Quantum back reaction of massive fields and self-consistent semiclassical extreme black holes and acceleration horizons

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We consider the effect of back reaction of quantized massive fields on the metric of extreme black holes (EBH's). We find the analytical approximate expression for the stress-energy tensor for a scalar (with an arbitrary coupling), spinor and vector fields near an event horizon. We show that, independent of a concrete type of EBH, the energy measured by a freely falling observer is finite on the horizon, so that quantum back reaction is consistent with the existence of EBH's. For the Reissner-Nordström EBH with a total mass M_{tot} and charge Q we show that for all cases of physical interest $M_{tot}<Q$. We also discuss different types of quantum-corrected Bertotti-Robinson spacetimes, find for them exact self-consistent solutions, and consider situations in which tiny quantum corrections lead to the qualitative change of the classical geometry and topology. In all cases one should start not from a classical background with further added quantum corrections but from the quantum-corrected self-consistent geometries from the very beginning.

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I. INTRODUCTION

Nowadays, the physical relevance and importance of the issue of extreme black holes (EBH's) does not need detailed clarification. Let us only mention briefly such issues as the end point of black hole evaporation, information loss, the black hole entropy, etc. In fact, the background of EBH's can serve as a promising testing area for potential predictions of (not yet constructed) quantum gravity in the semiclassical domain. Meanwhile, recently, the very fact of the existence of semiclassical black holes has become the subject of discussion [1]. In that paper Lowe presented strong arguments confirming the existence of semiclassical EBH's. These arguments, however, are of a phenomenological nature in that they tacitly assume that the components of the stress-energy tensor (SET) of a quantized field and their relevant combinations with the metric functions remain finite on the horizon. However, this is not obvious in advance. For example, in the background of the Reissner-Nordström (RN) EBH, this fact for massless radiation was established only by virtue of thorough numerical calculations [2].

In this situation it looks reasonable to elaborate general back reaction approach to EBH's similar to that [3] applied to Schwarzschild black holes. However, an attempt at moving in this direction immediately encounters the following difficulty, which reveals the crucial difference between nonextreme and extreme black holes in the given context. In the first case, it was sufficient to choose a fixed background and carry out calculations perturbatively, whereas in the second one the very nature of the background becomes not trivial. Say that for the classical Reissner-Nordström black hole with charge Q and mass M minor changes around the extreme relationship $M=Q$ can convert the EBH to a nonextreme hole or naked singularity. Correspondingly, one should be very careful in examining changes caused by quantum effects at the border of such different kinds of spacetime. Therefore, the emphasis in the back reaction program for EBH's (at least, at the first step) is shifted as compared to the Schwarzschild case: first of all, it is necessary to elucidate whether or not EBH's are compatible with back reaction. It looks natural to take a generic EBH metric, ''dressed'' by surrounding quantum fields, and elucidate whether or not back reaction is compatible with the property that the Hawking temperature $T_H=0$. In turn, this invokes information about the SET of quantized field spacetime of a generic spherically symmetrical EBH. For massless fields, this task is extremely difficult. Meanwhile, for massive fields recent progress in deriving general expressions for the SET [4,5] makes the task tractable. As the calculation of the SET is the key to the problem of existence of quantum-corrected EBH's, let us dwell upon this issue in more detail.

II. GENERAL FEATURES OF A SET OF MASSIVE FIELDS IN CURVED MANIFOLDS

According to the standard viewpoint, the renormalized stress-energy tensor of quantized fields evaluated in an appropriate state encodes all available information about quantum field theory in a curved background, and (in addition to the classical part) it serves as a source term of the semiclassical Einstein field equations. Unfortunately, mathematical complexities prevent an exact analytical treatment and in most physically interesting situations it cannot be expressed in terms of known special functions. Moreover, what is of principal interest in further applications is not the SET itself evaluated in the particular geometry, but rather its functional

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dependence on a wide class of metrics. Therefore, we are confronted with two serious problems: construction of the SET on the one hand, and studying the effect of the quantized field upon the spacetime geometry on the other. It is natural, therefore, that to address these problems, at least partially, one should employ approximate methods.

It seems that, for massive fields in the large mass limit considered in this paper, an approximation based on the DeWitt-Schwinger expansion is of the required generality, allowing one, in principle, to attack the problem of back reaction perturbatively. Moreover, in some situations (also considered here), it is even possible to construct *exact* solutions of the semiclassical field equations, or, what is more common, guided by physical considerations, guess the appropriate form of the line element. Although such a procedure is limited to especially simple geometries with a high degree of symmetry, the results obtained are of particular interest and importance.

For massive fields in a curved spacetime, the renormalized effective action W_R constructed by means of the DeWitt-Schwinger method is given by

$$W_R=\frac{1}{32\pi^2 m^2}\int d^4x g^{1/2}\sum_{n=3}^{\infty}\frac{(n-3)!}{(m^2)^{n-2}}[a_n], \tag{1}$$

where $[a_n]$ is the coincidence limit of the nth Hadamard-DeWitt coefficient and m is the mass of the field, and the first three terms of the DeWitt-Schwinger expansion have been absorbed by quadratic terms of the generalized classical gravitational action in the process of renormalization of the bare constants. As the complexity of the Hadamard-DeWitt coefficients rapidly grows with increasing n, the practical use of Eq. (1) is confined to the first order of W_R, which involves the integrated coincidence limit of the fourth Hadamard–DeWitt coefficient a_3 computed by Gilkey [6]. Being constructed from local, geometrical quantities, the first order effective action does not describe the process of particle creation, which is a nonlocal phenomenon; however, for sufficiently massive fields, the contribution of real particles may be neglected and the DeWitt-Schwinger W_R satisfactorily approximates the total effective action. It can be shown that for massive scalar, spinor, and vector fields the first order effective action can be compactly written in the form [7]

$$\begin{aligned}W_{ren}^{(1)}=&\frac{1}{192\pi^2 m^2}\int d^4x g^{1/2}(c_1^{(s)}R\Box R+c_2^{(s)}R_{\mu\nu}\Box R^{\mu\nu}\\&+c_3^{(s)}R^3+c_4^{(s)}RR_{\mu\nu}R^{\mu\nu}+c_5^{(s)}RR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\\&+c_6^{(s)}R^{\mu}_{\nu}R^{\nu}_{\rho}R^{\rho}_{\mu}+c_7^{(s)}R^{\mu\nu}R_{\rho\sigma}R^{\rho\ \sigma}_{\ \mu\ \nu}\\&+c_8^{(s)}R_{\mu\nu}R^{\mu}_{\lambda\rho\sigma}R^{\nu\lambda\rho\sigma}\\&+c_9^{(s)}R^{\mu\nu}_{\rho\sigma}R^{\lambda\gamma}_{\mu\nu}R^{\rho\sigma}_{\lambda\gamma}+c_{10}^{(s)}R^{\rho\ \sigma}_{\ \mu\ \nu}R^{\mu\ \nu}_{\ \lambda\ \gamma}R^{\lambda\ \gamma}_{\ \rho\ \sigma})\\=&\frac{1}{192\pi^2 m^2}\sum_{i=1}^{10}c_i^{(s)}W_i,\end{aligned} \tag{2}$$

TABLE I. The coefficients $c_i^{(s)}$ for the massive scalar, spinor, and vector fields. Note that to obtain the result for the massive neutral spinor field one has to multiply $W_{ren}^{(1)}$ by the factor 1/2.

	$s=0$	$s=1/2$	$s=1$
$c_1^{(s)}$	$\frac{1}{2}\xi^2-\frac{1}{5}\xi+\frac{1}{56}$	$-\frac{3}{140}$	$-\frac{27}{280}$
$c_2^{(s)}$	$\frac{1}{140}$	$\frac{1}{14}$	$\frac{9}{28}$
$c_3^{(s)}$	$(\frac{1}{6}-\xi)^3$	$\frac{1}{432}$	$-\frac{5}{72}$
$c_4^{(s)}$	$-\frac{1}{30}(\frac{1}{6}-\xi)$	$-\frac{1}{90}$	$\frac{31}{60}$
$c_5^{(s)}$	$\frac{1}{30}(\frac{1}{6}-\xi)$	$-\frac{7}{720}$	$-\frac{1}{10}$
$c_6^{(s)}$	$-\frac{8}{945}$	$-\frac{25}{378}$	$-\frac{52}{63}$
$c_7^{(s)}$	$\frac{2}{315}$	$\frac{47}{630}$	$-\frac{19}{105}$
$c_8^{(s)}$	$\frac{1}{1260}$	$\frac{19}{630}$	$\frac{61}{140}$
$c_9^{(s)}$	$\frac{17}{7560}$	$\frac{29}{3780}$	$-\frac{67}{2520}$
$c_{10}^{(s)}$	$-\frac{1}{270}$	$-\frac{1}{54}$	$\frac{1}{18}$

where the numerical coefficients depending on the spin of the field are listed in Table I. As the approximate stress-energy tensor is obtained by functional differentiation of Eq. (2) with respect to the metric tensor

$$T^{\mu\nu(q)}=\frac{1}{96\pi^2 m^2 g^{1/2}}\sum_{i=1}^{10}c_i^{(s)}\frac{\delta}{\delta g_{\mu\nu}}W_i, \tag{3}$$

one concludes that within the framework of the adopted approximation it is expressed as a linear combination of purely geometrical terms with the numerical coefficients depending on the spin of the field, and consequently is independent of boundary conditions. Since the calculations are carried out for Euclideanized geometry, the resulting Green functions bear close relationships to the temperature Green functions, and in black hole spacetimes in the absence of superradiant modes the SET thus obtained may be interpreted in terms of the thermal state. An alternative approach, consisting in the construction of appropriate Green functions by summing (integrating) WKB approximants of the mode functions of the scalar field equation with arbitrary coupling to a curvature, has been proposed in Ref. [8]. It was shown that, to obtain the lowest order terms in the DeWitt-Schwinger expansion, one has to employ the results of a sixth order WKB approximation. Moreover, detailed analyses, both analytical and numerical, of the stress-energy tensor of the quantized massive scalar field carried out in the Reissner-Nordström spacetime confirmed that the DeWitt-Schwinger approximation yields reasonable results as long as the mass of the field is sufficiently large [8]. Specifically, it was shown that for quantized scalars in the vicinity of the event horizon of a RN black hole, the approximation remains within a few percent of the exact (numerical) value if the condition $mM\geqslant 2$ holds.

General expressions for the first nonvanishing order of the SET of the massive scalar, spinor, and vector fields, which generalize earlier results of Frolov and Zel'nikov for vacuum type-D geometries [9], were constructed in [4] and [5]. They may be used in principle in any spacetime provided the temporal changes of the background are slow and the ratios of the Compton length to the characteristic radii of curvature are small. However, because of computational complexity, their practical use is limited to simple spacetimes. Happily, there are considerable simplifications for the class of metrics considered in this paper: spherically symmetric geometries with vanishing curvature scalar and spacetimes with maximally symmetric subspaces. On the other hand, however, in some physically important and computationally tractable cases, such as for example Kerr or Kerr-Newman spacetimes, there are superradiant modes, and the SET constructed along the lines of the DeWitt-Schwinger approximation must be interpreted with care. However, in spite of its inherent limitations, the DeWitt-Schwinger method is still the most general one not restricted to any particular type of symmetry.

In this paper we shall use the general results of [4] and [5] to evaluate the renormalized SET's of the massive scalar, spinor, and vector fields in the spacetime of extremal black holes. The calculations for a general metric turn out to be extremely complicated and are of little practical use. Fortunately, for the issue of the existence of quantum-corrected EBH's and the properties of corresponding self-consistent solutions of the Einstein equations, it is sufficient to expand the metric potentials in the vicinity of the event horizon into a Taylor series and examine the SET constructed for this simplified line element. Additionally, we shall construct and examine the SET in the Bertotti-Robinson-like spacetimes obtained by expanding the near-horizon geometry into a whole manifold.

III. SEMICLASSICAL EXTREME BLACK HOLES

A. Quantum back reaction and degenerate horizon

The metric under consideration reads

$$ds^2=-Udt^2+V^{-1}dr^2+r^2d\Omega^2, \tag{4}$$

where the form of $V(r)$ can be found from the 00 component of the Einstein equations. It is equal to

$$V=1-\frac{r_+}{r}-\frac{2\tilde{m}(r)}{r},$$
$$\tilde{m}(r)=4\pi\int_{r_+}^{r}dr' r'^2\rho(r'); \tag{5}$$

where $\rho=-T_0^0$ and the SET $T_\mu^\nu=T_\mu^{\nu(cl)}+T_\mu^{\nu(q)}$. Here the first term comes from a classical source, and the second one is due to the contribution of quantum fields and is to be understood as a quantum average with respect to the the Hartle-Hawking state, renormalized in a proper way. Let us assume that the role of the classical source is played by an electromagnetic field ($T_\mu^{\nu(cl)}\equiv T_\mu^{\nu(em)}$), so we deal with the quantum-corrected RN black hole. Correspondingly, $\tilde{m}(r)=m_{em}+m_q$, where $m_{em}=(Q^2/2)(1/r_+-1/r)$, $m^q=4\pi\int_{r_+}^{r}dr' r'^2\rho^q(r')$. Here it is implied that the event horizon is located at $r=r_+$. In this sense r_+ is the "exact" value of the horizon radius (to some extent the word "exact" is conditional since $T_\mu^{\nu(q)}$ is known in the one-loop approximation only). The function $U=Ve^{2\psi}$, where the concrete form of the function $\psi(r)$ can be found from the (rr) Einstein equation. Then the Hawking temperature

$$T_H=\frac{V'(r_+)}{4\pi}e^{\psi(r_+)}. \tag{6}$$

The explicit form of V is

$$V=1-\frac{2m(r)}{r}+\frac{Q^2}{r^2},$$
$$m(r)=\tilde{m}(r)+\frac{Q^2}{2r}+\frac{r_+}{2}=M+m^q(r). \tag{7}$$

It follows from the definitions that $m^q(r_+)=0$ (no room for radiation) and $m(r_+)=M$. The condition that r_+ is the root of $V(r)=0$ means that

$$g(r)\equiv r^2V=r^2-2m(r)r+Q^2=0. \tag{8}$$

If $\psi(r_+)$ is bounded on a horizon, the answer to the question of whether or not a black hole can reach the extreme state is determined by whether or not $V'(r_+)$ can become zero. In fact, the finiteness of $\psi(r_+)$ on the horizon, typical of a nonextreme black hole, becomes a nontrivial issue in the extreme case. It is equivalent to the problem of whether or not the energy measured by a freely falling observer remains finite on the horizon (see, e.g., [2]). Thorough numerical calculations showed that $\psi(r_+)$ is indeed finite for the RN EBH in the case when quantized fields are massless [10]. The behavior of ψ near the horizon is one of the key issues examined below in the present paper for the case of massive fields. Let us, however, put this matter aside for a moment and assume that $\psi(r_+)$ is indeed finite.

Equation (8) should be satisfied at $r=r_+$ independently of whether the horizon is extremal. Here $m(r)$ is an unknown function but near r_+ we can expand it as $m(r)=M+A(r-r_+)+\cdots$, where $A=4\pi r_+^2\rho^q(r_+)$ has the order $\varepsilon=\hbar/M^2$. It is more convenient to write

$$m(r)=M_0+Ar+\cdots, \tag{9}$$

where $M_0=M-Ar_+$. Now we get the equation

$$g(r)=r^2(1-2A)-2rM_0+Q^2=0, \tag{10}$$

whence the roots are

$$r_\pm=\frac{M_0}{1-2A}\pm\sqrt{\left(\frac{M_0}{1-2A}\right)^2-\frac{Q^2}{1-2A}}. \tag{11}$$

If one adjust parameters in such a way that $M_0^2=Q^2(1-2A)$, then

$$r_+=r_-=Q/\sqrt{1-2A}=\frac{M}{1-A} \tag{12}$$

is the double root of the function $g(r)$ and $M=M_0(1-A)/(1-2A)=Q(1-A)/\sqrt{1-2A}$.

On physical grounds, it is essential for the existence of EBH's that the *total energy* density on the horizon (including, in the RN case, the electromagnetic contribution) be positive (and large enough), so the sign of quantum contribution itself (if it is not too large) is not so crucial.

Let us also describe another seemingly "obvious" approach that, however, contains a hidden trap. In other (less successful) notations one can write the identity that follows from the substitution of $r=r_+$ into $g(r_+)=0$:

$$r_+^2-2Mr_++Q^2=0, \tag{13}$$

whence

$$r_+=M+\alpha\sqrt{M^2-Q^2}, \tag{14}$$

with $\alpha=1$ or $\alpha=-1$. At first glance, the choice $\alpha=1$ should correspond to the event horizon as is the case for classical RN black holes. However, when one inserts $r=r_+$ into the identity $g(r_+)=0$, it turns out that, as a matter of fact, one inserts into an equation its own root as a parameter of this very equation [by contrast, M_0, Q, and A are independent parameters and the roots $r_\pm$ are expressed in their terms directly according to Eq. (11)]. This procedure is not quite safe and can lead to the appearance of "spurious" solutions [1]. Therefore, one should verify its self-consistency to make sure that the root under consideration is a "true" one.

It is instructive to demonstrate this explicitly. Let us compare two different presentations of the same quantity—Eqs. (11) and (14). Then after some manipulations we have

$$\alpha\sqrt{M^2-Q^2}=\sqrt{Z}+\frac{M_0A}{1-2A}, \tag{15}$$

where $Z=[M_0/(1-2A)]^2-Q^2/(1-2A)$. If $A>0$, one should take $\alpha=1$ (it is supposed that A is not too large; in fact, $A\ll 1$). However, near the extreme state $Z\to 0$ and $A<0$, one should choose $\alpha=-1$.

Thus, quantum back reaction shifts the double root to a new position but does not change its character qualitatively [1]. In doing this, however, in sharp contrast with the classical case, the horizon for $A<0$ lies at $r_+=M-\sqrt{M^2-Q^2}$.

B. General approach to extreme black holes dressed by quantized massive fields

Meanwhile, as mentioned above, the fact that back reaction leaves the possibility for the existence of a double root of Eq. (10) is insufficient in itself for making conclusions about the existence of extreme quantum-corrected black holes. According to the rr and tt components of the Einstein equations,

$$\psi=4\pi\int_\infty^r drF(r), \quad F(r)=r\frac{T_1^1-T_0^0}{V}. \tag{16}$$

Here the lower limit of integration is set to infinity since it is supposed that spacetime infinity is flat and $\psi=0$. Below we will consider the case of massive fields only for which $T_\mu^\nu\to 0$ as $r\to\infty$ (for massless fields it is usually assumed that a system is enclosed in a finite cavity, otherwise $T_\mu^\nu\to\text{const}\neq 0$).

The key question is whether or not the quantity F is finite on the horizon. Further details depend on the possibility of power expansion of the metric near the horizon. As far as massless fields are concerned, the counterpart from two-dimensional black hole physics shows [11] that (i) for a generic fixed metric the function $F(r)$ diverges on the horizon which indicates a qualitative change of the metric of the extreme black hole under the influence of a quantum field, (ii) if, instead of fixing a metric in advance, one chooses it as a *self-consistent* solution of the field equations with back reaction taken into account, the existence of an extreme black hole is compatible with quantum back reaction, but (iii) the power expansion of the metric near the horizon fails to be analytic. On the other hand, numerical calculations for the four-dimensional RN background [2] showed that F remains finite on the horizon.

There are the following subtleties in our problem. As, by assumption, the quantum field is neutral, it does not screen a charge, so its value Q is the same for the original classical and the quantum-corrected backgrounds. Then it follows from Eq. (11) that $r_\pm<Q$, if $\rho^q(r_+)<0$. Had we chosen the classical background with such a relationship between parameters and tried to take into account quantum back reaction by building up the perturbation series, we would have obtained a physically meaningless result. Indeed, if the root of the equation $g(r)=0$ is less than Q, it corresponds in classical language to the Cauchy (not the event) horizon, where the stress-energy tensor of quantum fields is known to blow up. This obstacle shows clearly that, instead of using a standard scheme (pure classical background plus perturbative quantum corrections) we should start from the *self-consistent quantum-corrected background from the very beginning*.

Our strategy consists in the following. As the issue of the existence of extreme black holes demands knowledge of the behavior of the metric near the horizon only, let us consider the vicinity of the horizon of a generic EBH, expand the metric near the horizon into a power series, and examine whether or not the quantity F remains finite on the horizon. If F is finite (which means the finiteness of the SET in the orthonormal reference frame of a free falling observer [2]), one obtains a power expansion for the SET too, so the full self-consistent solution can be obtained by direct expansion into a Taylor series with respect to $r-r_+$.

For EBH's the conjectured power expansion in terms of $r-r_+$ looks like $V=a(r-r_+)^2+b(r-r_+)^3+\cdots$. For concrete calculations it is more convenient, however, to use, instead of r, the proper distance l from some fixed point. Then we have $dl/dr=-1/\sqrt{V}$. Substituting into this equation the power expansion for V near the horizon, we find

$$r-r_+=A_1e^{-l/\rho}+A_2e^{-2l/\rho}+\cdots, \tag{17}$$

where $\rho = a^{-1/2}$ and the integration constant l_0 is absorbed by the coefficients according to $A_1 = r_+ \exp(l_0/\rho)$, $A_2 = r_+^2 b/a \exp(2l_0/\rho)$.

We can write down

$$ds^2 = -dt^2 U(l) + dl^2 + r^2(l) d\Omega^2. \tag{18}$$

In what follows we assume that power expansion of U in terms of $r - r_+$ starts from the terms of $(r - r_+)^2$, as is typical for EBH's. In terms of l this function reads

$$U = e^{-2l/\rho} f(l), \quad f = f_0 + f_1 e^{-l/\rho} + f_2 e^{-2l/\rho}. \tag{19}$$

The expressions for the SET of massive fields in the metric (19) are very cumbersome. However, what is the most important for us is their general structure. It turns out that near the horizon

$$F = F_0 + F_1 e^{-l/\rho} + F_2 e^{-2l/\rho} + \cdots \tag{20}$$

with *finite* coefficients F_i $(i = 0,1,2, \ldots)$. The expressions for the components of the SET read

$$T_\mu^\nu = t_\mu^{\nu(0)} + t_\mu^{\nu(1)} e^{-l/\rho} + t_\mu^{\nu(2)} e^{-2l/\rho} + \cdots, \tag{21}$$

where explicit expressions for the coefficients t_μ^ν are listed in Appendix A for different kinds of field. It is essential that in all cases it turns out that $t_0^{0(0)} = t_1^{1(0)}$ and $t_0^{0(1)} = t_1^{1(1)}$, which just leads to the finiteness of F on the horizon.

It is worth stressing that the finiteness of F for the case of massive fields is shown for any EBH irrespective of whether or not its metric obeys the system of field equations and the type of theory to which these field equation correspond. Now this general result is applied for the most physically interesting case of a RN EBH, dressed by its quantum radiation.

C. Quantum-corrected RN extreme black hole

From the physical viewpoint, it is natural to fix the total mass measured by a distant observer at infinity (the microcanonical boundary condition). Then we have from Eq. (7)

$$M_{tot} = M + m^q, \quad m^q = -4\pi \int_{r_+}^{\infty} dr r^2 T_0^{0(q)},$$

$$M = \frac{r_+^2 + Q^2}{2r_+}. \tag{22}$$

The condition of extremality $V'(r_+) = 0$ entails

$$r_+^2 (1 - 2A) = Q^2. \tag{23}$$

In all cases $m^q = \alpha_s m^{-2} r_+^{-3}$, where $\alpha_0 = -\tilde{\alpha} 17/441$, $\alpha_{1/2} = -\tilde{\alpha} 19/147$, $\alpha_1 = -\tilde{\alpha} 107/441$, and $\tilde{\alpha} = 1/720\pi$. From Eqs. (22) and (23) it is seen that the corrections of first order in M cancel and we obtain

$$M_{tot} = Q + \alpha_s m^{-2} r_+^{-3}. \tag{24}$$

In the main approximation $\rho = r_+ = M = Q$. We see that for all physically relevant cases $\alpha_s < 0$ and $M_{tot} < Q$. Thus, a distant observer measuring by precise devices the total mass and charge of an extreme RN black hole, had he relied on classical notions only and neglected quantum back reaction completely, would have been led to the wrong conclusion that in fact the object under investigation is rather a naked singularity than a black hole. In other words, the quantum-corrected solution of the Einstein-Maxwell equations under discussion not only acquires some small corrections from back reaction of quantum fields but resides in a pure quantum domain where the existence of classical black holes (both extremal and nonextremal) is strictly forbidden.

One can also find the quantum-corrected position of the horizon in terms of physical parameters. Taking into account Eq. (12) and noting that $T_0^{0(q)}(r_+) = \eta_s m^{-2} r_+^{-6}$, $\eta_s = \mu_s/2880\pi^2$, $\mu_0 = 16/21 - 4(\xi - 1/6)$, $\mu_{1/2} = 37/14$, and $\mu_1 = 114/7$ [4], we obtain $A = 4\pi\eta_s m^{-2} r_+^{-4}$, $r_+ = Q(1 - 2A)^{-1/2} \approx Q(1 + A)$. Equivalently, in terms of the total mass, $r_+ = M_{tot}(1 + \beta_s m^{-2} M_{tot}^{-4})$, $\beta_s = 4\pi\eta_s - \alpha_s$. Here $\beta_0 = \tilde{\alpha}[353/441 - 4(\xi - 1/6)]$, $\beta_{1/2} = \tilde{\alpha} 815/294$, $\beta_1 = \tilde{\alpha} 7289/441$.

IV. QUANTUM-CORRECTED BERTOTTI-ROBINSON-LIKE SPACETIMES

A. Self-consistent solutions without cosmological term

It is obvious that it is impossible to find an exact solution of the back reaction equation for a realistic four-dimensional EBH in all space and this is the reason why we were forced to restrict ourselves to the treatment of the vicinity of the horizon only. Meanwhile, there exists another class of objects for which exact solutions (in the one-loop approximation) can indeed be found—metrics with acceleration horizons [Bertotti-Robinson (BR) spacetime and its modifications]. Such spacetimes have topology $(r,t) \times S_2$, where S_2 is a two-dimensional sphere, so that the coefficient standing at the angular part of a line element is constant. The physical relevance of such spacetimes stems, in particular, from the fact that they can serve as approximations to the true metric of the EBH in the vicinity of the horizon. Apart from this, such a metric appears in the limiting transition from nonextreme black holes to extreme ones [12–14]. The SET for the BR spacetime was studied in [15,4]. Now, however, we start not from the BR spacetime itself, but from its quantum-corrected version.

The general form of metric under consideration is

$$ds^2 = -U(l) dt^2 + dl^2 + r_0^2 d\Omega^2, \tag{25}$$

where it is assumed that there exists a horizon on which $U \to 0$. In the coordinates (x^1, θ, ϕ, t) the SET of the electromagnetic field is

$$8\pi T_\mu^{\nu(em)} = \frac{Q^2}{r_0^4}(-1,1,1,-1). \tag{26}$$

However, the expression for the SET of quantized fields in such a background is rather complicated and will not be written here. Fortunately, if we restrict ourselves to the BR-like spacetime and guess its quantum-corrected version, we obtain a very simple answer in a compact form. We found that this procedure is tractable for the following cases.

*Metric BR*1

$$ds^2 = -dt^2\rho^2 sh^2\frac{l}{\rho} + dl^2 + r_0^2 d\Omega^2, \tag{27}$$

$$G_\mu^\nu = \left(-\frac{1}{r_0^2},\frac{1}{\rho^2},\frac{1}{\rho^2},-\frac{1}{r_0^2}\right). \tag{28}$$

*Metric BR*2

$$ds^2 = -dt^2 \exp(-2l/\rho) + dl^2 + r_0^2 d\Omega^2. \tag{29}$$

The Einstein tensor has the same form as Eq. (28).

Metric $dS_2\times S_2$

$$ds^2 = -dt^2\sigma^2 \sin^2\frac{l}{\sigma} + dl^2 + r_0^2 d\Omega^2, \tag{30}$$

$$G_\mu^\nu = \left(-\frac{1}{r_0^2},-\frac{1}{\sigma^2},-\frac{1}{\sigma^2},-\frac{1}{r_0^2}\right). \tag{31}$$

Metric Rindler$_2\times S_2$

$$ds^2 = -dt^2 l^2 + dl^2 + r_0^2 d\Omega^2, \tag{32}$$

$$G_\mu^\nu = \left(-\frac{1}{r_0^2},0,0-\frac{1}{r_0^2}\right). \tag{33}$$

In all cases indicated above the SET of the quantum fields can be written as

$$8\pi T_\mu^{\nu(q)} = C(f_1,f_2,f_2,f_1), \tag{34}$$

where f_1, f_2 are simple constants depending on the curvatures of the two-dimensional maximally symmetric subspaces, $C = 1/12\pi^2 m^2$, and m is the mass of the field. The form of Eq. (34) follows from the fact that for Eqs. (27), (29), (30), and (32) the covariant derivatives of the Riemann tensor and its contractions vanish, which considerable simplifies the SET given by Eq. (3).

The fact that $T_2^{2(q)} = T_3^{3(q)}$ is a simple consequence of the symmetry of the metric with respect to rotations. The equality $T_0^{0(q)} = T_1^{1(q)}$ can be understood as follows: metrics of the type (25) can be obtained as a result of certain limiting transitions from black hole ones, in the process of which the near-horizon geometry expands into a whole manifold [14]; then the SET pick up their values from the horizon where the regularity condition demands just the validity of this equality.

It turns out that for all cases the functions f_1 and f_2 share a common general structure. In BR1 and BR2 $f_1 = \rho^{-6} r_0^{-6}(a_1 r_0^6 + b_1 r_0^4\rho^2 + c_1\rho^6)$, $f_2 = \rho^{-6} r_0^{-6}(a_2\rho^6 + b_2\rho^4 r_0^2 + c_2 r_0^6)$, where the coefficients a_i, b_i, and c_i ($i = 1,2$) are the same for both metrics.

For all values of the spin $a_2 = -a_1$, $b_2 = -b_1$, $c_2 = -c_1$.

In the scalar case ($s = 0$)

$$a_1 = \frac{1}{105}(8 - 84\xi + 420\xi^2 - 840\xi^3),$$

$$b_1 = -\frac{1}{105}(7 - 112\xi + 630\xi^2 - 1260\xi^3),$$

$$c_1 = \frac{1}{105}(4 - 42\xi + 210\xi^2 - 420\xi^3);$$

$$s = 1/2:\ \ a_1 = \frac{20}{420},\ \ b_1 = \frac{7}{420},\ \ c_1 = \frac{10}{420};$$

$$s = 1:\ \ a_1 = \frac{8}{35},\ \ b_1 = \frac{7}{35},\ \ c_1 = \frac{4}{35}.$$

For the metric $dS_2\times S_2$ [now $f_1 = \sigma^{-6} r_0^{-6}(a_1 r_0^6 + b_1 r_0^4\sigma^2 + c_1\sigma^6)$ and $f_2 = \sigma^{-6} r_0^{-6}(a_2\sigma^6 + b_2\sigma^4 r_0^2 + c_2 r_0^6)$]

$$a_2 = a_1,\ \ b_2 = b_1,\ \ c_2 = c_1;$$

$$s = 0:\ \ a_1 = \frac{1}{105}(-8 + 84\xi - 420\xi^2 + 840\xi^3),$$

$$b_1 = \frac{1}{105}(-7 + 112\xi - 630\xi^2 + 1260\xi^3),$$

$$c_1 = \frac{1}{105}(4 - 42\xi + 210\xi^2 - 420\xi^3);$$

$$s = 1/2:\ \ a_1 = -\frac{20}{420},\ \ b_1 = \frac{7}{420},\ \ c_1 = \frac{10}{420};$$

$$s = 1:\ \ a_1 = -\frac{8}{35},\ \ b_1 = \frac{7}{35},\ \ c_1 = \frac{4}{35}.$$

For the metric Rindler$_2\times S_2$

$$f_1 = a r_0^{-6},\ \ f_2 = -2a r_0^{-6},$$

$$a_0 = \frac{8 - 63\eta - 3780\eta^3}{945}\ (\eta = \xi - 1/6),\ \ a_{1/2} = \frac{1}{42},\ \ a_1 = \frac{4}{35}.$$

In the limits $\rho\to\infty$ and $\sigma\to\infty$ the metrics BR1 and $dS_2\times S_2$ turn into Rindler$_2\times S_2$. One can check that in these limits the function f_1 and f_2 go smoothly to their values for the metric Rindler$_2\times S_2$.

Let the classical background be of the BR1 or BR2 type metric [the metric (30) cannot appear on the pure classical level without a cosmological constant]. Then we obtain two independent equations from the Einstein ones:

$$-\frac{1}{r_0^2}=-\frac{Q^2}{r_0^4}+Cf_1, \tag{35}$$

$$\frac{1}{\rho^2}=\frac{Q^2}{r_0^4}+Cf_2. \tag{36}$$

Taking the sum of Eqs. (35) and (36) and noticing that $f_1+f_2=[(r_0^2-\rho^2)/r_0^6\rho^6]\chi$, where χ has the structure $\chi=\alpha_1 r_0^4+\alpha_2\rho^4+\alpha_3\rho^2 r_0^2$ (α_i are pure numbers), we obtain

$$(r_0^2-\rho^2)\left(1-\frac{C\chi}{r_0^4\rho^4}\right)=0. \tag{37}$$

Now take into account that $C\sim\lambda_{PL}^2\lambda^2$, where $\lambda=m^{-1}$ is the Compton length and λ_{PL} is the Planckian length. Then a simple estimate shows that the second factor in Eq. (37) can become zero (provided the proper signs appear in it) for $r_0\ll\lambda$ only, so far beyond the region of validity of the WKB approximation. Therefore, we will not discuss this possibility further and will assume that there is only one root of Eq. (37): $r_0=\rho$. This means that BR spacetime remains an exact solution of the semiclassical equations (cf. [15,16,14]). Making use of Eq. (35) and writing $f_1(r_0=\rho)=\gamma r_0^{-6}$, where $\gamma=a_1+b_1+c_1$, we find that $r_0^2=Q^2-C\gamma r_0^{-2}$, whence, in the same approximation,

$$r_0^2=Q^2(1-C\gamma Q^{-4}). \tag{38}$$

In Eq. (38) the second term in the parentheses represents only a small correction but accounting for this correction can be crucial in the following sense. Let $\gamma>0$. Then we have $Q>r_0$. Let us proceed, for definiteness, in the canonical ensemble approach in which the charge (rather than the potential on the boundary) should be fixed. First, if $Q\neq r_0$, the classical metric with an acceleration horizon of the type (35) or (29) is not possible at all. Instead, we would have the geometry of a RN black hole. Second, for $Q>r_0$ with r_0 being the horizon radius, we would have, moreover, a naked singularity. It is clear that the procedure in which the ground state is chosen as a classical geometry with a naked singularity with quantum corrections, calculated on such a background perturbatively, is physically unacceptable. Instead of this, we should from the very beginning use the quantum-corrected geometry and check the condition of self-consistency for the corresponding parameters. In our case it is the relative simplicity of the BR geometry that enables us to find the SET not only for the classical BR spacetime itself but also for the quantum-corrected version of it (below we will see that this is also the case even if the quantum-corrected metrics changes its form—for example, due to the cosmological term).

The values of γ for different values of field spin (the subscript indicates the value of spin s) are $\gamma_0=(5-14\xi)/105$, $\gamma_1=19/35$, and $\gamma_{1/2}=37/420$. Thus, for a spinor and vector field $\gamma>0$ as well as for the scalar case with minimal and conformal coupling.

B. Nonzero cosmological constant

Now for the BR1 and BR2 metrics the field equations read

$$\frac{1}{r_0^2}=\frac{Q^2}{r_0^4}-\Lambda-Cf_1 \tag{39}$$

and

$$\frac{1}{\rho^2}=\frac{Q^2}{r_0^4}+\Lambda+Cf_2. \tag{40}$$

For the BR3 case we have, instead of Eq. (40),

$$\frac{1}{\sigma^2}=-\frac{Q^2}{r_0^4}-\Lambda-Cf_2. \tag{41}$$

Then it follows from Eq. (39) that

$$\frac{1}{r_0^2}=\frac{1}{2Q^2}\pm\frac{1}{Q}\sqrt{\frac{1}{4Q^2}+\Lambda+Cf_1}. \tag{42}$$

The term with Cf_1 represents a small correction to the classical quantities, so that in the expression for f_1 one can replace r_0 and ρ by the classical values obtained for $C=0$. If $\Lambda>0$, only the solution with the $+$ sign should be taken. We will discuss the case $\Lambda=-|\Lambda|<0$, which is more interesting for us. Let Λ be very close to the value $\Lambda_0=-\frac{1}{4}Q^{-2}$ for which the radical in Eq. (42) becomes zero. In f_1 we can put in the main approximation $r_0=2^{1/2}Q$, $\rho=\infty$, neglecting corrections of the order of C. Classically, we would have the product of two-dimensional Rindler and a sphere (32), for which, according to the above results, $f_1=\bar{a}Q^{-6}$, $f_2=-2f_1$ with $\bar{a}=\frac{1}{8}a>0$ for spinor and vector fields as well as for the scalar case for both the conformal and minimal coupling. If $\Lambda-\Lambda_0<0$, *classical* constant curvature solutions of type (35) do not exist at all. However, if the difference $\Lambda-\Lambda_0$ is very small and such that $\Lambda-\Lambda_0+C\bar{a}Q^{-6}>0$, the solutions (42) do exist. Let us substitute the expression for r_0^2 into Eq. (40) for ρ. Then

$$\frac{1}{\rho^2}=\frac{1}{r_0^2}+2\Lambda+C(f_1+f_2)=2(\Lambda-\Lambda_0)+C(f_1+f_2)$$
$$\pm\frac{1}{Q}\sqrt{\Lambda-\Lambda_0+Cf_1}. \tag{43}$$

Consider two cases.

(1) $\Lambda=\Lambda_0$. Then we obtain two solutions. The first one is

$$\rho^2=\frac{Q^4}{\sqrt{C\bar{a}}}, \tag{44}$$

where the term $-\bar{a}Q^{-6}$ has been dropped. Thus, quantum corrections force the geometry to switch from Eq. (32) to Eqs. (27) or (29). The second solution is formally complex. In fact, this means that instead of Eq. (27) we have the geometry (30). Now $\sigma^2=Q^4/\sqrt{C\bar{a}}$.

(2) $\Lambda=\Lambda_0-Cf_1=\Lambda_0-C\bar{a}Q^{-6}$. Then we have the metric (30) with parameters

$$r_0^2=2Q^2, \quad \sigma^2=\frac{1}{C(f_1-f_2)}=\frac{Q^6}{3C\bar{a}}, \tag{45}$$

where we took into account the expression for the metric $\text{Rindler}_2\times S_2$.

V. CONCLUDING REMARKS

We have considered EBH's in equilibrium with quantized massive fields and demonstrated that for *any* EBH the components of the SET, measured by a free-falling observer, remain finite. If a metric obeys the Einstein equations, this entails that semiclassical EBH's do exist as their self-consistent solutions. The key point of our treatment consisted in restricting our analysis to the near-horizon geometry, which enabled us to avoid the complexity connected with the obvious impossibility of finding explicit self-consistent solutions in the whole domain.

We considered also BR-like spacetime, closely connected to the issue of EBH's, and showed that quantum-corrected BR spacetime remains as an exact solution of the one-loop field equations. In so doing, the relationship between the parameters of the solutions can be such that classically they completely are absent and only quantum effects make their existence (for fixed values of these parameters) possible. Apart from this, near some critical points in the space of solutions tiny quantum corrections can lead to a change of the type of BR spacetime, the scale of curvature remaining purely classical. Thus, quantum corrections not only slightly shift the values of relevant physical quantities but also lead to qualitative changes in the geometry and topology.

The questions about the near-horizon behavior of the SET and the self-consistent EBH for massless fields as well as the properties of self-consistent BR spacetimes deserve separate treatment.

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APPENDIX A: POWER EXPANSION FOR $T_\mu^{\nu(q)}$ OF MASSIVE FIELDS NEAR THE HORIZON OF A GENERIC EBH

In this Appendix we collect a number of formulas for the components of the SET of the massive scalar, spinor, and vector fields in the vicinity of the event horizon of a generic eternal black hole, which are used in this paper. Functionally differentiating W_i with respect to the metric tensor, performing the necessary symmetrizations and simplifications, and inserting the results obtained into Eq. (3) one obtains the general form of the renormalized SET of the quantized massive fields. As the resulting formulas are rather complicated, we shall not display them here, and the reader is referred to [4] and [5] for further details. Subsequently, constructing the components of the Riemann tensor, its contractions and necessary covariant derivatives for the line element (18), inserting the results obtained into the general expressions for the SET [5], combining them with the appropriate spin-dependent numerical coefficients $c_i^{(s)}$, and finally making use of the explicit form of $U(l)$ and $r(l)$ as given by Eqs. (19) and (17), and collecting the terms with like powers of $z=e^{-l/\rho}$, one has

$$T_\mu^{\nu(q)}=\sum_{i=0} t_\nu^{\nu(i)}z^i=\frac{1}{96\pi^2m^2}\sum_{i=0}\tilde{t}_\nu^{\nu(i)}z^i. \tag{A1}$$

Although the near-horizon power expansions of the line element (18) look rather simple, the complexity of the SET rapidly increases with increasing order of expansion, practically invalidating calculations of $t_\nu^{\nu(i)}$ for $i\geqslant 3$. Below the results for $i=0,1$ are listed.

Closer analysis of the coefficients $c_i^{(0)}$ given in Table I indicates that the general SET is a third-order polynomial in $\eta=\xi-1/6$, with coefficients given by purely local, geometrical terms:

$$\begin{aligned}\tilde{t}_0^{0(0)}&=\tilde{t}_1^{1(0)}\\&=\frac{8}{945}\frac{2r_+^6+\rho^6}{\rho^6r_+^6}-\frac{\eta(\rho^6-\rho r_+^4+2r_+^6)}{15\rho^6r_+^6}\\&\quad-4\frac{\eta^3(2r_+^6-3\rho^2r_+^4+\rho^6)}{\rho^6r_+^6},\end{aligned} \tag{A2}$$

$$\begin{aligned}\tilde{t}_0^{0(1)}&=\tilde{t}_1^{1(1)}\\&=-\frac{16}{315}\frac{A_1(r_+^6+\rho^6)}{r_+^7\rho^6}\\&\quad+\frac{2}{15}\frac{\eta A_1(3\rho^6-\rho^4r_+^2-\rho^2r_+^4+3r_+^6)}{\rho^6r_+^7}\\&\quad+24\frac{\eta^3A_1(\rho^6+r_+^6-\rho^2r_+^4-r_+^2\rho^4)}{r_+^7\rho^6},\end{aligned} \tag{A3}$$

$$\tilde{t}_2^{2(0)}=\tilde{t}_3^{3(0)} =\frac{4\eta^3(r_+^6+2\rho^6-3r_+^2\rho^4)}{r_+^6\rho^6}+\frac{\eta(\rho^6-\rho^4r_+^2+r_+^6)}{15\rho^6r_+^6} -\frac{2}{945}\frac{(8\rho^6+4r_+^6)}{r_+^6\rho^6}, \tag{A4}$$

$$\tilde{t}_2^{2(1)}=\tilde{t}_3^{3(1)} =\frac{2A_1}{315}\frac{16\rho^6+17r_+^6+7r_+^2\rho^4}{r_+^7\rho^6} -\frac{\eta}{30f_0\rho^6r_+^7}(24A_1f_0\rho^6+20A_1f_0\rho^4r_+^2+3f_1\rho^4r_+^3 +40A_1f_0\rho^2r_+^4+100A_1f_0r_+^6+15f_1r_+^7) +\frac{2\eta^2}{f_0r_+^5\rho^6}(4A_1f_0\rho^4+3f_1r_+^3\rho^2+16A_1r_+^4f_0 +16A_1r_+^2f_0\rho^2+6f_1r_+^5)-\frac{6\eta^3}{f_0r_+^7\rho^6}(20A_1r_+^2f_0\rho^4 +8\rho^6A_1f_0+8A_1r_+^4f_0\rho^2-36A_1r_+^6f_0+3f_1r_+^3\rho^4 +12f_1r_+^5\rho^2-15f_1r_+^7), \tag{A5}$$

Repeating the calculations with the coefficients $c_i^{(1/2)}$ one obtains

$$\tilde{t}_0^{0(0)}=\tilde{t}_1^{1(0)}=\frac{1}{420}\frac{7r_+^4\rho^2+10\rho^6+20r_+^6}{r_+^6\rho^6}, \tag{A6}$$

$$\tilde{t}_0^{0(1)}=\tilde{t}_1^{1(1)}=-\frac{1}{210}\frac{A_1(30\rho^6+7r_+^2\rho^4+7r_+^4\rho^2+30r_+^6)}{r_+^7\rho^6}, \tag{A7}$$

$$\tilde{t}_2^{2(0)}=\tilde{t}_3^{3(0)}=-\frac{1}{420}\frac{20\rho^6+7r_+^2\rho^4+10r_+^6}{r_+^6\rho^6}, \tag{A8}$$

$$\tilde{t}_2^{2(1)}=\tilde{t}_3^{3(1)}=-\frac{1}{840}\frac{21f_1r_+^3\rho^4-136A_1r_+^6f_0-240\rho^6A_1f_0-140A_1r_+^2f_0\rho^4}{f_0r_+^7\rho^6}. \tag{A9}$$

Finally, for the massive vector fields

$$\tilde{t}_0^{0(0)}=\tilde{t}_1^{1(0)}=\frac{1}{35}\frac{8r_+^6+7\rho^2r_+^4+4\rho^6}{\rho^6r_+^6}, \tag{A10}$$

$$\tilde{t}_0^{0(1)}=\tilde{t}_1^{1(1)}=-\frac{2}{35}\frac{A_1(12r_+^6+7\rho^2r_+^4+12\rho^6+7r_+^2\rho^4)}{r_+^7\rho^6}, \tag{A11}$$

$$\tilde{t}_2^{2(0)}=\tilde{t}_3^{3(0)}=-\frac{1}{35}\frac{7r_+^2\rho^4+8\rho^6+4r_+^6}{\rho^6r_+^6}, \tag{A12}$$

$$\tilde{t}_2^{2(1)}=\tilde{t}_3^{3(1)}=-\frac{1}{210f_0r_+^7\rho^6}(-280A_1r_+^4f_0\rho^2+63f_1r_+^3\rho^4 -420A_1r_+^2f_0\rho^4-124A_1r_+^6f_0-288\rho^6A_1f_0 +105f_1r_+^5\rho^2). \tag{A13}$$

The third-order coefficients of the expansion (A1), $\tilde{t}_\mu^{\nu(2)}$, are too lengthty to be presented here. On the other hand, however, of principal importance in the analyses of the regularity of F is the difference between the (00) and (11) components of the stress-energy tensor rather than the components themselves. The calculations give

$$T_0^{0(q)}-T_1^{1(q)}=\frac{1}{96\pi^2m^2}\beta z^2+\mathcal{O}(z^3), \tag{A14}$$

where

$$\beta^{(0)} = -\frac{1}{2520 f_0^2 \rho^6 r_+^6}(112 A_1^2 f_0^2 \rho^4 + 464 A_1^2 f_0^2 r_+^4 - 704 A_2 f_0^2 r_+^5 + 200 A_1 f_0 f_1 r_+^5 + 209 f_1^2 r_+^5 + 64 f_0 f_2 r_+^6)$$
$$+\frac{\eta}{30 f_0^2 \rho^6 r_+^6}(40 A_1^2 f_0^2 \rho^4 - 8 A_2 f_0^2 \rho^4 r_+ + 2 A_1 f_0 f_1 \rho^4 r_+ + 80 A_1^2 f_0^2 \rho^2 r_+^2 + 168 A_1^2 f_0^2 r_+^4 + 17 f_1^2 \rho^2 r_+^4 - 32 f_0 f_2 \rho^2 r_+^4$$
$$- 552 A_2 f_0^2 r_+^5 + 42 A_1 f_0 f_1 r_+^5 + 69 f_1^2 r_+^6 - 96 f_0 f_2 r_+^6) - \frac{\eta^2}{2 f_0^2 \rho^6 r_+^6}(16 A_1^2 f_0^2 \rho^4 + 176 A_1^2 f_0^2 \rho^2 r_+^2 - 96 A_2 f_0^2 \rho^2 r_+^3$$
$$+ 24 A_1 f_0 f_1 \rho^2 r_+^3 + 144 A_1^2 f_0^2 r_+^4 - 576 A_2 f_0^2 r_+^5 - 24 A_1 f_0 f_1 r_+^5 + 93 f_1^2 r_+^6 - 192 f_0 f_2 r_+^6) + \frac{6\eta^3}{f_0^2 \rho^6 r_+^6}(40 A_1^2 f_0^2 \rho^4$$
$$- 8 A_2 f_0^2 \rho^4 r_+ + 2 A_1 f_0 f_1 \rho^4 r_+ + 64 A_1^2 f_0^2 \rho^2 r_+^2 - 96 A_2 f_0^2 \rho^2 r_+^3 + 24 A_1 f_0 f_1 \rho^2 r_+^3 + 40 A_1^2 f_0 r_+^4 + 17 f_1 \rho^2 r_+^4 - 32 f_0 f_2 \rho^2 r_+^4$$
$$+ 104 A_2 f_0^2 r_+^5 + 46 A_1 f_0 f_1 r_+^5 - 8 f_1^2 r_+^6 + 32 f_0 f_2 r_+^6), \tag{A15}$$

$$\beta^{(1/2)} = -\frac{1}{840 f_0^2 \rho^6 r_+^5}(56 A_2 f_0^2 \rho^4 - 14 A_1 f_0 f_1 \rho^4 + 288 A_1^2 f_0^2 r_+^3 - 119 f_1^2 \rho^2 r_+^3 + 224 f_0 f_2 \rho^2 r_+^3 + 96 A_2 f_0^2 r_+^4 - 96 A_1 f_0 f_1 r_+^4$$
$$+ 318 f_1^2 r_+^5 - 192 f_0 f_2 r_+^5), \tag{A16}$$

and

$$\beta^{(1)} = -\frac{1}{420 f_0^2 \rho^6 r_+^5}(336 A_2 f_0^2 \rho^4 - 84 A_1 f_0 f_1 \rho^4$$
$$+ 280 A_1^2 f_0^2 \rho^2 r_+ + 1680 A_2 f_0^2 \rho^2 r_+^2 - 420 A_1 f_0 f_1 \rho^2 r_+^2$$
$$+ 1296 A_1^2 f_0^2 r_+^3 - 714 f_1^2 \rho^2 r_+^3 + 1344 f_0 f_2 \rho^2 r_+^3$$
$$+ 1440 A_2 f_0^2 r_+^4 - 1440 A_1 f_0 f_1 r_+^4 + 1935 f_1^2 r_+^5$$
$$- 2880 f_0 f_2 r_+^5), \tag{A17}$$

for the quantized massive scalar, neutral spinor, and vector fields, respectively.

APPENDIX B: $T_\mu^{\nu(q)}$ OF MASSIVE FIELDS IN THE SPACETIME OF EXTREMAL REISSNER-NORDSTRÖM BLACK HOLE

Although the stress-energy tensor of the massive fields in the spacetime of the extremal Reissner-Nordström black hole may be easily constructed by taking extremality limits in the results of Refs. [4] and [8], below we collect the formulas that have been used in the back reaction calculations. For the quantized massive scalar field with an arbitrary curvature coupling one has

$$T_0^{0(q)} = \frac{M^2\eta}{30240\pi^2 m^2 r^{12}}(34398 M^4 - 113904 r M^3$$
$$+ 139944 M^2 r^2 - 75600 r^3 M + 15120 r^4)$$
$$+ \frac{M^2}{30240\pi^2 m^2 r^{12}}(-1248 M^4 - 45 r^4 + 3084 r M^3$$
$$- 2509 M^2 r^2 + 726 r^3 M), \tag{B1}$$

$$T_1^{1(q)} = -\frac{M^2\eta}{30240\pi^2 m^2 r^{12}}(4914 M^4 - 21168 r M^3$$
$$+ 33432 M^2 r^2 - 23184 r^3 M + 6048 r^4)$$
$$- \frac{M^2}{30240\pi^2 m^2 r^{12}}(-444 M^4 - 477 r^4 + 1932 r M^3$$
$$- 2969 M^2 r^2 + 1950 r^3 M), \tag{B2}$$

and

$$T_2^{2(q)} = \frac{M^2\eta}{30240\pi^2 m^2 r^{12}}(44226 M^4 - 143136 r M^3$$
$$+ 172536 M^2 r^2 - 91728 r^3 M + 18144 r^4)$$
$$- \frac{M^2}{30240\pi^2 m^2 r^{12}}(3066 M^4 - 10356 r M^3$$
$$+ 12953 M^2 r^2 - 7086 r^3 M + 1431 r^4), \tag{B3}$$

whereas for the massive spinor field one obtains

$$T_0^{0(q)} = \frac{M^2}{40320\pi^2 m^2 r^{12}}(4917 M^4 - 21496 r M^3 + 32376 r^2 M^2$$
$$- 20080 M r^3 + 4320 r^4), \tag{B4}$$

$$T_1^{1(q)} = \frac{M^2}{40320\pi^2 m^2 r^{12}}(2253 M^4 - 8680 r M^3 + 12000 r^2 M^2$$
$$- 7120 M r^3 + 1584 r^4), \tag{B5}$$

and

$$T_2^{2(q)} = -\frac{M^2}{40320\pi^2 m^2 r^{12}}(9933M^4 - 23552Mr^3 + 42888r^2M^2 - 33984rM^3 + 4752r^4). \quad \text{(B6)}$$

Finally, for the massive vector field in the extremality limit one has

$$T_0^{0(q)} = \frac{M^2}{10080\pi^2 m^2 r^{12}}(31057M^4 - 107516rM^3 + 135391M^2r^2 - 72690r^3M + 13815r^4), \quad \text{(B7)}$$

$$T_1^{1(q)} = \frac{M^2}{10080\pi^2 m^2 r^{12}}(5365M^4 - 16996rM^3 + 19349M^2r^2 - 9398r^3M + 1737r^4), \quad \text{(B8)}$$

and

$$T_2^{2(q)} = -\frac{M^2}{10080\pi^2 m^2 r^{12}}(13979M^4 + 5211r^4 - 26854r^3M + 51789M^2r^2 - 44068rM^3). \quad \text{(B9)}$$

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