Constructing the fermion-boson vertex in three-dimensional QED

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We derive perturbative constraints on the transverse part of the fermion-boson vertex in massive three-dimensional QED through its one-loop evaluation in an arbitrary covariant gauge. Written in a particular form, these constraints naturally lead us to the first nonperturbative construction of the vertex, which is in complete agreement with its one-loop expansion in all momentum regimes. Without affecting its one-loop perturbative properties, we also construct an effective vertex in such a way that the unknown functions defining it have no dependence on the angle between the incoming and outgoing fermion momenta. Such a vertex should be useful for the numerical study of dynamical chiral symmetry breaking, leading to more reliable results.

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I. INTRODUCTION

Quantum electrodynamics in three dimensions (QED_3) is an attractive model to study the intricacies of Schwinger-Dyson equations (SDE's). Because of its simplicity as compared to quantum electrodynamics in four dimensions (QED_4) and quantum chromodynamics (QCD), the corresponding study of dynamical symmetry breaking is relatively neater in QED_3. There exist numerous works in this connection, e.g., [1–14]. An excellent review can be found in Ref. [15]. As is well known, the knowledge of the three-point vertex is crucial in such studies. In this respect, perturbation theory is a powerful point of reference as it is natural to believe that physically meaningful solutions of the Schwinger-Dyson equations must agree with perturbative results in the weak-coupling regime. This realization has been exploited in Refs. [16–19] to derive constraints on the fermion propagator and the three-point vertex in massless QED_3. In this paper, we extend this work to the more general massive case, based on Ref. [20].

The Ward-Takahashi identity (WTI) relates the three-point vertex to the fermion propagator. Using this relation, a part of the vertex, called longitudinal, can be expressed in terms of the fermion propagator [21]. We evaluate this propagator to one-loop and hence determine the longitudinal vertex to the same order. We also calculate the complete vertex to one-loop and a mere subtraction of the longitudinal part yields the transverse part, the one which is not fixed by the WTI. According to the choice of Ball and Chiu, which was later modified by Kızılersü, Reenders, and Pennington [22], the transverse vertex can be expressed in terms of eight independent spin structures. The vertex should be free of any kinematic singularities. Ball and Chiu chose the basis in such a way that the coefficient of each of the basis is independently free of kinematic singularities in the Feynman gauge. It was later shown by Kızılersü, Reenders, and Pennington [22] that a calculation similar to that of Ball and Chiu in an arbitrary covariant gauge does not have the same nice feature. Therefore, they proposed a modified basis whose coefficients are free of kinematic singularities in an arbitrary covariant gauge. The calculation in the present paper confirms that all the vectors of the modified basis also retain this feature for massive QED_3. The final result for the transverse vertex is written in terms of basic functions of the momenta in a form suitable for its extension to the nonperturbative domain, following the ideas of Curtis and Pennington [23].

Using perturbative constraints as a guide, we carry out a construction of the nonperturbative vertex, which has no explicit dependence on the coupling α. This vertex has an explicit dependence on the gauge parameter ξ. We demonstrate in the massless case that a vertex cannot be constructed without an explicit dependence on ξ. For practical purposes of the numerical study of dynamical chiral symmetry breaking, we also construct an effective vertex that shifts the angular dependence from the unknown fermion propagator functions to the known basic functions, without changing its perturbative properties at the one-loop level. We believe that this vertex should lead to a more realistic study of the dynamically generated masses through the corresponding SDE's.

II. LONGITUDINAL AND TRANSVERSE VERTEX TO ONE LOOP

A. The fermion propagator

One-loop fermion propagator can be obtained by evaluating the graph in Fig. 1. This graph corresponds to the following equation:

$$iS_F^{-1}(p)=iS_F^{0\,-1}(p)+e^2\int\frac{d^3k}{(2\pi)^3}\gamma^\mu S_F^0(k)\gamma^\nu\Delta^0_{\mu\nu}(q), \tag{2.1}$$

where $q=k-p$ and e is the QED coupling constant. The bare fermion and photon propagators are, respectively,

$$S_F^0(p)=\frac{1}{\not p-m},$$

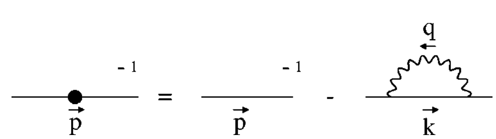

FIG. 1. One-loop correction to the fermion propagator.

$$\Delta^0_{\mu\nu}(q) = -[q^2 g_{\mu\nu} + (\xi-1)q_\mu q_\nu]/q^4, \tag{2.2}$$

where m is the bare mass of the fermion and ξ is the covariant gauge parameter. We define the full fermion propagator $S_F(p)$ in the most general form as

$$S_F(p) = \frac{F(p^2)}{\not p - \mathcal{M}(p^2)}. \tag{2.3}$$

Taking the trace of Eq. (2.1), having multiplied it with $\not p$ and with 1, respectively, one can obtain two independent equations. On simplifying, these equations can be written as

$$\frac{1}{F(p^2)} = 1 + i4\pi\alpha\xi\frac{1}{p^2}\int \frac{d^3k}{(2\pi)^3}\frac{1}{q^4(k^2-m^2)}[(k^2+p^2)k\cdot p - 2k^2p^2], \tag{2.4}$$

$$\frac{\mathcal{M}(p^2)}{F(p^2)} = m - i4\pi\alpha(\xi+2)\int\frac{d^3k}{(2\pi)^3}\frac{m}{q^2(k^2-m^2)}, \tag{2.5}$$

where $\alpha = e^2/4\pi$. On Wick rotating to the Euclidean space and carrying out angular and radial integrations, we arrive at

$$\frac{1}{F(p^2)} = 1 - \frac{\alpha\xi}{2p^2}[m - (m^2+p^2)I(p^2)],$$

$$\frac{\mathcal{M}(p^2)}{F(p^2)} = m[1+\alpha(\xi+2)I(p^2)], \tag{2.6}$$

where we have used the simplifying notation $I(p^2) = (1/\sqrt{-p^2})\arctan\sqrt{-p^2/m^2}$. Equations (2.3) and (2.6) form the complete fermion propagator at one loop.

B. Longitudinal vertex to one loop

The full vertex satisfies WTI:

$$q_\mu \Gamma^\mu(k,p) = S_F^{-1}(k) - S_F^{-1}(p). \tag{2.7}$$

This relation allows us to decompose the full vertex into longitudinal $[\Gamma^\mu_L(k,p)]$ and transverse $[\Gamma^\mu_T(k,p)]$ parts:

$$\Gamma^\mu(k,p) = \Gamma^\mu_L(k,p) + \Gamma^\mu_T(k,p), \tag{2.8}$$

where the transverse part satisfies

$$q_\mu \Gamma^\mu_T(k,p) = 0 \quad \text{and} \quad \Gamma^\mu_T(p,p) = 0 \tag{2.9}$$

and hence remains undetermined by WTI. Following the work of Ball and Chiu, we can define the longitudinal component of the vertex in terms of the fermion propagator as

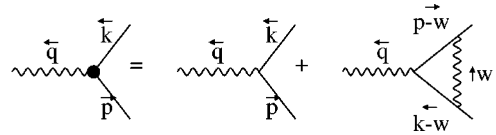

FIG. 2. One-loop correction to the vertex.

$$\Gamma^\mu_L = \frac{\gamma^\mu}{2}\left[\frac{1}{F(k^2)} + \frac{1}{F(p^2)}\right] + \frac{1}{2}\frac{(\not k+\not p)(k+p)^\mu}{(k^2-p^2)}\left[\frac{1}{F(k^2)} - \frac{1}{F(p^2)}\right] + \frac{(k+p)^\mu}{(k^2-p^2)}\left[\frac{\mathcal{M}(k^2)}{F(k^2)} - \frac{\mathcal{M}(p^2)}{F(p^2)}\right]. \tag{2.10}$$

On substituting Eq. (2.6) into the above expression, we obtain

$$\Gamma^\mu_L = \left[1 + \frac{\alpha\xi}{4}\sigma_1\right]\gamma^\mu + \frac{\alpha\xi}{4}\sigma_2[k^\mu\not k + p^\mu\not p + k^\mu\not p + p^\mu\not k] + \alpha(\xi+2)\sigma_3[k^\mu + p^\mu], \tag{2.11}$$

where

$$\sigma_1 = \frac{m^2+k^2}{k^2}I(k^2) + \frac{m^2+p^2}{p^2}I(p^2) - m\frac{k^2+p^2}{k^2p^2},$$

$$\sigma_2 = \frac{1}{k^2-p^2}\left[\frac{m^2+k^2}{k^2}I(k^2) - \frac{m^2+p^2}{p^2}I(p^2) + m\frac{k^2-p^2}{k^2p^2}\right],$$

$$\sigma_3 = m[I(k^2) - I(p^2)]. \tag{2.12}$$

Equations (2.11) and (2.12) give the longitudinal part of the fermion-photon vertex to one loop for the massive QED_3.

C. Transverse vertex to one loop

The vertex of Fig. 2 can be expressed as

$$\Gamma^\mu(k,p) = \gamma^\mu + \Lambda^\mu. \tag{2.13}$$

Using the Feynman rules, Λ^μ to $O(\alpha)$ is simply given by

$$-ie\Lambda^\mu = \int_M \frac{d^3w}{(2\pi)^3}(-ie\gamma^\alpha)iS_F^0(p-w)(-ie\gamma^\mu) \times iS_F^0(k-w)(-ie\gamma^\beta)i\Delta^0_{\alpha\beta}(w), \tag{2.14}$$

where the loop integral is to be performed in Minkowski space. Λ^μ can be expressed as

$$\Lambda^{\mu}=-\frac{i\,\alpha}{2\,\pi^2}\{[\gamma^{\alpha}\not p\gamma^{\mu}\not k\gamma_{\alpha}+m(4k^{\mu}+4p^{\mu}-\not p\gamma^{\mu}-\gamma^{\mu}\not k)$$
$$-m^2\gamma^{\mu}]J^{(0)}-[\gamma^{\alpha}\not p\gamma^{\mu}\gamma^{\nu}\gamma_{\alpha}+\gamma^{\alpha}\gamma^{\nu}\gamma^{\mu}\not k\gamma_{\alpha}$$
$$+6mg^{\mu\nu}]J^{(1)}_{\nu}+\gamma^{\alpha}\gamma^{\nu}\gamma^{\mu}\gamma^{\lambda}\gamma_{\alpha}J^{(2)}_{\nu\lambda}+(\xi-1)[\gamma^{\mu}K^{(0)}$$
$$-[\gamma^{\nu}\not p\gamma^{\mu}+\gamma^{\mu}\not k\gamma^{\nu}+2mg^{\mu\nu}]J^{(1)}_{\nu}+[\gamma^{\nu}\not p\gamma^{\mu}\not k\gamma^{\lambda}$$
$$+m(\gamma^{\nu}\not p\gamma^{\mu}\gamma^{\lambda}+\gamma^{\nu}\gamma^{\mu}\not k\gamma^{\lambda})+m^2\gamma^{\nu}\gamma^{\mu}\gamma^{\lambda}]I^{(2)}_{\nu\lambda}]\},\tag{2.15}$$

where the integrals $K^{(0)}$, $J^{(0)}$, $J^{(1)}_{\mu}$, $J^{(2)}_{\mu\nu}$, $I^{(0)}$, $I^{(1)}_{\mu}$, and $I^{(2)}_{\mu\nu}$ are

$$K^{(0)}=\int_M d^3w\frac{1}{[(p-w)^2-m^2][(k-w)^2-m^2]},$$

$$J^{(0)}=\int_M d^3w\frac{1}{w^2[(p-w)^2-m^2][(k-w)^2-m^2]},$$

$$J^{(1)}_{\mu}=\int_M d^3w\frac{w_{\mu}}{w^2[(p-w)^2-m^2][(k-w)^2-m^2]},$$

$$J^{(2)}_{\mu\nu}=\int_M d^3w\frac{w_{\mu}w_{\nu}}{w^2[(p-w)^2-m^2][(k-w)^2-m^2]},$$

$$I^{(0)}=\int_M d^3w\frac{1}{w^4[(p-w)^2-m^2][(k-w)^2-m^2]},$$

$$I^{(1)}_{\mu}=\int_M d^3w\frac{w_{\mu}}{w^4[(p-w)^2-m^2][(k-w)^2-m^2]},$$

$$I^{(2)}_{\mu\nu}=\int_M d^3w\frac{w_{\mu}w_{\nu}}{w^4[(p-w)^2-m^2][(k-w)^2-m^2]}.\tag{2.16}$$

We evaluate these integrals following the techniques developed in Refs. [16,17,21], and [22]. The results are tabulated in the Appendix, employing the notation $\Delta^2=(k\cdot p)^2-k^2p^2$ and $X_0=(2/i\pi^2)X^{(0)}$ for $X=I,J,K$. Having calculated the vertex to $O(\alpha)$, Eq. (2.15), we can subtract from it the longitudinal vertex, Eqs. (2.11) and (2.12), and obtain the transverse vertex to $O(\alpha)$. Following the scheme provided by Ball and Chiu [21], and modified later by Kızılersü, Reenders, and Pennington [22], the transverse vertex $\Gamma^{\mu}_T(k,p)$ can be written in terms of eight basis vectors as follows:

$$\Gamma^{\mu}_T(k,p)=\sum_{i=1}^{8}\tau_i(k^2,p^2,q^2)T^{\mu}_i(k,p),\tag{2.17}$$

where

$$T^{\mu}_1=[p^{\mu}(k\cdot q)-k^{\mu}(p\cdot q)],$$

$$T^{\mu}_2=[p^{\mu}(k\cdot q)-k^{\mu}(p\cdot q)](\not k+\not p),$$

$$T^{\mu}_3=q^2\gamma^{\mu}-q^{\mu}\not q,$$

$$T^{\mu}_4=q^2[\gamma^{\mu}(\not k+\not p)-k^{\mu}-p^{\mu}]-2(k-p)^{\mu}k^{\lambda}p^{\nu}\sigma_{\lambda\nu},$$

$$T^{\mu}_5=q_{\nu}\sigma^{\nu\mu},$$

$$T^{\mu}_6=-\gamma^{\mu}(k^2-p^2)+(k+p)^{\mu}\not q,$$

$$T^{\mu}_7=-\tfrac{1}{2}(k^2-p^2)[\gamma^{\mu}(\not k+\not p)-k^{\mu}-p^{\mu}]$$
$$+(k+p)^{\mu}k^{\lambda}p^{\nu}\sigma_{\lambda\nu},$$

$$T^{\mu}_8=-\gamma^{\mu}k^{\nu}p^{\lambda}\sigma_{\nu\lambda}+k^{\mu}\not p-p^{\mu}\not k,$$

with

$$\sigma_{\mu\nu}=\tfrac{1}{2}[\gamma_{\mu},\gamma_{\nu}].\tag{2.18}$$

After a lengthy but straightforward algebra, the coefficients τ_i can be identified. We prefer to write these out in the following form:

$$\tau_i(k,p)=\alpha g_i\left[\sum_{j=1}^{5}a_{ij}(k,p)I(l_j^2)+\frac{a_{i6}(k,p)}{k^2p^2}\right],$$
$$i=1,2,\ldots 8,\tag{2.19}$$

where $l_1^2=\eta_1^2\chi/4$, $l_2^2=\eta_2^2\chi/4$, $l_3^2=k^2$, $l_4^2=p^2$, and $l_5^2=q^2/4$. The functions η_1, η_2, and χ have been defined in the Appendix, Eqs. (A2). Similarly, the factors g_i are $-g_1=m\Delta^2g_2=2m\Delta^2g_3=2\Delta^2g_4=g_5=2m\Delta^2g_6=\Delta^2g_7=mg_8=m/4\Delta^2$. The coefficients $a_{ij}(k,p)$ have also been tabulated in the Appendix, Eqs. (A14). An important point to note is that these coefficients do not contain any trigonometric function, as it has been extracted out for raising the τ_i to a nonperturbative status. The τ_i have the required symmetry under the exchange of vectors k and p. All the τ_i are symmetric except τ_4 and τ_6, which are antisymmetric. Note that the form in which we write the transverse vertex makes it clear that each term in all the τ_i is either proportional to $\alpha I(l^2)$ or $\alpha/(k^2p^2)$. We shall see that this form provides us with a natural scheme to arrive at its simple nonperturbative extension.

A few comments in comparison with the work by Davydychev *et al.* [24], are as follows: (i) None of the τ_i we have calculated has kinematic singularity when $k^2\to p^2$. This clearly suggests that the choice of the τ_i suggested by Kızılersü, Reenders, and Pennington is preferred over the one of Ball and Chiu (in QED$_3$ as well) used by Davydychev *et al.* [24]. In particular, our τ_4 and τ_7 are independent of kinematic singularities. (ii) In three dimensions, their factorization of the common constant factor in Eq. (E 1) is singular. However, as the divergences completely cancel out, we find our expressions more suitable for writing the transverse vertex in three dimensions. (iii) With the way we express J_0, all the τ_i are written in terms of basic functions of k and p and a single trigonometric function of the form $I(l^2)$. This form plays a key role to enable us to make an easy transition to the possible nonperturbative structure of the vertex, as explained in Sec. III. Moreover, with the given form of J_0, a direct comparison can be made with the massless case.

III. NONPERTURBATIVE FORM OF THE VERTEX

A. On the gauge parameter dependence of the vertex

Let us first look at the τ_i in the simplified massless case, with the notation $k=\sqrt{-k^2}$, $p=\sqrt{-p^2}$, and $q=\sqrt{-q^2}$ [16,17],

$$\tau_2=\frac{\alpha\pi}{4}\frac{1}{kp(k+p)(k+p+q)^2}\left[1+(\xi-1)\frac{2k+2p+q}{q}\right], \tag{3.1}$$

$$\tau_3=\frac{\alpha\pi}{8}\frac{1}{kpq(k+p+q)^2}[4kp+3kq+3pq+2q^2 +(\xi-1)(2k^2+2p^2+kq+pq)], \tag{3.2}$$

$$\tau_6=\frac{\alpha\pi(2-\xi)}{8}\frac{k-p}{kp(k+p+q)^2}, \tag{3.3}$$

$$\tau_8=\frac{\alpha\pi(2+\xi)}{2}\frac{1}{kp(k+p+q)}. \tag{3.4}$$

It is interesting to note that the existence of the factor

$$\frac{k-p}{kp}=-\left(\frac{1}{k}-\frac{1}{p}\right)$$

in Eq. (3.3) puts τ_6 on a different footing as compared to the rest of the τ_i. The reason is that in the massless limit, the fermion propagator is simply

$$\frac{1}{F(p^2)}=1+\frac{\pi\alpha\xi}{4}\frac{1}{p},$$

implying

$$\frac{1}{F(k^2)}-\frac{1}{F(p^2)}\propto\left[\frac{1}{k}-\frac{1}{p}\right].$$

Therefore, the relation of τ_6 with the fermion propagator of the type $[1/F(k^2)-1/F(p^2)]$ seems to arise rather naturally:

$$\tau_6=-\frac{1}{2\xi}\frac{2-\xi}{(k+p+q)^2}\left[\frac{1}{F(k^2)}-\frac{1}{F(p^2)}\right], \tag{3.5}$$

as noticed first by Curtis and Pennington [23]. In the rest of the τ_i, the factor $1/k-1/p$ does not arise. However, one could introduce it by hand to arrive at the following expressions:

$$\tau_2=-\frac{1}{\xi}\frac{1}{(k^2-p^2)(k+p+q)^2}\left(1+(\xi-1)\frac{2k+2p+q}{q}\right)\times\left[\frac{1}{F(k^2)}-\frac{1}{F(p^2)}\right], \tag{3.6}$$

$$\tau_3=-\frac{1}{2\xi}\frac{1}{q(k-p)(k+p+q)^2}[4kp+3kq+3pq+2q^2 +(\xi-1)(2k^2+2p^2+kq+pq)]\left[\frac{1}{F(k^2)}-\frac{1}{F(p^2)}\right], \tag{3.7}$$

$$\tau_8=-\frac{2(2+\xi)}{\xi}\frac{1}{(k-p)(k+p+q)}\left[\frac{1}{F(k^2)}-\frac{1}{F(p^2)}\right]. \tag{3.8}$$

Equations (3.5)–(3.8) represent a nonperturbative vertex, which is in agreement with its complete one-loop expansion. This vertex has been constructed in accordance with the form advocated, e.g., in Refs. [2,23,25]. There are a couple of important points which need to be discussed here.

There is an explicit dependence on the gauge parameter ξ. A widespread practice has been to construct the vertex such that its gauge dependence solely arises through functions $F(k^2)$ and $F(p^2)$, and there is *no explicit appearance of the gauge parameter* ξ [2,3,10,23]. Here we show that at least in massless QED$_3$, such a construction is not possible. In this connection, it may be interesting to observe that (i) the work of Bashir and Pennington [25] for QED$_4$ gives one example of an explicitly gauge dependent vertex ensuring the gauge invariance of the critical coupling, above which chiral symmetry is dynamically broken; (ii) the work of Burden and Tjiang [10] contains a vertex *ansatz* in QED$_3$ consisting of a free parameter β, believed by the authors to be independent of the gauge parameter. The perturbative work of Bashir *et al.* [17] later proved that this β must be explicitly dependent upon the gauge parameter.

The one-loop calculation of the vertex $\Gamma^\mu(k,p)$ in QED$_4$ for $k^2\gg p^2$ reveals that its transverse part vanishes in the Landau gauge [23]. Motivated from this observation, Curtis and Pennington [23], followed by several others [3,10,25], proposed an *ansatz* for nonperturbative $\Gamma^\mu_T(k,p)$ such that it vanishes for $\xi=0$. Later, a complete one-loop calculation revealed that $\Gamma^\mu_T(k,p)\neq 0$ in the Landau gauge [22]. Accordingly, in a subsequent work, Bashir *et al.* removed the previously made assumption and presented the most general nonperturbative construction of the transverse vertex required by the multiplicative renormalizability of the fermion propagator in QED$_4$ [26]. A parallel one-loop calculation in QED$_3$ [16,17] yields identical results, i.e., $\Gamma^\mu_T(k,p)\neq 0$ for

$\xi = 0$. Therefore, the corresponding nonperturbative structure of the form $[1/F(k^2) - 1/F(p^2)]$ is not what we should aim for.[1]

We now show that the explicit dependence of the vertex on the gauge parameter ξ is unavoidable in massless QED$_3$. We notice that at the one-loop level, each of the τ_i can be written as

$$\tau_i(k,p,q) = \alpha\xi\, a_i(k,p,q) + \alpha\, b_i(k,p,q).$$

On the other hand, Eq. (2.6) yields the following form for F:

$$\frac{1}{F(p^2)} = 1 + \alpha\xi\, c_i(p).$$

If we want to write the nonperturbative form of the τ_i in terms of $1/F(p^2)$ and $1/F(k^2)$ alone and we do not expect an explicit presence of α, the only way to get rid of ξ dependence is to have

$$b_2 T_2^\mu + b_3 T_3^\mu + b_6 T_6^\mu + b_8 T_8^\mu = 0.$$

It is not possible as T_i^μ form a linearly independent set of basis vectors. Therefore, any construction of the three-point vertex will surely have an explicit dependence on the gauge parameter. Owing to these reasons, we emphasize that to demand the transverse vertex to be proportional to $[1/F(k^2) - 1/F(p^2)]$ is artificial (apart from τ_6) and is not required, as remarked earlier. Therefore, we do not pursue this line of action anymore. In the next section, we move on to construct the vertex for the massive case inspired from our perturbative results.

B. Nonperturbative vertex

As pointed out in the previous section, each term in all the τ_i is either proportional to the trigonometric function $\alpha I(l^2)$ or $\alpha/(k^2p^2)$. On the other hand, the perturbative expressions for $\mathcal{M}(p^2)$ and $F(p^2)$, Eqs. (2.6), permit us to write

$$\frac{1}{F(k^2)} - \frac{1}{F(p^2)} = \frac{\alpha}{k^2p^2}\,\frac{\xi}{2}\big[k^2\{m - (m^2+p^2)I(p^2)\} - p^2\{m - (m^2+k^2)I(k^2)\}\big] \tag{3.9}$$

and

$$\frac{\xi}{2(2+\xi)l^2 I(l^2)}\left[\frac{\mathcal{M}(l^2)}{F(l^2)} - m\right] - \left[1 - \frac{1}{F(l^2)}\right] = \frac{\xi(m^2+l^2)}{2l^2}\,\alpha I(l^2). \tag{3.10}$$

In the massless limit, Eq. (3.9) simply reduces to

$$\frac{1}{F(k^2)} - \frac{1}{F(p^2)} = \frac{\alpha\pi\xi}{4}\left[\frac{1}{k} - \frac{1}{p}\right]$$

in the Euclidean space, as expected. It was in fact an analogous massless expression in the limit when $k \gg p$ that inspired Curtis and Pennington [23], to propose their famous vertex in QED$_4$. Here, we are extending the reasoning to all the momentum regimes in the massive QED$_3$. Fortunate simultaneous occurrence of the factor $\alpha/(k^2p^2)$ in all eight equations (2.19) and (3.9), and the presence of the same trigonometric factor $I(l^2)$ in the expressions for the vertex as well as the propagator, one naturally arrives at the following nonperturbative form of τ_i:

$$\begin{aligned}\tau_i = g_i\Bigg\{ &\sum_{j=1}^{5}\left(\frac{2a_{ij}(k,p)\,l_j^2}{\xi(m^2+l_j^2)}\left[\frac{\xi}{2(\xi+2)l_j^2 I(l_j^2)}\left(\frac{\mathcal{M}(l_j^2)}{F(l_j^2)} - m\right) - \left(1 - \frac{1}{F(l_j^2)}\right)\right]\right) \\ &+ \frac{2a_{i6}(k,p)}{\xi[k^2\{m-(m^2+p^2)I(p^2)\} - p^2\{m-(m^2+k^2)I(k^2)\}]} \\ &\times\left[\frac{1}{F(k^2)} - \frac{1}{F(p^2)}\right]\Bigg\}.\end{aligned} \tag{3.11}$$

By construction, in the weak-coupling regime, this nonperturbative form of the transverse vertex reduces to its corresponding Feynman expansion at the one-loop level in an arbitrary covariant gauge and in all momentum regimes. We would like to emphasize that this is not a unique nonperturbative construction. However, it is probably the most natural and the simplest. A two-loop calculation similar to the one presented in our paper, and the Landau-Khalatnikov transformation law for the vertex should serve as tests of Eq. (3.11) or guides for improvement towards the hunt for the exact nonperturbative vertex. On practical side, the use of our perturbation theory motivated vertex in studies addressing important issues such as dynamical mass generation for fundamental fermions, should lead to more reliable results, attempting to preserve key features of gauge-field theories, e.g., gauge independence of physical observables. A computational difficulty to use the above vertex in such calculations, could arise as the unknown functions F and $\mathcal{M}$ depend on the angle between k and p. This would make it impossible to carry out angular integration analytically in the SDE for the fermion propagator. This problem can be circumvented by defining an effective vertex that shifts the angular dependence from the unknown functions F and $\mathcal{M}$ to the known basic functions of k and p. This can be done by rewriting the perturbative results, Eq. (2.19), as follows:

$$\tau_i(k,p) = \alpha g_i\left[b_{i1}(k,p)I(k^2) + b_{i2}(k,p)I(p^2) + \frac{a_{i6}(k,p)}{k^2p^2}\right], \tag{3.12}$$

[1] In fact the explicit presence of the gauge parameter in the nonperturbative form of the vertex tells us that the presence of the factor $[1/F(k^2) - 1/F(p^2)]$ is no longer a guarantee that the transverse vertex vanishes in the Landau gauge.

where

$$b_{i1}(k,p)=a_{i1}(k,p)\frac{I(l_1^2)}{I(l_3^2)}+a_{i3}(k,p)+\frac{1}{2}a_{i5}(k,p)\frac{I(l_5^2)}{I(l_3^2)}, \tag{3.13}$$

$$b_{i2}(k,p)=a_{i2}(k,p)\frac{I(l_2^2)}{I(l_4^2)}+a_{i4}(k,p)+\frac{1}{2}a_{i5}(k,p)\frac{I(l_5^2)}{I(l_4^2)}. \tag{3.14}$$

This form can now be raised to a nonperturbative level exactly as before, with the only difference that the functions F and $\mathcal{M}$ are independent of the angle between the momenta k and p:

$$\tau_i=g_i\Bigg\{\sum_{j=1}^{2}\left(\frac{2b_{ij}(k,p)\kappa_j^2}{\xi(m^2+\kappa_j^2)}\left[\frac{\xi}{2(\xi+2)\kappa_j^2 I(\kappa_j^2)}\left(\frac{\mathcal{M}(\kappa_j^2)}{F(\kappa_j^2)}-m\right)-\left(1-\frac{1}{F(\kappa_j^2)}\right)\right]\right)+\frac{2a_{i6}(k,p)}{\xi[k^2\{m-(m^2+p^2)I(p^2)\}-p^2\{m-(m^2+k^2)I(k^2)\}]}\times\left[\frac{1}{F(k^2)}-\frac{1}{F(p^2)}\right]\Bigg\}, \tag{3.15}$$

where $\kappa_1^2=k^2$ and $\kappa_2^2=p^2$.

IV. CONCLUSIONS

In this paper, we calculate the one-loop fermion-boson vertex in QED_3 in an arbitrary covariant gauge and write out the result in a form that naturally allows us to construct its nonperturbative counterpart. This is the first construction of the nonperturbative vertex, which agrees with its Feynman expansion in the weak-coupling regime at the one-loop level in all momentum regimes and in an arbitrary covariant gauge. For practical numerical purposes, we also suggest a simple effective vertex that shifts its angular dependence (angle between the incoming and outgoing fermion momenta) from the fermion functions to the known basic functions of the momenta involved, without affecting its perturbative properties at the one-loop level. Currently, work is underway to use this vertex in numerical calculations of dynamical mass generation for the fundamental fermions. We also plan to compare its gauge dependence with the one demanded by its Landau-Khalatnikov transformation [27,28] in a nonperturbative fashion.

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APPENDIX

The results of the integrals listed in Eqs. (2.16) are as follows:

$\mathbf{J^{(0)}}$:

$$J_0=\left[-\eta_1(k,p)I\left(\frac{\eta_1^2\chi}{4}\right)+\eta_2(k,p)I\left(\frac{\eta_2^2\chi}{4}\right)\right], \tag{A1}$$

with

$$\eta_1(k,p)=-\left\{\frac{m^2(k^2-p^2)(2m^2-k^2-p^2)+\chi}{\chi(m^2-k^2)}\right\},$$
$$\eta_2(k,p)=-\eta_1(p,k),$$
$$\chi=m^2(k^2-p^2)^2+q^2(k^2-m^2)(p^2-m^2). \tag{A2}$$

$\mathbf{K^{(0)}}$:

$$K^{(0)}=i\pi^2 I(q^2/4). \tag{A3}$$

$\mathbf{J_\mu^{(1)}}$:

$$J_\mu^{(1)}=\frac{i\pi^2}{2}\{k_\mu J_A(k,p)+p_\mu J_B(k,p)\}, \tag{A4}$$

where

$$J_A(k,p)=-\frac{2}{\Delta^2}\left\{[p^2(k^2-k\cdot p)-m^2(p^2-k\cdot p)]\frac{J_0}{4}+k\cdot p\, I(k^2)-p^2 I(p^2)+\frac{1}{2}(p^2-k\cdot p)I(q^2/4)\right\},$$
$$J_B(k,p)=J_A(p,k). \tag{A5}$$

$\mathbf{J_{\mu\nu}^{(2)}}$:

$$J_{\mu\nu}^{(2)}=\frac{i\pi^2}{2}\left\{\frac{g_{\mu\nu}}{3}K_0+\left(k_\mu k_\nu-g_{\mu\nu}\frac{k^2}{3}\right)J_C+\left(p_\mu k_\nu+k_\mu p_\nu-g_{\mu\nu}\frac{2k\cdot p}{3}\right)J_D+\left(p_\mu p_\nu-g_{\mu\nu}\frac{p^2}{3}\right)J_E\right\}, \tag{A6}$$

where

$$J_C(k,p)=\frac{1}{\Delta^2}\left\{[p^2(k\cdot p-2k^2)-m^2(k\cdot p-2p^2)]\frac{J_A}{2}-p^2(p^2-m^2)\frac{J_B}{2}+\frac{k\cdot p}{k^2}(m^2-k^2)I(k^2)+\frac{1}{2}(k\cdot p+p^2)I(q^2/4)-m\frac{k\cdot p}{k^2}\right\},$$

$$J_D(k,p)=\frac{1}{2\Delta^2}\Bigg\{[k^2(3k\cdot p-p^2)-m^2(3k\cdot p-k^2)]\frac{J_A}{2}+[p^2(3k\cdot p-k^2)-m^2(3k\cdot p-p^2)]\frac{J_B}{2}-(m^2-k^2)I(k^2)-(m^2-p^2)I(p^2)-\frac{1}{2}(k+p)^2I(q^2/4)+2m\Bigg\},$$

$$J_E(k,p)=J_C(p,k). \tag{A7}$$

$\mathbf{I^{(0)}}$:

$$I^{(0)}=\frac{1}{\chi}\{q^2(m^2+k\cdot p)J^{(0)}+i\pi^2 mL\}, \tag{A8}$$

where

$$L=\frac{q^2(k^2-m^2)-(k^2-p^2)(k^2+m^2)}{(k^2-m^2)^2}+\frac{q^2(p^2-m^2)+(k^2-p^2)(p^2+m^2)}{(p^2-m^2)^2}. \tag{A9}$$

$\mathbf{I_\mu^{(1)}}$:

$$I_\mu^{(1)}=\frac{i\pi^2}{2}[k_\mu I_A(k,p)+p_\mu I_B(k,p)], \tag{A10}$$

where

$$I_A(k,p)=\frac{2}{\Delta^2}\Bigg\{[k\cdot p(p^2-m^2)-p^2(k^2-m^2)]\frac{I_0}{4}+p\cdot q\frac{J_0}{4}+\frac{mp^2}{(m^2-p^2)^2}-\frac{mk\cdot p}{(m^2-k^2)^2}\Bigg\},$$

$$I_B(k,p)=I_A(p,k). \tag{A11}$$

$\mathbf{I_{\mu\nu}^{(2)}}$:

$$I_{\mu\nu}^{(2)}=\frac{i\pi^2}{2}\Bigg\{\frac{g_{\mu\nu}}{3}J_0+\left(k_\mu k_\nu-g_{\mu\nu}\frac{k^2}{3}\right)I_C+\left(p_\mu k_\nu+k_\mu p_\nu-g_{\mu\nu}\frac{2k\cdot p}{3}\right)I_D+\left(p_\mu p_\nu-g_{\mu\nu}\frac{p^2}{3}\right)I_E\Bigg\}, \tag{A12}$$

where

$$I_C(k,p)=\frac{1}{\Delta^2}\Bigg\{p^2J_0+[p^2(k\cdot p-2k^2)-m^2(k\cdot p-2p^2)]\frac{I_A}{2}-p^2(p^2-m^2)\frac{I_B}{2}+(k\cdot p-2p^2)\frac{J_A}{2}-p^2\frac{J_B}{2}-\frac{k\cdot p}{k^2}I(k^2)+\frac{mk\cdot p}{k^2(m^2-k^2)}\Bigg\},$$

$$I_D(k,p)=\frac{1}{2\Delta^2}\Bigg\{-2k\cdot p\,J_0+[k^2(3k\cdot p-p^2)-m^2(3k\cdot p-k^2)]\frac{I_A}{2}+[p^2(3k\cdot p-k^2)-m^2(3k\cdot p-p^2)]\frac{I_B}{2}+(3k\cdot p-k^2)\frac{J_A}{2}+(3k\cdot p-p^2)\frac{J_B}{2}+I(k^2)+I(p^2)-\frac{m}{m^2-k^2}-\frac{m}{m^2-p^2}\Bigg\},$$

$$I_E(k,p)=I_C(p,k). \tag{A13}$$

The coefficients a_{ij} in the one-loop perturbative expansion of the τ_i, Eq. (2.19), are tabulated below:

$$a_{11}(k,p)=-(\xi+2)\eta_1(m^2+k\cdot p),$$

$$a_{12}(k,p)=a_{11}(p,k),$$

$$a_{13}(k,p)=4(\xi+2)\frac{(k^2+k\cdot p)}{(k^2-p^2)},$$

$$a_{14}(k,p)=a_{13}(p,k),$$

$$a_{15}(k,p)=-2(\xi+2),$$

$$a_{16}(k,p)=0,$$

$$a_{21}(k,p)=-\eta_1\Bigg\{\Bigg[-\frac{q^2}{2}m^4+\{(k\cdot p)^2-(k^2+p^2)(k\cdot p)+k^2p^2\}m^2-\frac{q^2}{4}\{(k\cdot p)^2+k^2p^2\}\Bigg]+\frac{(\xi-1)}{2\chi}\times\Bigg[-q^4m^8-q^2\{(k\cdot p)^2+2(k^2+p^2)k\cdot p-5k^2p^2\}m^6+\frac{3}{2}q^2(k^2+p^2)\Delta^2m^4+\{2(k^4+p^4+k^2p^2)(k\cdot p)^3-7k^2p^2(k^2+p^2)(k\cdot p)^2+10k^4p^4k\cdot p-k^4p^4(k^2+p^2)\}m^2+\frac{1}{2}k^2p^2q^2\{(k^2+p^2)(k\cdot p)^2-4k^2p^2k\cdot p+k^2p^2(k^2+p^2)\}\Bigg]\Bigg\},$$

$$a_{22}(k,p)=a_{21}(p,k),$$

$$a_{23}(k,p)=\frac{1}{(k^2-p^2)}[\xi\{(k\cdot p)^3+k^2(k\cdot p)^2-3k^2p^2k\cdot p+2k^4k\cdot p+k^4p^2-2k^6\}m^2/k^2+(k\cdot p)^3+(2k^2-p^2)(k\cdot p)^2+k^2p^2k\cdot p-2k^4k\cdot p-k^2p^4+(\xi-1)\{(k\cdot p)^3+p^2(k\cdot p)^2-3k^2p^2k\cdot p+2k^4k\cdot p+k^2p^4-2k^4p^2\}],$$

$$a_{24}(k,p)=a_{23}(p,k),$$

$$a_{25}(k,p)=q^2(m^2+k\cdot p)+(\xi-1)[q^2m^2+(k\cdot p)^2-(k^2+p^2)k\cdot p+k^2p^2],$$

$$a_{26}(k,p)=m\Delta^2\left\{k\cdot p+\frac{(\xi-1)}{\chi}[q^2k\cdot pm^4+2(k^2+p^2)\Delta^2m^2-k^2p^2\{2(k\cdot p)^2+(k^2+p^2)k\cdot p-4k^2p^2\}]\right\},$$

$$a_{31}(k,p)=-\frac{\eta_1}{2}\left\{\left[\{-2(k\cdot p)^2+k^4+p^4\}m^4+2\{(k^2+p^2)\times(k\cdot p)^2+(k^2-p^2)^2k\cdot p-k^2p^2(k^2+p^2)\}m^2+\frac{1}{2}\{-4(k\cdot p)^4+(k^2+p^2)^2(k\cdot p)^2+k^2p^2(k^2-p^2)^2\}\right]+\frac{(\xi-1)}{\chi}\left[q^2\{-2(k\cdot p)^2+k^4+p^4\}m^8+2\{(k^2+p^2)[-2(k\cdot p)^3+(k^2+p^2)(k\cdot p)^2+(k^4+p^4)k\cdot p]-k^2p^2(3k^4+3p^4-2k^2p^2)\}m^6-\frac{3}{2}q^2\Delta^2(k^2-p^2)^2m^4-2\{(k^2+p^2)(k^4+p^4-4k^2p^2)(k\cdot p)^3-k^2p^2(k^4+p^4-6k^2p^2)(k\cdot p)^2+2k^4p^4(k^2+p^2)k\cdot p-k^4p^4(k^2+p^2)^2\}m^2-\frac{1}{2}k^2p^2q^2\{(k^4+p^4-6k^2p^2)(k\cdot p)^2+k^2p^2(k^2+p^2)^2\}\right]\right\},$$

$$a_{32}(k,p)=a_{31}(p,k),$$

$$a_{33}(k,p)=\xi\{(k\cdot p)^3-k^2(k\cdot p)^2-3k^2p^2k\cdot p+2k^4k\cdot p-k^4p^2+2k^6\}m^2/k^2+(\xi-2)\{(k\cdot p)^3-(2k^2-p^2)(k\cdot p)^2+k^2p^2k\cdot p-2k^4k\cdot p+k^2p^4\},$$

$$a_{34}(k,p)=a_{33}(k,p),$$

$$a_{35}(k,p)=-(k^4+p^4-2(k\cdot p)^2)[\xi m^2+(\xi-2)k\cdot p],$$

$$a_{36}(k,p)=-m\Delta^2\left\{k\cdot p(k^2+p^2)+2k^2p^2+\frac{(\xi-1)}{\chi}[q^2\{(k^2+p^2)k\cdot p+2k^2p^2\}m^4+2(k^2+p^2)^2\Delta^2m^2-k^2p^2(k+p)^2\{(k^2+p^2)k\cdot p-2k^2p^2\}]\right\},$$

$$a_{41}(k,p)=-\eta_1(\xi-1)\frac{(k^2-p^2)}{2\chi}\left[-q^4m^6+3q^2\{-(k^2+p^2)k\cdot p+2k^2p^2\}m^4+\{(k\cdot p)^2[4(k\cdot p)^2-3k^4-3p^4-26k^2p^2]+k^2p^2[24(k^2+p^2)k\cdot p-3k^4-3p^4-14k^2p^2]\}\frac{m^2}{2}+\frac{q^2}{2}\{(k^2+p^2)(k\cdot p)^3+2k^2p^2(k\cdot p)^2-3k^2p^2(k^2+p^2)k\cdot p+2k^4p^4\}\right],$$

$$a_{42}(k,p)=-a_{41}(p,k),$$

$$a_{43}(k,p)=\frac{(\xi-1)}{k^2}[(k^2+k\cdot p)(k\cdot p)^2+k^2(2k^2-3p^2)k\cdot p+k^4(p^2-2k^2)],$$

$$a_{44}(k,p)=-a_{43}(p,k),$$

$$a_{45}(k,p)=(\xi-1)(k^2-p^2)q^2,$$

$$a_{46}(k,p)=m(\xi-1)(k^2-p^2)\frac{\Delta^2}{\chi}[q^2k\cdot pm^2+2(k^2+p^2)\times(k\cdot p)^2-2k^2p^2k\cdot p-k^2p^2(k^2+p^2)],$$

$$a_{51}(k,p)=-\eta_1\left\{\Delta^2+\frac{(\xi-1)}{4\chi}[-2q^4m^6+6q^2\{2k^2p^2-(k^2+p^2)k\cdot p\}m^4-6k^2p^2q^4m^2-q^2\{(k^2-p^2)^2(k\cdot p)^2+2k^2p^2(k^2+p^2)k\cdot p-k^2p^2(k^2+p^2)^2\}]\right\},$$

$$a_{52}(k,p)=a_{51}(p,k),$$

$$a_{53}(k,p)=\frac{(\xi-1)}{k^2}[(k\cdot p)^2+2k^2k\cdot p-k^2(2k^2+p^2)],$$

$$a_{54}(k,p)=a_{53}(p,k),$$

$$a_{55}(k,p)=(\xi-1)q^2,$$

$$a_{56}(k,p)=-m(\xi-1)\frac{\Delta^2}{\chi}[q^2(k^2+p^2)m^2+2(k^4+p^4)k\cdot p-2k^2p^2(k^2+p^2)],$$

$$a_{61}(k,p)=-\eta_1\frac{(k^2-p^2)}{2}\bigg\{q^2m^4-2\{(k\cdot p)^2-(k^2+p^2)k\cdot p+k^2p^2\}m^2+\frac{q^2}{2}\{(k\cdot p)^2+k^2p^2\}+\frac{q^2(\xi-1)}{\chi}\bigg[q^2m^8+2\{(k\cdot p)^2+(k^2+p^2)k\cdot p-3k^2p^2\}m^6-\frac{3}{2}q^2\Delta^2m^4-2\{(k^2+p^2)(k\cdot p)^3-k^2p^2(k\cdot p)^2-k^4p^4\}m^2-\frac{1}{2}k^2p^2q^2\{(k\cdot p)^2+k^2p^2\}\bigg]\bigg\},$$

$$a_{62}(k,p)=-a_{61}(p,k),$$

$$a_{63}(k,p)=-[\xi\{(k^2+k\cdot p)(k\cdot p)^2+k^2(2k^2-3p^2)k\cdot p-k^4(2k^2-p^2)\}m^2/k^2+(\xi-2)\{(2k^2-p^2+k\cdot p)\times(k\cdot p)^2-k^2(2k^2-p^2)k\cdot p-k^2p^4\}],$$

$$a_{64}(k,p)=-a_{63}(p,k),$$

$$a_{65}(k,p)=-q^2(k^2-p^2)[\xi m^2-(\xi-2)k\cdot p],$$

$$a_{66}(k,p)=-m(k^2-p^2)\Delta^2\bigg[k\cdot p+\frac{(\xi-1)}{\chi}(q^2k\cdot pm^4+2(k^2+p^2)\Delta^2m^2-k^2p^2q^2k\cdot p)\bigg],$$

$$a_{71}(k,p)=-\frac{\eta_1(\xi-1)}{4\chi}[-2q^6m^6-6q^4\{(k^2+p^2)(k\cdot p)-k^2p^2\}m^4-3q^2\{[(k\cdot p)^2+k^2p^2](k^4+p^4+6k^2p^2)-8k^2p^2(k^2+p^2)k\cdot p\}m^2+q^2\{(k^2-p^2)^2(k\cdot p)^3+4k^2p^2(k^2+p^2)(k\cdot p)^2-k^2p^2(3k^4+3p^4+10k^2p^2)k\cdot p+4k^4p^4(k^2+p^2)\}],$$

$$a_{72}(k,p)=a_{71}(p,k),$$

$$a_{73}(k,p)=(\xi-1)\frac{(k^2-k\cdot p)}{k^2}[(k\cdot p)^2+4k^2k\cdot p-2k^4-3k^2p^2],$$

$$a_{74}(k,p)=a_{73}(p,k),$$

$$a_{75}(k,p)=(\xi-1)q^4,$$

$$a_{76}(k,p)=m(\xi-1)\frac{\Delta^2}{\chi}[q^2\{(k^2+p^2)k\cdot p-2k^2p^2\}m^2+2(k^4+p^4)(k\cdot p)^2-4k^2p^2(k^2+p^2)k\cdot p-k^2p^2(k^4+p^4-6k^2p^2)],$$

$$a_{81}(k,p)=-\eta_1\frac{(\xi+2)}{2}q^2(m^2+k\cdot p),$$

$$a_{82}(k,p)=a_{81}(p,k),$$

$$a_{83}(k,p)=2(\xi+2)k\cdot q,$$

$$a_{84}(k,p)=a_{83}(p,k),$$

$$a_{85}(k,p)=-(\xi+2)q^2,$$

$$a_{86}(k,p)=0. \tag{A14}$$

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