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Production of the *D*-wave $b\bar{b}$ states

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The first and second families of *D*-wave $b\bar{b}$ quarkonium states are expected to have masses near 10.16 and 10.44 GeV/c^2. The accuracy of these predictions is discussed, and the prospects of methods for producing these states in electron-positron collisions are updated. Direct scans in the e^+e^- center of mass can give rise to the $^3D_1$ states. The $1\,^3D_J$ states have also been searched for in electromagnetic cascades from $\Upsilon(3S)\to\gamma\chi_b'\to\gamma\gamma^3D_J$. The sample of $\Upsilon(3S)$ decays required to definitively observe the $1\,^3D_J$ states is found to be only somewhat greater than the world's present total.

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The bound states of heavy quarks have provided a key testing ground for quantum chromodynamics (QCD). Perturbative QCD (PQCD) describes the short-distance aspects of the interquark force and many of the decays of the states, while nonperturbative aspects are probed by the long-distance interaction and by details of fine-structure and hyperfine splittings [1–3]. Lattice methods [4] have much to say beyond purely perturbative physics, while attempts are made to extract the interquark interaction from nonrelativistic QCD (NRQCD) [5], an approach called "potential NRQCD" (pNRQCD) [6].

The $^3S_1$ states of charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) are easily accessible in both hadronic Drell-Yan processes and electron-positron collisions, since they couple directly to virtual photons. These states then decay electromagnetically to others with sizable branching ratios. In this way both the lowest charmonium state, the $\eta_c=c\bar{c}(^1S_0)$, and many *P*-wave $c\bar{c}$ and $b\bar{b}$ states have been discovered. With knowledge of the masses of the lowest $\chi_c=c\bar{c}(^3P_J)$ and the two lowest $\chi_b=b\bar{b}(^3P_J)$ families, the interquark interaction can be mapped out to an extent which permits the anticipation of other, as yet unseen, levels. Foremost among these levels are the $\eta_b=b\bar{b}(^1S_0)$ states, for which we have recently suggested observation strategies [7], and the *D*-wave $b\bar{b}$ levels. A candidate for the lowest $^3D_1$ $c\bar{c}$ level is $\psi(3770)$, which couples sufficiently strongly to e^+e^- to be useful as a copious source of charmed meson pairs.

In this paper we review some predictions of $b\bar{b}(n^3D_J)$ masses for $n=1$ and $n=2$, where n is the level number. These levels are expected to lie below the $B\bar{B}$ flavor threshold and thus to be quite narrow. A comprehensive treatment of them was presented in Ref. [8]. We compare the predictions of that work with others, estimate the likely errors in predictions of masses, and update the prospects for discovering some of the predicted levels. These questions have taken on renewed interest as a result of plans by the CLEO Collaboration [9] to examine some aspects of Υ spectroscopy, both via significant augmentation of the world's sample of $\Upsilon(3S)$ decays and via direct scans for $^3D_1$ states in the 10.13–10.17 and 10.42–10.46 GeV/c^2 mass range.

We shall show that most potential models predict a narrow range of values for the *D*-wave masses. The question has been raised of whether *any* potential description is valid for quarkonium and whether one must take into account color-octet degrees of freedom in the wave function [10,11]; discovery of the *D*-wave states within the range predicted by phenomenological potential models would not lay such doubts to rest, but would be one further point in favor of a description which, at least for the $b\bar{b}$ system, has been remarkably successful.

Some predictions of spin-weighted masses of the $n^3D_Jb\bar{b}$ levels [8,12–20] are summarized in Fig. 1 [21]. Most potentials give a center-of-gravity of the $1D$ levels within ±10 MeV/c^2 of 10 160 MeV/c^2, except for the more recent treatment of Eichten and Quigg [19], based on the Buchmüller-Tye [13] potential, which gives a value about 33 MeV/c^2 lower. However, their model also underestimates several other $b\bar{b}$ masses in comparison with experimental values [22], as shown in Table I. A quenched lattice calculation [20] gives a $1\,^1D_2$–$1\,^3S_1$ splitting of 761(20) MeV/c^2 for $\beta\equiv6/g^2_{\rm QCD}=6.0$ if the $1P$–$1S$ splitting is used to set the scale. This becomes 720(30) MeV/c^2 if the $2S$-$1S$ splitting is used to set the scale (see [23] in which it is pointed out that this scale is the one that makes the bare quark mass used closest to the b). The result with the $2S$–$1S$ scale is quoted in Fig. 1, resulting in a ^3D_J mass of 10 180 MeV if one assumes a small singlet-triplet splitting in the $1D$ levels. Potential NRQCD (pNRQCD) [6,11] (see also [24–26]) also can predict *D*-wave masses, but no values have been quoted yet. Such predictions (as well as calculations of radiative transition rates) could help distinguish between the mostly phenomenological approaches

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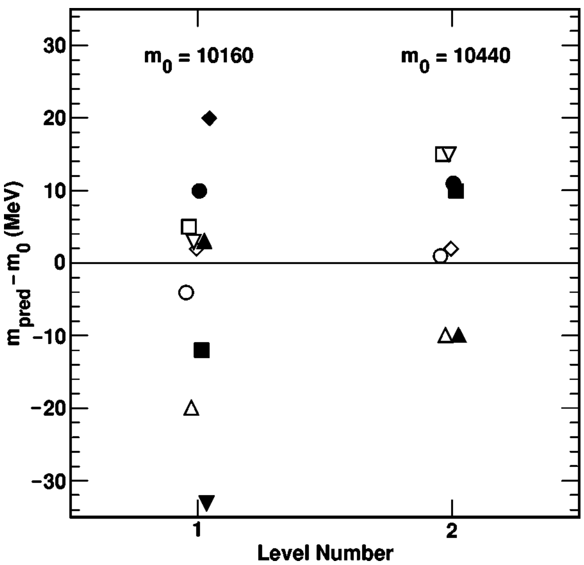

FIG. 1. Predictions for the spin-weighted averages of ^3D_J $b\bar{b}$ states. Open circle (○): KR [8] (inverse scattering); open square (□): Cornell [12] (QCD-based potential); open triangle (△): BT [13] (QCD-based potential; published masses only quoted to nearest 10 MeV); open inverted triangle (▽): Gupta *et al.* [14] (QCD-based potential); open diamond (◇): MR [15] (QCD-based potential); solid circle: MB [16] (relativistic corrections; mass of $^3D_1$ plotted); solid square: GI [17] (QCD-based potential, masses calculated to nearest MeV); solid triangle: Grant *et al.* [18] (power-law potential); solid inverted triangle: EQ [19] (QCD-based potential [13], quoted for $n=1$): solid diamond: lattice [20] (quenched approximation with $\beta=6.0$, quoted for $n=1$).

TABLE I. Deviations of predicted masses for $b\bar{b}$ levels from observed values, in MeV/c^2.

Reference	$\Upsilon(1S)$	$\chi_{bJ}(1P)$	$\Upsilon(2S)$	$\chi_{bJ}(2P)$	$\Upsilon(3S)$	$\Upsilon(4S)$
[8]	a	3	a	−1	a	a
[12]	a	23	a	11	3	50
[13][b]	a	−10	0	−10	0	40
[14]	2	0	−10	−2	0	c
[15]	a	6	a	−2	−5	27
[16]	a	23	a	7	0	40
[17][d]	5	−16	−20	−8	−1	55
[18][e]	−5	−9	19	−2	4	−7
[19]	4	−17	−16	−29	−16	22
[23]	a	−13±17	a	105±40	30±80	c
Expt. [22] [f]	9460	9900	10023	10260	10355	10580

[a]Input.
[b]Published masses quoted to nearest 10 MeV.
[c]Not quoted.
[d]Numbers in this table are based on masses calculated to 1 MeV, while those published in Ref. [17] were rounded to the nearest 10 MeV.
[e]Reference [18] quotes spin-averaged nS masses; values here are for $^3S_1$ states.
[f]Experimental masses quoted to nearest MeV/c^2.

TABLE II. Fine-structure splittings for spin-triplet states predicted in various models. Shown are deviations from spin-weighted centers of gravity, in MeV/c^2.

Reference	J	$1P$	$2P$	$1D$	$2D$
[8]	$L-1$	−40.4[a]	−28.9[a]	−6.6	−6.3
	L	−8.3[a]	−6.3[a]	−0.5	−0.6
	$L+1$	13.1[a]	9.5[a]	3.2	3.1
[14]	$L-1$	−32	−26	−8	−8
	L	−7	−6	−1	−1
	$L+1$	10	8	4	4
[15]	$L-1$	−29	−21	−11	−9
	L	−3	−2	−1	−1
	$L+1$	8	6	6	5
[16]	$L-1$	−56	−46	c	c
	L	−7	−6	c	c
	$L+1$	15	13	c	c
[17] [b]	$L-1$	−37	−26	−10	−9
	L	−8	−6	−1	−1
	$L+1$	13	9	7	5
[19]	$L-1$	−39	−32	−7	c
	L	−9	−7	−1	c
	$L+1$	13	11	3	c
[23]	$L-1$	−26	c	c	c
	L	−9	c	c	c
	$L+1$	10	c	c	c

[a]Input, based on experimental masses.
[b]Published masses quoted to nearest 10 MeV; the numbers in this table are calculated to 1 MeV.
[c]Not quoted.

noted here and the approach of those authors who derive potentials from NRQCD.

The difference of scales obtained using different experimental splittings is a feature of quenched lattice simulations. This difference should disappear once physical dynamical calculations can be done. In the meantime it is an additional source of uncertainty. The most recent unquenched calculations are given in Ref. [27], but clear signs of the effect of dynamical quarks are hard to see. For S-wave and P-wave lattice predictions, published results in the quenched approximation are given in Ref. [23]. These are quoted in Tables I and II using the $\beta=6.0$ results and setting the scale from the $2S$–$1S$ splitting, for consistency with the results quoted in Fig. 1 [28].

The predicted centers of gravity of the $2D$ levels range from 10 430 to 10 455 MeV/c^2. These levels are more likely to be affected by coupling [29] to $B^{(*)}\overline{B}^{(*)}$ systems above the flavor threshold of 10 560 MeV/c^2. The fine-structure splittings predicted in various models are summarized in Table II.

The production of the lowest D-wave $b\bar{b}$ states is most likely in cascade transitions from the $\Upsilon(3S)$. In Ref. [8], two means of studying these transitions were proposed. One can employ the three-photon inclusive transitions $3S \xrightarrow{\gamma} 2P \xrightarrow{\gamma} 1D \xrightarrow{\gamma} 1P$, or the four-photon transitions $3S \xrightarrow{\gamma} 2P \xrightarrow{\gamma} 1D$

TABLE III. Predicted numbers of $4\gamma e^+e^-$ events corresponding to $3S\xrightarrow{\gamma}2P\xrightarrow{\gamma}1D\xrightarrow{\gamma}1P\xrightarrow{\gamma}1S\rightarrow e^+e^-$ or $3S\xrightarrow{\gamma}2P\xrightarrow{\gamma}2S\xrightarrow{\gamma}1P\xrightarrow{\gamma}1S\rightarrow e^+e^-$ per 10^6 $\Upsilon(3S)$ decays. The numbers following spectroscopic symbols represent photon energies in MeV, in c.m. of decaying states.

$2\,^3P_J$ state	Next state	$1\,^3P_J$ state	$E(\gamma_4)$	Events
$2\,^3P_2$ (87)	$1\,^3D_3$ (107)	$1\,^3P_2$ (245)	443	7.8
	$1\,^3D_2$ (112)	$1\,^3P_2$ (240)	443	0.3
		$1\,^3P_1$ (261)	422	2.7
	$1\,^3D_1$ (119)	$1\,^3P_2$ (233)	443	0.0
		$1\,^3P_1$ (254)	422	0.1
		$1\,^3P_0$ (285)	391	0.0
	$2\,^3S_1$ (242)	$1\,^3P_2$ (110)	443	4.1
		$1\,^3P_1$ (131)	422	8.8
		$1\,^3P_0$ (162)	391	0.4
$2\,^3P_1$ (100)	$1\,^3D_2$ (99)	$1\,^3P_2$ (240)	443	2.5
		$1\,^3P_1$ (261)	422	20.1
	$1\,^3D_1$ (106)	$1\,^3P_2$ (233)	443	0.1
		$1\,^3P_1$ (254)	422	3.3
		$1\,^3P_0$ (285)	391	0.4
	$2\,^3S_1$ (229)	$1\,^3P_2$ (110)	443	7.5
		$1\,^3P_1$ (131)	422	15.9
		$1\,^3P_0$ (162)	391	0.7
$2\,^3P_0$ (123)	$1\,^3D_1$ (81)	$1\,^3P_2$ (233)	443	0.0
		$1\,^3P_1$ (261)	422	0.3
		$1\,^3P_0$ (285)	391	0.0
	$2\,^3S_1$ (210)	$1\,^3P_2$ (110)	443	0.3
		$1\,^3P_1$ (131)	422	0.7
		$1\,^3P_0$ (162)	391	0.0

$\xrightarrow{\gamma}1P\xrightarrow{\gamma}1S\rightarrow l^+l^-$, which should have considerably less background. Either process suffers from backgrounds due to $3S\xrightarrow{\gamma}2P\xrightarrow{\gamma}2S\xrightarrow{\gamma}1P$ and $3S\rightarrow 1S\pi^0\pi^0$. In four-photon processes one can eliminate events in which two pairs of photons are consistent in mass with two neutral pions. However, in such processes the cascade $3S\xrightarrow{\gamma}2P\xrightarrow{\gamma}2S\xrightarrow{\gamma}1P\xrightarrow{\gamma}1S\rightarrow l^+l^-$ leads to events in which the photons from $2P\xrightarrow{\gamma}2S$ are easily confused with those from $1D\xrightarrow{\gamma}1P$, while those from $2S\xrightarrow{\gamma}1P$ are easily confused with those from $2P\xrightarrow{\gamma}1D$.

Using the branching ratios predicted in Ref. [8] for the electromagnetic transitions from one $b\bar{b}$ state to another, and the measured branching ratio $\mathcal{B}(\Upsilon(1S)\rightarrow e^+e^-)=(2.38\pm 0.11)\%$ [22], one predicts the numbers of $4\gamma e^+e^-$ events per 10^6 $\Upsilon(3S)$ shown in Table III. The total numbers of events proceeding via $1\,^3D_J$ states (35.1) and via the $2\,^3S_1$ state (38.4) are approximately equal, with the dominant roles played by the transitions $3S\xrightarrow{\gamma}2\,^3P_1\xrightarrow{\gamma}1\,^3D_2\xrightarrow{\gamma}1\,^3P_1\xrightarrow{\gamma}1S$ (20.1 events) and $3S\xrightarrow{\gamma}2\,^3P_1\xrightarrow{\gamma}2^3S_1\xrightarrow{\gamma}1^3P_1\xrightarrow{\gamma}1S$ (15.9 events). Equal numbers of $4\gamma\mu^+\mu^-$ events are expected if muons can also be identified. The initial and final photon energies for these two sets of transitions are the same, since both sets of transitions proceed via $^3P_1$ states. However, the two intermediate pairs of energies are different: 99 and 261 MeV for the transitions via $1\,^3D_2$, and 131 and 229 MeV for the transitions via $2\,^3S_1$. Thus, it should not be too hard to distinguish the two processes from one another.

The branching ratio predictions of Ref. [8] were performed under the assumption that the hadronic widths of the D-wave states could be calculated purely using their color-singlet $b\bar{b}$ components. A more up-to-date calculation of hadronic widths, based on the inclusion of color-octet $b\bar{b}$ contributions [30], is probably called for, but is beyond the scope of the present paper. Even 100% augmentation of the hadronic widths of the $1\,^3D_J$ states would have little effect on their branching ratios for radiative decays, which are expected to be dominant. The partial widths for $2\,^3D_J$ states to decay non-electromagnetically were not calculated in Ref. [8] but were expected to be comparable to those for the $1\,^3D_J$ levels.

We conclude with a discussion of energy scans in e^+e^- annihilations for direct production of $b\bar{b}(^3D_1)$ states. The results of Table II indicate that these states are expected to lie between 6 and 11 MeV/c^2 below the D-wave spin-weighted centers of gravity. Taking account of the predictions in Fig. 1, one then expects the $1\,^3D_1$ level to lie between 10.13 and 10.17 GeV/c^2, while the $2\,^3D_1$ level should lie between 10.42 and 10.46 GeV/c^2. These predictions ignore coupled-channel distortions due to $B\bar{B}$ threshold [29], as mentioned.

As mentioned in Ref. [8], present limits on leptonic widths of the $1\,^3D_1$ and $2\,^3D_1$ states [31] are about a factor of 10 to 15 above the predicted values [15] of $\Gamma_{ee}(1\,^3D_1,2\,^3D_1)=(1.5,2.7)$ eV. [Reference [17] finds $\Gamma_{ee}(1\,^3D_1)=1.6$ eV.] The CUSB Collaboration's search [31] in the range from 10.34 to 10.52 GeV/c^2 sets 90% C.L. upper limits of $\Gamma_{ee}(2\,^3D_1)<40$ eV in this range on the basis of 5 pb^{-1} of integrated luminosity. Thus, an effective scan of this same range with 15 times the sensitivity would require at least 1 fb^{-1}. Similar estimates have been obtained by the CLEO Collaboration [32].

Once a $^3D_1$ state has been found in an energy scan, does it have any distinctive final states? For the $1\,^3D_1$ state, the dominant decay, with 60% branching ratio, was found [8] to be $\gamma+1^3P_0$, with $E_\gamma=285$ MeV. The $1\,^3P_0$ is expected to decay 97% of the time to hadrons. However, its hadronic decays do not appear to have the expected signature of a pair of nearly back-to-back jets [33]. For the $2\,^3D_1$ state, the corresponding photon energy for the dominant final state $\gamma+2\,^3P_0$ is predicted [8] to be 202 MeV. The branching ratio to $\gamma+1\,^3P_0$, leading to a 559 MeV photon, is expected to be about a factor of 5 lower.

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