breaking [19,20] has been studied so far within the usual mean-field approximation [29,30] and also by means of Schwinger-Dyson equations [31]. In the discrete light cone quantization formulation, the relation between fermion zero modes and

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vacuum degeneracy has been found within a two-dimensional supersymmetric $SU(N)$ gauge theory [32].

To simplify our discussion of SSB in the LF field theory, we will consider a version of the $O(2)$-symmetric sigma model with fermions [6,33] specified by the Lagrangian

$$\mathcal{L}=\bar{\psi}\left(\frac{i}{2}\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}-m\right)\psi+\frac{1}{2}(\partial_{\mu}\sigma\partial^{\mu}\sigma+\partial_{\mu}\pi\partial^{\mu}\pi)$$
$$-\frac{1}{2}\mu^2(\sigma^2+\pi^2)-g\bar{\psi}(\sigma+i\gamma^5\pi)\psi, \tag{1}$$

where the quartic self-interaction term for the scalar fields σ and π has been omitted, because it is not relevant for our purpose. Lagrangian (1) is invariant under the global $U(1)$ transformation $\psi\to\exp(-i\alpha)\psi$ and for $m=0$ also under the axial transformation:

$$\psi\to\exp(-i\beta\gamma^5)\psi, \quad \psi^{\dagger}\to\psi^{\dagger}\exp(i\beta\gamma^5), \tag{2}$$

$$\sigma\to\sigma\cos 2\beta-\pi\sin 2\beta,$$
$$\pi\to\sigma\sin 2\beta+\pi\cos 2\beta. \tag{3}$$

Rewriting the above Lagrangian in terms of the LF variables, for the LF Hamiltonian one finds

$$P^{-}=\int_V d^3\underline{x}[(\partial_k\sigma)^2+(\partial_k\pi)^2+\mu^2(\sigma^2+\pi^2)$$
$$+\psi_+^{\dagger}(m\gamma^0-i\alpha^k\partial_k)\psi_-+g\psi_+^{\dagger}\gamma^0$$
$$\times(\sigma+i\gamma^5\pi)\psi_-+\text{H.c.}], \tag{4}$$

where $d^3\underline{x}\equiv\frac{1}{2}dx^-d^2x^{\perp}$. Our notation is $x^{\pm}=x^0\pm x^3$, $p^{\mu}x_{\mu}=\frac{1}{2}p^-x^++\underline{p}\,\underline{x}$, $\underline{p}x=\frac{1}{2}p^+x^--x^{\perp}p^{\perp}$, $x^{\perp}p^{\perp}\equiv x^kp^k$, $\partial_{\perp}^2\equiv\partial_k\partial_k$, $k=1,2$ and x^+,p^- are the LF time and energy. Correspondingly, we define the Dirac matrices as $\gamma^{\pm}=\gamma^0\pm\gamma^3$ and $\alpha^k=\gamma^0\gamma^k$, the LF projection operators as $\Lambda_{\pm}=\frac{1}{2}\gamma^0\gamma^{\pm}$ and $\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$. $\Lambda_{\pm}$ separates the Fermi field into an independent component $\psi_+=\Lambda_+\psi$ and a dependent one $\psi_-=\Lambda_-\psi$.

The infrared-regularized formulation is achieved by enclosing the system into a three-dimensional box $-L\leqslant x^-\leqslant L, -L_{\perp}\leqslant x^k\leqslant L_{\perp}$ with volume $V=2L(2L_{\perp})^2$ and by imposing periodic boundary conditions for all fields in x^- and $x^{\perp}$. This leads to a decomposition of the fields into the zero-mode (subscript 0) and normal-mode (NM, subscript n) parts. One finds that $\psi_-,\psi_-^{\dagger},\sigma_0$, and π_0 are nondynamical fields with vanishing conjugate momenta, while $\psi_+,\psi_+^{\dagger},\sigma_n$, and π_n are dynamical. For a consistent quantization, one should apply the Dirac-Bergmann or a similar method suitable for systems with constraints. We will postpone this for a more detailed work [34], assuming here the standard (anti)commutators at $x^+=0$:

$$\{\psi_{+i}(\underline{x}),\psi_{+j}^{\dagger}(\underline{y})\}=\frac{1}{2}\delta_{ij}\delta^3(\underline{x}-\underline{y}), \quad i,j=1,4, \tag{5}$$

$$[\sigma_n(\underline{x}),\partial_-\sigma_n(\underline{y})]=[\pi_n(\underline{x}),\partial_-\pi_n(\underline{y})]=\frac{i}{4}\delta_n^3(\underline{x}-\underline{y}). \tag{6}$$

$\delta^3(\underline{x})=\delta_0^3+\delta_n^3(\underline{x})$ is the periodic delta function with $\delta_0^3=2/V$ being its global zero-mode part. Relation (5) can be derived using the expansion

$$\psi_+(\underline{x})=\sum_{\underline{p},s=\pm 1/2}\frac{u(s)}{\sqrt{V}}(b(\underline{p},s)e^{-i\underline{px}}+d^{\dagger}(\underline{p},-s)e^{i\underline{px}}), \tag{7}$$

$$\{b(\underline{p},s),b^{\dagger}(\underline{p}',s')\}$$
$$=\{d(\underline{p},s),d^{\dagger}(\underline{p}',s')\}=\delta_{s,s'}\delta_{\underline{p},\underline{p}'}. \tag{8}$$

The spinors in the representation with diagonal γ^5 are $u^{\dagger}(s=\frac{1}{2})=(1\ 0\ 0\ 0)$, $u^{\dagger}(s=-\frac{1}{2})=(0\ 0\ 0\ 1)$, where s is the LF helicity. The summations in Eq. (7) run over discrete momenta $p^+=2\pi L^{-1}n$, $n=0,1\ldots,\infty$, $p^k=\pi L_{\perp}^{-1}n^k$, $n^k=0,\pm 1,\ldots,\infty$. The modes with $p^+=0$ will be denoted by $b_0(p_{\perp},s)$, etc.

The nondynamical fields satisfy the constraints

$$2i\partial_-\psi_-=[m\gamma^0-i\alpha^k\partial_k+g\gamma^0(\sigma+i\gamma^5\pi)]\psi_+, \tag{9}$$

$$(\partial_{\perp}^2-\mu^2)\sigma_0=g\int_{-L}^{+L}\frac{dx^-}{2L}(\psi_+^{\dagger}\gamma^0\psi_-+\text{H.c.}), \tag{10}$$

$$(\partial_{\perp}^2-\mu^2)\pi_0=g\int_{-L}^{+L}\frac{dx^-}{2L}(i\psi_+^{\dagger}\gamma^0\gamma^5\psi_-+\text{H.c.}). \tag{11}$$

Note that at $x^+=0$, where the interacting ψ_+ field coincides with the free field, constraint (9) defines an *interacting* ψ_-. The ZM projection of Eq. (9),

$$g\left[\Sigma_0\psi_{+0}+\int_{-L}^{+L}\frac{dx^-}{2L}\Sigma_n\psi_{+n}\right]=(i\gamma^k\partial_k-m)\psi_{+0}, \tag{12}$$

where $\Sigma\equiv\sigma+i\pi\gamma^5$, puts no restrictions on ψ_{+0} [for free fermions, Eq. (12) allows no ZM for $m\neq 0$ and a *global* ZM, if $m=0$]. The NM solution of constraint (9) is

$$\psi_{-n}(\underline{x})=\frac{1}{4i}\int_{-L}^{+L}\frac{dy^-}{2}\epsilon_n(x^--y^-)\{(m\gamma^0-i\alpha^k\partial_k)\psi_{+n}(\underline{y})$$
$$+g\gamma^0[(\sigma_n(\underline{y})+i\pi_n(\underline{y})\gamma^5)$$
$$+(\sigma_0+i\pi_0\gamma^5)]\}\psi_+(\underline{y}), \tag{13}$$

where $\epsilon_n(x^--y^-)$ is the normal-mode part of the periodic sign function and $\underline{y}\equiv(y^-,x^{\perp})$.

In Eqs. (10) and (11), we have assumed the existence of the ψ_{-0} ZM, so the integrands are given by the diagonal combinations $\psi_{+0}\gamma^0\psi_{-0}+\psi_{+n}\gamma^0\psi_{-n}$, etc. Actually, by combining all three ZM constraints for $m=0$ into one,

$$\left\{\frac{g^2}{\partial_\perp^2-\mu^2}\left[\psi_{+0}^\dagger\gamma^0\psi_{-0}+\text{H.c.}-(\psi_{+0}^\dagger\gamma^0\gamma^5\psi_{-0}-\text{H.c.})\gamma^5 + \int_{-L}^{+L}\frac{dx^-}{2L}(\psi_{+n}^\dagger\gamma^0\psi_{-n}+\text{H.c.}-(\psi_{+n}^\dagger\gamma^0\gamma^5\psi_{-n} - \text{H.c.})\gamma^5)\right]\right\}\psi_{+0}=i\gamma^k\partial_k\psi_{+0}-g\int_{-L}^{+L}\frac{dx^-}{2L}\Sigma_n\psi_{+n}, \tag{14}$$

we find that nonzero ψ_{-0} is *required* for consistency: setting $\psi_{-0}=0$ in Eq. (14) reduces it into an operator relation among independent fields which cannot be satisfied. This can be verified explicitly using the lowest-order approximation to ψ_{-n} by setting $\sigma_0=\pi_0=0$ in Eq. (13).

While the free massive fermion Hamiltonian is, unlike the spacelike quantization, symmetric under the axial vector transformations [Eq. (16)] below [35], the mass term in the ψ_--constraint generates interaction terms which are proportional to mg and which, due to an extra γ^0, violate the axial symmetry explicitly. This is the reason why we shall set $m=0$ henceforth. Note however that the scalar fields have to be massive [6] to avoid infrared problems.

Using Eq. (9), the interacting part of P^- takes the form

$$P_{int}^-=\int_V d^3\underline{x}[\mu^2(\sigma_0^2+\pi_0^2)+(\partial_k\sigma_0)^2+(\partial_k\pi_0)^2] +ig\int_V d^3\underline{x}\psi_+^\dagger(\underline{x})\Sigma^\dagger(\underline{x})\int_{-L}^{+L}\frac{dy^-}{2}\frac{1}{2}\epsilon_n(x^--y^-) \times[i\gamma^k\partial_k\psi_{+n}(y^-,x^\perp)+\text{H.c.} -g\Sigma(y^-,x^\perp)\psi_+(y^-,x^\perp)]. \tag{15}$$

P_{int}^- and solution (13) are not closed expressions due to the presence of σ_0 and π_0, which in turn are given by their own constraints (10) and (11), depending on ψ_{-n}. However, this is not an obstacle for determining the symmetry properties of the Hamiltonian, which are of primary importance in the present approach. First we observe that the LF analog of the axial vector transformation [Eq. (2)] is

$$\psi_+(\underline{x})\rightarrow\exp(-i\beta\gamma^5)\psi_+(\underline{x}), \tag{16}$$

while the NM fields σ_n,π_n transform according to Eq. (3). As for the constrained variables, we shall demand that ψ_{-n} has a well defined transformation law, which is unambiguously fixed by the terms with α^k,σ_n and π_n in solution (13). It follows that $\sigma_0+i\pi_0\gamma^5$ will transform exactly as $\sigma_n+i\pi_n\gamma^5$ and that the whole ψ_{-n} as well as ψ_{-0} will transform for $m=0$ in the same way as ψ_+. As a result, we find that P_{int}^- is invariant under $U_A(1)$ transformations in addition to $U(1)$. The symmetries give rise to the conserved (normal-ordered) vector current $j^\mu=:\psi^\dagger\gamma^0\gamma^\mu\psi:$, $\partial_\mu j^\mu=0$ and the conserved axial-vector current $j_5^\mu=\psi^\dagger\gamma^0\gamma^\mu\gamma^5\psi+2(\sigma\partial^\mu\pi-\pi\partial^\mu\sigma)$ $(\mu=+,-,k)$:

$$\partial_\mu j_5^\mu=2m(i\psi_+^\dagger\gamma^0\gamma^5\psi_-+\text{H.c.})=0\quad\text{for}\quad m=0. \tag{17}$$

These symmetries are implemented by the unitary operators $U(\alpha)=\exp(i\alpha Q)$, $V(\beta)=\exp(i\beta Q^5)$:

$$\psi_+(\underline{x})\rightarrow e^{-i\alpha}\psi_+(\underline{x})=U(\alpha)\psi_+(\underline{x})U^\dagger(\alpha),$$
$$\psi_+(\underline{x})\rightarrow e^{-i\beta\gamma^5}\psi_+(\underline{x})=V(\beta)\psi_+(\underline{x})V^\dagger(\beta). \tag{18}$$

While the NM parts of the charge operators Q and Q^5,

$$Q=\int_V d^3\underline{x}j^+(\underline{x})=2\int_V d^3\underline{x}\psi_+^\dagger\psi_+, \tag{19}$$

$$Q^5=2\int_V d^3\underline{x}[\psi_+^\dagger\gamma^5\psi_+ +2(\sigma_n\partial_-\pi_n-\pi_n\partial_-\sigma_n)], \tag{20}$$

are diagonal in creation and annihilation operators, the ZM parts contain also off-diagonal terms:

$$Q_0=\sum_{p_\perp,s}[b_0^\dagger(p_\perp,s)b_0(p_\perp,s)-d_0^\dagger(p_\perp,s)d_0(p_\perp,s) +b_0^\dagger(p_\perp,s)d_0^\dagger(-p_\perp,-s)+d_0(p_\perp,s)b_0(-p_\perp,-s)], \tag{21}$$

$$Q_0^5=\sum_{p_\perp,s}2s[b_0^\dagger(p_\perp,s)b_0(p_\perp,s)+d_0^\dagger(p_\perp,s)d_0(p_\perp,s) +b_0^\dagger(p_\perp,s)d_0^\dagger(-p_\perp,-s)-d_0(p_\perp,s)b_0(-p_\perp,-s)]. \tag{22}$$

The commuting ZM charges Q_0 and Q_0^5 *do not* annihilate the LF vacuum $|0\rangle$ defined by $b(\underline{p},s)|0\rangle=d(\underline{p},s)|0\rangle=0$. However, their vacuum expectation values are zero, as they have to be. In this way, the vacuum of the model transforms under $U(\alpha),V(\beta)$ as $|0\rangle\rightarrow|\alpha\rangle=\exp(i\alpha Q_0)|0\rangle$, $|0\rangle\rightarrow|\beta\rangle=\exp(i\beta Q_0^5)|0\rangle$, where

$$|\beta\rangle=\exp\left(i\beta\sum_{p_\perp,s}2s[b_0^\dagger(p_\perp,s)d_0^\dagger(-p_\perp,-s)+\text{H.c.}]\right)|0\rangle,$$
$$|\alpha\rangle=\exp\left(i\alpha\sum_{p_\perp,s}[b_0^\dagger(p_\perp,s)d_0^\dagger(-p_\perp,-s)+\text{H.c.}]\right)|0\rangle. \tag{23}$$

The vacua contain an infinite number of ZM fermion-antifermion pairs with opposite helicities.

Thus the global symmetry of Hamiltonian (15) leads to an infinite set of translationally invariant states labeled by two real parameters. Since $U(\alpha)$ and $V(\beta)$ commute with P^-, the vacua are degenerate in the LF energy. The Fock space can be built from any of them since they are unitarily equivalent.

We are in a position now to demonstrate the existence of the Goldstone theorem in the considered model. We have all the ingredients for the usual proof of the theorem [21–25]: the existence of the conserved current j_5^μ, the operators

A, namely $\overline{\psi}\psi = \psi_+^\dagger \gamma^0 \psi_- + \psi_-^\dagger \gamma^0 \psi_+$ and $\overline{\psi}\gamma^5\psi = \psi_+^\dagger \gamma^0 \gamma^5 \psi_- + \psi_-^\dagger \gamma^0 \gamma^5 \psi_+$, which are noninvariant under the axial transformation

$$A \to V(\beta) A V^\dagger(\beta) \neq A \Rightarrow \delta A = -i\beta[Q^5, A] \neq 0, \quad (24)$$

and the property $Q^5|\alpha,\beta\rangle = Q_0^5|\alpha,\beta\rangle \neq 0$. Of course, the above Fermi bilinears are symmetric under $U(1)$, so the commutator $[Q,A]$ vanishes and there is no symmetry breaking associated with this symmetry.

In a little more detail, from the axial current conservation and the periodicity in x^- and $x^\perp$ we obtain the condition of the time independence of the vacuum expectation value of the commutator [Eq. (24)],

$$\partial_+ \langle \mathrm{vac}|[Q^5(x^+), A]|\mathrm{vac}\rangle = 0, \quad |\mathrm{vac}\rangle \equiv |\alpha,\beta\rangle, \quad (25)$$

in addition to

$$\langle \mathrm{vac}|[Q^5(x^+), A]|\mathrm{vac}\rangle \neq 0. \quad (26)$$

Note that relation (26) is only possible due to the fact that Q^5 does not annihilate the vacuum, and this crucially depends on the existence of the ZM part of Q^5. Inserting a complete set of four-momentum eigenstates into Eqs. (25) and (26) and using the translational invariance

$$e^{-iP_\mu x^\mu}|\mathrm{vac}\rangle = |\mathrm{vac}\rangle, \quad j_5^+(x) = e^{-iP_\mu x^\mu} j_5^+(0) e^{iP_\mu x^\mu}, \quad (27)$$

we arrive in the usual way [21,23,25,26] at the conclusion that there must exist a state $|n\rangle = |G\rangle$ such, that

$$\langle \mathrm{vac}|A|G\rangle\langle G|j_5^+(0)|\mathrm{vac}\rangle \neq 0, \quad (28)$$

with $P_G^- = 0$ for $P_G^+ = P_G^\perp = 0$. Thus $M_{\mathrm{G}}^2 = P^+_{\mathrm{G}} P^-_{\mathrm{G}} - (P^\perp_{\mathrm{G}})^2 = 0$. From the infinitesimal rotation of the Fock vacuum we have explicitly

$$Q_0^5|0\rangle = \sum_{p_\perp, s} 2s b_0^\dagger(p_\perp, s) d_0^\dagger(-p_\perp, -s)|0\rangle \equiv |G\rangle. \quad (29)$$

Using the transformation law of the $\psi_\pm$ fields and anticommutator (5), one can show that

$$[\overline{\psi}\psi, Q^5] = 2\overline{\psi}\gamma^5\psi, \quad [\overline{\psi}\gamma^5\psi, Q^5] = 2\overline{\psi}\psi \quad (30)$$

and thus relation (26) implies nonzero vacuum condensates $\langle \mathrm{vac}|A|\mathrm{vac}\rangle$. They will depend on the coupling constant through ψ_{-n}. To obtain quantitative results, one has to solve constraint (9) approximately [33].

To summarize, we have demonstrated that spontaneous symmetry breaking can occur in the *finite-volume* formulation of the fermionic LF field theory. While in contrast with the usual expectation within the spacelike field theory (see Ref. [36], for example), this is related to the explicit presence of a discrete infinity of dynamical fermion zero modes in the finite-volume LF quantization. One of the advantages of this infrared-regularized formulation is that one does not need to introduce test functions and complicated definitions of operators to obtain a mathematically rigorous framework [23]. For example, contrary to the standard infinite-volume formulation, the norm of the state $Q^5|\mathrm{vac}\rangle = Q_0^5|\mathrm{vac}\rangle$ is finite and *volume independent*. The issue of continuum limit and volume independence of the physical picture obtained in a finite volume requires a further study. We would like to stress however that the Goldstone boson state [Eq. (29)] is not an artifact of using boundary conditions but is a reliable prediction of the theory. Without boundary conditions, the light front theory is not mathematically well defined [1,5]. Quantization in a finite volume is then a natural next step which permits one to study the infrared phenomena in a consisent way.

In the usual treatment of fermionic theories [19,36,37], the considered vacua, related by a canonical transformation, are free-field vacua corresponding to fermion fields with different masses. In the LF picture, fields with different masses are unitarily equivalent [1] and if one neglects zero modes the vacuum is unique. Our approach relates the vacuum degeneracy to the unitary operators implementing the symmetries, making use of the ‘‘triviality’’ of the LF vacuum in the sector of normal Fourier modes.

Nevertheless, there are still a few aspects of the present approach that have to be understood better. First, one has to perform a full constrained quantization of the model to derive the (anti)commutation relations for all relevant (ZM) degrees of freedom. Also, the connection of our picture with the standard one, based on the mean-field approximation and the new vacuum with lower energy above the critical coupling, has to be clarified.

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