Structure of radiatively induced Lorentz and *CPT* violation in QED at finite temperature

L. Cervi and L. Griguolo
Dipartimento di Fisica, Universitá di Parma and INFN, Gruppo Collegato di Parma, Viale delle Scienze, 43100 Parma, Italy

D. Seminara
Dipartimento di Fisica, Universitá di Firenze and INFN, Sezione di Firenze, Largo Enrico Fermi, 50125 Firenze, Italy
(Received 16 April 2001; published 4 October 2001)

We obtain the induced Lorentz- and *CPT*-violating term in QED at finite temperature using the imaginary-time formalism and dimensional regularization. Its form resembles a Chern-Simons-like structure, but, unexpectedly, it does not depend on the temporal component of the fixed b_μ constant vector that is coupled to the axial-vector current. Nevertheless, Ward identities are respected and its coefficient vanishes at $T=0$, consistent with previous computations with the same regularization procedure, and it is a nontrivial function of temperature. We argue that at finite T a Chern-Simons-like Lorentz- and *CPT*-violating term is generically present, the value of its coefficient being unambiguously determined up to a T-independent constant, related to the zero-temperature renormalization conditions.

DOI: 10.1103/PhysRevD.64.105003 PACS number(s): 11.30.Cp, 11.10.Gh, 11.15.Tk, 11.30.Er

I. INTRODUCTION

The phenomenological consequences of breaking Lorentz and *CPT* invariance have been actively explored in recent years as they could be measurable low-energy effects of quantum gravity [1] or superstrings [2]. In QED this issue was examined some time ago in Ref. [3], while lately *CPT* and Lorentz noninvariant extensions of the standard model were scrutinized in Ref. [4]. As many breaking terms are allowed, most efforts have been focused on the possible constraints coming from experimental data [5] as well as from renormalizability requirements and anomaly cancellation. In this context, there arose a ‘‘theoretical’’ controversy on the possibility of generating, through radiative corrections, a Chern-Simons-like term in the effective action of QED. There Lorentz and *CTP* symmetries can be in fact destroyed by considering a term of the form

$$\mathcal{L}_{CS}=\frac{k_\mu}{2}\epsilon^{\mu\nu\alpha\beta}A_\nu F_{\alpha\beta}, \tag{1}$$

where k_μ is a constant vector. This breaking term, suggested in Ref. [3], predicts birefringence of the light in the vacuum and observations on distant galaxies put a very stringent bound on k_μ [5]. On the other hand the superstring inspired extensions of the standard model proposed in Ref. [4] contain, in the fermionic sector, a Lorentz- and *CPT*-violating axial-vector coupling

$$\mathcal{L}_b=b_\mu\bar{\psi}\gamma^\mu\gamma_5\psi, \tag{2}$$

with b_μ a constant, a prescribed four-vector that couples to the usual axial-vector current of QED. The interaction term $\mathcal{L}_b$ could generate, through radiative corrections, a nonvanishing value for k_μ [6]. If this were the case, the strong bounds on k_μ would translate into strong bounds for the noninvariant term (2). The aforementioned controversy arises from the fact that the calculation is plagued by a dependence on the regularization adopted. While some papers [7] claim that particular methods offer the correct result, others argue that the requirement of vector gauge invariance forces a vanishing induced term [8,9]. Recently this issue was also discussed in [10] in the heat kernel approach. A rather lucid discussion of the problem appeared in Ref. [11], where it was pointed out that the relevant form of the vectorial Ward identities may depend on how the vector b_μ is embedded into (or derived from) a more fundamental theory. As an example of that in [11] an axion-like model was proposed to generate b_μ as a vacuum expectation value (VEV) of a dynamical field: there, a weaker form of the vectorial Ward identity governs the appearance of the interaction (1) in the effective action, and, in particular, it is not strong enough to ensure the vanishing of its coefficient. In any case, when b_μ is considered a strictly constant nondynamical vector field, only vectorial Ward identities with vanishing axial momentum are relevant and they do not fix the actual value of the coefficient of the Chern-Simons (CS) term: it depends on the renormalization condition.

Our interest is instead devoted to a different feature of the problem: the purpose of this paper is in fact to study the effect of a thermal bath on the structure of the Chern-Simons like term (1), obtained by integrating out fermions coupled to the axial-vector b_μ. In particular our starting observation is that regularization ambiguities cannot modify temperature dependence, since they are related to the ultraviolet behavior of the theory, that is temperature independent. Renormalization conditions are usually implemented at $T=0$, where the parameters of the theory are defined, and consequently their temperature evolution is determined. Our one-loop computation may suffer, therefore, of the mentioned $T=0$ ambiguities while the functional form of the induced term and the temperature dependence of its coefficient are safe. We will use imaginary-time formalism and, for simplicity, dimensional regularization: in this scheme, where the vectorial Ward identities hold even at nonzero axial-vector momentum, a consistency check of our algebra is given by the vanishing of the induced term (1) in the limit $T\to 0$. The fact that dimensional regularization in its standard form does not allow the appearance of the CS term at $T=0$ has been already pointed out in [9]. On the contrary a *CPT*- and

Lorentz-violating Chern-Simons action is generically present at $T\neq 0$: while this fact may have some relevance for phenomenological application, potentially being active in the early universe, a serious question arises about the consistency of the effective theory. For time-like b_μ it was shown in [12] that the vacuum is unstable under pairs creation of tachyonic photon modes with finite vacuum decay rates and, recently, it was argued [13] that, in this case, unitarity itself may be in trouble (see also the original discussion in [3]). A more general analysis on the consistency of the theory at quantum level has been presented in [14], where both time-like and space-like cases appear to be problematic when microcausality and stability are examined. Rather surprisingly our computations show that the induced term does not depend on the temporal component of b_μ: we have not explored up to now the dynamical consequences of this fact in our finite-temperature context. Moreover, invariance under small (i.e., not wrapping around the compactified imaginary time [15]) vectorial gauge transformations of the induced term is easily shown due to the use of dimensional regularization.

II. THE STRUCTURE OF ONE-LOOP SELF-ENERGY AT $T\neq 0$

To begin with let us consider a modified QED action described by the Lagrangian density

$$\mathcal{L}=\bar{\psi}[i\partial\!\!\!/-m-\gamma_5 b\!\!\!/-eA\!\!\!/]\psi. \tag{3}$$

As discussed in Ref. [6] the b_μ linear contribution to the Chern-Simons term arises from the photon self-energy with one insertion of the axial-vector field,

$$\Pi_b^{\mu\nu}(p)=ib^\lambda[I_{\mu\nu\lambda}(p)+I_{\nu\mu\lambda}(-p)], \tag{4}$$

where $I_{\mu\nu\lambda}$ is given by the "triangle"-like graph with zero momentum axial vertex (we are working from now on directly in Euclidean space):

$$I_{\mu\nu\lambda}(p)=-ie^2\int\frac{d^4k}{(2\pi)^4}\times\frac{\mathrm{Tr}[\gamma_\mu(k\!\!\!/+im)\gamma_\lambda\gamma_5(k\!\!\!/+im)\gamma_\nu((k\!\!\!/+p\!\!\!/)+im)]}{(k^2+m^2)^2((k+p)^2+m^2)}. \tag{5}$$

The CPT- and Lorentz-violating Chern-Simons action is extracted from Eq. (5) by isolating, from the odd-parity part, the tensorial structure linear in the external momentum and by performing the limit $p^2\rightarrow 0$ in the scalar integral multiplying it. In Ref. [9] the explicit evaluation at $T=0$ of $\Pi_b^{\mu\nu}(p)$ in the dimensional regularization was presented. In particular it was noticed that the only algebraic properties of γ_5 used in the computation were: (a) the trace of γ_5 with an odd number of Dirac matrices vanishes and (b) the trace of γ_5 with an even number of Dirac matrices can be reduced using the Clifford algebra to the quantity $\mathrm{Tr}[\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta\gamma_5]$. Consistency also requires that $\mathrm{Tr}[\gamma_\mu\gamma_\nu\gamma_5]=\mathrm{Tr}[\gamma_5]=0$.

In this zero-temperature case, the linear p_μ dependence is easily extracted and the result can be presented as

$$-i\frac{e^2}{8\pi^2}b^\lambda p^\beta\,\mathrm{Tr}[\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\beta\gamma_5][\mathcal{F}_1(p^2/m^2)+\mathcal{F}_2(p^2/m^2)], \tag{6}$$

where the explicit form of $\mathcal{F}_1(p^2/m^2)$ and $\mathcal{F}_2(p^2/m^2)$ is given in [9]: evaluating $\mathcal{F}_1(p^2/m^2)$ and $\mathcal{F}_2(p^2/m^2)$ in D dimensions, taking the limit $D=4$ and expanding in p^2, it results that

$$\mathcal{F}_1(p^2/m^2)+\mathcal{F}_2(p^2/m^2)\simeq-\frac{p^2}{12m^2}, \tag{7}$$

showing the absence of a Chern-Simons contribution to the effective action. We agree with this computation but we want to recall a couple of remarks in order to better appreciate the finite T effects. First of all one can check that the cancellation of the leading order (constant) contribution to $\mathcal{F}_1(p^2/m^2)+\mathcal{F}_2(p^2/m^2)$ comes from a delicate balance between a "classical" term [proportional to m^2 in Eq. (5)] and an "anomalous" quantum term, deriving from the potential divergences.[1] As mentioned in Ref. [11], the massless case escapes this mechanism, therefore dimensional regularization gives a nonvanishing result. This fact is related to the loss of analyticity [8] of the "triangle"-like diagram at $m=0$ in the limit of vanishing axial-vector momentum. In the finite temperature case, where the analyticity properties in the external momenta are usually weaker, this observation suggests the concrete possibility that a nonzero result could appear even in the massive case. Second, we stress that the covariance of the momentum integration immediately selects the Chern-Simons tensorial structure and the dependence on p^2 of the coefficient function at zero temperature. This is no longer true at finite temperature, as we shall see in a while, due to explicit presence of the Matsubara frequencies: in particular the double limit $p_0\rightarrow 0$ and $p_i\rightarrow 0$ has to be performed very carefully.

Let us assume from now on that the system is in thermal equilibrium with a temperature $T=\beta^{-1}$, β being interpreted as the radius of the compactified Euclidean time. In this case we may use the Matsubara formalism that consists simply in taking $k_0=(n+\frac{1}{2})2\pi/\beta$ (antiperiodic boundary conditions for fermions requires semi-integers frequencies) and replacing $1/2\pi\int dk_0=1/\beta\Sigma_n$. The remaining $\int d^3\hat{k}$ integral is of course continued to D-spatial dimensions. The trace can still be performed in full generality and simple algebraic manipulations in the loop momenta (not involving shifts or symmetry properties) allow us to write Eq. (8) as follows:

[1]This mechanism of cancellation of a quantum contribution against a classical one is reminiscent of the analogous phenomena in three-dimension (3D) in the case of parity anomaly. This perfect balance between the two contributions is peculiar of the standard dimensional regularization. In other scheme this exact cancellation does not occur, leaving us with a nonvanishing CS term whose coefficient is, however, temperature independent.

$$I_{\mu\nu\lambda}(p)=-4i\frac{e^2}{\beta}\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}\frac{(2k_\mu\epsilon_{\nu\lambda\rho\sigma}p^\rho k^\sigma-2k_\nu\epsilon_{\mu\lambda\rho\sigma}p^\rho k^\sigma)-2(k\cdot p)\epsilon_{\mu\nu\lambda\rho}p^\rho+p^2\epsilon_{\mu\nu\lambda\rho}(k^\rho-p^\rho)}{(k^2+m^2)^2((k+p)^2+m^2)}$$
$$+4i\frac{e^2}{\beta}\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}\frac{\epsilon_{\mu\nu\lambda\rho}(k^\rho-p^\rho)}{(k^2+m^2)^2}. \tag{8}$$

The following step in the computation is to extract the tensorial structure, leaving us with the evaluation of scalar integrals. The second term in Eq. (8) is easily tamed, both the integral over the spatial components and the series over k_0 are antisymmetric in exchanging $k\to-k$ (we can find a region around $D=3$ where everything is convergent). It remains therefore

$$-4i\frac{e^2}{\beta}\epsilon_{\mu\nu\lambda\rho}p^\rho I_0, \tag{9}$$

where we have

$$I_0=\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}\frac{1}{(\hat{k}^2+k_0^2+m^2)^2}, \tag{10}$$

that exhibits the Chern-Simons-like structure. Let us discuss now the first contribution. We introduce Feynman parameters in order to perform the integral. To implement translations only on the space components of the loop momentum we decompose k_μ as follows:

$$k_\mu=\hat{k}_\mu+k_0\delta_{0\mu}. \tag{11}$$

Shifting $\hat{k}\to\hat{k}-x\hat{p}$ (where $\hat{p}_\mu$ is defined as above) in Eq. (8) and using the covariance under spacial rotations, allows us to conclude that

$$\hat{k}_\mu\hat{k}_\nu\to\frac{\hat{k}^2}{D}(\delta_{\mu\nu}-\delta_{\mu0}\delta_{\nu0}),$$

we arrive to the form

$$I_{\mu\nu\lambda}(p)=-4i\frac{e^2}{\beta}[\epsilon_{\mu\nu\lambda0}I_1+\epsilon_{\mu\nu\lambda\rho}p^\rho(2I_2+I_0)$$
$$+(p^\mu\epsilon_{\nu\lambda\rho0}p^\rho-p^\nu\epsilon_{\mu\lambda\rho0}p^\rho-p^2\epsilon_{\mu\nu\lambda0})I_3$$
$$+(\delta_{0\mu}\epsilon_{\nu\lambda\rho0}p^\rho-\delta_{0\nu}\epsilon_{\mu\lambda\rho0}p^\rho-p_0\epsilon_{\mu\nu\lambda0})I_4], \tag{12}$$

where

$$I_1=\int_0^1dx\,2(1-x)\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}$$
$$\times\frac{\left[p^2(1-2x)(k_0+xp_0)-\frac{2}{D}\hat{k}^2p_0+2p_0(k_0+xp_0)^2\right]}{[\hat{k}^2+(k_0+xp_0)^2+x(1-x)p^2+m^2]^3},$$

$$I_2=\int_0^1dx\,2(1-x)\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}$$
$$\times\frac{\left[-p_0(k_0+xp_0)-\frac{2}{D}\hat{k}^2-\frac{1}{2}p^2(1-x)\right]}{[\hat{k}^2+(k_0+xp_0)^2+x(1-x)p^2+m^2]^3},$$

$$I_3=-\int_0^1dx\,2(1-x)\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}$$
$$\times\frac{2x(k_0+xp_0)}{[\hat{k}^2+(k_0+xp_0)^2+x(1-x)p^2+m^2]^3},$$

$$I_4=\int_0^1dx\,2(1-x)\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}$$
$$\times\frac{\left(2(k_0+xp_0)^2-\frac{2}{D}\hat{k}^2\right)}{[\hat{k}^2+(k_0+xp_0)^2+x(1-x)p^2+m^2]^3}. \tag{13}$$

Working at finite temperature, more structures have been generated, a fact that is not unexpected due to the explicit breaking of four-dimensional covariance. The important point is that, nevertheless, the tensors must be transverse with respect to p_μ and p_ν, since the vectorial Ward identity is unaffected by the presence of the temperature. By inspection we see that the only potential trouble comes from I_1 ($\epsilon_{\mu\nu\lambda0}$ not being transverse). Luckily we can show that I_1 is exactly zero. To this purpose it is useful to rewrite I_1 as follows:

$$I_1=\int_0^1dx\,2(1-x)\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}\left\{\left[-\frac{k_0+xp_0}{2}\frac{d}{dx}\right.\right.$$
$$\left.\times\left(\frac{1}{[\hat{k}^2+(k_0+xp_0)^2+x(1-x)p^2+m^2]^2}\right)\right]$$
$$\left.-\frac{2p_0\hat{k}^2/D}{[\hat{k}^2+(k_0+xp_0)^2+x(1-x)p^2+m^2]^3}\right\}, \tag{14}$$

and integrating by part with respect to x (the boundary terms are zero) we get, after having performed the D-dimensional integral,

$$I_1 = -\frac{\Gamma\left(2-\frac{D}{2}\right)}{(4\pi)^{D/2}}\int_0^1 dx \times \sum_n \frac{(k_0+xp_0)}{[(k_0+xp_0)^2+x(1-x)p^2+m^2]^{2-(D/2)}}. \tag{15}$$

We can use now the explicit form of the Matsubara frequencies and the fact that p_0 is discrete ($p_0=2\pi/\beta l$): relabeling the sum in Eq. (15) as $n\rightarrow -n-1-l$ and making the change of variables $y=1-x$, one easily obtains that

$$I_1 = -I_1.$$

We stress that we did our computations in D dimensions, where everything is convergent and no limit on p has been performed. The same arguments apply to I_3, and we remain, therefore, with two independent tensorial structures: we need to evaluate I_0+2I_2 and I_4. Before entering the computations we remark that the emergence of a new, transverse tensorial structure was overlooked in Ref. [16], where the coefficient of the Chern-Simons term at finite T was obtained by simply evaluating the scalar integral, relevant at $T=0$, by introducing Matsubara frequencies for p_0. As we will see in the next section, our result disagrees with that.

III. THE *CPT*- AND LORENTZ-VIOLATING TERM AT $T\neq 0$

Let us evaluate the coefficients of the two independent structures in the small momentum limit: we remark that at finite temperature this procedure is rather delicate, due to the fact that, in general, the limits $p_0\rightarrow 0$ and $\hat{p}^2\rightarrow 0$ do not commute [17]. Here we shall take first $p_0\rightarrow 0$ and send $\hat{p}^2\rightarrow 0$. This limit is sometimes referred to as "static" and it allows for a comparison with the computations performed through the heat-kernel technique. More generally one could try the opposite one, after having continued the polarization tensor to the real time [18]. In principle a different result could be obtained, an explicit example of this being the computations of the induced Chern-Simons term at finite temperature presented in [19]. The sum I_0+2I_2 becomes, when the D-dimensional integrals have been computed:

$$I_0+2I_2 = -\frac{\Gamma\left(2-\frac{D}{2}\right)}{(4\pi)^{D/2}}\int_0^1 dx 2(1-x) \times \sum_n \left[\left(1-\frac{4}{D}\right)\frac{1}{[k_0^2+m^2]^{2-(D/2)}} + \frac{4}{D}\,\frac{2-D/2}{\Gamma(3)}\,\frac{1}{[k_0^2+m^2]^{2-(D/2)}}\right]. \tag{16}$$

The result is zero identically in arbitrary dimension: one can check that when the dependence on $\hat{p}^2$ is retained the corrections are regular, and of order $\hat{p}^2$ (and of course the coefficient depends on T). We see that the zero-temperature tensorial structure still has a vanishing coefficient in the small momentum limit when $T\neq 0$. The only possible contribution to Lorentz and CPT violation could therefore arise from I_4, i.e., from the noncovariant structure. The relevant term to be calculated is

$$I_4 = \int_0^1 dx 2(1-x)\sum_n \int \frac{d^D\hat{k}}{(2\pi)^D}\frac{\left(2k_0^2-\frac{2}{D}\hat{k}^2\right)}{[\hat{k}^2+k_0^2+m^2]^3}. \tag{17}$$

The D-dimensional integration leads to

$$I_4 = \frac{\Gamma\left(2-\frac{D}{2}\right)}{(4\pi)^{D/2}}\int_0^1 dx(1-x)\sum_n \left[(3-D)\frac{1}{[k_0^2+m^2]^{2-(D/2)}} + (D-4)\frac{m^2}{[k_0^2+m^2]^{3-(D/2)}}\right]. \tag{18}$$

At this point we need an explicit representation for the sum over the Matsubara frequencies: we use the following result [20], valid when $1/2<\lambda<1$

$$\sum_n [(n+b)^2+a^2]^{-\lambda} = \frac{\sqrt{\pi}\Gamma(\lambda-1/2)}{\Gamma(\lambda)(a^2)^{\lambda-1/2}} + 4\sin(\pi\lambda)\int_{|a|}^{\infty}\frac{dz}{(z^2-a^2)^\lambda} \times \mathrm{Re}\left(\frac{1}{\exp 2\pi(z+ib)-1}\right). \tag{19}$$

We cannot apply directly this formula to our case: at the end we want to take the $D=3$ limit, and it is clear that, evaluating the second contribution to I_4 in Eq. (18), the integral in Eq. (19) does not converge there in the limit $D=3$. It is not difficult anyway to perform the analytical continuation in Eq. (19) using the relation

$$\int_{|a|}^{\infty}\frac{dz}{(z^2-a^2)^\lambda}\mathrm{Re}\left(\frac{1}{\exp 2\pi(z+ib)-1}\right) = \frac{1}{2a^2}\,\frac{3-2\lambda}{1-\lambda}\int_{|a|}^{\infty}\frac{dz}{(z^2-a^2)^{\lambda-1}}\mathrm{Re}\left(\frac{1}{\exp 2\pi(z+ib)-1}\right) - \frac{1}{4a^2}\,\frac{1}{(2-\lambda)(1-\lambda)}\int_{|a|}^{\infty}\frac{dz}{(z^2-a^2)^{\lambda-2}}\,\frac{d^2}{dz^2} \times \left[\mathrm{Re}\left(\frac{1}{\exp 2\pi(z+ib)-1}\right)\right]. \tag{20}$$

Equation (18) can now be explicitly evaluated at $D=3$: we see that the potential singularity at $D=3$, coming from the first contribution, cancels (notice the factor $D-3$ in front), the finite residue (that would be temperature independent)

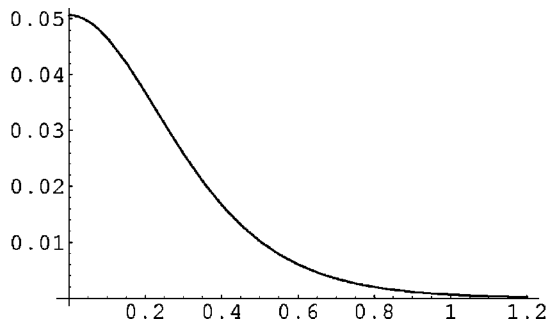

FIG. 1. Plot of $F(\xi)$.

cancels with an analogous term coming from the second contribution, leaving us with the final result

$$I_4=2\beta\int_{|\xi|}^{\infty}dz(z^2-\xi^2)^{1/2}\frac{\tanh(\pi z)}{\cosh^2(\pi z)}=2\,\beta F(\xi),\quad(21)$$

where we have defined $\xi=\beta m/2\pi$. The behavior of $F(\xi)$ is displayed in Fig. 1. Equation (21) is the main result of our paper: it shows that for $\beta\neq\infty$ $(T\neq 0)$ a CPT- and Lorentz-violating term appears. In momentum space it can be written as

$$\Pi_b^{\mu\nu}(p)=4e^2F(\xi)b^\lambda(\delta_{0\mu}\epsilon_{\nu\lambda\rho 0}p^\rho-\delta_{0\nu}\epsilon_{\mu\lambda\rho 0}p^\rho-p_0\epsilon_{\mu\nu\lambda 0})+O(p^2).\quad(22)$$

Several comments are now in order. First of all when $T=0$ $F(\xi)$ vanishes, recovering therefore the fact that, using dimensional regularization, no CPT- and Lorentz-violating Chern-Simons-like term is present in the effective action. The opposite limit $(T\rightarrow\infty)$ is otherwise finite $[F(0)=1/2\pi^2]$. In Ref. [16] a similar behavior was found for the temperature evolution of the coefficient of the Chern-Simons-like term but there the $T=0$ boundary condition was taken so that at $T=\infty$ the symmetries were restored. Moreover, at variance with our result, the Chern-Simons term there was implicitly assumed to be related to the covariant tensorial structure, a fact that from our computation turns out to be incorrect. The second point is that our induced action does not depend on the temporal component of b_μ; in Refs. [12–14] the consistency of the theory at quantum level, when the Chern-Simons-like action is present, was discussed. It would be interesting to address the problem of stability in the finite-temperature situation considering our induced term. Another observation is related to the dependence on ξ: we see that the limit $\beta\rightarrow 0$ is the same as $m\rightarrow 0$ (since the only dependence on the mass and on the temperature appears through ξ). This suggests that the presence at finite temperature of a nonzero CS-like term is related to the loss of analyticity in external momenta, bypassing therefore the argument of Coleman and Glashow [8] against it (analyticity was also assumed in Ref. [9]). It is interesting to write down the induced term in configuration space

$$S_{``CS"}=4ie^2F(\xi)\int d^4x\, b_i[A_0\epsilon^{ijk}F_{jk}-2\epsilon^{ijk}A_jF_{0k}],\quad(23)$$

gauge invariance is achieved via Bianchi identity up to a total derivative.

The asymmetrical behavior played in the above action by the "spacial" and "temporal" component of b_μ (the latter being completely absent) might seem strange. A source of this asymmetry can be surely traced back to the presence of a thermal bath which, selecting a specified frame, provides an additional Lorentz violation. However, below, with the help of the analogous problem in two dimensions, we would like to suggest that the origin of this term at finite temperature may be related to a deeper geometrical reason. In $D=2$, in fact, its structure and its coefficient can be easily understood through the interplay of the global part of the Quillen anomaly, which controls the obstruction to the chiral splitting, and the presence of a nontrivial cycle (finite temperature in the above language).

There, the relevant Green's function is the one-point function with one b_μ insertion (we study the massless case for simplicity). Using dimensional regularization we see that the zero-temperature computation gives a vanishing result because $\int d^Dk k_\mu k_\nu/k^2=0$ (we remark that having b_μ constant implies that the external momentum has to be null). But turning on the temperature the situation changes drastically. The relevant integral is

$$\Pi_\mu^b=-ieb^\lambda\,\mathrm{Tr}[\gamma_\mu\gamma_\alpha\gamma_\lambda\gamma_5\gamma_\nu]\frac{1}{\beta}\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}\frac{k^\alpha k^\nu}{k^4},\quad(24)$$

that is equivalent, after using Dirac algebra to

$$2eb^\lambda\Bigg[-\epsilon_{\lambda\mu}\frac{1}{\beta}\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}\frac{1}{[k_0^2+\hat{k}^2]}+2\epsilon_{\lambda\alpha}\delta_{0\mu}\delta_{0\alpha}\frac{1}{\beta}\times\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}\frac{k_0^2}{[\hat{k}^2+k_0^2]^2}+2\epsilon_{\lambda\alpha}\frac{1}{\beta}\times\sum_n\int\frac{d^D\hat{k}}{(2\pi)^D}\frac{\hat{k}_\mu\hat{k}^\alpha}{[k_0^2+\hat{k}^2]^2}\Bigg].\quad(25)$$

It is not difficult to see that the D-dimensional integration and the analytical continuation of the sum gives a finite result for the b_1 component (due to a cancellation between a pole and a zero as $D=1$) leaving us with

$$\Pi_\mu^b=\frac{16}{\pi}eb^\lambda\epsilon_{\lambda\alpha}\delta_{0\mu},\quad(26)$$

while one can easily show that the b_0 component has zero coefficient after performing the $\hat{k}$ integration. We remark that here no external momentum limit has been done, therefore the result is exact. The above term has a natural interpretation in configuration space as

$$i\int d^2x b_1 A_0(x), \tag{27}$$

that is the analog of $D=4$ (the "complete" CS-like term would be here $\epsilon^{\mu\nu}b_\mu A_\nu$). The term appearing in Eq. (27) is nothing but a remnant of the holomorphic anomaly on the torus [21]. In fact, when nontrivial cycles are present in the base-space, the effective action acquires a subtle dependence on the harmonic part of the gauge potentials. In particular when both vector and axial gauge fields are coupled, requiring gauge invariance implies that an anomalous phase has to be present in order to cure the transformation properties of the modulus of the Dirac determinant. This phase can be derived from general algebraic-geometrical arguments [21], being related to the Quillen anomaly, or by an explicit ζ-function computation [22] of the relevant functional determinants. The asymmetric character of the phase has to be ascribed to the anomalous modular transformation properties of chiral partition functions. We can now understand the appearance of this term at finite temperature: b_μ is basically an harmonic one-form axially coupled and therefore being able to interact with the harmonic component of A_0. The complete anomalous phase requires a quadratic part in b_μ that can be easily recovered by computing the Feynman graph with two b_μ insertions. This discussion may suggest that the four-dimensional term could have an appealing mathematical interpretation.

ACKNOWLEDGMENTS

We warmly thank Professor S. Deser, Professor R. Jackiw, and Professor V. A. Kostelecky for suggestions and discussions.

[1] R. M. Wald, Phys. Rev. D **21**, 2742 (1980); S. W. Hawking, *ibid.* **32**, 2489 (1985).
[2] V. A. Kostelecky and S. Samuel, Phys. Rev. D **39**, 683 (1989); V.A. Kostelecky and R. Potting, Nucl. Phys. **B359**, 545 (1991).
[3] S. M. Carroll, G. B. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).
[4] D. Colladay and V. A. Kostelecky, Phys. Rev. D **55**, 6760 (1997); **58**, 116002 (1998).
[5] M. Goldhaber and V. Trimble, J. Astrophys. Astron. **17**, 17 (1996); S. M. Carroll and G. B. Field, Phys. Rev. Lett. **79**, 2394 (1997); D. Bear *et al.*, *ibid.* **85**, 5038 (2000).
[6] R. Jackiw and V. A. Kostelecky, Phys. Rev. Lett. **82**, 3572 (1999).
[7] L. Chan, hep-ph/9907349 (1999).
[8] S. Coleman and S. Glashow, Phys. Lett. B **405**, 249 (1997); Phys. Rev. D **59**, 116008 (1999).
[9] G. Bonneau, Nucl. Phys. **B593**, 398 (2001).
[10] Yu A. Sitenko, hep-th/0103215.
[11] M. Perez-Victoria, J. High Energy Phys. **04**, 032 (2001).
[12] A. A. Andrianov and R. Soldati, Phys. Lett. B **435**, 449 (1998).
[13] C. Adam and F. R. Klinkhamer, hep-ph/0101087.
[14] V. A. Kostelecky and R. Lehnert, Phys. Rev. D **63**, 065008 (2001).
[15] S. Deser, L. Griguolo, and D. Seminara, Phys. Rev. Lett. **79**, 1976 (1997); Phys. Rev. D **57**, 7444 (1998).
[16] J. R. Nascimento, R. F. Ribeiro, and N. F. Svaiter, hep-th/0012039.
[17] D. J. Gross, R. D. Pisarski, and L. G. Yaffe, Rev. Mod. Phys. **53**, 43 (1981).
[18] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974).
[19] Y. Kao and M. Yang, Phys. Rev. D **47**, 730 (1993).
[20] L. H. Ford, Phys. Rev. D **21**, 933 (1980).
[21] L. Alvarez-Gaumé, G. Moore, and C. Vafa, Commun. Math. Phys. **106**, 1 (1986).
[22] I. Sachs and A. Wipf, Ann. Phys. (N.Y.) **249**, 380 (1996); L. Griguolo and D. Seminara, Nucl. Phys. **B495**, 400 (1997).