## Early-universe constraints on a time-varying fine structure constant

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Higher-dimensional theories have the remarkable feature of predicting a time (and hence redshift) dependence of the "fundamental" four dimensional constants on cosmological time scales. In this paper, we update the bounds on a possible variation of the fine structure constant  $\alpha$  at the time of big bang nucleosynthesis  $(z \sim 10^{10})$  and cosmic microwave background (CMB)  $(z \sim 10^3)$ . Using the recently released high-resolution CMB anisotropy data and the latest estimates of primordial abundances of <sup>4</sup>He and D, we do not find evidence for a varying  $\alpha$  at more than a one-sigma level at either epoch.

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## I. INTRODUCTION

In the last few years it has been pointed out that the fundamental energy scale of gravity does not need to be the Planck scale, but rather it could be a lower scale maybe not far from the electroweak one [1-3]. In this framework, where the hierarchy problem is definitely solved, the Newton constant turns out to be so small because the gravitational force spreads into some higher-dimensional space which may be compact or have an infinite volume. One remarkable feature of higher-dimensional particle physics theories is that, in this framework, the coupling constants in the fourdimensional subspace are merely effective quantities. Furthermore, from what is known about the dynamics of these extra dimensions, one expects these effective constants to be time and/or space varying quantities on cosmological time scales, and this represents an interesting signature of these models which would be worth testing. The best example of such a quantity is the fine structure constant  $\alpha$ , which is expected to be time-varying in a wide class of theories.

There is quite a large number of experimental constraints on the value of  $\alpha$ . These measurements cover a wide range of time scales (see [4] for a review of this subject), starting from present-day laboratories  $(z \sim 0)$ , geophysical tests  $(z \ll 1)$ , and quasars  $(z \sim 1-3)$ , until cosmic microwave background (CMB)  $(z \sim 10^3)$  and big bang nucleosynthesis (BBN)  $(z \sim 10^{10})$  bounds.

We define  $\Delta \alpha / \alpha \equiv \alpha(z) / \alpha - 1$ , with  $\alpha$  the present value for the fine structure constant. By using atomic clocks one gets a strong limit on the time variation of the fine structure constant,  $|\Delta \alpha / \alpha| \leq 10^{-14}$  over a period of 140 days [5]. The best geophysical constraint comes from measurements of isotope ratios in the Oklo natural reactor, which give  $|\Delta \alpha / \alpha| \leq 10^{-7}$  over a period of 1.8 billion years [6], corresponding to  $z \sim 0.1$ . The fine splitting of quasar doublet absorption lines probes higher redshifts. This is the method of Ref. [4] which finds  $\Delta \alpha / \alpha = [-4.6 \pm 4.3 \text{ (stat)} \pm 1.4 \text{ (sys)}] \times 10^{-5}$  for redshifts  $z \sim 2-4$ , but is subject to uncertainties associated with laboratory wavelength determinations and other systematic effects.

On the other hand the analysis of Ref. [7] gives a  $4\sigma$  evidence for a time variation of  $\alpha$ ,  $\Delta \alpha / \alpha = (-0.72 \pm 0.18) \times 10^{-5}$ , for the redshift range  $z \sim 0.5 - 3.5$ . This positive result was obtained using a many-multiplet method, which is claimed to achieve an order of magnitude greater precision than the alkali doublet one. Some of the initial ambiguities of the method have been tackled by the authors with an improved technique, in which a range of ions is considered, with varying dependence on  $\alpha$ , which helps reduce possible problems such as varying isotope ratios, calibration errors and possible Doppler shifts between different populations of ions [8–11].

A deeper knowledge of the primordial universe at the time of photon decoupling is emerging from recent results on cosmic microwave background (CMB) temperature anisotropies [12–16], and on this ground, a new bound on  $|\Delta \alpha/\alpha|$  at  $z \sim 10^3$  has been obtained [17,18] by using the first release of data by BOOMERANG and MAXIMA [12,14]. As observed by many authors, the analysis of cosmological implications of this first bunch of data seemed to prefer values for the baryon fraction sensibly larger than the big bang nucleosynthesis (BBN) requirement [19–21]. This preliminary analysis thus suggested that likely *new physics* was active in the early universe, and in this direction many different mechanisms have been proposed in the literature [19–30].

In this paper we update the constraints on  $|\Delta \alpha / \alpha|$  from CMB by using the latest available observational data [13,16] which now single out a value for the baryon fraction in perfect agreement with the BBN one [31].

At higher redshift,  $z \sim 10^{10}$ , BBN can provide strong bounds on a possible deviation of the value of fine structure constant from the present-day one. In this paper we obtain new constraints on  $|\Delta \alpha / \alpha|$  at the time of BBN, based on the new data on primordial chemical abundances and on a new and more precise BBN code recently developed [32,19]. This represents an update of the analysis of Ref. [33].

### **II. BBN DATA ANALYSIS**

The high level of predictivity of *standard* nucleosynthesis (SBBN), which yields abundances for D, <sup>3</sup>He, <sup>4</sup>He, and <sup>7</sup>Li as a function of the baryon fraction  $\Omega_b h^2$  only,<sup>1</sup> makes the comparison of the theoretical BBN predictions with experimental data a crucial test for Hot Big Bang models.

The measurements of the deuterium Ly- $\alpha$  features in several quasar absorption systems at high redshift (z>2) give a relative deuterium abundance D/H= $(3.0\pm0.4)\times10^{-5}$  [34]. Deuterium has a relevant role in BBN since it mainly fixes the baryon fraction. For the <sup>4</sup>He mass fraction,  $Y_P$ , the key results come from the study of H II regions in Blue Compact Galaxies. This has been performed in two different analyses, which give rather different values for  $Y_P$ . The value found in Ref. [35] is  $Y_P=0.234\pm0.002$  and it is quite smaller than  $Y_P=0.244\pm0.002$  obtained in Ref. [36]. However, even though both analyses use large samples of objects, the analysis of Ref. [36] is based on a more homogeneous set of measurements. Moreover, some of the most metal-poor objects used in [35] seem to suffer from stellar absorption. For these reasons we use the value  $Y_P=0.244\pm0.002$ .

Inferring the <sup>7</sup>Li primordial abundance is a rather difficult task, since it is strongly affected by stellar processes. The value reported in Ref. [37], <sup>7</sup>Li/H= $1.23^{+0.68}_{-0.32} \times 10^{-10}$ , is based on the measurement of <sup>7</sup>Li in the halos of old stars. It should represent well its primordial abundance since in Ref. [37] it is also taken into account production and depletion mechanisms due to cosmic rays and stellar dynamics, respectively. This result, which is compatible with other similar analyses [38-40], is unfortunately smaller by a factor 2 to 3 than what is typically predicted by BBN. Moreover, stellar models where strong <sup>7</sup>Li depletion mechanisms are present have been extensively discussed in the literature and are also supported by the observation of old stars where no <sup>7</sup>Li at all is present in the halo. For this reason, at the moment the <sup>7</sup>Li primordial abundance cannot be safely used in a BBN analysis in order to impose bounds on the baryon fraction or other parameters.

The predictions of SBBN [19,41,42], for three standard light neutrinos,  $N_{\nu}=3$ , once compared with the above experimental observations through a likelihood analysis, yield a baryon fraction which is in the range  $\Omega_b h^2 = 0.019^{+0.004}_{-0.004}$  (at

95% C.L.), in fair agreement with the value  $\Omega_b h^2 = 0.020 \pm 0.002$  (at 95% C.L.) of Ref. [43].<sup>2</sup> SBBN for the central value  $\Omega_b h^2 = 0.019$  gives  $D/H = 3.26 \times 10^{-5}$ ,  $Y_P = 0.2467$  and  $^7\text{Li}/\text{H} = 3.31 \times 10^{-10}$ , which corresponds to  $\chi^2 = 2.1$ .

A variation of the value of the fine structure function  $\alpha$ does not have negligible effects on SBBN abundance predictions. This issue has been already investigated in the literature [33] in order to fix bounds on  $\Delta \alpha / \alpha$  at  $z \sim 10^{10}$ . The effect on BBN of a varying  $\alpha$  is essentially twofold, affecting both the neutron-proton mass difference and the Coulomb barrier in nuclear reactions. The mass difference between neutron and proton,  $\Delta m$ , since it fixes the neutron to proton ratio at decoupling, provides the initial condition at the onset of BBN. The  $\alpha$  dependence of  $\Delta m$  can be derived phenomenologically, as done in Ref. [44],  $\Delta m \approx 2.05$  $-0.76(1 + \Delta \alpha / \alpha)$  MeV, whereas the dependence on  $\alpha$  of the most important nuclear reactions involved in BBN has been carefully evaluated and reported in Table I of Ref. [33]. Both effects have been implemented in a high accuracy BBN code [19,32], to produce the light element abundances as functions of  $\Omega_b h^2$  and  $\Delta \alpha / \alpha$ .

It is important to mention that, in general, if we consider models where the electromagnetic coupling is a time dependent parameter, it is reasonable to expect that all fundamental parameters, the Yukawa couplings, the strong coupling constant and Weinberg angle, and the vacuum expectation value (VEV) for the Higgs field, v, may be functions of time as well. In particular, a different value for the Fermi constant,  $G_F \sim v^{-2}$ , would change all rates of weak processes and result in different BBN predictions.<sup>3</sup> Nevertheless we have chosen to keep constant the values for all these parameters. This represents the simplest scenario which may account for Quasar measurements and provides the most restrictive bounds on  $\Delta \alpha / \alpha$ . On the other hand, since, in the general case, the time dependence of fundamental parameters is model dependent, a completely general analysis clearly loses any predictivity.

By using the BBN results on D and <sup>4</sup>He abundances we have performed a likelihood analysis in the plane  $\Omega_b h^2 - \Delta \alpha / \alpha$ . The 68% and 95% C.L. contours are reported in Fig. 1. By marginalizing with respect to  $\Omega_b h^2$  and  $\Delta \alpha / \alpha$  one gets the allowed intervals at 95% C.L.,  $\Omega_b h^2 = 0.020^{+0.005}_{-0.003}$  and  $\Delta \alpha / \alpha = (-7 \pm 9) \times 10^{-3}$ , respectively. This result, which is compatible with the bound found in [33], shows that BBN does not clearly favor a value of  $\Delta \alpha / \alpha \neq 0$  at more than  $1 - \sigma$  level. For the maximum of the likelihood we find D/H= $2.98 \times 10^{-5}$ ,  $Y_P = 0.2440$ , and <sup>7</sup>Li/H= $3.86 \times 10^{-10}$ , with a  $\chi^2 = 1.3 \times 10^{-3}$ . The agreement with experimental observation is much improved, due to the additional free parameter  $\Delta \alpha / \alpha$ .

<sup>&</sup>lt;sup>1</sup>We assume here the standard scenario with only the three active neutrinos, and photons, contributing to the relativistic energy density.

<sup>&</sup>lt;sup>2</sup>Note that our wider range for the baryon fraction is basically due to the larger uncertainty on the deuterium abundance we use in our analysis (see Ref. [34]) with respect to the experimental data used in [43].

<sup>&</sup>lt;sup>3</sup>Notice that  $G_F$  does not depend on the electroweak couplings.



FIG. 1. The dashed and solid lines represent the 68% and 95% C.L. contours, respectively, for the CMB and BBN likelihood analysis. Crosses correspond to the maxima of the likelihood functions.

#### **III. CMB DATA ANALYSIS**

We have performed a similar likelihood analysis for the recently released BOOMERANG [13] and DASI [16] data, as well as the COBE data, with the additional free parameter  $\Delta \alpha / \alpha$ . Our analysis method follows the procedure described in [45] taking into account the effects of the beam and calibration uncertainties for the BOOMERANG data. For the DASI data we consider the publicly available correlation matrices and window functions.

The calibration uncertainty is taken into account by adding a Gaussian term  $\chi^2_{cal} = (1 - A_{cal})^2 / \sigma^2_{cal}$  to the computed  $\chi^2$  for each theoretical spectrum.  $A_{cal}$  is a calibration parameter and  $\sigma_{cal} = 0.23$ , 0.08 for BOOMERANG and DASI respectively.

The calculation of the angular power spectrum  $C_l$  follows [17,46–49] and was obtained using a modified CMBFAST algorithm which allows a varying  $\alpha$  parameter. The space of model parameters spans  $0.1 \leq \Omega_m \leq 1.0$ ,  $0.009 \leq \Omega_b h^2 \leq 0.036$ ,  $0.4 \leq h \leq 0.9$ ,  $-0.2 \leq \Delta \alpha / \alpha \leq 0.1$ ,  $0.7 \leq n_s \leq 1.3$ . The basic grid of models was obtained considering parameter step sizes of 0.1 for  $\Omega_m$ ; 0.003 for  $\Omega_b h^2$ ; 0.05 for *h*; 0.01 for  $\Delta \alpha / \alpha$  and finally 0.05 for the tilt  $n_s$ . When necessary the grid resolution is increased by using interpolation procedures. All our models have  $\Omega_{total} = 1$ . We assume the presence of a classical cosmological constant when necessary to achieve such a result. We also assume an age of the universe prior to  $t_0 > 10$  Gyr.

Performing the marginalization over one of the two parameters  $\Delta \alpha / \alpha$  and  $\Omega_b h^2$  gives respectively  $\Omega_b h^2 = 0.020^{+0.004}_{-0.004}$  and  $\Delta \alpha / \alpha = -0.05^{+0.07}_{-0.04}$  at 68% C.L. The CMB results on  $\alpha$  can be further constrained by the inclusion of external priors on cosmological parameters. Assuming  $h = 0.72 \pm 0.08$  [50], for example, yields  $\Delta \alpha / \alpha = -0.02^{+0.03}_{-0.04}$ , while assuming  $\Omega_b h^2 = 0.019^{+0.004}_{-0.002}$  gives  $\Delta \alpha / \alpha = -0.03 \pm 0.05$ . To compare CMB results with BBN constraints on

 $\Delta \alpha / \alpha$  we use this latter BBN prior result. This is perfectly justified in view of the excellent agreement between the two determinations of  $\Omega_b h^2$  from CMB and BBN. The 68% and 95 % C.L. regions in the plane  $\Omega_b h^2 - \Delta \alpha / \alpha$  are shown in Fig. 1. It is worth pointing out that the upper limit found for  $\Delta \alpha / \alpha$  would be weakened by the inclusion in the CMB analysis of re-ionization.

#### **IV. CONCLUSIONS**

The analysis of the  $\alpha$ -dependence of two relevant cosmological observables like the anisotropy of CMB and the light element primordial abundances does not support evidence for variations of the fine structure constant at more than the one-sigma level at either epoch. This is the first time a joint analysis for the two epochs has been done, and as such it is quite a robust result.

A few comments are nevertheless in order. The most noticeable aspect of our results is the apparent disagreement with earlier work of some of the present authors [17]. However, the discrepancy is trivially explained by the use of different CMB datasets. Indeed, the earlier release of BOOMER-ANG and MAXIMA [12,14] data which were used by [17] singled out a quite large value for the baryonic fraction with respect to the BBN prediction. Thus, in that scenario a smaller value of  $\alpha$  with respect to the present-day one, was a possible way to lower the value of  $\Omega_b h^2$ , making it compatible with BBN.

In the new release of data from BOOMERANG and DASI, this baryon discrepancy has been eliminated, at the price of a sligtly lower spectral index ( $n_s \sim 0.9$  as opposed to the previously preferred  $n_s \sim 1.00$ ). In this context, and given the intrinsic degeneracies in the problem, a slightly negative  $\alpha$ while still able to marginally improve the fits is no longer a significant advantage. As was emphasized in [17], more data and an independent knowledge of other cosmological parameters will be needed in order to obtain a more precise "measurement" of  $\alpha$  from the CMB. We point out, however, that a recent re-analysis of the old MAXIMA-I dataset with an increased pixel resolution produced results still in better agreement with a high baryon fraction [15].

Hence, from the observational viewpoint, the only current strong evidence for a varying  $\alpha$  seems to be the four-sigma detection using quasar data at redshifts  $z \sim 1-3$ . It should be said that even imposing fairly strong constraints at redshift  $z \sim 10^{10}$  and  $z \sim 10^{3}$ , our results cannot strictly be extrapolated for the whole cosmological period in between these epochs. Indeed, two-metric models exist where  $\alpha$  and other constants suffer "temporary" variations for fairly limited time periods, the case in point being the epoch of equal matter and radiation densities.

What our results, together with the quasar data, do strongly rule out is any cosmological model where  $\alpha$  behaves as a simple and smooth power law function of say the scale factor or cosmic time. If there were indeed any variations of  $\alpha$  in the past, then they are likely to have been fairly "sharp," most likely as a side-effect of phase transitions or other dramatic events in the history of the universe.

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