Dynamical electroweak breaking and latticized extra dimensions

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Using gauge invariant effective Lagrangians in $1+3$ dimensions describing the standard model in $1+4$ dimensions, we explore dynamical electroweak symmetry breaking. The top quark seesaw model arises naturally, as well as the full Cabibbo-Kobayashi-Maskawa structure. We include a discussion of the effects of warping, and indicate how other dynamical schemes may also be realized.

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I. INTRODUCTION

Recently we introduced the low energy effective Lagrangian in $1+3$ dimensions for the standard model in a D dimensional Yang-Mills gauge theory [1]. Gauge fields, fermions, and Higgs scalars propagate in the bulk which is latticized [2]. The extra dimensions, when described by the transverse lattice technique [3] become a prescription for writing down an extension of the standard model in $1+3$ dimensions. As Kaluza-Klein (KK) modes are discovered, they carry a hidden copy of the gauge group they represent through the hidden local symmetry of vector mesons [4]. Thus, the enlargement of the gauge group into the bulk is realized as one climbs the KK tower. An independent approach, very similar to ours, was proposed in [5].

Our approach emphasizes the importance of a gauge invariant description of an infrared truncation of the theory. There is significant utility in mapping the D-dimensional theory into an equivalent $1+3$ theory as a model building tool. We are led to a chain of standard model gauge groups (this element of the scheme has a heritage; see e.g., [6]) and ''linking-Higgs fields'' which, in the broken phase, are the Wilson links in the extra dimensions, allowing hopping from one lattice brane to another [1,2]. These linking-Higgs fields can be viewed as a valid UV description of the extra-dimensional theory up to the quartic Landau poles of our Higgs potential, where something like a superstring phase transition probably occurs [7].

We will henceforth refer to our approach of describing the D theory with the $1+3$ dynamics as a *remodeled* extra dimensional theory. The *remodeling*, or latticization of compact extra dimensions to produce an effective $1+3$ Lagrangian with new dynamics, is in a sense the analogue of descending in supersymmetry from a full superspace action to an action in pure space-time. Just as supersymmetry acts as an organizing principle and dictates constraints on the spacetime theory, so too an extra dimensional theory *remodeled* into $1+3$ dictates a certain structure and dynamics. Moreover, we can map the physical questions we wish to address, e.g., dynamics, topology, electroweak symmetry breaking, etc., into conventional methods familiar to $1+3$ model builders. Everything is manifestly gauge invariant and renormalizable. Casting a given theory with new dynamics into *remodeled* extra dimensions can yield insights and avenues for extension of the new dynamics.

Our present task is to explore dynamical fermion bilinear condensate formation for the breaking of electroweak symmetry in the context of remodeled extra dimensions. A striking aspect of the standard model in the latticized bulk construction is that it provides the essential ingredients of a top-color model [8]. Indeed, top color is a dynamical gauge theory basis for top quark condensation [9] and involves rather uniquely the imbedding of $SU(3)\rightarrow SU(3)_1 \times SU(3)_2 \ldots$. Here the third generation feels the stronger $SU(3)_1$ interaction, while the first and second generations feel the weaker $SU(3)_2$. Such an imbedding, or enlargement of the $SU(3)$ gauge group is a natural consequence of extra dimensions with localized fermions [1,2]. Indeed, top color viewed as a remodeled extra dimensional theory anticipates the fermionic generations arising in a localized way in extra dimensions [6,10].

Extra dimensional models with gauge fields in the bulk [11], or their remodeled counterparts, are inherently strongly coupled. We show that the inherent strong coupling expected in these models can naturally provide a dynamical condensation of $\langle \bar{t}t \rangle$. In the remodeled description this is on a firm footing since the dynamics can be approximated reliably by a Nambu–Jona-Lasinio (NJL) model. We should say that *a priori* nothing precludes the addition of more physics, e.g., supersymmetry or technicolor, etc. We pursue top-color and top seesaw models at present because the remodeling of the $1+4$ standard model supplies all the ingredients for free.

If we wanted to construct a pure top-color model, or a model such as top-color assisted technicolor [8,12], we also require a ''tilting'' mechanism to block the formation of a $\langle \bar{b}b \rangle$. Again, the standard model in the latticized bulk provides the desired extra weak hypercharge imbedding $U(1)_Y \rightarrow U(1)_{Y1} \times U(1)_{Y2} \ldots$ needed to tilt in the direction of the top condensate. The fact that the top–anti-top channel is the most attractive channel in a standard model configura-

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tion then drives the formation of the top condensate alone.

In the present paper, however, we will explore a further aspect of the dynamics of a remodeled $1+4$ theory with the standard model gauge structure propagating in the bulk. We will show that the top seesaw model [13], which may indeed be the best and most natural model of dynamical electroweak symmetry breaking, arises completely and naturally from extra dimensions. In a top seesaw model a top condensate forms with the natural electroweak mass gap, $\mu \sim 600$ GeV, but there exist additional vectorlike partners to the t_R quark, usually designated by χ_R and χ_L. These objects form heavier Dirac mass combinations such as $M\overline{\chi}\chi$ and $m'\overline{\chi}_L t_R$, and taken together the physical top mass is given by $m_{top} = m'\mu/M$. The top seesaw affords a way to make a heavy top quark, and explain all of the electroweak breaking with a minimum amount of fine tuning. It has a heavy Higgs boson ~ 1 TeV, yet is in full consistency with the $S-T$ error ellipse constraints [14,15]. Remarkably, the vectorlike χ quarks of the top seesaw are also available for free from extra dimensions. These are simply the ''roaming'' t_R quark in the bulk, away from the domain wall that localizes its chiral zero mode t_R.

The possibility of generating top condensation (or other) schemes in the context of extra dimensions has been developed previously in explicit continuum extra dimensions [16]; indeed, Dobrescu [17] first observed that dynamical electroweak symmetry breaking was a likely consequence of the strong coupling of QCD in extra dimensions. The geometric reasoning we inherit from extra dimensions leads us to a systematic way of extending the models. Remodeled extra dimensions has led us in the present paper to the first theory of flavor physics from the top quark seesaw, with Cabibbo-Kobayashi-Maskawa (CKM) structure and light fermion masses. We also show that one can readily construct a viable 4th generation scheme along these lines. All fermions are condensed by the $SU(3)\times U(1)_Y$ structure on the 4th generation brane, and one can postulate Majorana masses for the ν_{Ri} as well, allowing the Gell-Mann–Ramond–Slansky–Yanagida neutrino seesaw (see, e.g., [19] and references therein).

Our present discussion will be largely schematic. We will describe the structure of the theory, and in a later work we will present the full phenomenology [18]. To make the present discussion as transparent as possible we will ''thin the degrees of freedom.'' Normally, we would approximate the bulk with a very large number of branes and interlinking Higgs fields. Presently, however, we will describe reduced n-brane models, in which n is small, typically $n = 2,3,4,5 \ldots$. In our minimal top seesaw scheme we have $n=4$, i.e., there is one brane per generation and one extra spectator brane (required for technical reasons). Hence, in this case all of the bulk is approximated by a transverse lattice with four branes. The gauge group we consider in $1+3$ dimensions for the n-brane model is $SU(3)^n \times SU(2)_L^n \times U(1)_Y^n$ and we have $n-1$ link-Higgs fields per gauge group. Thus, we keep only the zero modes and $n-1$ Kaluza-Klein modes for each gauge field. We will also keep some of the vectorlike KK modes of the fermions, in particular for the third generation. The masses of the vectorlike KK fermions are controlled by the mechanism that produces the chiral fermions on the branes [20,21] and these can be lifted to arbitrarily large Dirac masses, independent of the compactification scale.

The thinning of degrees of freedom is a mathematical approximation to the full theory. It is presumably derived from the fine-grained theory by a Kadanoff-style renormalization group. As a result, we expect many renormalization effects, and e.g., any translational invariance that may be softly broken by background fields of the short-distance theory can be lost in the thinned degrees of freedom of the effective theory. Our residual engineering freedom, leading to any given scheme, arises largely from the localization of the chiral fermions and the freedom to renormalize the linking-Higgs vacuum expectation values (VEV's) and gauge couplings in a non-translationally invariant way. How all of this ultimately interfaces with flavor physics constraints, e.g., flavor changing neutral current constraints [22], etc., remains to be examined in detail [18].

Thus our models can be viewed as transverse lattice descriptions of a standard model in $1+4$ dimensions in which the gauge fields and fermions and Higgs bosons all live in the bulk [11,10,16] with thinned degrees of freedom. Alternatively, they are a new class of $1+3$ models with top-color [8,9] and Top Seesaw [13] dynamics. The two pictures are equivalent through *remodeling*.

II. EFFECTIVE LAGRANGIANS IN WARPED LATTICIZED BACKGROUNDS

We begin with some essential preliminaries on latticized extra dimensions. We wish to describe the low energy effective Lagrangian of, e.g., the standard model in $1+4$ dimensions using the transverse lattice, but we include presently effects that break translational invariance in x^5. We begin with the QCD content and allow a general background geometry described by a metric with dependence upon the extra dimension x^5.

Consider the pure gauge Lagrangian in $1+3$ dimensions for $N+1$ copies of QCD:

$$\mathcal{L}_{QCD} = -\sum_{j=0}^{N} \frac{1}{4\tilde{g}_j^2} G^B_{j\mu\nu} G_j^{B\mu\nu} + \sum_{j=1}^{N} D_\mu \Phi_j^\dagger D^\mu \Phi_j \qquad (1)$$

in which we have $N+1$ gauge groups $SU(3)_j$ with gauge couplings $\tilde{g}_j$ *that depend upon* j and N link-Higgs fields, Φ_j forming $(\overline{3}_{j-1},3_j)$ representations. The covariant derivative is defined as $D_\mu = \partial_\mu + i\Sigma_{j=0}^N A^B_{j\mu} T_j^B$. T_j^B are the generators of the ith $SU(3)_i$ gauge symmetry, where B is the color index. Thus, $[T_i, T_j]=0$ for $i \neq j$; T_j^B annihilates a field that is singlet under the $SU(3)_j$; when the covariant derivative acts upon Φ_j we have a commutator of the gauge part with Φ_j, $T_j^{B\dagger}$ acting on the left and T_{j-1}^B acting on the right; the jth field strength is determined as usual, $G^B_{j\mu\nu} \propto \mathrm{tr} T_j^B [D_\mu, D_\nu]$, etc.

We treat the Φ_j as explicit Higgs fields. Renormalizable potentials can be constructed for each of the link-Higgs

fields, and we can always arrange the parameters in the potential such that the diagonal components of each Φ_j develop a common vacuum expectation value v_j, while the Higgs and $U(1)$ pseudo Nambu-Goldstone bosons (PNGB's) are arbitrarily heavy (for the perturbative unitarity constraint on this limit see Ref. [1]). Hence, each Φ_j becomes effectively a nonlinear σ model field [1,2]:

$$\Phi_j \to v_j \exp(i\phi_j^B T_j^B / v). \tag{2}$$

In our previous discussion [1,2], we assumed that $\tilde{g}_j$ and v_j were common for all $N+1$ gauge groups and N links, i.e., independent of j. This corresponds to a translationally invariant extra dimension with physical parameters independent of x^5.

In general, we must consider non-uniform $\tilde{g}_j$ and v_j in the remodeled theory. These correspond to a large variety of possible effects. For example, we may have an extra dimension with non-trivial background metric and a space dependent gauge coupling. These effects can arise from a bulk cosmological constant, background space dependent dilaton field, or from other fields and the finite renormalization effects due to localization of these fields. Alternatively, a background scalar field with nontrivial dependence upon x^5, $\varphi(x^5)$, and coupled to the gauge kinetic term, $(G^B_{\mu\nu})^2$, will give finite x^5 dependent renormalization of $\tilde{g}$.

Let us consider presently the case of a warped geometry, where the metric will contain an overall warp factor or background dilaton field, e.g., a Randall-Sundrum model [23]. The effect of the dilaton field can be seen through the implicit identification of the link-Higgs fields Φ_n with the Wilson lines:

$$\Phi_j(x^\mu) = \exp\left(i \int_{(j-1)a}^{ja} dy A_5^B(x^\mu, y) T^B \right), \tag{3}$$

where a is the lattice spacing. One finds

$$D_\mu \Phi_j^\dagger D^\mu \Phi_j \to \frac{1}{2} a^2 v_j^2 G^B_{(j-1/2)\mu 5} G^{B\mu 5}_{(j-1/2)}. \tag{4}$$

Let us compare this with the $1+4$ dimensional action for the gauge field in the background metric:

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2. \tag{5}$$

We have, for the gauge action,

$$\begin{aligned} \mathcal{L}_G &= \int d^5x \sqrt{g} g^{MN} g^{PQ} \frac{-1}{4 g_5^2(y)} G^B_{MP} G^B_{NQ} \\ &= \int d^4x \int dy \frac{-1}{4 g_5^2(y)} (\eta^{\lambda\kappa} G^B_{\mu\nu} G^{B\mu\nu} \\ &\quad - 2 e^{-2\sigma(y)} G^B_{\mu 5} G^{B\mu 5}), \end{aligned} \tag{6}$$

where the indices μ, ν are raised and lowered by the Minkowskian metric $\eta_{\mu\nu}$. We thus can see by comparison that the gauge coupling $\tilde{g}_j$ is related to the 5-dimensional gauge coupling by $\tilde{g}_j^2 = g_5^2(ja)/a$ (assuming that g_5 is smoothly varying) and v_j is simply related to the warp factor by

$$\tilde{g}_{j-1/2} v_j a = e^{-\sigma[(j-1/2)a]}, \tag{7}$$

where

$$\tilde{g}_{j-1/2} \equiv \frac{g_5[(j-1/2)a]}{\sqrt{a}}. \tag{8}$$

For smoothly varying $\tilde{g}_j$ and v_j, we can make the following interpolation:

$$\tilde{g}^2_{j-1/2} = \tilde{g}_{j-1} \tilde{g}_j, \quad \tilde{g}_j^2 v_j v_{j+1} a^2 = e^{-2\sigma(ja)} \equiv e^{-2\sigma_j}. \tag{9}$$

An example with 3 lattice points is described in Appendix A.

It is also straightforward to obtain the transverse lattice Lagrangian for scalar and fermion fields under the warped background metric. The action for a scalar field under the background (5) is given by [24]

$$\begin{aligned} &\int d^5x \sqrt{g} (g^{MN} \partial_M H^\dagger \partial_N H - m_H^2 H^\dagger H) \\ &\quad = \int d^4x \int dy (e^{-2\sigma(y)} \partial_\mu H^\dagger \partial^\mu H - e^{-4\sigma(y)} \partial_5 H^\dagger \partial^5 H \\ &\qquad - e^{-4\sigma(y)} m_H^2 H^\dagger H). \end{aligned} \tag{10}$$

After discretization, we have

$$\begin{aligned} &\int d^4x \sum_{j=0}^{N} \left(e^{-2\sigma_j} \partial_\mu H_j^\dagger \partial^\mu H_j - e^{-4\sigma_{j-1/2}} \frac{1}{a^2} \left| H_j - \frac{\Phi_j}{v_j} H_{j-1} \right|^2 \right. \\ &\qquad \left. - e^{-4\sigma_j} m_H^2 H_j^\dagger H_j \right). \end{aligned} \tag{11}$$

We can rescale $e^{-\sigma_j} H \to H$; the Lagrangian is then given by

$$\begin{aligned} \mathcal{L}_S = \sum_{j=0}^{N} \Bigg\{ &\partial_\mu H_j^\dagger \partial^\mu H_j - (m_H^2 e^{-2\sigma_j} \\ &+ \tilde{g}^2_{j-1/2} v_j^2 e^{2(\sigma_j - \sigma_{j-1/2})}) |H_j|^2 \\ &- \tilde{g}^2_{j-1/2} v_j^2 e^{-2(\sigma_{j-1/2} - \sigma_{j-1})} \left| \frac{\Phi_j}{v_j} H_{j-1} \right|^2 \\ &+ \left(\tilde{g}^2_{j-1/2} v_j^2 e^{(\sigma_j + \sigma_{j-1} - 2\sigma_{j-1/2})} H_j^\dagger \frac{\Phi_j}{v_j} H_{j-1} + \text{H.c.} \right) \Bigg\}. \end{aligned} \tag{12}$$

As discussed in the previous paper [1], the aliphatic model corresponds to the S^1/Z_2 orbifold compactification of the extra dimension. The even field under Z_2 corresponds to the boundary condition $H_{-1} = H_0$, and the odd field under Z_2 corresponds to the boundary conditions $H_{-1} = H_N = 0$. The

mass parameter m_H^2 should be replaced by m_{Hj}^2 if it depends on y, which can come from a y-dependent VEV of some field or the renormalization effects.

The action of a fermion under the background (5) is given by [25–27]

$$\int d^4x \int dy\, e^{-(3/2)\sigma}\overline{\Psi}\left(i\gamma^\mu\partial_\mu - \gamma_5 e^{-\sigma}\partial_5 - \frac{1}{2}\gamma_5(\partial_5 e^{-\sigma})\right) e^{-(3/2)\sigma}\Psi - e^{-4\sigma}m_\Psi\,\overline{\Psi}\,\Psi. \tag{13}$$

After rescaling and discretization, the fermion Lagrangian is given by

$$\begin{aligned}\mathcal{L}_F &= \sum_{j=0}^{N}\left\{ \overline{\Psi}_j i\gamma^\mu\partial_\mu\Psi_j \right.\\ &+\left[\frac{e^{-\sigma_{j+1/2}}}{a}\overline{\Psi}_{jR}\left(\frac{\Phi_{j+1}^\dagger}{v_{j+1}}\Psi_{(j+1)L}-\Psi_{jL}\right)+\text{H.c.}\right]\\ &-\frac{1}{2a}(e^{-\sigma_{j+1/2}}-e^{-\sigma_{j-1/2}})(\overline{\Psi}_{jL}\Psi_{jR}-\overline{\Psi}_{jR}\Psi_{jL})\\ &\left.-e^{-\sigma_j}m_{\Psi j}(\overline{\Psi}_{jL}\Psi_{jR}+\overline{\Psi}_{jR}\Psi_{jL})\right\}\\ &=\sum_{j=0}^{N}\left\{\overline{\Psi}_j i\gamma^\mu\partial_\mu\Psi_j+\left(\tilde{g}_{j-1/2}v_j\,\overline{\Psi}_{jL}\frac{\Phi_j}{v_j}\Psi_{(j-1)R}+\text{H.c.}\right)\right.\\ &\left.-\left(\frac{1}{2}(\tilde{g}_{j-1/2}v_j+\tilde{g}_{j+1/2}v_{j+1})+e^{-\sigma_j}m_{\Psi j}\right)\overline{\Psi}_j\Psi_j\right\},\end{aligned} \tag{14}$$

where we have used the relation (7) and imposed the boundary conditions $\Psi_{-1R}=\Psi_{NR}=0$ and $\Psi_{(N+1)L}=\Psi_{NL}$, corresponding to having Ψ_R (Ψ_L) odd (even) under Z_2. There is one more Ψ_L than Ψ_R at lattice N, so there is a massless left-handed chiral fermion left. The gauge anomaly must be canceled by including additional chiral fermions. Reversing the Z_2 parity of Ψ_L and Ψ_R gives rise a massless right-handed fermion. This can be obtained by imposing the boundary conditions $\Psi_{-1R}=\Psi_{0R}$, $\Psi_{0L}=\Psi_{(N+1)L}=0$. Alternatively, we can make the changes $L\leftrightarrow R$, $\tilde{g}\rightarrow-\tilde{g}$, $(a\rightarrow -a)$ in the Lagrangian (14) (corresponding to an opposite sign for the Wilson term that is included to avoid the fermion doubling problem) and impose the boundary conditions $\Psi_{-1L}=\Psi_{NL}=0$, $\Psi_{(N+1)R}=\Psi_{NR}$.

III. TOP QUARK SEESAW FROM REMODELED EXTRA DIMENSIONS

We consider a sequence of n-brane schemes. We put one generation of fermions and a copy of $SU(2)_L\times U(1)_Y\times SU(3)$ on each brane. In addition, we have $n-1$ link-Higgs fields (chiral fields, one for each gauge group). In the end we will have a set of links from a spectator brane to the up brane (which is defined as the brane on which the chiral up quark is localized), one set from up to charm, and another from charm to top. We will thus have constructed an ''aliphatic model,'' as in [1,2]. There are the usual zero-mode gauge fields and the $n-1$ KK modes, which are determined exactly. No Nambu-Goldstone boson zero modes occur as is usually the case in technicolor-like models (indeed these models have nothing to do with technicolor).

We are assuming throughout that we have an underlying Jackiw-Rebbi mechanism [20] to trap the fermionic chiral modes at the specific locations in the bulk. This involves scalar fields in $1+4$, $\varphi(x^5)_q$, which couple to $\overline{\psi}_q\psi_q$ and have domain wall configurations on which chiral zero-mode solutions exist. Away from the domain wall the fermions are vectorlike and have large Dirac masses. (The Jackiw-Rebbi mechanism on a discrete lattice is described in Appendix B.) For the remodeled description of matter fields, we exploit the fact that the chiral fermions can always be engineered on any given brane, with arbitrarily massive vectorlike KK modes partners on all branes, so we need keep only the chiral zero modes and the lower mass vectorlike fermions. Indeed, it is an advantage of the remodeled $1+3$ formalism that we can do this; in a sense the chiral generations are put in by hand in the remodeled theory, and we retain only the minimal relevant information that defines the low energy effective Lagrangian.

We require a mechanism to make the bare $\tilde{g}_3$ coupling of $SU(3)_{C,j}$ critically strong on the top brane j, such that the top quark will condense. Of course, with N branes (of equal couplings) the bare $\tilde{g}_3$ coupling is already $\sqrt{N}$ times stronger than the QCD physical coupling. The freedom exists to choose an arbitrarily strong bare $\tilde{g}_3$ on brane j for a variety of reasons as described in Sec. II. For example, if the kink field that localizes the chiral fermions couples to $(G^B_{\mu\nu})^2$, it can give finite renormalization to the top brane gauge coupling constants and trigger the formation of the condensate (see below). Any non-universal translational invariance breaking in x^5 may provide such a mechanism.

The vectorlike fermions of the top seesaw arise in a simple way: they are the roaming t_R (and/or t_L) in the bulk. In a sense it is remarkable that all of the ingredients are present. In addition, we get top flavor [28], with the copies of the $SU(2)_L$ gauge groups. Here arises a novel problem first noted in Ref. [8]. With large $SU(2)_L$ couplings the instanton mediated baryon number violation mechanism of 't Hooft becomes potentially problematic.

Finally, we ask: how is CKM matrix generated? We can put generational linking terms in by hand, which presumably arise from an underlying mechanism of overlapping wave functions for split fermions [10]. In our remodeled formulation we get no more or less information out than is put in by localizing the fermions in the bulk in the first place.

A. The schematic top seesaw

Let us first briefly review the top seesaw model. In a schematic form of the top seesaw model, QCD is embedded into the gauge groups $SU(3)_1\times SU(3)_2$, with gauge cou-

plings $\tilde{g}_{3,1}$ and $\tilde{g}_{3,2}$ respectively. The relevant fermion transforms under these gauge groups are (anomalies are dealt with by extension to include the b quark) [13]

$$T_L: (\mathbf{3},\mathbf{1}), \quad \chi_R: (\mathbf{3},\mathbf{1}), \quad t_R, \chi_L: (\mathbf{1},\mathbf{3}), \tag{15}$$

where $T_L=(t_L,b_L)$ is the third generation left-handed $SU(2)_L$ doublet, and χ_L,χ_R,t_R are $SU(2)_L$ singlets. We include a scalar field, Φ, transforming as $(\mathbf{3},\overline{\mathbf{3}})$, and it develops a diagonal VEV, $\langle\Phi^i_j\rangle=v\,\delta^i_j$, which breaks the top color to QCD,

$$SU(3)_1\times SU(3)_2\to SU(3)_{QCD}. \tag{16}$$

The massive gauge bosons (colorons) have mass

$$M^2=(\tilde{g}^2_{3,1}+\tilde{g}^2_{3,2})\,v^2. \tag{17}$$

Since χ_L,t_R have the same gauge quantum numbers, we can write down an explicit Dirac mass term:

$$\mu_{\chi t}\,\overline{\chi}_L t_R+\text{H.c.} \tag{18}$$

A second Dirac mass term between χ_L and χ_R can be induced from the Yukawa coupling to Φ,

$$\xi\,\overline{\chi}_R\Phi\chi_L+\text{H.c.}\to\mu_{\chi\chi}\,\overline{\chi}_R\chi_L+\text{H.c.} \tag{19}$$

These masses are assumed to be in the TeV range and have the order $\mu_{\chi t}<\mu_{\chi\chi}<M$. Below the scale M, various 4-fermion interactions are generated after integrating out the heavy gauge bosons. We assume that $\tilde{g}_{3,1}$ is supercritical and $\gg\tilde{g}_{3,2}$. A $\overline{T}_L\chi_R$ condensate will form and break the electroweak symmetry. To obtain the correct electroweak breaking scale, $\overline{t}_L\chi_R$ should have a dynamical mass $m_{t\chi}\sim 600$ GeV [9]. The mass matrix for the $t_{L,R},\chi_{L,R}$ is then [13]

$$(\overline{t}_L,\overline{\chi}_L)\begin{pmatrix}0 & m_{t\chi}\\ \mu_{\chi t} & \mu_{\chi\chi}\end{pmatrix}\begin{pmatrix}t_R\\ \chi_R\end{pmatrix}. \tag{20}$$

The light eigenstate is identified as the top quark and has a mass

$$m_t\sim m_{t\chi}\frac{\mu_{\chi t}}{\mu_{\chi\chi}}. \tag{21}$$

The top quark mass is correctly produced for $\mu_{\chi\chi}/\mu_{\chi t}\sim 3.5$. The model thus produces an acceptable dynamical electroweak symmetry breaking and a composite Higgs boson (composed of $\sim\overline{t}_L\chi_R$) with a fairly natural scale of the new physics (the QCD imbedding scale) of $\Lambda\sim$ few TeV.

B. Top seesaw from remodeled extra dimensions

All of the ingredients of a top-color scenario, in particular the top quark seesaw, are present in an extra-dimensional scheme. We assume that we have only the fermions T and t in $1+4$ dimensions. The χ fields will appear automatically as the vectorlike KK mode components of these fields. The fermions are coupled in $1+4$ to a background field as

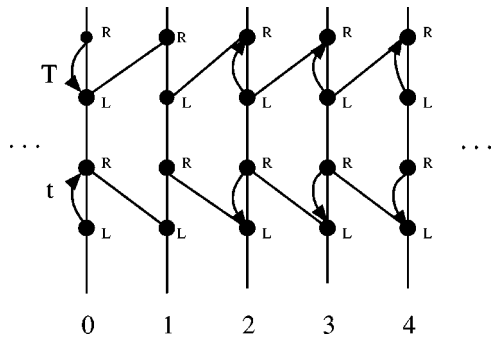

FIG. 1. A left-handed chiral zero mode T_L (right-handed t_R) is localized on brane 1, by coupling to a kink in a background field $\varphi(x^5)$ which gives a negative (positive) Dirac mass to the right of brane 1 and negative (positive) mass to the left of brane 1. We denote the negative (positive) Dirac mass by the up-arrow (down-arrow) curved links on each brane. The trapping Dirac mass, with a coarse grain lattice, can alternatively be added to link terms on one side of the zero mode as in Appendix B. We use the definition of the derivative for T_L with linking Higgs fields with $L,j-1$ hopping to R,j, represented by the diagonal links between nearest neighbor branes, and $\sim-\overline{T}_{R,j}T_{L,j}$ are vertical links on a given brane, Eq. (B1). For t_R we use the first definition Eq. (B2). We keep only the lowest lying vectorlike modes in the picture.

$\varphi(x^5)\overline{T}T$ and $\varphi(x^5)\overline{t}t$ and we assume that $\varphi(x^5)$ produces a domain wall kink at x^5_0 which we identify in our latticized approximation as brane 1 in the figures below. Before the formation of the top condensate the top quark configuration on the lattice branes is depicted in Fig. 1.

The basic idea underlying the formation of a condensate is to allow a particular gauge coupling constant to become supercritical on a particular brane. In Fig. 2 we show the formation of the condensate $\langle\overline{T}_L t_R\rangle$ on brane 1 where we assume that the $SU(3)_1$ coupling constant, $\tilde{g}_{3,1}$ is supercritical, i.e., in the NJL model approximation to QCD $3\tilde{g}^2_{3,1}/8\pi^2>1$.

A trigger mechanism in the $1+4$ theory for supercritical coupling at the location of the trapping domain wall can arise

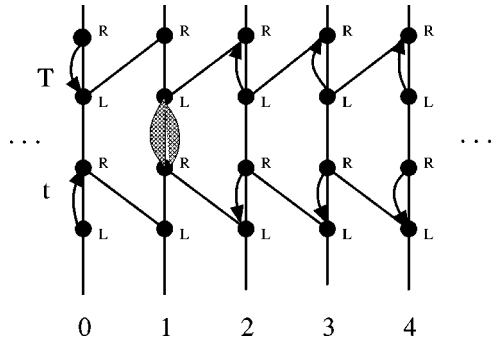

FIG. 2. A condensate $\langle\overline{T}_L t_R\rangle$ forms on brane 1 when the $SU(3)_1$ coupling constant $\tilde{g}_{3,1}$ is supercritical. This can be triggered from $\varphi(x^5)(G^2_{\mu\nu})$ in the $1+4$ underlying theory, but is a free parameter choice in the $1+3$ effective Lagrangian. Note that we have chosen for convenience the other hopping convention for the T_L field (Appendix B).

from a coupling of φ to the squared field strength $(G^a_{\mu\nu})^2$ such that the gauge Lagrangian in $1+4$ becomes

$$\left[-\frac{1}{4g_3^2}-\frac{\lambda\varphi^2}{4M^2}\right](G^a_{\mu\nu})^2. \tag{22}$$

Such a coupling will always be induced by the fermion fields that couple to the gauge fields. We assume $\varphi\to M$ ($\varphi\to -M$) for $x^5\to R$ ($x^5\to 0$). For $\lambda>0$ the action is well behaved, and off the domain wall the effective coupling constant, $\overline{g_3^2}=g_3^2/(1+\lambda)$ is suppressed. On the domain wall the effective coupling is then g_3^2 which we assume is supercritical. Moreover, the condensate is generally suppressed, for fixed coupling g_3^2 in the NJL model approximation for fields with large Dirac masses, so we expect only the chiral fields to pair up. In fact, one need not appeal to the trigger mechanism alluded to above, but it is a useful way to suppress $\overline{g_3^2}$ elsewhere in the bulk, and such operators are expected on general grounds when we construct the renormalized effective Lagrangian with fewer degrees of freedom.

In our latticized $1+3$ description the varying coupling constants $\tilde{g}_{3,j}$ and Dirac mass terms can be put in "by hand" as defining parameters.

To derive the top seesaw model from extra dimensions, we thin the degrees of freedom of the extra dimension to 2-branes. There is an $SU(3)_j$ on each brane. The scalar field Φ which breaks $SU(3)_1\times SU(3)_2$ down to $SU(3)_{QCD}$ is now just the link-Higgs field and has exactly the right structure for top-color breaking. The $SU(2)_L$ doublet and singlet quarks propagate in the $1+4$ bulk, and in the latticized scheme are represented by the fields T_j, t_j, $j=1,2$ on the two branes:

$$\begin{array}{lll} SU(3)_1 & SU(3)_2 & \\ T_{1L} & T_{2R} & T_{2L} \\ t_{1R} & t_{2L} & t_{2R} \end{array} \tag{23}$$

We have projected out the chiral partners T_{1R} and t_{1L} by coupling to the background localizing field with a domain wall kink at brane 1, which produces chiral fermions. The kinetic terms in the extra dimension give rise the mass terms in the $1+3$ effective Lagrangian that interconnect T_{1L} to T_{2R} etc. The backgound localizing field φ also produces the Dirac masses that interlink, e.g. T_{2R} and T_{2L}, etc. So, in the two brane approximation we have

$$m_{T_{22}}\overline{T}_{2L}T_{2R}+m_{T_{12}}\overline{T}_{1L}\frac{\Phi^\dagger}{v}T_{2R}+m_{t_{22}}\overline{t}_{2L}t_{2R} + m_{t_{12}}\overline{t}_{1R}\frac{\Phi^\dagger}{v}t_{2L}+\text{H.c.}, \tag{24}$$

with

$$m_{T_{12}}=-m_{t_{12}}\approx-\sqrt{\tilde{g}_{3,1}\tilde{g}_{3,2}}\,v,\quad m_{T_{22}}\sim\tilde{g}_{3,2}v+h_T\varphi,$$
$$m_{t_{22}}\sim\tilde{g}_{3,2}v+h_t\varphi.h_T\gg h_t. \tag{25}$$

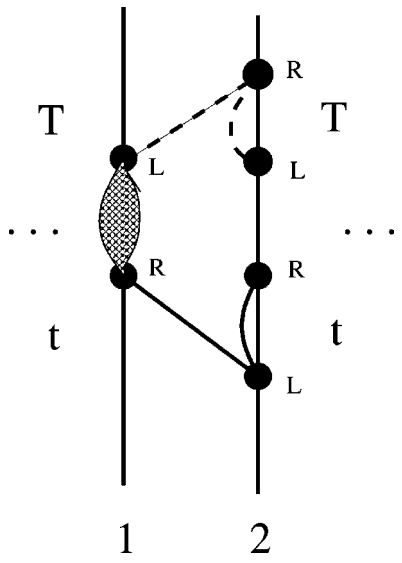

FIG. 3. Two brane approximation. In the limit that T_2 decouples this is just the original top seesaw model of [13].

This configuration is shown in Fig. 3.

We can see explicitly that this matches onto the schematic top seesaw model. To match we first assume that the φ contribution to the $\overline{T}_{2L}T_{2R}$ mass term is so large that T_{2L}, T_{2R} decouple. Then, the 2-brane model is identical to the schematic top seesaw model described above through the following identification:

$$T_L=T_{1L},\quad \chi_R=t_{1R},\quad \chi_L=t_{2L},\quad t_R=t_{2R},$$
$$\mu_{\chi t}=m_{t_{22}},\quad \mu_{\chi\chi}=m_{t_{12}}. \tag{26}$$

For a supercritical gauge coupling $\tilde{g}_{3,1}$, the $\langle\overline{T}_{1L}t_{1R}\rangle$ condensate will form, breaking the electroweak symmetry. The top quark mass is then obtained from the seesaw mechanism.

C. The light generations and flavor physics

We now consider all three fermionic generations of the standard model in the latticized bulk. We discuss the issue of how we can generate light quark masses and mixings in a generalized geometric top seesaw scenario.

Clearly, in order to generate light fermion masses from the third generation condensates, some flavor mixing terms must be present. Small masses and mixings can be generated in $1+4$ models by the overlap of the Higgs and fermion wave functions in the extra dimension [10] and/or small flavor mixing effects arising from localization. We examine this mechanism in the latticized extra dimension with the simplest flavor mixing mass terms. We find that the light generation fermion masses are generated radiatively in this picture.

There is a copy of the $SU(3)\times SU(2)_L\times U(1)_Y$ standard model gauge group on each brane, with gauge couplings $\tilde{g}_{a,j}$ respectively, where $a=1,2,3$, is the gauge group index and $j=0,1,2,3$ is the brane index. There are link fields $\Phi_{a,j}$, $a=1,2,3$, $j=1,2,3$, which break the full $SU(3)^4\times SU(2)^4_L$

$\times U(1)_Y^4$ gauge group down to the standard model $SU(3) \times SU(2)_L \times U(1)_Y$.

We will denote the 3 generation $SU(2)_L$ doublet quarks with uppercase letters (T,C,U) and $SU(2)_L$ singlet fermions with lower case letters (t,c,u). We assume that the third generation fermions propagate on all branes, with the localization removing the right-handed $SU(2)_L$ doublets and the left-handed singlets on brane 0. Hence the third generation fields T_j and t_j, etc., carry the brane index j, while the C,c (U,u) are localized on brane 1 (2). The localization of the top, charm and up quarks is accomplished with additional $\varphi(x^5)_t$, $\varphi(x^5)_c$ and $\varphi(x^5)_u$ fields that produce domain walls in the underlying $1+4$ theory.

If we assume that only the $\tilde{g}_{3,0}$ $SU(3)$ coupling constant is supercritical, this then drives the formation of the condensate $\langle \overline{T}_{L0} t_{R0} \rangle$, breaking the electroweak symmetry. The top quark mass is then obtained from the generalized seesaw mechanism. In general the left- and the right-handed top quark zero modes are linear combinations of T_{Lj} and t_{Rj} (and C_L, U_L, c_R, u_R after including flavor mixings):

$$T_L^{(0)} = \sum_{j=0}^{3} \alpha_{T_j} T_{Lj} (+ \alpha_C C_L + \alpha_U U_L),$$

$$t_R^{(0)} = \sum_{j=0}^{3} \alpha_{t_j} t_{Rj} (+ \alpha_c c_R + \alpha_u u_R), \tag{27}$$

where $\alpha_{T_j}, \alpha_{t_j}$ are coefficients determined by the direct and link mass terms among $T_{L,R}$'s and $t_{L,R}$'s. The top quark mass is suppressed by the mixings α_{T_0} and α_{t_0}:

$$m_t \sim \alpha_{T_0} \alpha_{t_0} \times 600 \text{ GeV}. \tag{28}$$

Let us first thin the degrees of freedom of the extra dimension to a 3-brane model, and we consider first the generation of the charm quark mass. This configuration is as shown in Fig. 4.

To generate the charm quark mass, we include flavor-mixing mass terms. In the underlying $1+4$ theory we might suppose that these can arise on a given brane from couplings of the form, e.g., $\epsilon \varphi(x^5)_t \overline{C}_L \psi_R$. In the $1+3$ theory this is a common Dirac mass on brane 1 that mixes all fermions with equivalent quantum numbers. However, the direct contact mass terms $\overline{C}_L T_{R1}$, $\overline{t}_{L1} c_R$, can all be rotated away by redefinitions of the fields T_{R1} and t_{L1}. This can be seen by considering the overall mass term on brane 1, $m' \overline{t}_{L1} c_R + M \overline{t}_{L1} t_{R1}$, where the second term is just the mass of the Dirac (non-chiral) vectorlike t quark on brane 1. Thus, by redefining $c_R \to (-m' t_{R1} + M c_R)/\sqrt{M^2+m'^2}$ and $t_{R1} \to (M t_{R1} + m' c_R)/\sqrt{M^2+m'^2}$ we eliminate the direct charm quark mass term. The important point is that this redefinition involves only fields on the common brane 1, and there is no residual kinetic term mixing since all of the kinetic terms involve the same gauge fields $A^B_{\mu,1}$. In order to generate a surviving mass term for the charm quark we thus need additional terms to frustrate the chiral redefinition. Such terms are seen to be present when we consider brane 2.

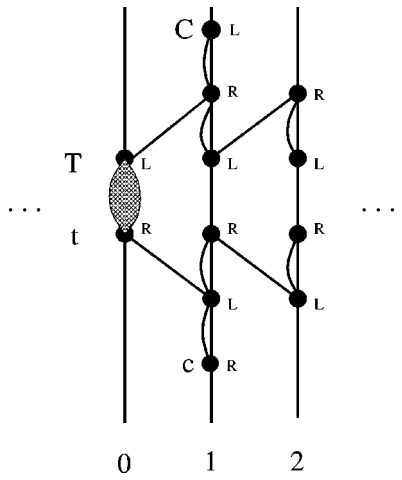

FIG. 4. Three brane approximation incorporating charm, where $C=(c,s)_L$ is a doublet zero mode, and $c=c_R$ is a singlet zero mode, both trapped on brane 1 (we assume that the vectorlike partners of C and c are decoupled). The Dirac flavor mixing between $\overline{C}_L T_{R1}$ and $\overline{c}_R t_{L1}$ can be rotated away by redefinitions of T_{R1} and t_{L1}.

Note that when we rotate away the direct charm mass terms on brane 1, we in general will obtain the linking mass terms

$$\overline{C}_L \left(\prod_a \Phi_{a,2} \right) T_{R2}, \quad \overline{t}_{L2} \left(\prod_a \Phi^\dagger_{a,2} \right) c_R, \tag{29}$$

where the $(\Pi_a \Phi_{a,i})$ is a product of linking-Higgs terms, as shown in Fig. 5. These terms can be viewed as mass terms, but in reality they are all higher dimension operators; since we are in the broken phase in which the Φ's all have VEV's we can only approximately describe these terms as though

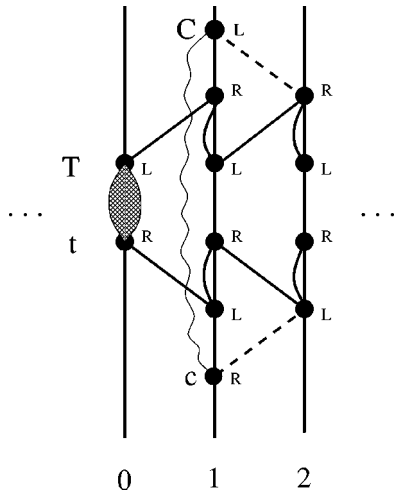

FIG. 5. The flavor mixing (dashed lines) between $\overline{C}_L \Pi(\Phi/v) T_{R2}$ and $\overline{c}_R \Pi(\Phi/v) t_{L2}$ cannot be rotated away by redefinitions of T_{R2} and t_{L2} without generating effective kinetic term mixing which leads to non-zero-mode flavor changing gluon vertices (in the broken phase where $\Phi \to v$; this mixing is actually a higher dimension operator). The charm quark mass is thus generated when radiative corrections are included (wavy line).

they are mass terms. However, even with the flavor mixing linking (higher dimension) mass terms and the direct mass terms, *at tree level* (neglecting the gauge interactions on branes 1 and 2) we can again perform a field redefinition as before and we still fail to generate a charm quark electroweak mass and only the top quark retains a nonzero electroweak mass. This can be seen readily as the electroweak symmetry breaking (EWSB) condensate couples only to T_{L0} and t_{R0}. We can rewrite T_{L0}, t_{R0} in terms of the redefined eigenstates of the electroweak preserving masses:

$$T_{L0} = \beta_3 T_L^{(0)} + \beta_2 C_L^{(0)} + \beta_1 U_L^{(0)} + \text{heavy states},$$
$$t_{R0} = \gamma_3 t_R^{(0)} + \gamma_2 c_R^{(0)} + \gamma_1 u_R^{(0)} + \text{heavy states}. \tag{30}$$

After decoupling the heavy vector-like states, the 3×3 up-type quark mass matrix M_U

$$M_{Uij} \propto \beta_i \gamma_j \tag{31}$$

is of rank 1.

However, the result of the field redefinition is that now *off-diagonal couplings to the gluons on branes 1 and 2 are generated*. When we now take into account the gauge interactions on branes 1 and 2, the charm quark does indeed obtain a nonzero mass from radiative corrections as shown in Fig. 5. For this to occur we require the interference with the linking mass terms, because otherwise the gauge radiative corrections produce only multiplicative corrections to the (zero) mass on a given brane.

More explicitly, we now have the interbrane mass term of the form, e.g., $m' \bar{t}_{L2} c_R$ for the charm quark. This implies that on brane 2 there is the overall mass term $m' \bar{t}_{L2} c_R + M \bar{t}_{L2} t_{R2}$ where the second term is the mass of the Dirac vectorlike t quark. Thus, redefining $t_{R2} \to \cos\theta t_{R2} + \sin\theta c_R$ and $c_R \to -\sin\theta t_{R2} + \cos\theta c_R$ we can eliminate the direct charm quark mass term. However, in kinetic terms we have

$$\bar{c}_R (i\partial\!\!\!/ - A\!\!\!/_1) c_R + \bar{t}_{R2} (i\partial\!\!\!/ - A\!\!\!/_2) t_{R2}. \tag{32}$$

Upon performing the redefinitions we generate off-diagonal transitions:

$$\bar{c}_R (i\partial\!\!\!/ - \tilde{A}\!\!\!/_1) c_R + \bar{t}_{R2} (i\partial\!\!\!/ - \tilde{A}\!\!\!/_2) t_{R2} + \kappa (\bar{c}_R (A\!\!\!/_1 - A\!\!\!/_2) t_{R2} + \text{H.c.}) + \ldots \tag{33}$$

where $\tilde{A}_{1(2)} = \cos^2\theta A_{1(2)} + \sin^2\theta A_{2(1)}$ and $\kappa = \sin\theta\cos\theta$, and the ellipsis represents diagonal terms. On the left-hand side of Fig. 6 we also generate off-diagonal couplings of the form $\kappa' [\bar{C}_L (A\!\!\!/_1 - A\!\!\!/_2) T_{L2} + \text{H.c.}]$. In evaluating the induced charm quark mass and mixing it is useful to remain in the current eigenbasis in which the gluon interactions are diagonal. We emphasize that this effect is different than that described in [22] in which localization produces off-diagonal flavor transitions among fermions coupled to KK mode vector bosons.

These off-diagonal kinetic terms, we emphasize, are higher dimension operators involving the link-Higgs fields. They take the apparent $d=4$ form only as a result of working in the broken phase of the Φ's. However, the result is that we have now generated an interaction that acts like *extended technicolor* [29]. When we include the radiative effects of the gluons we generate charm quark mass. In Fig. 6 we illustrate the diagram in the basis in which the gluon couplings are diagonal (the current eigenbasis). We also generate radiative mixing between charm and top masses through diagrams as in Fig. 7 where now the mixing of the gluonic gauge groups on different branes must be included.

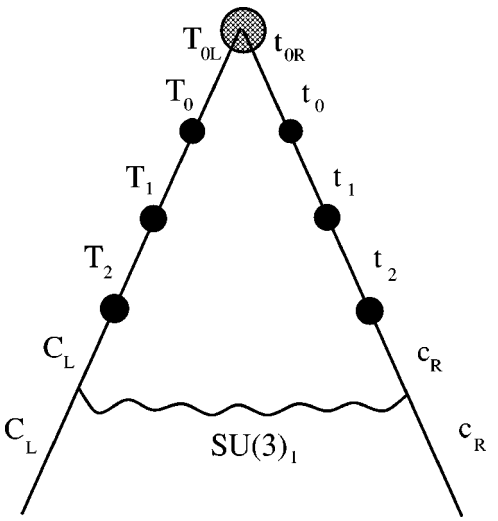

FIG. 6. The radiative correction diagram in the current eigenbasis for the induced charm mass.

The extension of the scheme to include the up quark mass generation and the mixing is shown in Fig. 8. Again, we require nearest neighbor mixing between branes which produces vanishing mass in tree approximation, but off-diagonal gluon vertices in the broken phase due to kinetic term mixing. The full mass matrix is regenerated when radiative corrections are included.

One can understand the origin of the mass matrix in the language of the ''shining Higgs VEV profile'' as discussed in our previous paper [1]. The gauge interactions on branes 1 and 2 are subcritical, so the Higgs bound states formed on these branes have positive squared masses. However, due to the links with brane 0, the composite Higgs fields on branes 1 and 2 will receive tadpole terms as shown in Fig. 9, and therefore obtain nonzero VEV's.

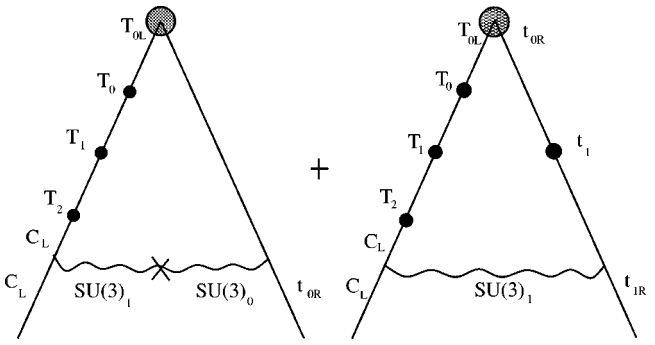

FIG. 7. The radiative correction diagrams in the current eigenbasis for the induced charm$_L$-top$_R$ mixing mass term.

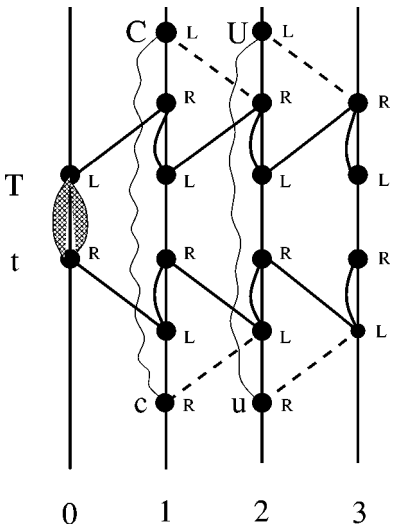

FIG. 8. The extension to include the up quark in a 4-brane model with radiatively generated mass and mixing.

From the shining and the flavor mixing effects, the final Higgs VEV will contain some small components of $\bar{C}_L c_R$ and $\bar{U}_L u_R$ after diagonalization, which are responsible for generating the charm and up quark masses.

To generate the down-type quark mass matrix requires a mechanism to first generate the *b*-quark mass. One possibility is to condense the *b* quark as in the case of the top quark, and exploit a larger seesaw. This encounters generally a large degree of fine-tuning; to have the large seesaw suppression of the physical *b* mass requires a larger vectorlike Dirac mass for the roaming *b* quarks, and this can turn off the condensate except for large supercritical coupling. An alternative and less fine-tuned approach is to exploit $SU(3)_0$ instantons on brane 0 which produce a 't Hooft determinant containing terms like $\sim \bar{b}_L b_R \bar{t}_L t_R + \ldots$. Then the nonzero $\langle \bar{t}t \rangle$ induces the *b*-quark mass. The magnitude of this condensate can be controlled by seesaw with the vectorlike *b* quarks. In any case, the dynamical and phenomenological details of the generation of the *b* quark mass are a top seesaw modeling issue, and will be described in detail in a forthcoming paper by He, Hill, and Tait [15]. For our present purposes we can simply assume that an induced *b*-quark mass or $\langle \bar{b}b \rangle$ can be arranged for brane 0. The full model then takes the form of Fig. 8 with (u,c,t) replaced by (d,s,b). This produces a second species of Higgs boson, the *b* Higgs boson H_b which then shines through the bulk.

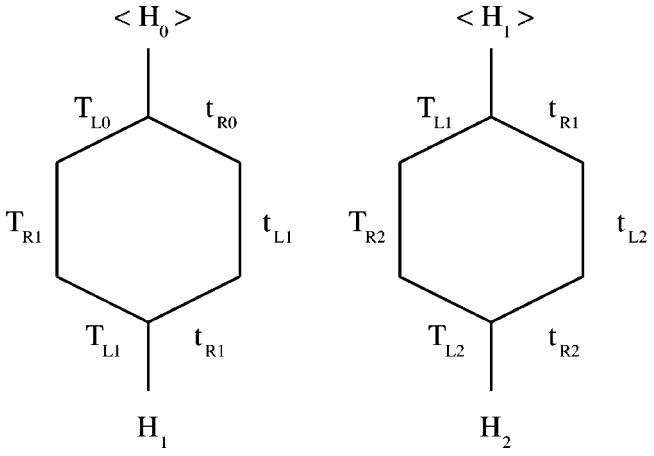

FIG. 9. The formation of composite Higgs fields on each brane and their propagation to subsequent branes. This sets up a tadpole on each brane which exponentially attenuates away from the brane 0 of the top condensate.

Top color does not specifically address the issue of leptons. We can in principle use $U(1)_{Y0}$ on the brane 0 to condense the τ lepton, and a corresponding seesaw to produce the physical m_τ. Alternatively, any new physics that produces the higher dimension operator $\bar{t}t\bar{\tau}\tau$ structure will suffice to give the τ lepton a mass. Having produced the $\langle \bar{\tau}\tau \rangle \neq 0$ on brane 0, we again repeat the construction to provide the masses for μ and *e*. In the lepton case the $U(1)_Y$ radiative corrections replace the gluonic radiative corrections. The neutrinos do not condense since $U(1)_Y$ does not produce a nontrivial 't Hooft determinant, and we do not presently address the origin of the small neutrino Majorana masses.

D. Fourth generation condensates

A fourth generation scheme may have advantages for the lepton mass generation mechanism. Here we imagine condensing on a fourth generation brane $=0$ the three attractive channel condensates of $\langle \bar{T}T \rangle$ and $\langle \bar{B}B \rangle$ from $SU(3)_0$ and $\langle \bar{E}E \rangle$ from a strong $U(1)_{Y0}$. The effective Higgs bosons are heavy, ~ 1 TeV. The model has an acceptably small positive $S \sim 2/3\pi$ and built in custodial $SU(2)$ breaking from the $U(1)_{Y0}$ which may provide an acceptably large positive T parameter contribution to bring the theory into the $S-T$ error ellipse.

The structure of the quark sector for a 5-brane version of the model is shown in Fig. 10. We use the strong $SU(3)_0$ [with strong $U(1)_{Y0}$ corrections] to form a $\langle \bar{B}B \rangle \lesssim \langle \bar{T}T \rangle$ condensate on brane 0 of the fourth generation quarks. Thus, the $SU(3)$ interaction overwhelms the tilting effect of the $U(1)_Y$. The quarks of the fourth generation roam through the bulk and propagate the composite Higgs bosons. The three lighter generations feel the condensate as in the top seesaw scheme of Sec. III, as seen in Fig. 10 which shows explicitly the quark sector. The masses and CKM structure are then generated radiatively in analogy to the top seesaw scheme.

In Fig. 11 we illustrate how the lepton sector can be dynamically generated. Here there is a $U(1)_{Y0}$ condensate of $\langle \bar{E}E \rangle$ on brane 0 which produces the leptonic Higgs boson. As before, the fermion masses are generated by linking flavor changing terms.

We also include the right-handed neutrino N_R. We emphasize that this need not be chiral, i.e., it need not be a localized chiral zero mode associated with a brane 0 kink. The N_R is a gauge singlet so we can write down the Majorana mass term, $\bar{N}_R^C N_R$, which presumably comes from external physics, e.g., it may come from the effective Planck scale (we can certainly complicate the picture by including all pos-

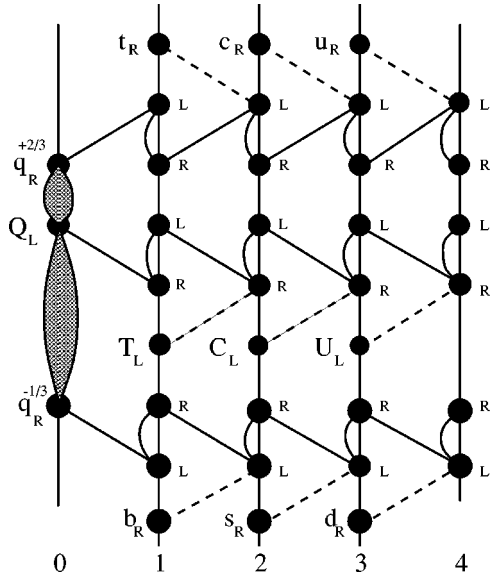

FIG. 10. The fourth generation condensate generating the up and down type quark masses. We position the kinetic term partner Q_R (which is nonchiral) next to the chiral zero mode Q_L.

sible allowed Majorana and Dirac links within and between branes, e.g. $\overline{N_j^C}N_{j+1}$, etc.).

To produce small Majorana mass terms for the known neutrinos we first require a mechanism to generate a Dirac mass or condensate for the 4th generation neutrino, $\sim M\overline{\nu_L}N_R$ on brane 0. This requires one of two options: (a) A condensate such as $\langle\overline{L}N_R\rangle\neq 0$ must form involving yet a new strong gauge group, such as $U(1)_{B-L}$ *or* (b) a higher dimension operator exists which allows the bilinear $\overline{L}N_R$ to feel the electroweak condensates, as in

$$\frac{1}{M^2}\overline{L_L^a}N_R(\overline{B}_R T_L^a), \quad \frac{1}{M^2}\overline{L_L^a}N_R(\overline{E}_R L_L)^C. \tag{34}$$

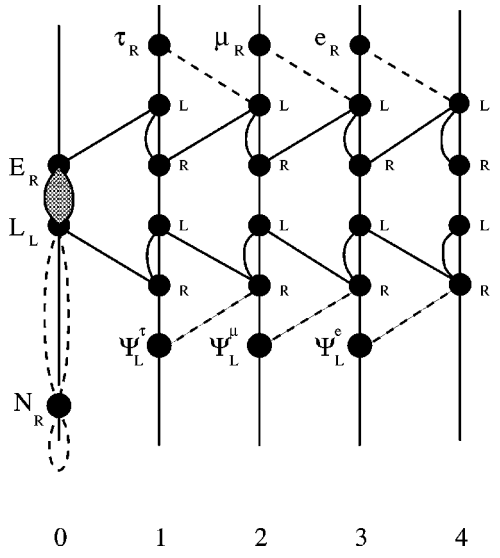

FIG. 11. The fourth generation condensate generating the lepton masses. A single N_R (which can be part of a vectorlike Dirac pair, and need not be chiral) is shown and an external mechanism gives it a Majorana mass. This is communicated to the left-handed neutrinos through the condensate with L_L.

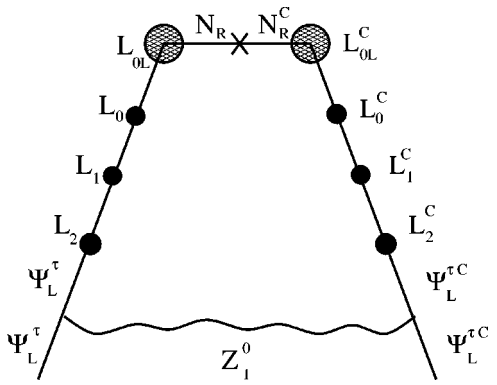

FIG. 12. The ν_τ Majorana mass is radiatively induced by feeling the condensate and Majorana mass of N_R. This corresponds to an effective term $(\overline{\psi_L}\cdot H)^2$ where H is the Higgs boson through mixing with quarks and charged leptons, or the neutrino condensate effective bound-state Higgs boson $\sim\overline{N_R}L_L$.

Once the master 4th generation Dirac mass is established we can invoke the seesaw. We depict this in Fig. 11. The Feynman diagram of Fig. 12 then shows the formation of the radiatively induced Majorana mass (by, e.g., Z exchange) for the ν_τ. Similar mixings and mass terms arise for the first and second generation neutrinos in analogy with the quark and charged lepton masses.

We mention that one should be wary of the possibility of enhanced proton decay coming from the 't Hooft process with strong $SU(2)_L$ gauge groups located on various branes. This is an issue for ''top flavor'' models which we defer to another session. Moreover, as discussed in Ref. [22], the KK gauge bosons can induce flavor-changing effects in the split fermion generation models. This puts a strong constraint on the KK gauge boson masses. In our model, the first two generations are localized away from the EWSB brane. Flavor-changing effects involving the first two generations from heavy gauge boson exchanges can be suppressed if the link VEV's associated with the first two generation branes are much larger than the weak scale (since they are not directly related to EWSB). We have not yet touched upon the vacuum alignment of the VEV's of the composite Higgs boson, possibly through mechanisms such as radiative corrections or higher order couplings. Such questions have been discussed in earlier works on dynamical electroweak symmetry breaking, e.g. [15]; we will put off discussion on our specific model for future work [18].

IV. DISCUSSION AND CONCLUSION

In conclusion, we have given a description of a fairly complete extension of the standard model with dynamical electroweak symmetry breaking. This arises from the bulk $1+4$ dimensions as a $1+3$ dimensional effective theory af-

ter remodeling. We have kept only a small number of lattice slices (branes) as a minimal approximation with thinned degrees of freedom.

A dynamical electroweak symmetry breaking scheme emerges naturally in this description, as first anticipated by Dobrescu [17]. We see immediately the emergence of an imbedding of QCD as in $SU(3)\rightarrow SU(3)_1\times SU(3)_2\ldots$, and the appearance of vectorlike partners of the elementary fermions such as the top quark. With fermionic localization we can have flavor dependent couplings to these gauge groups, and trigger the formation of condensates using localization background fields or warped geometry.

These elements are all part of the structure of top color, [8] and the top seesaw [13], and we are thus led naturally to this class of extra-dimensional models in which the electroweak symmetry is broken dynamically. However, one can go beyond these schemes to, e.g., a fourth generation scheme which is somewhat more reminiscent of technicolor and may have direct advantages for the lepton sector masses.

One can always discard the notion of extra dimensions and view this as an extension of the standard model within $1+3$ dimensions with extra discrete symmetries; however the specific structures we have considered are almost compelled by extra dimensions. The connection to extra dimensions is made through *remodeling* [1,2], bulk inhabitation of gauge fields [11], the transverse lattice [3], hidden local symmetries [4], etc., and may be viewed as a manifestly gauge invariant low energy effective theory for an extension of the standard model in $1+4$.

Remodeling is a remarkable model building tool, and a system of new organizational principles. Remodeling has guided our thinking in producing the present sketch of a full theory of flavor physics based upon the top seesaw, something which has not been previously done. Much work remains to sort out the model and to check that the systematics of experimental constraints can be accommodated [15,18], and to see if the model survives as a natural scheme without a great deal of fine-tuning. It is already encouraging that the top seesaw model is a strong dynamics that is consistent with experimental $S-T$ constraints.

ACKNOWLEDGMENTS

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APPENDIX A: THREE BRANE EXAMPLE OF GAUGE FIELDS WITHOUT TRANSLATIONAL INVARIANCE

We now wish to address the impact of the breaking of translational invariance in x^5 on the physics of the effective $1+3$ Lagrangian. It is generally advantageous to thin the degrees of freedom in the lattice description of the extra dimensions. We can construct a coarse grain n-brane model with $n\ll N$ as a crude approximation to a fine grained N-brane model. Such a description can be improved in principle by a block-spin renormalization group, which is beyond the scope of our present discussion.

Consider, for example, a 3-brane model. The effective $1+3$ Lagrangian now contains 3 copies of the standard model gauge group and link fields interpolating each of the $SU(3)_C$, $SU(2)_W$, and $U(1)_Y$ groups in the aliphatic configuration. The pure gauge Lagrangian in $1+3$ dimensions for 3 copies of QCD is given by

$$\mathcal{L}_{QCD}=-\frac{1}{4}\sum_{j=0}^{2} G^B_{i\mu\nu}G^{B\mu\nu}_j+\sum_{j=1}^{2} D_\mu\Phi^\dagger_j D^\mu\Phi_j, \quad \text{(A1)}$$

where $G^B_{i\mu\nu}$ has been rescaled so that the gauge coupling $\tilde{g}_{3,j}$ appears in the covariant derivative. The electroweak gauge Lagrangian can be written down analogously.

After substituting the VEV's of the link fields,

$$\Phi_j\rightarrow v_j\exp(i\phi^B_j T^B_j/v), \quad \text{(A2)}$$

the Φ_j kinetic terms lead to a mass-squared matrix for the gauge fields:

$$\sum_{j=1}^{2}\frac{1}{2}v_i^2(\tilde{g}_{3,(i-1)}A^B_{(i-1)\mu}-\tilde{g}_{3,i}A^B_{i\mu})^2. \quad \text{(A3)}$$

This mass-squared matrix can be written as a 3×3 matrix sandwiched between the column vector $A=(A^B_{0\mu},A^B_{1\mu},A^B_{2\mu})$ and its transpose, as A^TMA, where

$$M=\frac{1}{2}\begin{pmatrix} (\tilde{g}_{3,0})^2v_1^2 & -(\tilde{g}_{3,0}\tilde{g}_{3,1})v_1^2 & 0 \\ -(\tilde{g}_{3,0}\tilde{g}_{3,1})v_1^2 & (\tilde{g}_{3,1})^2(v_1^2+v_2^2) & -(\tilde{g}_{3,1}\tilde{g}_{3,2})v_2^2 \\ 0 & -(\tilde{g}_{3,1}\tilde{g}_{3,2})v_2^2 & (\tilde{g}_{3,2})^2v_2^2 \end{pmatrix}, \quad \text{(A4)}$$

where we have kept the full set of effects of j dependence in v_j and $\tilde{g}_{3,j}$.

We can diagonalize the mass matrix as

$$A_{j\mu}=\sum_{n=0}^{2} a_{jn}\tilde{A}^n_\mu. \quad \text{(A5)}$$

The a_{jn} form a normalized eigenvector $(\vec{a}_n)$ associated with the nth eigenvalue. The eigenvectors and the corresponding eigenstate masses for common $\tilde{g}_3$ and v, which corresponds to the flat extra dimension case, were obtained in the previous papers [1,2],

$$\vec{a}_0 = \frac{1}{\sqrt{3}}(1,1,1),$$

$$\vec{a}_1 = \sqrt{\frac{2}{3}}\left(\cos\frac{\pi}{6}, \cos\frac{3\pi}{6}, \cos\frac{5\pi}{6}\right) = \frac{1}{\sqrt{2}}(1,0,-1),$$

$$\vec{a}_2 = \sqrt{\frac{2}{3}}\left(\cos\frac{2\pi}{6}, \cos\frac{6\pi}{6}, \cos\frac{10\pi}{6}\right) = \frac{1}{\sqrt{6}}(1,-2,1), \tag{A6}$$

$$(M_0, M_1, M_2) = 2\tilde{g}_3 v\left(0, \sin\frac{\pi}{6}, \sin\frac{\pi}{3}\right) = \tilde{g}_3 v(0,1,\sqrt{3}). \tag{A7}$$

The expressions for the eigenvectors and eigenvalues for general $\tilde{g}_{3,i}$ and v_i are more complicated. However, if $\tilde{g}_{3,0}v_1, \tilde{g}_{3,1}v_1 \gg \tilde{g}_{3,1}v_2, \tilde{g}_{3,2}v_2$, we have a sequential decoupling. In this case the $SU(3)_0 \times SU(3)_1$ is first broken down to the diagonal $SU(3)'$ by Φ_1, then $SU(3)' \times SU(3)_2$ is broken by Φ_2 to $SU(3)_{QCD}$ at a lower scale. In this case, the eigenstates and their masses are given approximately by

$$\vec{a}_2 \approx \frac{1}{\sqrt{\tilde{g}_{3,0}^2 + \tilde{g}_{3,1}^2}}(\tilde{g}_{3,0},\ -\tilde{g}_{3,1},\ 0),$$

$$\vec{a}_1 \approx \frac{1}{\sqrt{(\tilde{g}_{3,0}^2\tilde{g}_{3,1}^2 + \tilde{g}_{3,1}^2\tilde{g}_{3,2}^2 + \tilde{g}_{3,0}^2\tilde{g}_{3,2}^2)(\tilde{g}_{3,0}^2 + \tilde{g}_{3,1}^2)}}[\tilde{g}_{3,0}\tilde{g}_{3,1}^2,\ \tilde{g}_{3,0}^2\tilde{g}_{3,1},\ \tilde{g}_{3,2}(\tilde{g}_{3,0}^2 + \tilde{g}_{3,1}^2)],$$

$$\vec{a}_0 = \frac{1}{\sqrt{\tilde{g}_{3,0}^2\tilde{g}_{3,1}^2 + \tilde{g}_{3,1}^2\tilde{g}_{3,2}^2 + \tilde{g}_{3,0}^2\tilde{g}_{3,2}^2}}(\tilde{g}_{3,1}\tilde{g}_{3,2},\ \tilde{g}_{3,0}\tilde{g}_{3,2},\ \tilde{g}_{3,0}\tilde{g}_{3,1}), \tag{A8}$$

$$M_2^2 \approx (\tilde{g}_{3,0}^2 + \tilde{g}_{3,1}^2)\, v_1^2, \quad M_1^2 \approx \frac{\tilde{g}_{3,0}^2\tilde{g}_{3,1}^2 + \tilde{g}_{3,1}^2\tilde{g}_{3,2}^2 + \tilde{g}_{3,0}^2\tilde{g}_{3,2}^2}{\tilde{g}_{3,0}^2 + \tilde{g}_{3,1}^2} v_2^2, \quad M_0^2 = 0. \tag{A9}$$

Note that unlike the translationally invariant case, here the massive states do not necessarily correspond to the lowest eigenstates in the continuum limit.

APPENDIX B: CHIRAL FERMIONS AND A DISCRETIZED VERSION OF THE JACKIW-REBBI DOMAIN WALL

In $1+4$ dimensions free fermions are vectorlike. Chiral fermion zero modes can be obtained by using domain wall kinks in a background field which couples to the fermion like a mass term. This can trap a chiral zero mode at the kink [20]. This mechanism can be generalized to the lattice action [21]. We now discuss the chiral fermions in the discretized version of the Jackiw-Rebbi domain wall.

We first consider an infinite fifth dimension [i.e., there are infinite number of $SU(3)$'s for QCD] and for simplicity, we assume that $\tilde{g}$ and v are constant. From Eq. (14) we see that the kinetic term in the fifth dimension appears as fermion mass terms on the lattice:

$$i\bar{\Psi}\gamma^5\partial_5\Psi \sim \tilde{g}v\,\bar{\Psi}_{iL}\Psi_{iR} - \tilde{g}v\,\bar{\Psi}_{iL}\Psi_{(i-1)R} + \text{H.c.} \tag{B1}$$

Note that the derivative hops $[i,L] \to [(i-1),R]$. We can equally well represent the derivative as

$$i\bar{\Psi}\gamma^5\partial_5\Psi \sim \tilde{g}v\,\bar{\Psi}_{iL}\Psi_{(i+1)R} - \tilde{g}v\,\bar{\Psi}_{iL}\Psi_{iR} + \text{H.c.}, \tag{B2}$$

hopping $[(i-1),L] \to [i,R]$.

The mass matrix between $\bar{\Psi}_L$ and Ψ_R, $\bar{\Psi}_L M_f \Psi_R$ in the first convention is

$$M_f=\tilde{g}v\begin{pmatrix} \ddots & \ddots & \cdots & & & \\ \ddots & 1 & 0 & \cdots & & \\ \cdots & -1 & 1 & 0 & \cdots & \\ & 0 & -1 & 1 & 0 & \cdots \\ & \vdots & 0 & -1 & 1 & \ddots \\ & & \vdots & \vdots & \ddots & \ddots \end{pmatrix}. \quad \text{(B3)}$$

A left-handed chiral zero mode can be localized at $y=y_k$ by a kink fermion mass term which has $m_\Psi(y<y_k)>0$ and $m_\Psi(y>y_k)<0$. In the discrete version, one can add positive and negative masses to the diagonal term, $-m\overline{\Psi}_{iL}\Psi_{iR}$ for $i<k$ and $i>k$ respectively as in Ref. [21]. For example, with the kink at $k=3$ we have

$$M_f=\begin{pmatrix} \tilde{g}v+m & 0 & \cdots & & \\ -\tilde{g}v & \tilde{g}v+m & 0 & \cdots & \\ 0 & -\tilde{g}v & \tilde{g}v & 0 & \cdots \\ \vdots & 0 & -\tilde{g}v & \tilde{g}v-m & \ddots \\ & \vdots & \ddots & \ddots & \ddots \end{pmatrix}. \quad \text{(B4)}$$

The *approximate* solution for a zero mode is then

$$\psi_L\propto\sum_i \epsilon^{|i-k-1/2|}\Psi_{iL}, \quad \epsilon\approx\frac{\tilde{g}v}{\tilde{g}v+m}<1, \quad \text{(B5)}$$

where in this case we require $m\ll\tilde{g}v$ and hence localization of the zero mode requires a fine grain lattice such that $|\tilde{g}v-m|<\tilde{g}v$.

Alternatively, and more efficiently for a coarse grain lattice, we can give a positive mass to the diagonal mass term $m\overline{\Psi}_{iL}\Psi_{iR}$, $m>0$ for $i<k$, and a negative mass to the off-diagonal mass term $-m\overline{\Psi}_{iL}\Phi_i\Psi_{(i-1)R}/v$ for $i>k$. As in the previous example, we now have

$$M_f=\begin{pmatrix} \tilde{g}v+m & 0 & \cdots & & \\ -\tilde{g}v & \tilde{g}v+m & 0 & \cdots & \\ 0 & -\tilde{g}v & \tilde{g}v & 0 & \cdots \\ \vdots & 0 & -\tilde{g}v-m & \tilde{g}v & \ddots \\ & \vdots & \ddots & \ddots & \ddots \end{pmatrix}. \quad \text{(B6)}$$

This enhances the diagonal links for $i<k$ and the off-diagonal links for $i>k$. A left-handed chiral zero mode then arises centered around Ψ_{kL} which has the "weakest links." One can easily check that the state

$$\psi_L\propto\sum_i \epsilon^{|i-k|}\Psi_{iL}, \quad \epsilon=\frac{\tilde{g}v}{\tilde{g}v+m}<1 \quad \text{(B7)}$$

is a zero mode, while there is no normalizable right-handed zero mode. The width of the zero mode becomes narrower for smaller ϵ. In the limit $m\gg\tilde{g}v$, the zero mode is effectively localized only on the lattice point k. Similarly, a right-handed chiral mode can be localized by considering the opposite mass profile, and the method can easily be adapted to the opposite derivative (hopping) definition.

If we compactify the extra dimension with the periodic boundary condition, there will be another zero mode with the opposite chirality localized at the anti-kink of the mass term. In general, the pair of zero modes will receive a small mass due to the tunneling between the finite distance of the kink–anti-kink separation unless some fine-tuning is made. With the S^1/Z_2 orbifold compactification, however, one of the zero modes will be projected out. One can see that in the discrete aliphatic model, the boundary conditions remove one chiral fermion at the end of the lattice point, so there must be a chiral fermion left massless due to the mismatch of the numbers of the left-handed and right-handed fermions. The massless chiral fermion can be localized anywhere on the lattice using the discrete domain wall mass term.

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