New *CP* violation in neutrino oscillations

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Measurements of *CP*-violating observables in neutrino oscillation experiments have been studied in the literature as a way to determine the *CP*-violating phase in the mixing matrix for leptons. Here we show that such observables also probe new neutrino interactions in the production or detection processes. Genuine *CP* violation and fake *CP* violation due to matter effects are sensitive to the imaginary and real parts of new couplings. The dependence of the *CP* asymmetry on the source-detector distance is different from the standard one and, in particular, enhanced at short distances. We estimate that future neutrino factories will be able to probe in this way new interactions that are up to four orders of magnitude weaker than the weak interactions. We discuss the possible implications for models of new physics.

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I. NEW *CP* VIOLATION IN NEUTRINO INTERACTIONS

In the future, neutrino oscillation experiments will search for *CP*-violating effects [1–22]. The standard model, extended to include masses for light, active neutrinos, predicts that *CP* is violated in neutrino oscillations through a single phase in the mixing matrix for leptons. This effect is suppressed by small mixing angles and small mass differences.

It is not unlikely, however, that the high-energy physics that is responsible for neutrino masses and mixing involves also new neutrino interactions. Such interactions provide new sources of *CP* violation. In this work we study *CP*-violating effects due to contributions from new neutrino interactions to the production and/or detection processes in neutrino oscillation experiments. We investigate the following questions:

(i) How would new, *CP*-violating neutrino interactions manifest themselves in neutrino oscillations?

(ii) Are the effects qualitatively different from the standard models ones? In particular, can we use the time (or, equivalently, distance) dependence of the transition probability to distinguish between standard model and new *CP* violation?

(iii) How large can the effects be? In particular, do the new interactions suffer from suppression factors related to mixing angles and mass differences?

(iv) Can the new *CP* violation be observed in proposed experiments? What would be the optimal setting for these observations?

(v) Which models of new physics can be probed in this way?

The plan of this paper goes as follows. In Sec. II we present a parametrization of the new physics effects that are of interest to us and explain the counting of independent *CP*-violating phases in our framework. In Sec. III we evaluate the new physics effects on the transition probability in neutrino vacuum oscillation experiments. (A full expression for the transition probability, without any approximations concerning mixing angles and mass differences, is given in the Appendix.) In Sec. IV we investigate the resulting *CP* asymmetry and compare the new physics contribution to the standard one (that is, the contribution to the asymmetry from lepton mixing). In Secs. V and VI we evaluate the new physics effects on, respectively, the transition probability and *CP* asymmetry, in neutrino matter oscillations. In Sec. VII we study how these effects can be observed in future neutrino factory experiments. In particular, we estimate a lower bound on the strength of the new interactions that can be observed in these experiments. This lower bound is compared to existing model–independent upper bounds in Sec. VIII. We summarize our results and discuss some of the implications that would arise if a signal is experimentally observed in Sec. IX.

II. NOTATION AND FORMALISM

In this section we give a model-independent parametrization of the new physics effects on production and detection processes in neutrino oscillation experiments. We put special emphasis on *CP*-violating phases.

We denote by $|\nu_i\rangle$, $i=1$, 2, and 3, the three neutrino mass eigenstates. We denote by $|\nu_\alpha\rangle$ the weak interaction partners of the charged lepton mass eigenstates α^- ($\alpha=e,\mu,\tau$):

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle. \tag{2.1}$$

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Whenever we use an explicit parametrization of the lepton mixing matrix [23,24], we will use the most conventional one,

$$U \equiv U_{23}U_{13}U_{12} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$$\times \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (2.2)$$

with $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$. Alternatively, a convention-independent definition of the phase δ that we will use in our calculations is given by

$$\delta \equiv \arg\left(\frac{U_{e3}U_{\mu3}^*}{U_{e1}U_{\mu1}^*}\right). \quad (2.3)$$

We consider new, possibly CP-violating, physics in the production and/or detection process. Such effects were previously studied in Ref. [25], and we closely follow the formalism of that paper. Most of the analysis in Ref. [25], however, was carried out assuming CP conservation. We parametrize the new physics interaction in the source and in the detector by two sets of effective four-fermion couplings $(G_{NP}^s)_{\alpha\beta}$ and $(G_{NP}^d)_{\alpha\beta}$, where $\alpha,\beta = e,\mu,\tau$. Here $(G_{NP}^s)_{\alpha\beta}$ refers to processes in the source where a ν_β is produced in conjunction with an incoming α^- or an outgoing α^+ charged lepton, while $(G_{NP}^d)_{\alpha\beta}$ refers to processes in the detector where an incoming ν_β produces an α^- charged lepton. While the $SU(2)_L$ gauge symmetry requires that the four-fermion couplings of the charged current weak interactions be proportional to $G_F\delta_{\alpha\beta}$, new interactions allow couplings with $\alpha \neq \beta$. Phenomenological constraints imply that the new interaction is suppressed with respect to the weak interaction:

$$|(G_{NP}^s)_{\alpha\beta}| \ll G_F, \quad |(G_{NP}^d)_{\alpha\beta}| \ll G_F. \quad (2.4)$$

For the sake of concreteness, we consider the production and detection processes that are relevant to neutrino factories. We therefore study an appearance experiment where neutrinos are produced in the process $\mu^+ \to e^+\nu_\alpha\bar{\nu}_{\alpha'}$ and detected by the process $\nu_\beta d \to \mu^- u$, and antineutrinos are produced and detected by the corresponding charge-conjugate processes. Our results can be modified to any other neutrino oscillation experiment in a straightforward way. The relevant couplings are then $(G_{NP}^s)_{e\beta}$ and $(G_{NP}^d)_{\mu\beta}$. It is convenient to define small dimensionless quantities $\epsilon_{\alpha\beta}^{s,d}$ in the following ways:

$$\epsilon_{e\beta}^s \equiv \frac{(G_{NP}^s)_{e\beta}}{\sqrt{|G_F + (G_{NP}^s)_{ee}|^2 + |(G_{NP}^s)_{e\mu}|^2 + |(G_{NP}^s)_{e\tau}|^2}},$$

$$\epsilon_{\mu\beta}^d \equiv \frac{(G_{NP}^d)_{\mu\beta}}{\sqrt{|G_F + (G_{NP}^d)_{\mu\mu}|^2 + |(G_{NP}^d)_{\mu e}|^2 + |(G_{NP}^d)_{\mu\tau}|^2}}. \quad (2.5)$$

Since we assume that $|\epsilon_{\alpha\beta}^{s,d}| \ll 1$, we will only evaluate their effects to leading (linear) order. New flavor-conserving interactions affect neutrino oscillations only at $\mathcal{O}(|\epsilon|^2)$, and will be neglected from here on. [More precisely, the leading effects from flavor-diagonal couplings are proportional to ϵ (flavor-diagonal) $\times \epsilon$ (flavor-changing) and can therefore be safely neglected.]

We use an explicit parametrization for only two of the ϵ's, with the following convention:

$$\epsilon_{e\mu}^s \equiv |\epsilon_{e\mu}^s|e^{i\delta_\epsilon}, \quad \epsilon_{\mu e}^{d*} \equiv |\epsilon_{\mu e}^{d*}|e^{i\delta_\epsilon'}. \quad (2.6)$$

Alternatively, we can define the phases δ_ϵ and δ_ϵ' in a convention-independent way:

$$\delta_\epsilon \equiv \arg\left(\frac{\epsilon_{e\mu}^s}{U_{e1}U_{\mu1}^*}\right), \quad \delta_\epsilon' \equiv \arg\left(\frac{\epsilon_{\mu e}^{d*}}{U_{e1}U_{\mu1}^*}\right). \quad (2.7)$$

We would like to conclude this section with a comment on the number of independent CP-violating phases in our framework. It is well known that the three-generation mixing matrix for leptons depends, in the case of Majorana neutrinos, on three phases. Two of these, related to the fact that there is no freedom in redefining the phases of neutrino fields, do not affect neutrino oscillations and are therefore irrelevant to our discussion. The other one is analogous to the Kobayashi-Maskawa phase of the mixing matrix for quarks. The freedom of redefining the phases of charged lepton fields is fully used to reduce the number of relevant phases to one. Consequently, it is impossible to remove any phases from the $\epsilon_{\alpha\beta}^{s,d}$ parameters. Each of these parameters introduces a new, independent CP-violating phase.

For example, when we discuss $\nu_e \to \nu_\mu$ oscillations, our results will depend on $\epsilon_{e\mu}^s$ and $\epsilon_{\mu e}^d$, and the $U_{ei}U_{\mu i}^*$ ($i = 1,2,3$) mixing parameters. This set of parameters depends on three independent phases, one of which is the δ of
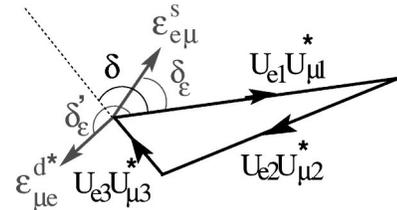

FIG. 1. The neutrino parameters that dominate $P_{e\mu}$ in the complex plane. We show the relevant unitarity triangle, which is the geometrical presentation of the relation $U_{e1}U_{\mu1}^* + U_{e2}U_{\mu2}^* + U_{e3}U_{\mu3}^* = 0$, and the two parameters that describe the new physics in the production, $\epsilon_{e\mu}^s$, and in the detector, $\epsilon_{\mu e}^{d*}$. The three independent phases defined in the text, δ, δ_ϵ, and δ_ϵ', are shown explicitly. The standard convention puts $U_{e1}U_{\mu1}^*$ on the real axis.

Eq. (2.3), while the other two can be chosen to be δ_ϵ and δ'_ϵ of Eq. (2.7). This situation is illustrated in Fig. 1, where we show in the complex plane the unitarity triangle and the $\epsilon^{s,d}$ parameters that are most relevant to $\nu_e \to \nu_\mu$ oscillations.

III. TRANSITION PROBABILITY IN VACUUM

In this section we derive the expression for the transition probability in neutrino oscillation experiments as a function of the mixing matrix parameters and the new physics parameters. We denote by ν_e^s the neutrino state that is produced in the source in conjunction with an e^+, and by ν_μ^d the neutrino state that is signaled by μ^- production in the detector:

$$|\nu_e^s\rangle = \sum_i [U_{ei} + \epsilon_{e\mu}^s U_{\mu i} + \epsilon_{e\tau}^s U_{\tau i}]|\nu_i\rangle,$$

$$(3.1)$$

$$|\nu_\mu^d\rangle = \sum_i [U_{\mu i} + \epsilon_{\mu e}^d U_{ei} + \epsilon_{\mu \tau}^d U_{\tau i}]|\nu_i\rangle.$$

[Note that the norm of the states so defined is one up to effects of $\mathcal{O}(|\epsilon|^2)$, which we consistently neglect.] We obtain the following expression for the transition probability $P_{e\mu} = |\langle \nu_\mu^d | \nu_e^s(t)\rangle|^2$, where $\nu_e^s(t)$ is the time-evolved state that was purely ν_e^s at time $t = 0$:

$$P_{e\mu} = \left| \sum_i e^{-iE_i t}[U_{ei}U_{\mu i}^* + \epsilon_{e\mu}^s|U_{\mu i}|^2 + \epsilon_{\mu e}^{d*}|U_{ei}|^2 + \epsilon_{e\tau}^s U_{\tau i}U_{\mu i}^* + \epsilon_{\mu \tau}^{d*}U_{\mu i}U_{\tau i}^*] \right|^2. \qquad (3.2)$$

Our results will be given in terms of Δm_{ij}^2, Δ_{ij}, and x_{ij}, which are defined as

$$\Delta m_{ij}^2 \equiv m_i^2 - m_j^2, \quad \Delta_{ij} \equiv \Delta m_{ij}^2/(2E), \quad x_{ij} \equiv \Delta_{ij}L/2, \qquad (3.3)$$

where E is the neutrino energy and L is the distance between the source and the detector.

Equation (3.2) will be the starting point of our calculations. The full expression for $P_{e\mu}$ in vacuum is given in the Appendix, and has been used for our numerical calculations described below. To understand the essential features of our analysis it is, however, more useful to do the following. First we separate $P_{e\mu}$ into a standard model piece $P_{e\mu}^{SM}$ and a new physics piece $P_{e\mu}^{NP}$. What we mean by $P_{e\mu}^{SM}$ is $P_{e\mu}(\epsilon_{\alpha\beta}^{s,d} = 0)$. This is the contribution to $P_{e\mu}$ from the standard model extended to include neutrino masses but no new interactions. In contrast, $P_{e\mu}^{NP}$ contains all the $\epsilon_{\alpha\beta}^{s,d}$–dependent terms. Second, since the atmospheric and reactor neutrino data imply that $|U_{e3}|$ is small and the solar neutrino data imply that $\Delta m_{12}^2/\Delta m_{13}^2$ is small, we expand $P_{e\mu}^{SM}$ to second order and $P_{e\mu}^{NP}$ to first order in $|U_{e3}|$ and Δm_{12}^2.

For $P_{e\mu}^{SM}$ we obtain

$$P_{e\mu}^{SM} = 4x_{21}^2|U_{e2}|^2|U_{\mu 2}|^2 + 4\sin^2 x_{31}|U_{e3}|^2|U_{\mu 3}|^2$$
$$+ 4x_{21}\sin 2x_{31} \text{Re}(U_{e2}U_{e3}^* U_{\mu 2}^* U_{\mu 3})$$
$$- 8x_{21}\sin^2 x_{31} \text{Im}(U_{e2}U_{e3}^* U_{\mu 2}^* U_{\mu 3}). \qquad (3.4)$$

The first term is the well known transition probability in the two-generation case. The second term gives the well known transition probability in the approximation that $\Delta m_{12}^2 = 0$. The last term is a manifestation of the standard model CP violation.

For $P_{e\mu}^{NP}$ we obtain

$$P_{e\mu}^{NP} = -4\sin^2 x_{31} \text{Re}[U_{e3}^* U_{\mu 3}(\epsilon_{\mu e}^{d*} + \epsilon_{e\mu}^s(1 - 2|U_{\mu 3}|^2)$$
$$- 2\epsilon_{e\tau}^s U_{\mu 3}^* U_{\tau 3})]$$
$$+ 4x_{21}\sin 2x_{31} \text{Re}[U_{e2}^* U_{\mu 2}(\epsilon_{e\mu}^s|U_{\mu 3}|^2 + \epsilon_{e\tau}^s U_{\mu 3}^* U_{\tau 3})]$$
$$- 4x_{21} \text{Im}[U_{e2}^* U_{\mu 2}(\epsilon_{\mu e}^{d*} + \epsilon_{e\mu}^s(1 - |U_{\mu 3}|^2)$$
$$- \epsilon_{e\tau}^s U_{\mu 3}^* U_{\tau 3})] - 2\sin 2x_{31} \text{Im}[U_{e3}^* U_{\mu 3}(\epsilon_{\mu e}^{d*} + \epsilon_{e\mu}^s)]$$
$$- 4x_{21}\cos 2x_{31} \text{Im}[U_{e2}^* U_{\mu 2}(\epsilon_{e\mu}^s|U_{\mu 3}|^2$$
$$+ \epsilon_{e\tau}^s U_{\mu 3}^* U_{\tau 3})]. \qquad (3.5)$$

The last three terms in this expression are CP violating and would be the basis for our results.

IV. *CP* VIOLATION IN VACUUM OSCILLATIONS

To measure CP violation, one will need to compare the transition probability $P_{e\mu}$ evaluated in Sec. III to that of the CP-conjugate process $P_{\bar{e}\bar{\mu}}$. The latter will be measured in oscillation experiments where antineutrinos are produced in the source in conjunction with e^- and detected through μ^+ production. It is clear that a CP transformation relates the production processes, $\mu^- \to e^- \bar{\nu}_\alpha \nu_{\alpha'}$ and $\mu^+ \to e^+ \nu_\alpha \bar{\nu}_{\alpha'}$. As concerns the detection processes, $\bar{\nu}_\beta u \to \mu^+ d$ and $\nu_\beta d \to \mu^- u$, the situation is less straightforward. We have $G_{\beta\mu}^d \propto \langle p\mu^-|\bar{\mu}^- \bar{u} \nu_\beta d|\nu_\beta n\rangle$ and $G_{\bar{\beta}\bar{\mu}}^d \propto \langle n\mu^+|\bar{\mu}^+ \bar{d} \bar{\nu}_\beta u|\bar{\nu}_\beta p\rangle$. The relation is through CP and crossing symmetry, but for a four-fermion interaction this is equivalent to a CP transformation.

CP transformation of the Lagrangian takes the elements of the mixing matrix and the ϵ terms into their complex conjugates. It is then straightforward to obtain the transition probability for antineutrino oscillations. Our interest lies in the CP asymmetry,

$$A_{CP} = \frac{P_-}{P_+}, \quad (4.1)$$

where

$$P_\pm = P_{e\mu} \pm P_{\bar{e}\bar{\mu}}. \quad (4.2)$$

We quote below the leading contributions for "short" distances, $x_{31} \ll 1$. In some of the observables, we consider two limiting cases for $|U_{e3}|$:

the "large" s_{13} limit:

$$x_{21}/x_{31} \ll |(U_{e3}U_{\mu3})/(U_{e2}U_{\mu2})|, \quad (4.3)$$

the small s_{13} limit:

$$x_{21}/x_{31} \gg |(U_{e3}U_{\mu3})/(U_{e2}U_{\mu2})|.$$

The CP conserving rate P_+ is always dominated by the standard model. It is given by

$$P_+ = \begin{cases} 8x_{31}^2|U_{e3}U_{\mu3}|^2 & \text{large } s_{13} \\ 8x_{21}^2|U_{e2}U_{\mu2}|^2 & \text{small } s_{13}. \end{cases} \quad (4.4)$$

The CP-violating difference between the transition probabilities within the standard model can be obtained from Eq. (3.4):

$$P_-^{\mathrm{SM}} = -16x_{21}x_{31}^2 \, \mathrm{Im}(U_{e2}U_{\mu2}^*U_{e3}^*U_{\mu3}). \quad (4.5)$$

As is well known, CP violation within the standard model is suppressed by both the small mixing angle $|U_{e3}|$ and the small mass-squared difference Δm_{12}^2. More generally, it is proportional to the Jarlskog measure of CP violation, $J = \mathrm{Im}(U_{e2}U_{\mu2}^*U_{e3}^*U_{\mu3})$. For short distances $(x_{21}, x_{31} \ll 1)$, the dependence of P_-^{SM} on the distance is L^3. Since it is CP violating, it should be odd in L. The absence of a term linear in L comes from the fact that the standard model requires for CP to be violated, that all three mass-squared differences do not vanish, that is, $P_- \propto \Delta_{21}\Delta_{31}\Delta_{32}$. In the limit $x_{21}/x_{31} \ll |(U_{e3}U_{\mu3})/(U_{e2}U_{\mu2})|$, we obtain the following standard model asymmetry:

$$A_{CP}^{\mathrm{SM}} = -2x_{21} \, \mathrm{Im}\left(\frac{U_{e2}U_{\mu2}^*}{U_{e3}U_{\mu3}^*}\right). \quad (4.6)$$

In the small s_{13} limit, the standard CP violation is unobservably small.

The CP-violating difference between the transition probabilities that arises from the new physics interactions can be obtained from Eq. (3.5):

$$P_-^{\mathrm{NP}} = \begin{cases} -8x_{31} \, \mathrm{Im}[U_{e3}^*U_{\mu3}(\epsilon_{\mu e}^{d*} + \epsilon_{e\mu}^s)] & \text{large } s_{13} \\ -8x_{21} \, \mathrm{Im}[U_{e2}^*U_{\mu2}(\epsilon_{\mu e}^{d*} + \epsilon_{e\mu}^s)] & \text{small } s_{13}. \end{cases} \quad (4.7)$$

We learn that CP violation beyond the weak interactions requires only that either $|U_{e3}|$ or Δm_{21}^2 be different from zero, but not necessarily both. Also the dependence on the

distance is different: for short distances, $P_-^{\mathrm{NP}} \propto L$. From Eqs. (4.4) and (4.7) we obtain the following new physics contribution to the CP asymmetry:

$$A_{CP}^{\mathrm{NP}} = \begin{cases} -\dfrac{1}{x_{31}} \, \mathrm{Im}\left(\dfrac{\epsilon_{\mu e}^{d*} + \epsilon_{e\mu}^s}{U_{e3}U_{\mu3}^*}\right) & \text{large } s_{13} \\ -\dfrac{1}{x_{21}} \, \mathrm{Im}\left(\dfrac{\epsilon_{\mu e}^{d*} + \epsilon_{e\mu}^s}{U_{e2}U_{\mu2}^*}\right) & \text{small } s_{13}. \end{cases} \quad (4.8)$$

The apparent divergence of A_{CP}^{NP} for small L is only due to the approximations that we used. Specifically, there is an $\mathcal{O}(|\epsilon|^2)$ contribution to P_+ that is constant in L [25], namely $P_+ = \mathcal{O}(|\epsilon|^2)$ for $L \to 0$. In contrast, $P_- = 0$ in the $L \to 0$ limit to all orders in $|\epsilon|$.

Equations (4.7) and (4.8) lead to several interesting conclusions:

(i) It is possible that, in CP-violating observables, the new physics contributions compete with or even dominate over the standard model ones in spite of the superweakness of the interactions ($|\epsilon| \ll 1$). Given that for the proposed experiments $x_{31} \lesssim 1$, it is sufficient that

$$\max(|\epsilon_{e\mu}^s|, |\epsilon_{\mu e}^d|) \gtrsim \min(|U_{e3}|, x_{21}) \quad (4.9)$$

for the new contribution to the CP-violating difference P_- to be larger than the standard one.

(ii) The different distance dependence of P_-^{NP} and P_-^{SM} will allow, in principle, an unambiguous distinction to be made between new physics contributions of the type described here and the contribution from lepton mixing.

(iii) The $1/L$ dependence of A_{CP}^{NP} suggests that the optimal baseline to observe CP violation from new physics is shorter than the one optimized for the standard model.

We carried out a numerical calculation of the probabilities $P_\pm$ and asymmetry A_{CP} as a function of the distance between the source and the detector. We use $E_\nu = 20$ GeV, which is the range of neutrino energy expected in neutrino factories. For the neutrino parameters, we take $\Delta m_{31}^2 = 3 \times 10^{-3}$ eV^2 and $\tan^2\theta_{23} = 1$, consistent with the atmospheric neutrino measurements [26], and $\Delta m_{21}^2 = 10^{-4}$ eV^2 and $\tan^2\theta_{12} = 1$, consistent at present with the large mixing angle (LMA) solution of the solar neutrino problem [26,27]. As concerns the third mixing angle and CP-violating phase in the lepton mixing matrix, we consider two cases. First, we take $s_{13} = 0.2$, close to the upper bound from CHOOZ [28,29,26], and $\delta = \pi/2$. This set of parameters is the one that maximizes the standard CP asymmetry. Second, we take $s_{13} = 0$, in which case there is no standard CP violation in the lepton mixing. As concerns the effects of new physics, we demonstrate them by taking only $|\epsilon_{e\mu}^s| \neq 0$. With our first set of mixing parameters (maximal standard CP violation), we take $|\epsilon_{e\mu}^s| = 10^{-3}$ and $\delta_\epsilon = 0$. With our second set of mixing parameters (zero standard CP violation), we take $|\epsilon_{e\mu}^s| = 10^{-4}$ and $\delta_\epsilon = \pi/2$. Our choice of CP-violating phases can be easily understood on the basis of Eq. (4.8): in the large s_{13}
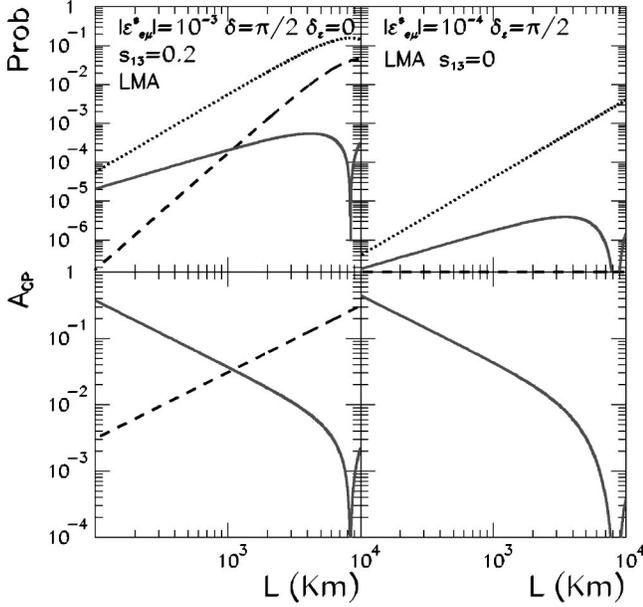
FIG. 2. Transition probabilities and CP asymmetries in vacuum as a function of the distance. In the upper panels the curves correspond to P_+^{SM} (dotted), P_-^{SM} (dashed) and P_-^{NP} (solid). In the lower panels the curves correspond to A_{CP}^{NP} (solid) and A_{CP}^{SM} (dashed). In the left panels, $s_{13}=0.2$, $\delta=\pi/2$, $|\epsilon_{e\mu}^s|=10^{-3}$, and $\delta_\epsilon=0$. In the right panels, $s_{13}=0$, $|\epsilon_{e\mu}^s|=10^{-4}$, and $\delta_\epsilon=\pi/2$. In all curves $E_\nu=20$ GeV, $\Delta m_{13}^2=3\times10^{-3}$ eV^2, $\tan^2\theta_{23}=1$, $\Delta m_{21}^2=10^{-4}$ eV^2, and $\tan^2\theta_{12}=1$.

limit, the CP asymmetry depends on $\arg[\epsilon_{e\mu}^s/(U_{e3}U_{\mu3}^*)]=\delta_\epsilon-\delta$, while in the small s_{13} limit it depends on $\arg[\epsilon_{e\mu}^s/(U_{e2}U_{\mu2}^*)]=\delta_\epsilon$. We use the full expression for the transition probabilities that is presented in the Appendix. Consequently, the only approximation that we make is that we omit effects of $\mathcal{O}(|\epsilon|^2)$.

The results of this calculation are presented in Fig. 2. The left panels correspond to the first case (maximal standard CP violation) and the right ones to the second (zero standard CP violation). For each case we present, as a function of the distance between the source and the detector, P_+ (dotted line), P_-^{SM} and A_{CP}^{SM} (dashed lines in, respectively, upper and lower panels), and P_-^{NP} and A_{CP}^{NP} (solid lines in, respectively, upper and lower panels).

We learn a few interesting facts:

(i) The new physics contribution to CP violation can dominate over even the maximal standard CP violation for values of $|\epsilon|$ as small as 10^{-4}. This is particularly valid for distances shorter than 1000 km.

(ii) The approximations that lead to Eqs. (4.7) and (4.8) are good for $L\lesssim5000$ km.

(iii) As anticipated from our approximate expressions, for short enough distances, P_-^{NP} grows linearly with distances and A_{CP}^{NP} is strongly enhanced at short distances.

(iv) In the large s_{13} case, the new CP violation is sensitive mainly to the phase difference $\delta-\delta_\epsilon$ and is almost independent of the solar neutrino parameters.

(v) In the very small s_{13} limit, the new CP violation is

proportional to $\sin\delta_\epsilon$. The rate P_-^{NP} is suppressed by the solar neutrino mass difference and mixing angle.

V. TRANSITION PROBABILITY IN MATTER

Since long-baseline experiments involve the propagation of neutrinos in the matter of Earth, it is important to understand matter effects on our results. For our purposes, it is sufficient to study the case of constant matter density. Then the matter contribution to the effective ν_e mass, $A=\sqrt{2}G_F N_e$, is constant.

One obtains the transition probability in matter by replacing the mass-squared differences Δ_{ij} and mixing angles $U_{\alpha i}$ with their effective values in matter, Δ_{ij}^m and $U_{\alpha i}^m$. The full expression for $P_{e\mu}$ in matter can then be written in terms of x_{ij}^m and $U_{\alpha i}^m$ by a straightforward modification of the vacuum probability given the Appendix. To understand the matter effects it is, however, more useful to take into account the smallness of $|U_{e3}|$ and x_{12}. We will expand the transition probability in these parameters to second order for $P_{e\mu}^{SM}$ and to first order for $P_{e\mu}^{NP}$.

For the standard model case, we obtain

$$
\begin{aligned}
P_{e\mu}^{SM}=&\ 4\left(\frac{\Delta_{21}}{A}\right)^2\sin^2\left(\frac{AL}{2}\right)|U_{e2}U_{\mu2}|^2 \\
&+4\left(\frac{\Delta_{31}}{B}\right)^2\sin^2\left(\frac{BL}{2}\right)|U_{e3}U_{\mu3}|^2+8\left(\frac{\Delta_{21}}{A}\right) \\
&\times\left(\frac{\Delta_{31}}{B}\right)\sin\left(\frac{AL}{2}\right)\sin\left(\frac{BL}{2}\right) \\
&\times\{\cos x_{31}\,\mathrm{Re}[\,U_{e3}^*U_{\mu3}U_{e2}U_{\mu2}^*\,] \\
&-\sin x_{31}\,\mathrm{Im}[\,U_{e3}^*U_{\mu3}U_{e2}U_{\mu2}^*\,]\},
\end{aligned}
\tag{5.1}
$$

where

$$
B=\Delta_{31}-A. \tag{5.2}
$$

Again, the first term is the full result for two generations, and the second is the full result for the case of $\Delta_{21}=0$. The last term violates CP. In the limit $A=0$, Eq. (3.4) is reproduced. Note that our definition of B is such that B changes sign according to whether Δ_{31} is larger or smaller than A. This is different from the usual convention where $B=|\Delta_{31}-A|$. The standard model results are an even function of B and either definition can be used. But for the new physics results given below, the choice of convention is important.

For the new physics contribution, we find

$$P_{e\mu}^{NP} = 4\left(\frac{\Delta_{21}}{A}\right)\sin^2\left(\frac{AL}{2}\right)\text{Re}[U_{e2}^*U_{\mu2}(\epsilon_{\mu e}^{d*} - \epsilon_{e\mu}^s(1-2|U_{\mu3}|^2) + 2\epsilon_{e\tau}^s U_{\mu3}^* U_{\tau3})] - 4\left(\frac{\Delta_{31}}{B}\right)\sin^2\left(\frac{BL}{2}\right)\text{Re}[U_{e3}^*U_{\mu3}(\epsilon_{\mu e}^{d*} + \epsilon_{e\mu}^s$$

$$\times(1-2|U_{\mu3}|^2) - 2\epsilon_{e\tau}^s U_{\mu3}^* U_{\tau3})] - 2\left(\frac{\Delta_{21}}{A}\right)\sin(AL)\text{Im}[U_{e2}^*U_{\mu2}(\epsilon_{\mu e}^{*d} + \epsilon_{e\mu}^s)] - 2\left(\frac{\Delta_{31}}{B}\right)\sin(BL)\text{Im}[U_{e3}^*U_{\mu3}(\epsilon_{\mu e}^{*d} + \epsilon_{e\mu}^s)]$$

$$-8\sin\left(\frac{AL}{2}\right)\sin\left(\frac{BL}{2}\right)\cos x_{31}\left\{\left(\frac{\Delta_{31}}{B}\right)\text{Re}[U_{e3}^*U_{\mu3}(\epsilon_{e\mu}^s(1-|U_{\mu3}|^2) - \epsilon_{e\tau}^s U_{\mu3}^* U_{\tau3})] - \left(\frac{\Delta_{21}}{A}\right)\text{Re}[U_{e2}^*U_{\mu2}(\epsilon_{e\mu}^s|U_{\mu3}|^2$$

$$+\epsilon_{e\tau}^s U_{\mu3}^* U_{\tau3})]\right\} + 8\sin\left(\frac{AL}{2}\right)\sin\left(\frac{BL}{2}\right)\sin x_{31}\left\{\left(\frac{\Delta_{31}}{B}\right)\text{Im}[U_{e3}^*U_{\mu3}(\epsilon_{e\mu}^s(1-|U_{\mu3}|^2) - \epsilon_{e\tau}^s U_{\mu3}^* U_{\tau3})]$$

$$-\left(\frac{\Delta_{21}}{A}\right)\text{Im}[U_{e2}^*U_{\mu2}(\epsilon_{e\mu}^s|U_{\mu3}|^2 + \epsilon_{e\tau}^s U_{\mu3}^* U_{\tau3})]\right\}. \tag{5.3}$$

Unlike the case of vacuum oscillation, P_- will receive contributions from both CP-violating terms (proportional to the imaginary parts of various combinations of parameters) as CP conserving terms (proportional to the real parts).

Note that, in addition to the effects of new neutrino interactions in the source and in the detector, there could be other, independent effects due to new neutrino interactions with matter during their propagation [30–32]. Such effects have been studied in the context of solar and atmospheric neutrinos (see e.g., Refs. [33–35]) but we neglect them here.

VI. CP VIOLATION IN MATTER OSCILLATIONS

Since matter in Earth is not CP symmetric, there will be contributions to A_{CP} even in the case when there is no CP violation. It is our purpose in this section to evaluate these contributions and, in particular, the fake asymmetry that is related to the real part of ϵ. We denote the matter-related contribution to P_- by $P_-^m \equiv P_-(A) - P_-(A=0)$. Since the leading contributions to P_+ are the same as in the vacuum case [Eq. (4.4)], we can similarly define the matter-related contribution to A_{CP}: $A_{CP}^m \equiv P_-^m/P_+$. Note that in the evaluation of $P_{e\mu}^-$ from the expressions that we found for $P_{e\mu}$ we need not only to replace $U_{\alpha i}$ and $\epsilon_{\alpha\beta}^{s,d}$ with their complex conjugates, but also A with $-A$.

For the standard model, we obtain from Eq. (5.1), in the small x_{31} and large s_{13} limits,

$$(P_-^m)^{SM} = \frac{16}{3}x_{31}^4\left(\frac{A}{\Delta_{31}}\right)|U_{e3}U_{\mu3}|^2. \tag{6.1}$$

In the small s_{13} limit $(x_{21}/x_{31} \gg |(U_{e3}U_{\mu3})/(U_{e2}U_{\mu2})|)$ the standard model effect is unobservably small, and we do not consider it here. Taking into account that [see Eq. (4.4)] $P_+ \approx 8x_{31}^2|U_{e3}U_{\mu3}|^2$, we obtain

$$(A_{CP}^m)^{SM} = \frac{2}{3}x_{31}^2\left(\frac{A}{\Delta_{31}}\right). \tag{6.2}$$

For the new physics contribution, we obtain from Eq. (5.3), in the small x_{31} limit,

$$(P_-^m)^{NP} = \begin{cases} 8x_{31}^2\dfrac{A}{\Delta_{31}}\text{Re}[U_{e3}^*U_{\mu3}(\epsilon_{\mu e}^{d*} - \epsilon_{e\mu}^s)] & \text{large } s_{13} \\[12pt] 8x_{21}^2\dfrac{A}{\Delta_{21}}\text{Re}[U_{e2}^*U_{\mu2}(\epsilon_{\mu e}^{d*} - \epsilon_{e\mu}^s)] & \text{small } s_{13}, \end{cases} \tag{6.3}$$

and

$$(A_{CP}^m)^{NP} = \begin{cases} \dfrac{A}{\Delta_{31}}\text{Re}\left(\dfrac{\epsilon_{\mu e}^{d*} - \epsilon_{e\mu}^s}{U_{e3}U_{\mu3}^*}\right) & \text{large } s_{13} \\[16pt] \dfrac{A}{\Delta_{21}}\text{Re}\left(\dfrac{\epsilon_{\mu e}^{d*} - \epsilon_{e\mu}^s}{U_{e2}U_{\mu2}^*}\right) & \text{small } s_{13}. \end{cases} \tag{6.4}$$

We would like to make a few comments regarding our results here:

(i) Each of the four contributions has a different dependence on the distance. In the short distance limit, we have

$$(P_-^m)^{SM} \propto L^4, \quad P_-^{SM} \propto L^3, \quad (P_-^m)^{NP} \propto L^2, \quad P_-^{NP} \propto L, \tag{6.5}$$

and, equivalently,

$$(A_{CP}^m)^{SM} \propto L^2, \quad A_{CP}^{SM} \propto L, \quad (A_{CP}^m)^{NP} \propto L^0, \quad A_{CP}^{NP} \propto 1/L. \tag{6.6}$$

One can then distinguish between the various contributions, at least in principle.

(ii) If the phases of the ϵ's are of order 1, then the genuine CP asymmetry will be larger (at short distances) than the fake one.

(iii) It is interesting to note that the search for CP violation in neutrino oscillations will allow us to constrain both $\text{Re}(\epsilon)$ and $\text{Im}(\epsilon)$.

We carried out a numerical calculation of the probabilities $P_\pm^m$ and asymmetry A_{CP}^m as a function of the distance between the source and the detector. We again use $E_\nu = 20$ GeV, $\Delta m_{31}^2 = 3\times10^{-3}$ eV^2, $\tan^2\theta_{23} = 1$, $\Delta m_{21}^2 = 10^{-4}$ eV^2, $\tan^2\theta_{12} = 1$, and $s_{13} = 0.2$ or 0. For the new physics parameters, we take $|\epsilon_{e\mu}^s| = 10^{-3}$. To isolate the mat-
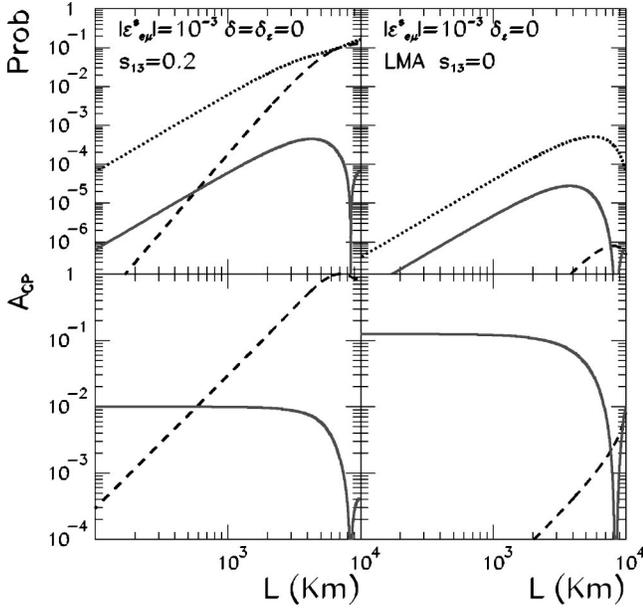
FIG. 3. Transition probabilities and fake *CP* asymmetries in matter as a function of the distance. All *CP*-violating phases are set to zero. In the upper panels the curves correspond to $P_+^{\rm SM}$ (dotted), $(P_-^m)^{\rm SM}$ (dashed) and $(P_-^m)^{\rm NP}$ (solid). In the lower panels the curves correspond to $(A_{CP}^m)^{\rm NP}$ (solid) and $(A_{CP}^m)^{\rm SM}$ (dashed). In the left panels $s_{13}=0.2$, and in the right panels $s_{13}=0$. In all curves $E_\nu = 20$ GeV, $\Delta m_{31}^2 = 3\times10^{-3}$ eV^2, $\tan^2\theta_{23}=1$, $\Delta m_{21}^2 = 10^{-4}$ eV^2, $\tan^2\theta_{12}=1$, $\delta=\delta_\epsilon=0$, and $|\epsilon_{e\mu}^s|=10^{-3}$.

ter effects we now, however, switch off all genuine *CP* violation, that is, we take $\delta=\delta_\epsilon=0$ in both cases.

The results of this calculation are presented in Fig. 3. The left panels correspond to the first case (large s_{13}) and the right ones to the second (vanishing s_{13}). For each case we present, as a function of the distance between the source and the detector, P_+ (dotted line), $(P_-^m)^{\rm SM}$ and $(A_{CP}^m)^{\rm SM}$ (dashed lines in, respectively, upper and lower panels), and $(P_-^m)^{\rm NP}$ and $(A_{CP}^m)^{\rm NP}$ (solid lines in, respectively, upper and lower panels).

We learn a few interesting facts:

(i) The new physics contribution to the fake *CP* violation can dominate over the standard contribution for values of $|\epsilon|$ as small as 10^{-4}. This is particularly valid for distances shorter than 500 km.

(ii) As anticipated from our approximate expressions, for short enough distances $(P_-^m)^{\rm NP}$ grows quadratically with distances and $(A_{CP}^m)^{\rm NP}$ is independent of the distance.

(iii) Both the standard contribution and the new contribution to P_-^m are suppressed by a small s_{13}. The s_{13} suppression is however stronger for P_+ than it is for $(P_-^m)^{\rm NP}$. Consequently, the new physics contribution to $(A_{CP}^m)^{\rm NP}$ becomes very large for vanishing s_{13}.

In reality, the measured P_- and A_{CP} will be affected by both genuine *CP*-violating contributions and matter-induced contributions. This situation is illustrated in Fig. 4. We present P_+ (dotted curve), $P_-^{\rm SM}$ and $A_{CP}^{\rm SM}$ (dashed curves in, respectively, upper and lower panels), and $P_-^{\rm NP}$ and $A_{CP}^{\rm NP}$ (solid curves in, respectively, upper and lower panels), as a

function of the distance. For the neutrino parameters, we always take $\Delta m_{31}^2 = 3\times10^{-3}$ eV^2 and $\tan\theta_{23}=1$, consistent with the atmospheric neutrino data. For the other parameters, we take three cases: (a) Left panel: we take the LMA parameters ($\Delta m_{21}^2 = 10^{-4}$ eV^2 and $\tan\theta_{12}=1$), "large" $s_{13}=0.2$ and maximal phase $\delta=\pi/2$. This choice of parameters gives maximal standard *CP* violation. For the new physics parameters we take $|\epsilon_{e\mu}^s|=10^{-3}$ and $\delta_\epsilon=0$. (The reason for the choice of phase is that the dominant contributions depend on $\delta-\delta_\epsilon$.) (b) Middle panel: we take the small mixing angle (SMA) parameters ($\Delta m_{21}^2 = 5.2\times10^{-6}$ eV^2 [26,27], $\tan^2\theta_{12} = 7.5\times10^{-4}$), $s_{13}=0.2$, $\delta=\pi/2$, $|\epsilon_{e\mu}^s|=10^{-3}$, and $\delta_\epsilon=0$. Here the standard *CP* violation is unobservably small, but the standard matter effects are still large. (c) Right panel: we take the LMA parameters and $s_{13}=0$. With a vanishing s_{13}, the total transition probability is highly suppressed as is the standard matter effect, and standard *CP* violation vanishes. For the new physics parameters we take $|\epsilon_{e\mu}^s|=10^{-4}$ and $\delta_\epsilon=\pi/2$. We take a smaller $|\epsilon_{e\mu}^s|$ so that our approximation will not break down.

We would like to emphasize the following points:

(i) Similar three cases will be the basis, in the next section, for our analysis of the sensitivity of *CP*-violating observables measured in neutrino factories to new physics effects (see Fig. 5).

(ii) With large s_{13}, the dependence of the new physics effects (and of the standard matter-induced effects) on the solar neutrino parameters is very weak.

(iii) A small or even vanishing s_{13} will suppress all the rates and will introduce a strong dependence on the solar neutrino parameters. The new physics contributions to A_{CP} will be, however, only slightly affected, because both the standard *CP* conserving rate and the new physics *CP*-violating rate are suppressed in the same way.

(iv) With large s_{13}, the new physics *CP*-violating effects are dominated by the combination $\delta-\delta_\epsilon$. With small (but not vanishing) s_{13}, the dependence is on both $\delta-\delta_\epsilon$ and δ_ϵ.

(v) For distances shorter than 800 km, the effects of $|\epsilon| \gtrsim 10^{-3}$ are always dominant. For distances shorter than 300 km, the new physics dominates even for $|\epsilon|\sim10^{-4}$.

VII. LONG-BASELINE EXPERIMENTS

We would like to quantify the sensitivity of a neutrino factory to the *CP*-violating effects from new neutrino interactions. For this purpose, we consider the measurement of the following integrated asymmetry [5]:

$$\overline{A_{CP}} = \frac{N[\mu^-]/N_0[e^-]|_+ - N[\mu^+]/N_0[e^+]|_-}{N[\mu^-]/N_0[e^-]|_+ + N[\mu^+]/N_0[e^+]|_-}. \quad (7.1)$$

Here $N[\mu^-]/N_0[e^-]|_+$ refers to an oscillation experiment that has μ^+ decay as its production process: $N[\mu^-]$ is the measured number of wrong-sign muons while $N_0[e^-]$ is the expected number of ν_e charge current (CC) interaction events (in the absence of oscillations). Similarly, $N[\mu^+]/N_0[e^+]|_-$ refers to an oscillation experiment that has μ^- decay as its production process: $N[\mu^+]$ is the mea-
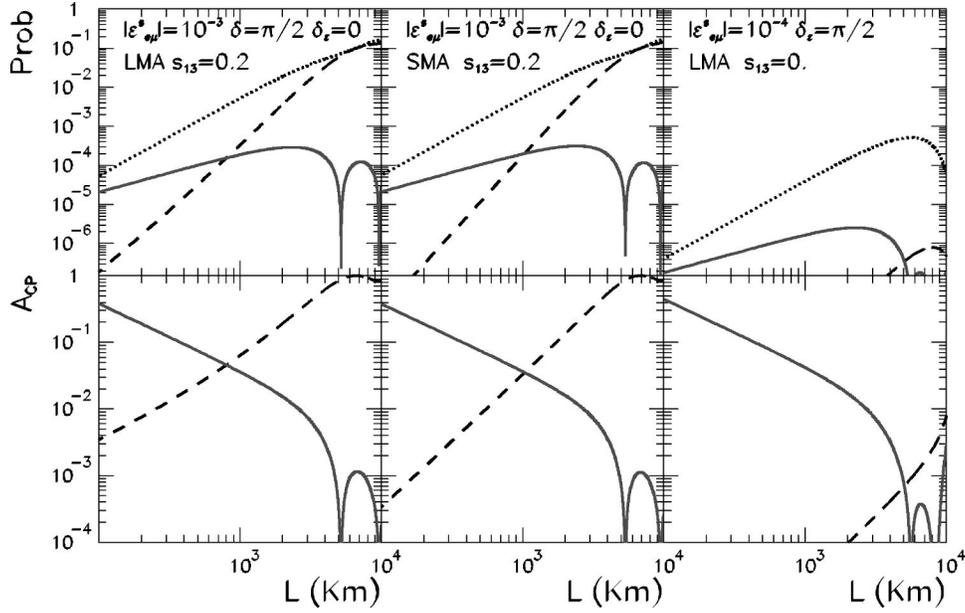
FIG. 4. Transition probabilities and fake CP asymmetries in matter as a function of the distance. In the upper panels the curves correspond to P_+^{SM} (dotted), P_-^{SM} (dashed) and P_-^{NP} (solid). In the lower panels the curves correspond to A_{CP}^{NP} (solid) and A_{CP}^{SM} (dashed). In all curves $E_\nu = 20$ GeV, $\Delta m_{31}^2 = 3 \times 10^{-3}$ eV^2, and $\tan^2 \theta_{23} = 1$. In the left panels $\Delta m_{21}^2 = 10^{-4}$ eV^2, $\tan^2 \theta_{12} = 1$, $s_{13} = 0.2$, $\delta = \pi/2$, $|\epsilon_{e\mu}^s| = 10^{-3}$, and $\delta_\epsilon = 0$. In the middle panels $\Delta m_{21}^2 = 5.2 \times 10^{-6}$ eV^2, $\tan^2 \theta_{12} = 7.5 \times 10^{-4}$, $s_{13} = 0.2$, $\delta = \pi/2$, $|\epsilon_{e\mu}^s| = 10^{-3}$, and $\delta_\epsilon = 0$. In the right panels $\Delta m_{21}^2 = 10^{-4}$ eV^2, $\tan \theta_{12} = 1$, $s_{13} = 0$, $|\epsilon_{e\mu}^s| = 10^{-4}$, and $\delta_\epsilon = \pi/2$.

sured number of wrong-sign muons while $N_0[e^+]$ is the expected number of $\overline{\nu_e}$ CC interaction events (again, in the absence of oscillations). The measured number of wrong-sign muon events can be expressed as

$$N[\mu^-]\big|_+ = \frac{N_\mu N_T}{\pi m_\mu^2} \frac{E_\mu}{L^2} \int dE_\nu f_\nu(E_\nu) \sigma_{CC}(E_\nu) P_{e\mu}(E_\nu),$$

(7.2)

where N_T is the number of protons in the target detector, N_μ is the number of useful muon decays, E_μ is the muon energy, and m_μ is the muon mass. The function $f_\nu(E_\nu)$ is the energy distribution of the produced neutrinos. We assume that the muons are not polarized, in which case $f_\nu(E_\nu) = 12x^2(1-x)$ with $x = E_\nu/E_\mu$. Finally, $\sigma_{CC}(E_\nu)$ is the neutrino-nucleon interaction cross section which, in the interesting range of energies, can be taken to be proportional to the neutrino energy: $\sigma_{CC} = \sigma_0 E_\nu$ with $\sigma_0 = 0.67 \times 10^{-38}$ cm^2/GeV for neutrinos and $\sigma_0 = 0.34 \times 10^{-38}$ cm^2/GeV for antineutrinos. The expression for $N[e^-]\big|_+$ is obtained by an integral similar to Eq. (7.2), except that $P_{e\mu}$ is replaced by 1.

We define A_{CP}^{NP} as the contribution from new physics (that is, ϵ-dependent) to the integrated CP asymmetry. We take into account both genuine CP-violating and matter-induced contributions. [In the limit of a real lepton mixing matrix, that is, no standard CP violation, the first contributions are proportional to Im(ϵ) and the latter to Re(ϵ).] We define ΔA to be the statistical error on $\overline{A_{CP}}$. In order to quantify the significance of the signal due to new physics, we compute the ratio $\overline{A_{CP}^{NP}}/\Delta A$.

The statistical error ΔA scales with distance and energy as follows:

$$\Delta A \simeq \frac{1}{\sqrt{N[\mu^+]\big|_- + N[\mu^-]\big|_+}} \propto \frac{1}{\sqrt{P_+^{SM} N_{CC}}} \propto \frac{1}{\sqrt{E_\nu}}.$$

(7.3)

To find this scaling, we took into account that the number of CC interactions scales as $N_{CC} \propto E_\nu^3/L^2$ while, for $L \lesssim 3000$ km, $P_+^{SM} \propto L^2/E_\nu^2$. Consequently, the dependence of ΔA on the distance is very weak. Given our results for A_{CP}^{NP}, we obtain the following scaling with distance of the signal-to-noise ratio:

$$\overline{A_{CP}^{NP}}/\Delta A \propto \begin{cases} 1/L & \text{genuine } CP\text{-violating effects} \\ \text{const}(L) & \text{matter induced effects.} \end{cases}$$

(7.4)

This behavior is illustrated in Fig. 5 where we display the signal-to-noise ratio $\overline{A_{CP}^{NP}}/\Delta A$ as a function of the distance. For simplicity, we consider only the effect of $\epsilon_{e\mu}^s$. The standard CP violation is presented only in the upper panel, where it corresponds to maximal A_{CP}^{SM} (LMA parameters are $\Delta m_{21}^2 = 10^{-4}$ eV^2 and $\tan \theta_{12} = 1$, large s_{13} and $\delta = \pi/2$), while the middle panel has unobservably small A_{CP}^{SM} (SMA parameters are $\Delta m_{21}^2 = 5.2 \times 10^{-6}$ eV^2 and $\tan^2 \theta_{12} = 7.5 \times 10^{-4}$), and the lower panel has zero A_{CP}^{SM} ($s_{13} = 0$). As concerns the new CP violation, the dashed line corresponds to the case with maximal CP-violating phase ($\delta_\epsilon = \pi/2$) and the solid line corresponds to purely matter-induced asymme-
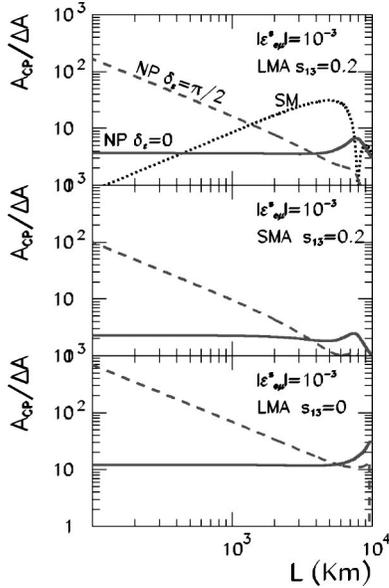
FIG. 5. The signal-to-noise ratio $\overline{A_{CP}^{\mathrm{NP}}}/\Delta A$ as a function of the distance L. We considered the following parameters for the experiment: $E_\mu = 50$ GeV, 10^{21} μ^- decays and a 40-kt detector, and the neutrino parameters $\delta = 0$, $\Delta m_{31}^2 = 3 \times 10^{-3}$ eV^2, $\tan \theta_{23} = 1$. In the upper and lower (middle) panels we use the LMA (SMA) parameters. In the upper two (lower) panels we use $s_{13} = 0.2(0)$. For the new physics we take $|\epsilon_{e\mu}^s| = 10^{-3}$ and $\delta_\epsilon = 0$ or $\pi/2$. In the upper panel, the dotted curve gives the SM matter-subtracted asymmetry $A_{CP}^{\mathrm{SM}}(\delta = \pi/2) - A_{CP}^{\mathrm{SM}}(\delta = 0)$.

try ($\delta_\epsilon = 0$). In our calculations we have assumed a total of 10^{21} useful μ^- decays with energy $E_\mu = 50$ GeV and a 40-kt detector.

It is clear from the figure that the maximal sensitivity to new, *CP*-violating contributions to the production or detection processes will be achieved with shorter distances, while the sensitivity to *CP* conserving contributions through matter induced effects is almost independent of distance.

A truly short baseline experiment can potentially probe the $\mathcal{O}(|\epsilon|^2)$ *CP* conserving effects. But in this case, due to the small signal, systematic errors will dominate over the statistical ones discussed above. It is unlikely that $|\epsilon|$ smaller than $\mathcal{O}(10^{-3})$ can be signaled in such a measurement.

We next investigate the sensitivity to the size of the new physics interaction that can be achieved by the measurement of the integrated *CP* asymmetry. In Fig. 6, we show the regions in the $[\mathrm{Re}(\epsilon_{e\mu}^s), \mathrm{Im}(\epsilon_{e\mu}^s)]$ plane that will lead to $\overline{A_{CP}^{\mathrm{NP}}}/\Delta A = 3$ (darker-shadow region) and $\overline{A_{CP}^{\mathrm{NP}}}/\Delta A = 1$ (lighter-shadow regions) at $L = 732$ km, the shorter baseline discussed for an oscillation experiment at a neutrino factory. We have assumed a total of 10^{21} useful μ^- decays with energy $E_\mu = 50$ GeV and a 40-kt detector. In all panels we have $\delta = 0$ (no standard *CP* violation), $\Delta m_{31}^2 = 3 \times 10^{-3}$ eV^2 and $\tan \theta_{23} = 1$, and the LMA parameters $\Delta m_{21}^2 = 10^{-4}$ eV^2, and $\tan \theta_{12} = 1$. In the left panels we have $s_{13} = 0.2$ and in the right ones $s_{13} = 0$. In the upper panels $\mathrm{Im}(\epsilon_{e\mu}^s) > 0$, which, for our choice of parameters, results in a constructive interference between the matter-induced and

CP-violating effects, while in the lower panels $\mathrm{Im}(\epsilon_{e\mu}^s) < 0$, which results in a destructive interference.

In order to illustrate the expected improvement in sensitivity to the new physics when the baseline is better optimized for this particular purpose, we plot in Fig. 7 the corresponding regions when the measurement of the integrated *CP* asymmetry is performed at a distance of $L = 200$ km.

We would like to emphasize the following three points:

(i) Figure 6 shows that $|\epsilon|$ in the range $3 \times 10^{-5} - 10^{-4}$ would lead to a "3σ" effect.

(ii) A shorter distance will improve the sensitivity to the new *CP* violation. Figure 7 shows that, for $\delta = 0$, in which case *CP*-violating effects are proportional to $\mathrm{Im}(\epsilon)$, an improvement by a factor of about 3 in the sensitivity to $\mathrm{Im}(\epsilon)$ is expected. In contrast, the sensitivity to $\mathrm{Re}(\epsilon)$ is not affected by the choice of baseline since the new physics contribution to the matter-induced asymmetry is independent of L.

(iii) A nonvanishing standard *CP*-violating phase, $\delta \neq 0$, together with a "large" s_{13}, will change the interference pattern between the matter-induced and *CP*-violating contributions from new physics. The reason is that now some of the contributions depend on $\delta_\epsilon - \delta$, so that $\mathrm{Re}(\epsilon)$ and $\mathrm{Im}(\epsilon)$ do not correspond to matter-induced and *CP*-violating effects in any simple way.

VIII. PHENOMENOLOGICAL CONSTRAINTS

The measurements of $P_{e\mu}$ and $P_{\bar{e}\bar{\mu}}$ are sensitive to the four effective couplings, $\epsilon_{e\mu}^s$, $\epsilon_{e\tau}^s$, $\epsilon_{\mu e}^d$ and $\epsilon_{\mu\tau}^d$. These dimensionless couplings represent new flavor-changing (FC) neutrino interactions. They are subject to various phenomenological constraints. In this section, we present these bounds in order to compare them with the experimental sensitivity that we estimated in the previous section.

Before we present the bounds, we would like to clarify a subtlety that concerns the ϵ^s couplings. Each of these couplings stands for several different processes. Specifically, $\epsilon_{e\mu}^s$ gives the amplitude for $\mu^+ \to e^+ \nu_\mu \bar{\nu}_\sigma$ decays with $\sigma = e, \mu, \tau$ and, similarly, $\epsilon_{e\tau}^s$ gives the amplitude for $\mu^+ \to e^+ \nu_\tau \bar{\nu}_\sigma$ decays with $\sigma = e, \mu, \tau$. The index σ is irrelevant for the analysis of $P_{e\mu}$ ($P_{\bar{e}\bar{\mu}}$), where we are only interested in the neutrino (antineutrino) interactions. This is the reason that we did not distinguish between the three possible production processes for each of the ϵ^s's. Most of the bounds that we discuss below do, however, depend on σ. It is important to understand that it is the *weakest* of the bounds which applies model independently.

We consider three types of upper bounds:

(i) There is a generic bound of $\mathcal{O}(0.1)$ on the purely leptonic couplings $\epsilon_{\alpha\beta}^s$ from universality in lepton decays and a somewhat weaker bound of $\mathcal{O}(0.2)$ on the semihadronic couplings $\epsilon_{\alpha\beta}^d$ from universality in pion decays [36]. While universality is experimentally confirmed to high accuracy, these bounds are rather weak because deviations from universality are $\mathcal{O}(\epsilon^2)$.

(ii) By $SU(2)_L$ symmetry, the couplings are related to FC charged lepton interactions. The latter have not been ob-
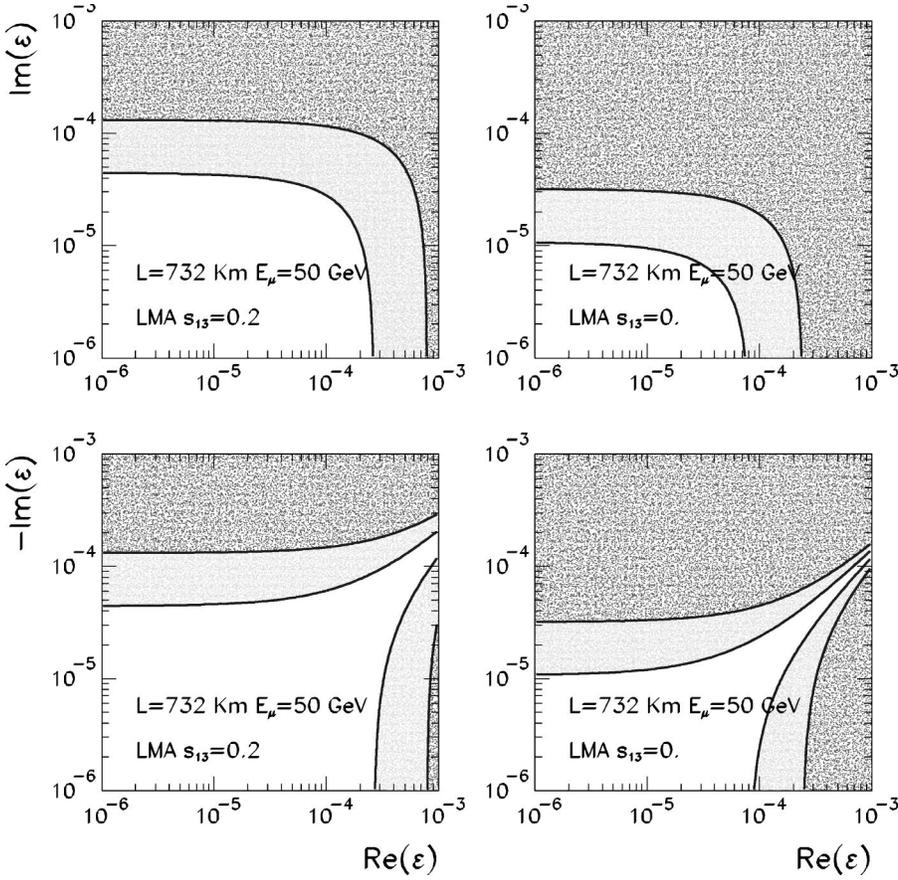
FIG. 6. Regions in the plane of $[\mathrm{Re}(\epsilon_{e\mu}^{s}), \mathrm{Im}(\epsilon_{e\mu}^{s})]$ that give $A_{CP}^{\mathrm{NP}}/\Delta A = 3$ (darker shadow) and 1 (light shadow). For the experiment, we take $L = 732$ km, $E_{\mu} = 50$ GeV, 10^{21} μ^{-} decays and a 40-kt detector. For the neutrino parameters, we take $\delta = 0$, $\Delta m_{31}^{2} = 3 \times 10^{-3}$ eV^2, $\tan^{2}\theta_{23} = 1$, $\Delta m_{21}^{2} = 10^{-4}$ eV^2, and $\tan^{2}\theta_{12} = 1$. In the left (right) panels we have $s_{13} = 0.2$ 0).

served, and there are experimental constraints on their strength. There could be $SU(2)_{L}$ breaking effects that would somewhat enhance the neutrino couplings with respect to the corresponding charged lepton couplings. These effects are discussed in detail in Refs. [37,36] where it is shown that they are constrained (by electroweak precision data) to be small. Since our purpose is only to obtain order-of-magnitude estimates of the bounds, we neglect the possible $SU(2)_{L}$ breaking effects.

(iii) For some cases, the ϵ^{s} coupling contributes at the loop level to the $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ decays. The question of how to extract reliable bounds from loop processes in an effective theory involves many subtleties. A calculation in the spirit of Ref. [38] yields very weak bounds. Instead, we quote below the bounds in specific full high energy models. We emphasize however that, in contrast to the bounds from $SU(2)_{L}$ related charged lepton tree-level decays, the bounds that we quote for the loop processes may be violated in models other than the ones that we consider.

The $\epsilon_{e\mu}^{s}$ coupling gives the amplitude for $\mu^{-} \rightarrow e^{-} \bar{\nu}_{\mu} \nu_{\sigma}$ decays with $\sigma = e, \mu, \tau$. For $\sigma = e$, there is a bound from muonium-antimuonium oscillations [37]:

$$|\epsilon_{e\mu}^{s}| \lesssim 3.0 \times 10^{-3} \quad (\sigma = e). \tag{8.1}$$

For $\sigma = \tau$, we derive a bound from the $\tau^{+} \rightarrow e^{-} \mu^{+} \mu^{+}$ decay:

$$|\epsilon_{e\mu}^{s}| \lesssim 2.9 \times 10^{-3} \quad (\sigma = \tau). \tag{8.2}$$

For $\sigma = \mu$, there is no $SU(2)_{L}$-related tree-level charged lepton decay. Instead, by closing the neutrino lines into a loop, the four-Fermi coupling contributes to the $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ decays. We quote here the bound in a specific full high energy model: if the effective $\mu_{L} \bar{e}_{L} \nu_{\mu} \bar{\nu}_{\mu}$ coupling is induced by an intermediate scalar triplet, the constraint from the $\mu \rightarrow e\gamma$ decay reads (see, for example, Ref. [39])

$$|\epsilon_{e\mu}^{s}| \lesssim 5 \times 10^{-5} \quad (\sigma = \mu). \tag{8.3}$$

We emphasize again that the bound in Eq. (8.3) is model dependent, in contrast to those of Eqs. (8.1) and (8.2).

The $\epsilon_{e\tau}^{s}$ coupling gives the amplitude for $\mu^{-} \rightarrow e^{-} \bar{\nu}_{\tau} \nu_{\sigma}$ decays with $\sigma = e, \mu, \tau$. For $\sigma = e$, there is a bound from the $\tau^{-} \rightarrow \mu^{+} e^{-} e^{-}$ decay [37]:

$$|\epsilon_{e\tau}^{s}| \lesssim 2.9 \times 10^{-3} \quad (\sigma = e). \tag{8.4}$$

For $\sigma = \mu$, we derive a bound from the $\tau^{-} \rightarrow \mu^{+} \mu^{-} e^{-}$ decay:

$$|\epsilon_{e\tau}^{s}| \lesssim 3.1 \times 10^{-3} \quad (\sigma = \mu). \tag{8.5}$$

For $\sigma = \tau$ there is no $SU(2)_{L}$-related tree level charged lepton decay, but there is a direct one-loop contribution to $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$. We again quote the bound in a specific
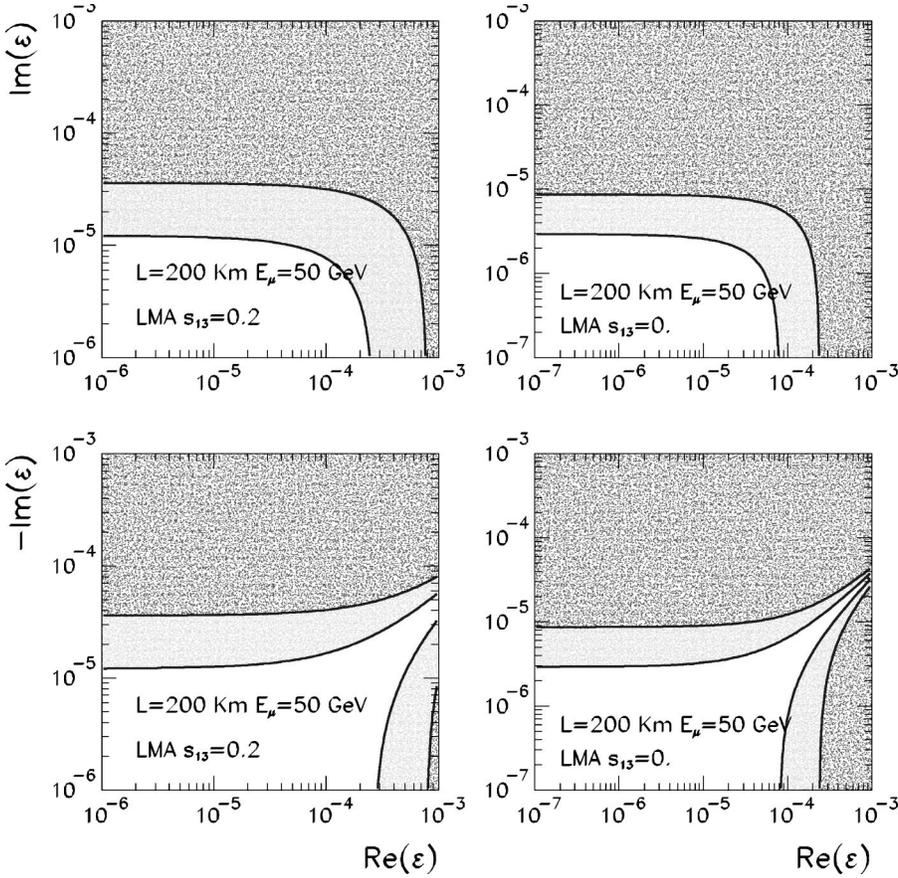
FIG. 7. Regions in the plane of $[\mathrm{Re}(\epsilon_{e\mu}^{s}), \mathrm{Im}(\epsilon_{e\mu}^{s})]$ that give $A_{CP}^{\mathrm{NP}}/\Delta A = 3$ (darker shadow) and 1 (light shadow). For the experiment, we take $L = 200$ km, $E_{\mu} = 50$ GeV, 10^{21} μ^{-} decays and a 40-kt detector. For the neutrino parameters, we take $\delta = 0$, $\Delta m_{31}^{2} = 3 \times 10^{-3}$ eV^{2}, $\tan^{2}\theta_{23} = 1$, $\Delta m_{21}^{2} = 10^{-4}$ eV^{2}, and $\tan^{2}\theta_{12} = 1$. In the left (right) panels we have $s_{13} = 0.2$ (0). Note that the scales in the right panels are different from the left panels and from Fig. 6.

full high energy model: if the effective $\mu_{L}\overline{e_{L}}\nu_{\tau}\overline{\nu_{\tau}}$ coupling is induced by an intermediate scalar singlet, the constraint from the $\mu \to e\gamma$ decay reads (see, for example, Refs. [40,41])

$$|\epsilon_{e\tau}^{s}| \lesssim 3.5 \times 10^{-4} \quad (\sigma = \tau). \tag{8.6}$$

Note that, within the effective theory, the contributions to $\mu \to e\gamma$ from $\epsilon_{e\mu}^{s}$ of Eq. (8.3) and $\epsilon_{e\tau}^{s}$ of Eq. (8.6) are equal. The factor of $\mathcal{O}(7)$ difference in the respective bounds [which reflects a ratio of $\mathcal{O}(50)$ between the contributions to the rate] demonstrates their model dependence.

The $\epsilon_{\mu e}^{d}$ coupling gives the amplitude for $\nu_{e}d \to \mu^{-}u$. It is constrained by muon conversion [37]:

$$|\epsilon_{\mu e}^{d}| \lesssim 2.1 \times 10^{-6}. \tag{8.7}$$

The $\epsilon_{\mu\tau}^{d}$ coupling gives the amplitude for $\nu_{\tau}d \to \mu^{-}u$. It is constrained by the $\tau^{-} \to \mu^{-}\rho$ decay [36]:

$$|\epsilon_{\mu\tau}^{d}| \lesssim 10^{-2}. \tag{8.8}$$

The bound on $|\epsilon_{\mu\tau}^{d}|$ is the weakest that we obtain. Moreover, it is not unlikely that it is indeed the largest of the couplings since it is the only one not to involve a first-generation lepton. For precisely the same reason, however, its contribution

to $P_{e\mu}$ is suppressed by an additional power of $|U_{e3}|$, which is the reason that it is omitted in our approximate expressions.

To summarize, we expect that all the ϵ's that play a role in the transition probabilities of interest are of $\mathcal{O}(10^{-3})$ or smaller. In Sec. VII, we learned that proposed experiments might probe these couplings down to values as small as $\mathcal{O}(10^{-4})$. This means that the possibility to measure new neutrino interactions through *CP* violation in neutrino oscillation experiments is open. Conversely, such future experiments can improve the existing bounds on FC neutrino interactions which, at present, come from rare charged lepton decays.

IX. CONCLUSIONS AND DISCUSSION

We summarize the main points of our study:

(i) *CP*-violating observables are particularly sensitive to new physics. The reason is that the standard *CP* violation that comes from the lepton mixing matrix gives effects that are particularly suppressed by small mass differences and mixing angles. Some of these suppression factors do not apply to new contributions.

(ii) The fact that matter effects contribute to *CP*-violating observables means that these observables are sensitive to both the *CP* conserving and *CP*-violating contributions from new physics.

(iii) The effects of new physics in the production and detection processes depend on the source-detector distance in a way that is different from the standard one. One consequence of this situation is that, at least in principle, it is possible to disentangle standard and new effects. Another consequence is that in short distance experiments the new effects are enhanced.

(iv) Our rough estimate is that future neutrino factories will be able to probe, through CP-violating observables, effects from new interactions that are up to about four orders of magnitude weaker than the weak interactions.

(v) The sensitivity to new physics effects is better than most of the existing model-independent bounds.

We would like to mention that a similar (and, for specific models, even stronger) level of sensitivity may be achieved by other experiments that search for lepton flavor violation. Particularly promising are those involving muon decay and conversion (for a recent review, see Ref. [46]): for example, a future experiment at PSI will be sensitive to $B(\mu \to e \gamma)$ at the 10^{-14} level [47], and the MECO Collaboration has proposed an experiment to probe $\mu - e$ conversion down to 5×10^{-17}, four orders of magnitude beyond present sensitivities [48]. If these experiments observe a signal, the search for related CP violation will become of particular importance.

What type of new physics will be implied in case that a signal is observed? The ϵ couplings represent effective four-fermion interactions coming from the exchange of heavy particles related to new physics. If the new physics takes place at some high scale Λ_{NP}, then one can set an upper bound:

$$\epsilon_{\alpha\beta}^{s,d} \lesssim \frac{m_Z^2}{\Lambda_{NP}^2}. \qquad (9.1)$$

The source of this bound is in the definition of ϵ, which is the ratio of the four-fermion operator to G_F, and the fact that it is maximal when the new physics contribution comes at tree level and the couplings are of order 1. Since the expected experimental sensitivity is to $|\epsilon| \gtrsim \mathcal{O}(10^{-4})$, we learn that we can probe models with

$$\Lambda_{NP} \lesssim 10 \text{ TeV}. \qquad (9.2)$$

If the new physics contributes to the relevant processes only at the loop level, there is another suppression factor in $|\epsilon|$ of order $1/16\pi^2$. This would mean that such models can be probed only if $\Lambda_{NP} \lesssim 1$ TeV. Finally, if the flavor changing nature of the interaction introduces a suppression factor, e.g., $|\epsilon_{e\mu}^s| \sim m_\mu / \Lambda_{NP}$, that by itself would be enough to make it unobservable in near future experiments. We thus learn that CP violation in neutrino oscillation experiments will explore models with a scale that is, at most, 1–2 orders of magnitude above the electroweak breaking scale, and where the flavor structure is different from the standard model.

Another point concerns the Dirac structure of the four-Fermi interaction. We did not present this explicitly in our discussion of the $G_{NP}^{s,d}$ couplings. However, it is implicitly assumed in our discussion that the Dirac structure is the same as that of the weak interactions, i.e., a $(V\text{-}A)(V\text{-}A)$ structure. The reason for that is that the effects that we discuss are a consequence of interference between weak and new interactions. A different Dirac structure would give strong suppression factors related to the charged lepton masses. While our formalism would still apply, these suppression factors would make the related effects practically unobservable.

We conclude that a signal is likely to imply new physics at a relatively low scale (up to 1–10 TeV) with new sources of flavor (and, perhaps, CP) violation. We know of several well motivated extensions of the standard model that can, in principle, induce large enough couplings. In particular, we have in mind loop contributions involving sleptons and gauginos in supersymmetric models, tree contributions involving charged singlet sleptons in supersymmetric models without R parity, and tree contributions involving a triplet scalar in left-right symmetric models. In another class of relevant models, such as the model of Ref. [42], active neutrinos mix with singlet neutrinos. (Here there can be Z-mediated contributions to the non-standard couplings, and the phenomenological constraints are different [43,44].) A detailed analysis of new neutrino interactions within relevant extensions of the standard model is beyond the scope of this paper, but preliminary results show that large enough couplings are allowed and in some cases even predicted [45].

APPENDIX: TRANSITION PROBABILITY IN VACUUM

Neglecting terms of $\mathcal{O}(\epsilon^2)$ and with no other approximations, we obtain the following expression for $P_{e\mu}$:

$$\begin{aligned}
P_{e\mu} =\ & 4\sin^2 x_{21}\{|U_{\mu2}|^2|U_{e2}|^2 - \mathrm{Re}[\,\epsilon^s_{e\mu}(U^*_{e1}U_{\mu1}|U_{\mu2}|^2 + U^*_{e2}U_{\mu2}|U_{\mu1}|^2) + \epsilon^d_{\mu e}(U^*_{\mu2}U_{e2}|U_{e1}|^2 + U^*_{\mu1}U_{e1}|U_{e2}|^2) \\
& + \epsilon^s_{e\tau}(U^*_{e2}U_{\mu2}U^*_{\mu1}U_{\tau1} + U^*_{e1}U_{\mu1}U^*_{\mu2}U_{\tau2}) + \epsilon^d_{\mu\tau}(U^*_{\mu2}U_{e2}U^*_{e1}U_{\tau1} + U^*_{\mu1}U_{e1}U^*_{e2}U_{\tau2}) - U^*_{e2}U_{e3}U^*_{\mu3}U_{\mu2}]\} \\
& + 2\sin 2x_{21}\,\mathrm{Im}[\,\epsilon^s_{e\mu}(U^*_{e1}U_{\mu1}|U_{\mu2}|^2 - U^*_{e2}U_{\mu2}|U_{\mu1}|^2) + \epsilon^d_{\mu e}(U^*_{\mu2}U_{e2}|U_{e1}|^2 - U^*_{\mu1}U_{e1}|U_{e2}|^2) \\
& + \epsilon^s_{e\tau}(U^*_{e2}U_{\mu2}U^*_{\mu1}U_{\tau1} - U^*_{e1}U_{\mu1}U^*_{\mu2}U_{\tau2}) + \epsilon^d_{\mu\tau}(U^*_{\mu2}U_{e2}U^*_{e1}U_{\tau1} - U^*_{\mu1}U_{e1}U^*_{e2}U_{\tau2}) + U^*_{e2}U_{e3}U^*_{\mu3}U_{\mu2}] \\
& + 4\sin^2 x_{31}\{|U_{\mu3}|^2|U_{e3}|^2 - \mathrm{Re}[\,\epsilon^s_{e\mu}(U^*_{e1}U_{\mu1}|U_{\mu3}|^2 + U^*_{e3}U_{\mu3}|U_{\mu1}|^2) + \epsilon^d_{\mu e}(U^*_{\mu3}U_{e3}|U_{e1}|^2 + U^*_{\mu1}U_{e1}|U_{e3}|^2) \\
& + \epsilon^s_{e\tau}(U^*_{e1}U_{\mu1}U^*_{\mu3}U_{\tau3} + U^*_{e3}U_{\mu3}U^*_{\mu1}U_{\tau1}) + \epsilon^d_{\mu\tau}(U^*_{\mu3}U_{e3}U^*_{e1}U_{\tau1} + U^*_{\mu1}U_{e1}U^*_{e3}U_{\tau3}) - U^*_{e2}U_{e3}U^*_{\mu3}U_{\mu2}]\} \\
& + 2\sin 2x_{31}\,\mathrm{Im}[\,\epsilon^s_{e\mu}(U^*_{e1}U_{\mu1}|U_{\mu3}|^2 - U^*_{e3}U_{\mu3}|U_{\mu1}|^2) + \epsilon^d_{\mu e}(U^*_{\mu3}U_{e3}|U_{e1}|^2 - U^*_{\mu1}U_{e1}|U_{e3}|^2) + \epsilon^s_{e\tau}(U^*_{e1}U_{\mu1}U^*_{\mu3}U_{\tau3} \\
& - U^*_{e3}U_{\mu3}U^*_{\mu1}U_{\tau1}) + \epsilon^d_{\mu\tau}(U^*_{\mu3}U_{e3}U^*_{e1}U_{\tau1} - U^*_{\mu1}U_{e1}U^*_{e3}U_{\tau3}) - U^*_{e2}U_{e3}U^*_{\mu3}U_{\mu2}] - 4\sin^2 x_{32} \\
& \times \mathrm{Re}[\,\epsilon^s_{e\mu}(U^*_{e2}U_{\mu2}|U_{\mu3}|^2 + U^*_{e3}U_{\mu3}|U_{\mu2}|^2) + \epsilon^d_{\mu e}(U^*_{\mu3}U_{e3}|U_{e2}|^2 + U^*_{\mu2}U_{e2}|U_{e3}|^2) + \epsilon^s_{e\tau}(U^*_{e2}U_{\mu2}U^*_{\mu3}U_{\tau3} \\
& + U^*_{e3}U_{\mu3}U^*_{\mu2}U_{\tau2}) + \epsilon^d_{\mu\tau}(U^*_{\mu3}U_{e3}U^*_{e2}U_{\tau2} + U^*_{\mu2}U_{e2}U^*_{e3}U_{\tau3}) + U^*_{e2}U_{e3}U^*_{\mu3}U_{\mu2}] + 2\sin 2x_{32} \\
& \times \mathrm{Im}[\,\epsilon^s_{e\mu}(U^*_{e2}U_{\mu2}|U_{\mu3}|^2 - U^*_{e3}U_{\mu3}|U_{\mu2}|^2) + \epsilon^d_{\mu e}(U^*_{\mu3}U_{e3}|U_{e2}|^2 - U^*_{\mu2}U_{e2}|U_{e3}|^2) + \epsilon^s_{e\tau}(U^*_{e2}U_{\mu2}U^*_{\mu3}U_{\tau3} \\
& - U^*_{e3}U_{\mu3}U^*_{\mu2}U_{\tau2}) + \epsilon^d_{\mu\tau}(U^*_{\mu3}U_{e3}U^*_{e2}U_{\tau2} - U^*_{\mu2}U_{e2}U^*_{e3}U_{\tau3}) + U^*_{e2}U_{e3}U^*_{\mu3}U_{\mu2}].
\end{aligned} \tag{A1}$$

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