Skyrme model and nonleptonic hyperon decays

G. Duplančić,[1] H. Pašagić,[2] M. Praszłowicz,[3] and J. Trampetić[1]
[1]Theoretical Physics Division, R. Bošković Institute, P.O. Box 180, 10002 Zagreb, Croatia
[2]Faculty of Transport and Traffic Engineering, University of Zagreb, P.O. Box 195, 10000 Zagreb, Croatia
[3]M. Smoluchowski Institute of Physics, Jagellonian University, Reymonta 4, 30-059 Kraków, Poland
(Received 24 April 2001; published 27 September 2001)

This Brief Report is an attempt to explain both *s*- and *p*-wave nonleptonic hyperon decays by means of the QCD enhanced effective weak Hamiltonian supplemented by the SU(3) Skyrme model used to estimate nonperturbative matrix elements. The model has only one free parameter, namely, the Skyrme charge *e*, which is fixed through the experimental values of the octet-decuplet mass splitting Δ and the axial vector coupling constant g_A. Such a dynamical approach produces nonleptonic hyperon decay amplitudes that agree with experimental data reasonably well.

DOI: 10.1103/PhysRevD.64.097502 PACS number(s): 12.39.Dc, 12.39.Fe, 13.30.Eg

In the Skyrme model, baryons emerge as soliton configurations of pseudoscalar mesons [1–5]. The extension of the model to the strange sector, in order to account for a large strange quark mass, requires that appropriate chiral symmetry breaking terms are included. The resulting effective Lagrangian can be treated by starting from a flavor symmetric formulation in which the kaon fields arise from rigid rotations of the classical pion field [3,6,7]. The associated collective coordinates are canonically quantized to generate states that possess quantum numbers of the physical strange baryons [3,5,6]. It turns out that the resulting collective Hamiltonian can be diagonalized exactly even in the presence of the flavor symmetry breaking (SB) [4]. This approach leads to a good description of hyperon masses, charge radii, magnetic moments, etc. [5]. It should be noted that in the first phenomenological applications of the Skyrme model one attempted to fit absolute baryon masses, which required a ridiculously small pion decay constant [2,7]. Nowadays it is understood that there exist $1/N_c$ corrections to the total baryon masses that are not fully under control and therefore only mass *splittings* can be reliably reproduced. In this approach f_π is kept at its experimental value. Hence the results for nonleptonic hyperon decays (NHD) [8] need to be updated accordingly.

Both *s*- and *p*-wave NHD amplitudes were quite successfully predicted by using quark models with QCD enhancement factors [9–11]. Note that there are not only current-algebra and ground-state exchange pole-diagram terms, but there exist other important contributions to both *s* and *p* waves. The so-called *factorizable* contributions and/or kaon poles were estimated in [9,10]. Pole-diagram contributions to *p* waves from the $(1/2^+)$-Rooper type of resonances and to *s*-waves through the $(1/2^-)$-resonance exchange were calculated in [12].

This Brief Report is an attempt to test whether the effective weak Hamiltonian and the extended SU(3) Skyrme model are able to predict both *s*- and *p*-wave NHD amplitudes. *The minimal number of couplings* Skyrme model is used to estimate only the nonperturbative matrix elements of the 4-quark operators [8]. All remaining quantities entering the expressions for the decay amplitudes such as mass differences, coupling constants, etc., are taken from experiment. This approach uses only one free parameter, i.e., the Skyrme charge *e*. In order to avoid the unnecessary numerical burden, throughout this Brief Report we use the arctan ansatz for the Skyrme profile function [13].

The starting point of our analysis of NHD in the framework of the standard model [9] is the effective weak Hamiltonian in the form of the current $\otimes$ current interaction, enhanced by QCD. It is obtained by integrating out heavy-quark and *W*-boson fields. This Hamiltonian contains the 4-quark operators O_i and the well-known Wilson coefficients [9,10]. For the most recent values, see Ref. [11]. For the purpose of this paper, we use the Wilson coefficients from Ref. [10]: $c_1=-1.90-0.61\zeta$, $c_2=0.14+0.020\zeta$, $c_3=c_4/5$, $c_4=0.49+0.005\zeta$, with $\zeta=V_{td}^*V_{ts}/V_{ud}^*V_{us}$. Without QCD short-distance corrections, the Wilson coefficients would be $c_1=-1$, $c_2=1/5$, $c_3=2/15$, and $c_4=2/3$. In this paper we simply consider both possibilities and compare the results.

Note that there exists a different approach of Refs. [14,15] in which meson-baryon couplings are directly obtained from the chiral Lagrangian. There, the effective phenomenological constants extracted from experiment take into account all QCD effects hidden in the structure of the effective Hamiltonian (including the *enhancement factors* embodied in the values of the c_i constants). This approach gives comparable results for the *s*-waves but fails for the *p*-waves [15].

The techniques used to describe NHD $(1/2^+\to1/2^++0^-$ reactions) are known as a modified current-algebra (CA) approach. The general form is

$$\langle\pi(q)B'(p')|H_w^{eff}|B(p)\rangle=\overline{u}(p')[\mathcal{A}(q)+\gamma_5\mathcal{B}(q)]u(p)$$

$$=\frac{-i}{2f_\pi}\langle B'(p')|\hat{H}_w|B(p)\rangle|_{q=0}+\mathcal{P}(q)+\mathcal{S}(q). \qquad (1)$$

Here the first term is the CA contribution, the second is the modified pole term, and the third is a term that vanishes in the soft-meson limit. The $\mathcal{P}(q)$ term contains the contribution from the surface term, the soft-meson Born-term contraction, and the baryon pole term, which are combined in a well-known way [9,10]. It represents a continuation of the CA result from the soft-meson limit. Further continuation is contained in the factorizable term $\mathcal{S}(q)$, which is proportional to the meson four-momenta.

The parity-violating amplitudes $\mathcal{A}$ receive contributions A^c from CA commutator terms, factorizable terms $\mathcal{S}(q)$, and pole terms from the $(1/2^-)$-resonance exchange. The main contributions to the $\mathcal{B}$ amplitudes come from the baryon pole terms $\mathcal{P}(q)$, including both the ground state and the radially excited states.

The current-algebra A^c and baryon-pole $B^{\mathcal{P}}$ amplitudes are well known from the literature. They contain weak matrix elements defined as $a_{BB'}=\langle B'|H_w^{PC}|B\rangle$, which have the following general structure:

$$a_{BB'}=\sqrt{2}G_F V_{ud}^* V_{us}\langle B'|c_i O_i^{PC}|B\rangle. \tag{2}$$

The factorizable term $\mathcal{S}(q)$ is calculated by inserting vacuum states. It is therefore a factorized product of two current matrix elements, where the first two-quark current is sandwiched between baryon states, while the second two-quark current is responsible for pion emission.

The CA and the baryon-pole terms contain the 4-quark operator matrix elements, which are nonperturbative quantities. This is exactly the point at which the Skyrme model can be used. Each of the operators O_i from Eq. (2) contains four types of operators, namely, $\overline{d}u\overline{u}s$, $\overline{d}s\overline{u}u$, $\overline{d}s\overline{d}d$, $\overline{d}s\overline{s}s$, and takes the form of the product of two Noether SU(3) currents, which can be found in Refs. [8,16]. In our calculations we use four operators $\hat{O}_i$. The first of them is

$$\hat{O}_1=\frac{1}{4}\overline{q}_L\gamma_\mu(\lambda_1-i\lambda_2)q_L\overline{q}_L\gamma^\mu(\lambda_4+i\lambda_5)q_L, \tag{3}$$

where the SU(3) properties are expressed explicitly in terms of the Gell-Mann λ-matrices. The connection with the effective Hamiltonian operators O_i is obvious.

In order to estimate the matrix elements entering Eq. (2), we take the SU(3) extended Skyrme Lagrangian [5,16]:

$$\mathcal{L}=\mathcal{L}_{SK}^{(1)}+\mathcal{L}_{SK}^{(2)}+\mathcal{L}_{SB}+\mathcal{L}_{WZ},$$
$$\mathcal{L}_{SK}^{(1)}=\frac{f_\pi^2}{4}\int d^4x\,\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger),\quad \text{etc.} \tag{4}$$

where $\mathcal{L}_{SK}^{(1,2)}$, $\mathcal{L}_{SB}$, and $\mathcal{L}_{WZ}$ denote the σ-model, Skyrme, symmetry breaking (SB), and Wess-Zumino (WZ) terms, respectively. For $U(x)\in\mathrm{SU}(2)$, the SB and WZ terms vanish. The $f_\pi=93$ MeV is the pion decay constant. Here the space-time-dependent matrix field $U(\vec{r},t)\in\mathrm{SU}(3)$ takes the form

$$U(\vec{r},t)=A(t)\mathcal{U}(\vec{r})A^\dagger(t), \tag{5}$$

where $\mathcal{U}(\vec{r})$ is the SU(3) matrix in which the Skyrme SU(2) ansatz is embedded:

$$\mathcal{U}(\vec{r})=\begin{pmatrix}\exp(i\vec{\tau}\cdot\vec{n}F(r)) & 0\\ 0 & 1\end{pmatrix}. \tag{6}$$

The time-dependent collective coordinate matrix $A(t)\in\mathrm{SU}(3)$ defines the generalized velocities $A^\dagger(t)\dot{A}(t)=i/2\Sigma_{\alpha=1}^8\lambda_\alpha\dot{a}^\alpha$ and the profile function $F(r)$ is interpreted as a chiral angle that parametrizes the soliton.

In this work we use the arctan ansatz for $F(r)$ [13]:

$$F(r)=2\arctan[(r_0/r)^2]. \tag{7}$$

Here r_0—the soliton size—is the variational parameter and the second power of r_0/r is determined by the long-distance behavior of the massless equations of motion. After rescaling $x=ref_\pi$, one obtains $r_0/r=x_0/x$. The quantity x_0 has the meaning of a *dimensionless* size of a soliton and it is determined by minimizing the classical mass E_{cl}. All relevant integrals involving the profile function turn into an integral representation of the Euler beta functions, which can be evaluated *analytically*. The accuracy of this method with respect to the numerical calculations is of the order of a few percent. In the chiral limit of the SU(2) Skyrme model, we obtain $x_0=\sqrt{15}/4$ and the arctan ansatz reproduces nucleon static properties well [7,17]. Moreover we gain an insight into how different quantities depend on the soliton size, which in turn is a function of the symmetry breaker and e.

In the SU(3) extended Lagrangian (4) we have a new set of parameters, namely, $\hat{x}=36.4$, $\beta'=-2.98\times10^{-5}$ GeV^2, $\delta'=4.16\times10^{-5}$ GeV^4, determined from the masses and decay constants of the pseudoscalar mesons [5]. Owing to the presence of the β' and δ' terms in E_{cl}, x_0 becomes a function of e, f_π, β', and δ', and it is equal to

$$x'^2_0=\frac{15}{8}\left[1+\frac{6\beta'}{f_\pi^2}+\sqrt{\left(1+\frac{6\beta'}{f_\pi^2}\right)^2+\frac{30\delta'}{e^2f_\pi^4}}\right]^{-1}, \tag{8}$$

where we use the symbol x_0' to distinguish it from the SU(2) case. After introducing the SB terms into the Lagrangian (4), one can either treat them as a perturbation [7] or one can try to sum up the perturbation series by numerically diagonalizing the resulting Hamiltonian [4].

The fitting procedure employed in this work is based on taking the physical values for f_π and f_K which takes care of the SU(3) symmetry breaking. In fact the SB affects the calculations in three different ways: (a) through the soliton size x_0'; (b) via the explicit SB in the currents; (c) through the admixture of the higher SU(3) representations in the baryon wave functions. It was shown in Ref. [16] that the latter contributions to NHD are small, since the higher representations enter with small weights. Our estimate shows that they are of the order of 15%. This uncertainty is of the order of the accuracy of the model which is reflected in the variation of the Skyrme parameter e depending on which static property is used in the fitting procedure. In the remainder of this paper we use the SU(3) symmetric wave functions.

For the evaluation of NHD, the important baryon static properties are the octet-decuplet mass splitting Δ and the axial decay coupling constant g_A. The value of the only free parameter $e\approx4$ was successfully adjusted to the mass difference Δ of the low-lying $1/2^+$ and $3/2^+$ baryons [5]. However, if we fix Δ, the constant g_A is underestimated. This is a well-known problem of the Skyrme model, which can be cured in the more sophisticated chiral models involving quarks [18].

Therefore, we determine two values of the charge e through fixing Δ and g_A to their experimental values. The arctan ansatz gives

$$\Delta=\frac{3}{2\lambda_c(x_0')}, \tag{9}$$

$$g_A=\frac{14\pi}{15e^2}(2x_0'^2+\pi)+(1-\hat{x})\frac{16\pi\beta'}{225e^2f_\pi^2}x_0'^2 + \frac{7\sqrt{2}N_c}{192ef_\pi}\frac{x_0'}{\lambda_s(x_0')}, \tag{10}$$

where

$$\lambda_c(x_0')=\frac{\sqrt{2}\pi^2}{3e^3f_\pi}\left[6\left(1+2\frac{\beta'}{f_\pi^2}\right)x_0'^3+\frac{25}{4}x_0'\right], \tag{11}$$

$$\lambda_s(x_0')=\frac{\sqrt{2}\pi^2}{4e^3f_\pi}\left[4\left(1-2(1+2\hat{x})\frac{\beta'}{f_\pi^2}\right)x_0'^3+\frac{9}{4}x_0'\right]. \tag{12}$$

The quantity $\lambda_c(x_0')$ represents the rotation moment of inertia in coordinate space, while the $\lambda_s(x_0')$ is the moment of inertia for flavor rotations in the direction of the strange degrees of freedom, except for the eighth direction [5,7]. The static kaon fluctuations were omitted [19] in the derivations of Eqs. (9)–(12).

For the Lagrangian $\mathcal{L}$, we calculate the matrix element of the product of two $(V-A)$ currents between the octet states using of the Clebsch-Gordan decomposition [8]:

$$\langle B_2|\hat{O}^{(SK)}|B_1\rangle=\Phi^{SK}\times\sum_R C_R \tag{13}$$

where Φ^{SK} is a dynamical constant and C_R denotes the pertinent sum of the SU(3) Clebsch-Gordan coefficients in the intermediate representation R. The total matrix element is simply a sum $\langle\hat{O}_i^{(SK)}+\hat{O}_i^{(WZ)}+\hat{O}_i^{(SB)}\rangle$, with $i=1,\ldots,4$. The quantities Φ are given by the overlap integrals of the profile function. Using the arctan anzatz (7), we obtain analytical expressions for the integrals as functions of x_0':

$$\Phi^{SK}=3\sqrt{2}\pi^2\left(2x_0'+\frac{15}{2x_0'}+\frac{847}{64}\frac{1}{x_0'^3}\right)\frac{f_\pi^3}{e},$$

$$\Phi^{WZ}=\frac{231}{512}\frac{\sqrt{2}}{\pi^2}\frac{1}{x_0'^3}(ef_\pi)^3, \tag{14}$$

$$\Phi^{SB}=(1-\hat{x})\beta'\frac{4\pi^2}{\sqrt{2}}\left(x_0'+\frac{45}{8x_0'}\right)\frac{f_\pi}{e}.$$

For the $\hat{O}_1$ operator, $R=8_{a,s}$ or 27; then

TABLE I. The s-wave ($\mathcal{A}$) and p-wave ($\mathcal{B}$) NHD amplitudes. Choices (off, on) correspond to the amplitudes without and with inclusion of short-distance corrections, respectively. For the sake of comparison, we have added the constituent quark-model evaluation of the A^c and $B^{\mathcal{P}}$ amplitudes [9,10].

Amplitude (10^{-7})		(Λ_-^0)	(Ξ_-^-)	(Σ_0^+)	(Σ_+^+)
$A^c(0)$	off	2.02	−2.94	−2.28	0.02
	on	3.84	−5.56	−4.34	0.04
$A^{\mathcal{S}}(m_\pi^2)$	off	0.03	−0.57	−0.49	0
[9]	on	−0.42	0.25	−0.01	0
$\mathcal{A}(m_\pi^2)$	off	2.05	−3.51	−2.77	0.02
(this work)	on	3.42	−5.31	−4.35	0.04
Expt. [22]		3.35	−4.85	−3.27	0.13
$A^c(0)$	off	0.78	−1.86	−1.36	0
CQM [9]	on	1.49	−3.53	−2.59	0
$B^{\mathcal{P}}_{(1/2^+)}(m_\pi^2)$	off	20.1	21.8	13.7	14.8
	on	38.1	41.4	25.9	28.2
$B^{\mathcal{S}}(m_\pi^2)$	off	3.6	−1.5	−0.4	0
[9]	on	6.0	−2.4	0.4	0
$\mathcal{B}(m_\pi^2)$	off	23.7	20.3	13.3	14.8
(this work)	on	43.4	38.2	26.3	28.2
Expt. [22]		22.3	17.4	26.6	42.2
$B^{\mathcal{P}}_{(1/2^+)}(m_\pi^2)$	off	2.9	7.8	7.3	10.4
CQM [9]	on	5.6	14.8	13.9	19.7

$$\langle p\uparrow|\hat{O}_1|\Sigma^+\uparrow\rangle=-\tfrac{1}{4}\left(\tfrac{2}{25}\Big|_8+\tfrac{1}{675}\Big|_{27}\right)\Phi^{SK} -\tfrac{1}{4}\left(\tfrac{2}{25}\Big|_8-\tfrac{1}{75}\Big|_{27}\right)\Phi^{WZ} -\tfrac{1}{4}\left(\tfrac{7}{75}\Big|_8+\tfrac{17}{1050}\Big|_{27}\right)\Phi^{SB}. \tag{15}$$

The 27-piece is very small, which is an important proof of the octet dominance.

By fixing Δ and g_A to their experimental values, we obtain $e=4.228$ and $e=3.385$, respectively. In further calculations of the NHD amplitudes, we use the mean value $e=3.81$ [20] and $x_0'|_{e=3.81}=0.8782$, i.e., 10% less than in the massless case. For $f_\pi=93$ MeV and $e=3.81$, we obtain the following numerical values of the integrals (14) in units of GeV^3:

$$\Phi^{SK}=0.264,\quad \Phi^{WZ}=0.004,\quad \Phi^{SB}=0.005. \tag{16}$$

From Eqs. (15) and (16) we find the following structure for a typical matrix element:

$$\langle p\uparrow|\hat{O}_1|\Sigma^+\uparrow\rangle=(-20.37\Phi^{SK}-16.67\Phi^{WZ}-27.38\Phi^{SB})10^{-3} =(-5.38|_{SK}-0.06|_{WZ}-0.14|_{SB})10^{-3}\ \mathrm{GeV}^3. \tag{17}$$

It is clear that on top of the octet dominance we also find the dominance of the Skyrme Lagrangian currents over the WZ and SB currents in the evaluation of a typical weak matrix element between two hyperon states. For $e\approx 4$, the SB and WZ terms are of comparable size and their coherent contribution to Eq. (17) is below 4%. We see therefore from Eqs. (14) that within this accuracy the result for Eq. (17) scales like $1/e$ (up to 5% due to the small variations of the soliton size which weakly depends on e). The change of e between 3.385 and 4.228 produces 14% variations of the amplitudes $A^c(0)$ and $B^{\mathcal{P}}(m_\pi^2)$ around their mean values given in Table I.

In this work we have added factorizable, $A^{\mathcal{S}}(m_\pi^2)$ and $B^{\mathcal{S}}(m_\pi^2)$, contributions to the Skyrme model amplitudes $A^c(0)$ and $B^{\mathcal{P}}(m_\pi^2)$. The complete results are given in Table I. Comparison of the total amplitudes $\mathcal{A}(m_\pi^2)$ and $\mathcal{B}(m_\pi^2)$ with experiment shows the following:

(a) Short-distance corrections to the effective weak Hamiltonian are without a doubt very important.

(b) Signs and order of magnitudes of all amplitudes are always correctly reproduced.

(c) s waves are in good agreement with experiment.

(d) The Pati-Woo theorem violation [21] and the 27-contaminations are found to be small. It is clear that the nonvanishing $\mathcal{A}(\Sigma_+^+)$ amplitude is still too small, in good accord with small values of the 27-contamination [23], and that additional contributions are needed [12].

(e) p waves are subject to some uncertainties. Namely, in Ref. [14] it was shown that, in the Skyrme model, a contact term appeared and should be added to the results for p-waves. That has been taken care of in Ref. [15] and is not present in our approach. In our opinion, $\mathcal{B}(m_\pi^2)$ amplitudes are not fully described by our formulas; nevertheless, they agree with experiment reasonably well.

(f) Finally, the factorizable contributions are small, and represent the fine tuning to the total amplitudes.

To conclude, we would like to emphasize the fact that the pure Skyrme model Lagrangian $\mathcal{L}$ cannot explain nonleptonic hyperon decays [15,16]. However, the QCD-corrected weak Hamiltonian H_w^{eff}, together with the inclusion of other possible types of contribution to the total amplitudes [K, K^*-poles, and/or factorization; $(1/2^{\pm *})$-poles, etc.] supplemented by the Skyrme model, leads to a correct answer. This includes the explanation of the octet dominance, the $|\Delta I|=1/2$ selection rule, $\mathcal{A}(\Sigma_+^+)\neq 0$, and the p/s-wave puzzle. Nevertheless, this is certainly a matter for another series of studies.

One of us (J.T.) would like to thank J. Wess for many useful discussions and the Ludwig Maximillians Universität, München, Section Physik, where part of this work was done, for its hospitality. This work was supported by the Ministry of Science and Technology of the Republic of Croatia under Contract No. 00980102. M.P. was supported by the Polish KBN Grant PB 2 P03B 019 17.

[1] T. H. R. Skyrme, Proc. R. Soc. London, Ser. A **260**, 127 (1961).

[2] G. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).

[3] E. Guadagnini, Nucl. Phys. **B236**, 35 (1984).

[4] H. Yabu and K. Ando, Nucl. Phys. **B301**, 601 (1988).

[5] H. Weigel, Int. J. Mod. Phys. A **11**, 2419 (1996); J. Schechter and H. Weigel, hep-ph/9907554.

[6] P. O. Mazur, M. A. Nowak, and M. Praszałowicz, Phys. Lett. **147B**, 137 (1984); S. Jain and S. Wadia, Nucl. Phys. **B258**, 713 (1985).

[7] M. Praszałowicz, Phys. Lett. **158B**, 264 (1985); Acta Phys. Pol. B **22**, 525 (1991).

[8] J. Trampetić, Phys. Lett. **144B**, 250 (1984); M. Praszałowicz and J. Trampetić, *ibid.* **161B**, 169 (1985); Fizika (Zagreb) **18**, 391 (1986).

[9] D. Tadić and J. Trampetić, Phys. Rev. D **23**, 144 (1981); J. F. Donoghue, H. Golowich, and B. R. Holstein, Phys. Rep. **131**, 319 (1986).

[10] J.F. Donoghue, H. Golowich, and B. R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, England, 1994).

[11] A. J. Buras, hep-ph/9806471.

[12] M. Milošević, D. Tadić, and J. Trampetić, Nucl. Phys. **B207**, 461 (1982); S. Pakvasa and J. Trampetić, Phys. Lett. **126B**, 122 (1983).

[13] D. I. Diakonov, V. Yu. Petrov, and M. Praszałowicz, Nucl. Phys. **B323**, 53 (1989).

[14] J. F. Donoghue, H. Golowich, and Y.-C. R. Lin, Phys. Rev. D **32**, 1733 (1985); **33**, 2728(E) (1986).

[15] D. G. Dumm, A. J. Garcia, and N. N. Scoccola, Phys. Rev. D **62**, 014001 (2000).

[16] N. N. Scoccola, Phys. Lett. B **428**, 8 (1998).

[17] In the chiral limit of the SU(2) Skyrme model, for $e=5.45$, $f_\pi=65$ MeV [2], arctan ansatz gives $\langle r^2\rangle_{I=0}^{1/2}=0.61$ fm, $\langle r^2\rangle_{M,I=0}^{1/2}=0.83$ fm, $\mu_p=1.88$, $\mu_n=-1.30$, and $g_A=0.71$. These nucleon static properties are in very good agreement with their numerical evaluation presented in Table I of Ref. [2].

[18] M. Wakamatsu and T. Watabe, Phys. Lett. B **312**, 184 (1993); Ch. V. Christov *et al. ibid.*, **325**, 467 (1994); A. Blotz, M. Praszałowicz, and K. Goeke, *ibid.* **317**, 195 (1993).

[19] Using the arctan ansatz without static kaon fluctuations we obtain for $e=4$: $E\equiv E_{cl}(x_0')=1.94$ GeV, $\alpha^2\equiv\lambda_c(x_0')=7.55$ GeV^{-1}, $\beta^2\equiv\lambda_s(x_0')=3.62$ GeV^{-1}, and $\gamma(x_0')=1.62$ GeV. These quantities are in good agreement with the values (2.48) from Ref. [5]. Note, for example, that the value of 3.62 GeV^{-1} for β^2 is very close to the value 3.52 GeV^{-1} mentioned in the discussion after Eq. (2.48) of Ref. [5]. This is another proof of the accuracy of the arctan ansatz.

[20] $\Delta(x_0')|_{e=3.81}=174$ MeV and $g_A(x_0')|_{e=3.81}=1.04$.

[21] J. C. Pati and C. W. Woo, Phys. Rev. D **3**, 2920 (1971).

[22] Particle Data Group, D. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[23] The $\mathcal{L}_{2+4}^{(27)}$ from Ref. [15] with the arctan ansatz as the profile function gives $A_{(\Sigma_+^+)}^{(27)}(x_0')|_{e=4.1}=2.5\times 10^{-9}$.