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Nonperturbative QCD vacuum effects in nonlocal quark dynamics

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A straightforward calculation reveals a fundamental nonlocality in the leading heavy $Q\bar{Q}$ interaction arising from nonperterbative gluon field correlations in the model of a fluctuating QCD vacuum. In light of this, quarkonium spin splitting ratio predictions which have supported the scalar confinement ansatz are reconsidered.

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I. NONLOCAL DYNAMICS

A great deal of work has gone toward the effort to describe the spectrum and high energy scattering of hadrons as bound states of quarks and gluons. It remains an open and interesting problem. Part of the difficulty derives from the historical identification of particle field theory with its familiar perturbation expansion in QED, while it is a rather nonperturbative phenomena such as confinement that in QCD plays the more prominent role. Nonperturbative phenomenology has developed over the past two decades with the aim in view of describing essential features of the fundamental field theory for a given arrangement by preserving its effective degrees of freedom and freezing or integrating out those degrees less relevant. (Thus the $Q\bar{Q}$ four point function averaged over gluon field configurations leads to the static Wilson loop [1] and simple linear confinement.) There is unfortunately no unique program by which the reduction is carried out. Two competing approaches among several are those of potential models and sum rule methods; the former assumes the form of a local interaction in terms of quark coordinates while the gluon field itself enters more directly into the latter.

Voloshin and Leutwyler separately demonstrated some time ago [2] that large scale fluctuations of the QCD vacuum are not amenable to purely local description. The finding was corroborated by the leading relativistic interaction Hamiltonian of Eichten and Feinberg [3] (see also Lemma in [4]) and subsequently refined at the hands of Marquard and Dosch [5] who considered two extreme regions for the vacuum field's correlation length relative to the correlation length of heavy quarks in a meson. Roughly, for $T_g \ll T_q$ a local description is derived, while for $T_g \gg T_q$ a nonlocality appears making the sum rule approach more suitable (additional refinements along the same lines have been made in the more recent articles of reference [6]).

Potential model builders have generally taken this statement to validate the use of local potentials when sufficiently heavy quarks are involved. This is quite right. The question of sufficiency however has nowhere in the literature been addressed. A quantitative sense of this validity in terms familiar to the language of potential models is gained by imagining the interaction Hamiltonian to be doubly expanded in the ratios of the two time scales each with a common scale, say Λ_{QCD}, appropriate to the above limits. To any specified order in T_q then (corresponding to an order in $1/m_q$) the limits now read, $T_g \to 0$ and $T_g \to \infty$, for local and nonlocal descriptions respectively. Finite $T_g \approx 1.0\ \text{GeV}^{-1}$ [7] therefore requires a generalization of the analysis in line with intermediate correlations.

This is readily carried out beginning from the Schwinger function of a singlet $Q\bar{Q}$ pair in an external color field modeled on the idea of a heavy meson [5]. In the dipole approximation [8]

$$G \sim \int [d\mathbf{x}] \exp\left[-\frac{g_s^2}{36} \int_0^T \int_0^T \mathbf{x}(t_1) \cdot \mathbf{x}(t_2) \times \langle \mathbf{E}^a(t_1) \cdot \mathbf{E}^a(t_2) \rangle_E \, dt_1 dt_2 \right] \tag{1}$$

$$\approx \int [d\mathbf{x}] \exp\left[\int_0^T dt \int_{-\infty}^t d\tau \left(\mathbf{x}^2 - \tau \mathbf{x} \cdot \frac{\mathbf{p}}{m} \right) \zeta \right] \tag{2}$$

with

$$\zeta = -\frac{g_s^2}{36} \langle \mathbf{E}^a(t) \cdot \mathbf{E}^a(t_2) \rangle_E, \qquad \tau \equiv t - t_2, \tag{3}$$

assuming the correlator falls off rapidly for large Euclidean time differences and $\mathbf{x}(t_2) \approx \mathbf{x}(t) - \tau \dot{\mathbf{x}}(t)$.

Hence the nonlocality enters into the interaction Hamiltonian of Eq. (2) at lowest order in quark motion. When the fields correlate adiabatically, e.g., corresponding to a stochastic delta correlation (or white noise), so that $\tau(\mathbf{p}/m)\zeta \approx 0$, a local potential emerges, *but only in this limit.* All other correlations lead to nonlocal dynamics, the degree of which measured by gluon degrees of freedom, as they occur in the fluctuating vacuum. For a general discussion of the phenomena in the context of the flux tube picture see Isgur [9].

A convenient parametrization for the field's statistical distribution is provided by the stochastic vacuum model [10]:

$$\zeta = -\frac{\beta}{12} \left[D(\tau) + D_1(\tau) + \tau^2 \frac{\partial D_1}{\partial \tau^2} \right] \tag{4}$$

with

$$D, D_1 \sim \exp(-|\tau|/T_g). \tag{5}$$

The model calculation estimates nonlocal contributions to Eq. (2) at $\approx 46\%$.

II. QUARKONIUM SPIN SPLITTINGS

The qualitative success of the nonrelativistic potential model with linear confinement does not entirely carry over when introduced into relativistic kinematics. This problem has long been thought to be related to the neglect in the formalism of gluon field momentum. Examples in the study of Regge behavior are found in [11]. These nonlocal effects might also be relevant to the question of the Lorentz structure of confinement. Evaluation of heavy $Q\bar{Q}$ spin splittings for a given local potential leads to a widely known argument in favor of dominant scalar confinement. The quantity of interest is the ratio of χ-state masses [12]

$$r \equiv \frac{M_2 - M_1}{M_1 - M_0} \approx 0.5(\text{expt}) \tag{6}$$

with expected values of less than 0.8 or greater than 2.0 for linear scalar coupling, ranging from 0.8 to 1.4 for the vector case. All other structures are ruled out by their incompatibility with the nonrelativistic limit, assuming a pure Lorentz source. Here the analysis is reconsidered, taking into account possible nonlocal effects.

In the conventional treatment [13] one begins with the expansion of a general interaction kernel over five Lorentz invariant amplitudes (scalar, pseudoscalar, vector, axialvector and tensor) in the $q \otimes \bar{q}$ Dirac space

$$K = V_s 1 \otimes 1 + V_{ps}\gamma^5 \otimes \gamma_5 + V_v \gamma^\mu \otimes \gamma_\mu + V_{av}\gamma^\mu\gamma^5 \otimes \gamma_\mu\gamma_5 + \tfrac{1}{2} V_t \sigma^{\mu\nu} \otimes \sigma_{\mu\nu}. \tag{7}$$

A $O(m^{-2})$ potential is then derived in the usual way by a reduction of either the transformation matrix or Bethe-Salpeter equation. Both routes however require the use of free quark propagators of whose adequacy there is some doubt particularly in the present nonperturbative [14] nonlocal [15] context. A more suitable approach might be the introduction of a nonlocal minimal coupling, e.g., via the instantaneous Salpeter equation. The question will not be gone into here. For comparison with local potential results and for the sake of simplicity we proceed in the usual way as described above. The lowest order nonlocal contribution to each amplitude in Eq. (7) in the center of momentum we here parametrize as

$$V_{nl} = \{\Theta_{ij}(\mathbf{r}), p_i p_j\}_{symm}, \tag{8}$$

with

$$\Theta_{ij}(\mathbf{r}) = \theta_1(r)\delta_{ij} + \theta_2(r)\hat{r}_i\hat{r}_j \tag{9}$$

where $\mathbf{r}$ is the relative coordinate. A similar parametrization is used in [16] where the nonlocality is proposed to resolve the outstanding small baryon splitting puzzle. Upon augmenting the above by the local funnel potential, $-\frac{4}{3}(\alpha_s/r) + ar$, and taking the simplifying, $\theta_1 = -\theta_2 \equiv \theta$, a straightforward but lengthy calculation yields

$$r_{scalar} = \frac{1}{5}\,\frac{8\alpha_s\langle r^{-3}\rangle - \frac{5}{2}a\langle r^{-1}\rangle + 5\langle R_1\rangle}{2\alpha_s\langle r^{-3}\rangle - \frac{1}{4}\ a\langle r^{-1}\rangle + \frac{1}{2}\langle R_1\rangle}, \tag{10}$$

$$r_{vector} = \frac{1}{5}\,\frac{8\alpha_s\langle r^{-3}\rangle + 7a\langle r^{-1}\rangle + \frac{5}{2}\langle \mathcal{R}_{(2-1)}\rangle}{2\alpha_s\langle r^{-3}\rangle +\ a\langle r^{-1}\rangle + \frac{1}{2}\langle \mathcal{R}_{(1-0)}\rangle}, \tag{11}$$

with R and $\mathcal{R}$ given in the Appendix. It is not possible to establish numerical ranges for these expressions analogous to those found for the case of local interactions without more information on the nonlocality itself. The analysis as it stands is indeterminate, favoring no Lorentz structure for confinement over any other.

III. SUMMARY AND DISCUSSION

The point emphasized here has been that nonperturbative gluon degrees of freedom generally arise in the hadronic bound state as nonlocal quark dynamics. This has been shown explicitly for the $Q\bar{Q}$ dipole approximation first considered in the analysis of Ref. [5], reconsidered here. A corollary is that the interaction Hamiltonian becomes nonlocal whenever quark motion is taken into account, e.g., in the leading relativistic corrections. Effects in quarkonium spin splitting ratio predictions have been evaluated using a nonlocal parametrization.

It should be noted for completeness that because the pseudoscalar, axialvector, and tensor Lorentz structures fail to reduce individually to suitable static limits, does not exclude them *a priori* from contributing at higher orders. On the contrary. The nonrelativistic limit suggests nothing beyond what might be said of itself. Hence the *local* analysis of [13] remains indeterminate also. While scalar confinement may or may not be the most simple ansatz there are in any event other possibilities: vector + pseudoscalar and vector + axialvector + tensor among other combination spin structures for a confining local interaction, each yield in acceptable ranges for the χ-triplet ratio.

What is needed of course is a better understanding of the mechanism by which nonperturbative via nonlocal forces enter into the QCD interaction and a reliable means to estimate the effect. The Wilson loop occurs naturally in gauge invariant formulations of the bound state. It (and so interactions derived from it) is manifestly nonlocal for nonstatic quarks. Its evaluation in the minimal area law, the stochastic vacuum model, and dual QCD has recently been carried out by Brambilla and Vario [17]. These are first order approximations in three mutually distinct expansions of the Wilson loop—no one contained entirely within another.

Minimal substitution of the relativistic flux tube model [18] into the linear Dirac equation [19] is an example of a promising nonlocal model with many attractive features: appropriate Regge structure and spin orbit sign, to name two. As a model however it is to be measured against both observation and the fundamental theory. On the other hand differences between evaluations of the Wilson loop within any one of the above mentioned approximations [20] is amenable to unambiguous, mathematical resolution. These points seem to have been overlooked in Ref. [21].

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APPENDIX

$$\mathcal{R}_{(2-1)} = -6R_1 - \frac{1}{5}R_2 - \frac{1}{10}R_3\left(2\frac{\partial}{\partial r} + \frac{3}{r}\right) + \frac{1}{10}R_4\left(\frac{1}{r}\frac{\partial}{\partial r} - \frac{4}{r^2}\right), \tag{A1}$$

$$\mathcal{R}_{(1-0)} = -3R_1 + \frac{1}{2}R_2 + \frac{1}{4}R_3\left(2\frac{\partial}{\partial r} + \frac{3}{r}\right) - \frac{1}{4}R_4\left(\frac{1}{r}\frac{\partial}{\partial r} - \frac{4}{r^2}\right), \tag{A2}$$

with

$$R_1 = \frac{1}{r^2}\left[\theta\left(\frac{\partial^2}{\partial r^2} + \frac{2}{3}\frac{1}{r^2}\right) + \theta'\left(\frac{\partial}{\partial r} - \frac{1}{r}\right) - \frac{1}{3}\theta''\right], \tag{A3}$$

$$R_2 = \frac{2}{r}\left[\frac{1}{r}\theta\left(3\frac{\partial^2}{\partial r^2} + \frac{5}{r}\frac{\partial}{\partial r} + \frac{8}{3}\frac{1}{r^2}\right) - 2\theta'\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{2}{3}\frac{1}{r^2}\right) + \frac{1}{3}\frac{1}{r}\theta'' + \frac{1}{3}\theta'''\right], \tag{A4}$$

$$R_3 = \frac{4}{r}\left[-\frac{1}{r}\theta\left(\frac{\partial}{\partial r} + \frac{3}{2}\frac{1}{r}\right) + \theta'\frac{\partial}{\partial r}\right], \tag{A5}$$

$$R_4 = -\frac{2}{r^2}\theta. \tag{A6}$$

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