Neutrino, lepton, and quark masses in supersymmetry

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The recently proposed model of neutrino mass with no new physics beyond the TeV energy scale is shown to admit a natural and realistic supersymmetric realization, when combined with another recently proposed model of quark masses in the context of a softly broken U(1) symmetry. Four Higgs doublets are required, but two must have masses at the TeV scale. New characteristic experimental predictions of this synthesis are discussed.

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In the minimal standard model of fundamental particle interactions, neutrinos are massless. In the minimal supersymmetric standard model (MSSM), they are still massless, because of the imposition of additive lepton-number conservation. Although the assignment of lepton number(s) is by no means unique [1], a minimal scenario for neutrino mass is to assume the conservation of a discrete Z_2 (odd-even) symmetry which is odd for all leptons and even for all others. By the addition of three neutral singlet lepton superfields N_i with allowed large Majorana masses, the usual doublet neutrinos ν_i will then obtain small masses through the famous seesaw mechanism [2].

The conventional wisdom is that m_N must be very large, say of order 10^{13} GeV or greater, for m_ν to be much less than 1 eV. However, it has been shown recently [3] that $m_N \sim 1$ TeV is possible (and natural) if there exists a second Higgs doublet with $m^2>0$ so that its vacuum expectation value (VEV) is naturally small, say of order 1 MeV. This is achieved by an appropriate assignment of additive lepton number which is softly broken in the scalar sector. More recently, a model of quark masses is proposed [4], where the smallness of m_u, m_d, m_s compared to m_c, m_b, m_t and the pattern of the charged-current mixing matrix may be understood in a similar way. In this paper the two proposals are shown to be naturally combined in a supersymmetric model with four Higgs doublets, in the context of a *single* softly broken U(1) symmetry.

The gauge group is the standard one: i.e. $SU(3)_C \times SU(2)_L \times U(1)_Y$. The particle content is the usual three families of quark and lepton superfields, with the addition of three neutral singlet superfields N_i and four (instead of two) Higgs superfields. Each matter superfield (all defined to be left-handed) transforms under an assumed global U(1) symmetry as follows:

$$0:(t,b),t^c,b^c,s^c,d^c,N_i,(h_1^0,h_1^-),(h_2^+,h_2^0) \tag{1}$$

$$1:(\nu_i,l_i),c^c,(h_3^0,h_3^-) \tag{2}$$

$$-1:(c,s),(u,d),\tau^c,(h_4^+,h_4^0) \tag{3}$$

$$2:u^c \tag{4}$$

$$-2:\mu^c,e^c \tag{5}$$

Let $h^0_{1,2,3,4}$ acquire VEVs equal to $v_{1,2,3,4}$ respectively, then the quark mass matrices are given by [4]

$$\mathcal{M}_u = \begin{bmatrix} f_u v_4 & 0 & 0 \\ f_{cu} v_4 & f_c v_2 & 0 \\ 0 & f_{tc} v_4 & f_t v_2 \end{bmatrix},$$

$$\mathcal{M}_d = \begin{bmatrix} f_d v_3 & f_{ds} v_3 & f_{db} v_3 \\ 0 & f_s v_3 & f_{sb} v_3 \\ 0 & 0 & f_b v_1 \end{bmatrix}, \tag{6}$$

where the freedom to rotate among (c,s) and (u,d) has been used to set the uc^c element to zero and the freedom to rotate among (b^c,s^c,d^c) has been used to set the 3 lower off-diagonal entries of $\mathcal{M}_d$ to zero. Similarly, the charged-lepton mass matrix is given by

$$\mathcal{M}_l = \begin{bmatrix} f_e v_3 & 0 & 0 \\ 0 & f_\mu v_3 & 0 \\ f_{\tau e} v_3 & f_{\tau\mu} v_3 & f_\tau v_1 \end{bmatrix}, \tag{7}$$

whereas the neutrino mass matrix linking ν_i to N_j is proportional to v_4, but otherwise arbitrary.

If the assumed U(1) symmetry is unbroken, then $v_3 = v_4 = 0$. This means that $m_u = m_d = m_s = 0$ and $m_e = m_\mu = m_{\nu_i} = 0$, i.e. only $t,b,c,$ and τ are massive. [Of course N_j have allowed large Majorana masses, but there would be no Dirac mass matrix linking them to ν_i.] To see how v_3 and v_4 become nonzero but small, consider the Higgs sector of this model. The terms H_1H_2 and H_3H_4 are allowed by U(1) invariance, thus guaranteeing that appropriately large Higgsino masses are present in the 6×6 (instead of the usual 4×4) neutralino mass matrix. The terms H_1H_4 and H_2H_3 break U(1) softly, thus it is natural for their coefficients to be small [5], which allow $v_4 \ll v_1$ if $m_4^2>0$ while $m_1^2<0$ and $v_3 \ll v_2$ if $m_3^2>0$ while $m_2^2<0$, as explained in Refs. [3,4]. [The $L_iH_{2,4}$ terms are forbidden by the unbroken Z_2 lepton parity discussed earlier.]

Since $m_t = f_t v_2$ and $m_b = f_b v_1$, the natural magnitude of v_2 is 10^2 GeV and that of v_1 is a few GeV. Hence it is natural as well for $v_3 \sim 10^2$ MeV and $v_4 \sim$ a few MeV. A glance at Eqs. (6) and (7) shows that these are indeed very realistic values. Since $m_\nu \simeq f^2 v_4^2/m_N$, this also means that

$m_N\sim$ a few TeV is realistic, as shown in Ref. [3]. Note that Eqs. (29), (31), (32), (33), and (35) of Ref. [4] are unchanged (except of course m_2 and v_2 there are redefined as m_3 and v_3 here) because $f_b v_1 = m_b$ even though v_1 here is numerically much smaller. Hence the constraints due to flavor-changing neutral currents (FCNC) in the *down* sector are all satisfied provided that

$$m_3 > 3.23\left(\frac{0.3\ \text{GeV}}{v_3}\right)\text{TeV}, \tag{8}$$

i.e. Eq. (30) of Ref. [4]. In the case of $D^0-\overline{D}^0$ mixing, Eq. (34) of Ref. [4] becomes

$$\frac{\Delta m_{D^0}}{m_{D^0}} \simeq \frac{B_D f_D^2 v_2^2}{3m_4^2} f_c^2 f_{cu}^2 \frac{m_u}{m_c^3} < 2.5\times 10^{-14}. \tag{9}$$

Using $f_D = 150$ MeV, $B_D = 0.8$, $f_c v_2 = m_c = 1.25$ GeV, and $m_u = 4$ MeV, this implies

$$m_4 > 2.77\left(\frac{f_{cu}}{0.1}\right)\text{TeV}. \tag{10}$$

The Higgs potential of this model is given by

$$V=\sum_i m_i^2 H_i^\dagger H_i + [m_{12}^2 H_1 H_2 + m_{34}^2 H_3 H_4 + m_{14}^2 H_1 H_4 + m_{23}^2 H_2 H_3 + \text{H.c.}] + \frac{1}{2} g_1^2\left[-\frac{1}{2}H_1^\dagger H_1 + \frac{1}{2}H_2^\dagger H_2 - \frac{1}{2}H_3^\dagger H_3 + \frac{1}{2}H_4^\dagger H_4\right]^2 + \frac{1}{2} g_2^2 \sum_\alpha \left|\sum_i H_i^\dagger \tau_\alpha H_i\right|^2, \tag{11}$$

where τ_α ($\alpha=1,2,3$) are the usual SU(2) representation matrices. Let $\langle h_i^0\rangle = v_i$, then the minimum of V is

$$V_{min}=\sum_i m_i^2 v_i^2 + 2m_{12}^2 v_1 v_2 + 2m_{34}^2 v_3 v_4 + 2m_{14}^2 v_1 v_4 + 2m_{23}^2 v_2 v_3 + \frac{1}{8}(g_1^2+g_2^2)(v_1^2 - v_2^2 + v_3^2 - v_4^2)^2, \tag{12}$$

where all parameters have been assumed real for simplicity. The 4 equations of constraint are

$$0 = m_1^2 v_1 + m_{12}^2 v_2 + m_{14}^2 v_4 + \frac{1}{4}(g_1^2+g_2^2) \times v_1(v_1^2 - v_2^2 + v_3^2 - v_4^2), \tag{13}$$

$$0 = m_2^2 v_2 + m_{12}^2 v_1 + m_{23}^2 v_3 - \frac{1}{4}(g_1^2+g_2^2) \times v_2(v_1^2 - v_2^2 + v_3^2 - v_4^2), \tag{14}$$

$$0 = m_3^2 v_3 + m_{34}^2 v_4 + m_{23}^2 v_2 + \frac{1}{4}(g_1^2+g_2^2) \times v_3(v_1^2 - v_2^2 + v_3^2 - v_4^2), \tag{15}$$

$$0 = m_4^2 v_4 + m_{34}^2 v_3 + m_{14}^2 v_1 - \frac{1}{4}(g_1^2+g_2^2) \times v_4(v_1^2 - v_2^2 + v_3^2 - v_4^2). \tag{16}$$

A solution with $v_4 \gg v_3 \gg v_1 \gg v_2$ is then possible with the result

$$v_2 \simeq \frac{-m_2^2}{\frac{1}{4}(g_1^2+g_2^2)}, \quad v_1 \simeq \frac{-m_{12}^2 v_2}{m_1^2 + m_2^2}, \tag{17}$$

and

$$v_3 \simeq \frac{-m_{23}^2 v_2}{m_3^2 - \frac{1}{4}(g_1^2+g_2^2) v_2^2}, \quad v_4 \simeq \frac{-m_{14}^2 v_1 - m_{34}^2 v_3}{m_4^2 + \frac{1}{4}(g_1^2+g_2^2) v_2^2}. \tag{18}$$

The $H_{1,2}$ doublets are essentially those of the MSSM, while H_3 and H_4 have masses m_3 and m_4 respectively at the TeV scale, as constrained phenomenologically by Eqs. (8) and (10). Once produced, the dominant decays of $H_{1,2}$ are the same as in the MSSM, i.e. into t,b,c and τ states. Their decay branching fractions into light fermions depend on $H_1 H_4$ and $H_2 H_3$ mixing, but since they are very much suppressed, it will be difficult to distinguish them from those of the MSSM. If H_3 and H_4 are produced, then their decays will be the decisive evidence of this model. As discussed in Ref. [3], the decays

$$h_4^+ \to l_i^+ N_j, \quad \text{then} \quad N_j \to l_k^\pm W^\mp, \tag{19}$$

will determine the relative magnitude of each element of the neutrino mass matrix. The difference in the present model is that H_4 also couples to $(u,d)u^c$, $(c,s)u^c$, and $(t,b)c^c$. This means that the three-body decay of N is actually dominant [6], i.e.

$$N \to \nu(l) + 2 \ \text{quark jets}. \tag{20}$$

Of course, this still carries the relevant information on the neutrino mass matrix by the flavor of the charged lepton in the final state.

In the model of Ref. [4], lepton flavor is assumed conserved, but it cannot be maintained in the presence of neutrino oscillations. Here H_3 couples to both quarks and leptons together with H_1 according to $\mathcal{M}_l$ of Eq. (7). Following the discussion given in Ref. [4], the FCNC effects in the charged-lepton sector are thus contained in the term

$$f_\tau \overline{\tau_L} \tau_R\left[\overline{h_1^0} - \frac{v_1}{v_3}\overline{h_3^0}\right] + \text{H.c.}, \tag{21}$$

where $\tau_{L,R}$ are not mass eigenstates and have to be rotated using Eq. (7). The analog of Eq. (28) of Ref. [4] is then

$$\left[\frac{v_3}{v_1}\overline{h_1^0}-\overline{h_3^0}\right]\left[f_{\tau\mu}\left(\overline{\tau}_L\mu_R+\frac{m_\mu}{m_\tau}\overline{\mu}_L\tau_R\right)+f_{\tau e}\left(\overline{\tau}_L e_R+\frac{m_e}{m_\tau}\overline{e}_L\tau_R\right)+\frac{f_{\tau\mu}f_{\tau e}v_3}{m_\tau^2}(m_\mu\overline{\mu}_L e_R+m_e\overline{e}_L\mu_R)\right]+\text{H.c.} \tag{22}$$

The most stringent bounds on $f_{\tau\mu}$ and $f_{\tau e}$ come from $\tau\to\mu\mu\mu$ and $\tau\to e\mu\mu$ through h_3^0 exchange. Using $m_3=3.23$ TeV, $v_3=0.3$ GeV, and $f_{\tau e}=1$, the fraction

$$\frac{\Gamma(\tau\to e\mu\mu)}{\Gamma(\tau\to\nu_\tau e\nu_e)}\simeq\frac{f_{\tau e}^2 f_\mu^2}{32G_F^2 m_3^4}=2.6\times10^{-7}, \tag{23}$$

which is well below the experimental upper bound of $1.8\times10^{-6}/0.1783=1.0\times10^{-5}$. Similarly, for $f_{\tau\mu}=1$, the analogous fraction is also 2.6×10^{-7} and well below the experimental upper bound of $1.9\times10^{-6}/0.1737=1.1\times10^{-5}$. Once produced, the decays of h_3^0 are into $s\overline{s}$, $\mu^-\mu^+$, as well as distinct FCNC final states such as $\tau^\pm\mu^\mp$, $\tau^\pm e^\mp$, and $s\overline{b}+b\overline{s}$.

In conclusion, it has been shown that a supersymmetric extension of the standard model with four Higgs doublets has the following desirable features. (i) Only heavy quarks (i.e. t, b, c) and the one heavy lepton (τ) are massive under the assumed global U(1) symmetry. (ii) As the U(1) symmetry is broken softly, the two extra Higgs doublets also acquire nonzero (but small) vacuum expectation values, and all the light quarks and leptons become massive. (iii) The pattern of the quark charged-current mixing matrix is obtained naturally. (iv) Small Majorana neutrino masses are obtained with three singlet superfields N_i at the TeV energy scale. (v) The two extra Higgs doublets are also at the TeV scale with observable decays which are characteristic of this model.

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[1] E. Ma and D. Ng, Phys. Rev. D **41**, R1005 (1990).
[2] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979), p. 95; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
[3] E. Ma, Phys. Rev. Lett. **86**, 2502 (2001).
[4] E. Ma, Phys. Lett. B **516**, 165 (2001).
[5] G. 't Hooft, in *Recent Developments in Gauge Theories: Proceedings of the NATO Advanced Study Institute*, Cargese, 1979, edited by G. 't Hooft *et al.* (Plenum, New York, 1980).
[6] A known example of possible three-body dominance, i.e., $h^+\to W^+b\overline{b}$ over $h^+\to c\overline{s}$ and $h^+\to\tau^+\nu_\tau$, was discussed by E. Ma, D.P. Roy, and J. Wudka, Phys. Rev. Lett. **80**, 1162 (1998).