$K_L \to \pi^0 \gamma\gamma$ and the bound on the CP-conserving $K_L \to \pi^0 e^+ e^-$

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It has been known for many years that there is a CP-conserving component for the decay mode $K_L \to \pi^0 e^+ e^-$ and that its magnitude can be obtained from a measurement of the amplitudes in the $K_L \to \pi^0 \gamma\gamma$ decay mode. We point out that the usual description of the latter in terms of a single parameter a_V is not sufficient to extract the former in a model independent manner. We further show that there exist known physics contributions to $K_L \to \pi^0 \gamma\gamma$ that cannot be described in terms of the single parameter a_V. We conclude that a model independent analysis requires the experimental extraction of three parameters.

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I. INTRODUCTION

The mode $K_L \to \pi^0 \gamma\gamma$ has been the subject of intense study both as a test of chiral perturbation theory [1] and as the source of a CP-conserving amplitude for $K_L \to \pi^0 e^+ e^-$ [2–8]. It has been known since the first experimental results appeared [9] that lowest order (p^4) chiral perturbation theory is not sufficient to explain simultaneously the observed rate and spectrum. For some time now, it has become standard to use a theoretical description which incorporates certain nonanalytic terms at next to leading order (p^6) [10,11], as well as one parameter a_V [11]. This parameter arises in vector meson dominance models for this decay [12], but it does *not* parametrize the most general analytic amplitude at next to leading order in chiral perturbation theory, p^6. Instead, at order p^6 the amplitude is described by three independent parameters: α_1, α_2 and β in the notation of Cohen *et al.* [11].

Nevertheless, the parametrization of the amplitudes for $K_L \to \pi^0 \gamma\gamma$ in terms of a_V alone has been retained in the literature. In this paper we wish to point out that this is insufficient if one wants to extract a model independent bound on the CP-conserving component of $K_L \to \pi^0 e^+ e^-$ from experiment. This is something which should be considered by the forthcoming experimental analyses of the mode $K_L \to \pi^0 \gamma\gamma$ by the KTeV and NA48 collaborations. Within the framework of chiral perturbation theory the new data should be analyzed in terms of α_1, α_2 and β. The issue of whether vector mesons dominate this decay mode is an experimental question, and should *not* be an input in the analysis of data. As a further motivation for the more general fit, we show in this paper that there exists known physics, the $f_2(1270)$, which affects the $K_L \to \pi^0 \gamma\gamma$ amplitude at a level comparable to that of vector mesons, and which cannot be parametrized in terms of the single constant a_V. It should be no surprise that the $f_2(1270)$ can play an important role in this decay mode, given its prominence in the reaction $\gamma\gamma \to \pi^0 \pi^0$ [13].

Of particular importance is the determination of the CP-conserving contribution to $K_L \to \pi^0 e^+ e^-$. This contribution is completely dominated by one of the two amplitudes present in $K_L \to \pi^0 \gamma\gamma$. In fact, it is of phenomenological relevance only when it arises from the amplitude in which the two photons are in a relative D-wave [4]. For this reason an accurate determination of both amplitudes is crucial. The model independent analysis we advocate here permits the extraction of the necessary information directly from the data, whereas the usual analysis in terms of a_V forces correlations between the two $K_L \to \pi^0 \gamma\gamma$ amplitudes which may or may not be present in the data.

II. $K_L \to \pi^0 \gamma\gamma$ AMPLITUDES AND FIT

In this section we review the parametrization of the $K_L \to \pi^0 \gamma\gamma$ amplitude with terms of order up to p^6 in chiral perturbation theory and we compare fits to the KTeV data from 1999 in terms of a_V and in the general parametrization.

The most general form of the $K \to \pi\gamma\gamma$ amplitude contains four independent invariant amplitudes A, B, C and D and has been described in the literature before [3]. For the case of $K_L \to \pi^0 \gamma\gamma$, and in the limit of CP conservation, only two of these amplitudes come into play:

$$\mathcal{M}[K_L(p_K) \to \pi^0(p_\pi)\gamma(q_1)\gamma(q_2)] = \frac{G_8 \alpha_{EM}}{4\pi}\epsilon_\mu(q_1)\epsilon_\nu(q_2) \times \Bigg[A(q_2^\mu q_1^\nu - q_1\cdot q_2 g^{\mu\nu}) + 2\frac{B}{m_K^2}(p_K\cdot q_1 q_2^\mu p_K^\nu + p_K\cdot q_2 q_1^\nu p_K^\mu - q_1\cdot q_2 p_K^\mu p_K^\nu - p_K\cdot q_1 p_K\cdot q_2 g^{\mu\nu}) \Bigg], \quad (1)$$

where $G_8 = 9.1\times 10^{-6}\ \mathrm{GeV}^{-2}$ and $\alpha_{EM} \approx 1/137$ is the usual electromagnetic fine structure constant. In chiral perturbation theory with terms of order up to p^6, the amplitudes A and B take the form [11]

$$A(z)=4F\left(\frac{z}{r_\pi^2}\right)\frac{a_1(z)}{z}+4\frac{F(z)}{z}(1+r_\pi^2-z)$$
$$+\frac{a_2M_K^2}{\Lambda_\chi^2}\left\{\frac{4r_\pi^2}{z}F\left(\frac{z}{r_\pi^2}\right)+\frac{2}{3}\left(2+\frac{z}{r_\pi^2}\right)\left[\frac{1}{6}+R\left(\frac{z}{r_\pi^2}\right)\right]\right.$$
$$-\frac{2}{3}\log\frac{m_\pi^2}{M_\rho^2}-2\frac{r_\pi^2}{z^2}(z+1-r_\pi^2)^2\left[\frac{z}{12r_\pi^2}+F\left(\frac{z}{r_\pi^2}\right)\right.$$
$$\left.+\frac{z}{r_\pi^2}R\left(\frac{z}{r_\pi^2}\right)\right]+8\frac{r_\pi^2}{z^2}y^2\left[\frac{z}{12r_\pi^2}+F\left(\frac{z}{r_\pi^2}\right)+\frac{z}{2r_\pi^2}F\left(\frac{z}{r_\pi^2}\right)\right.$$
$$\left.\left.+3\frac{z}{r_\pi^2}R\left(\frac{z}{r_\pi^2}\right)\right]\right\}+\alpha_1(z-r_\pi^2)+\alpha_2,$$

$$B(z)=\frac{a_2M_K^2}{\Lambda_\chi^2}\left\{\frac{4r_\pi^2}{z}F\left(\frac{z}{r_\pi^2}\right)+\frac{2}{3}\left(10-\frac{z}{r_\pi^2}\right)\left[\frac{1}{6}+R\left(\frac{z}{r_\pi^2}\right)\right]\right.$$
$$\left.+\frac{2}{3}\log\frac{m_\pi^2}{m_\rho^2}\right\}+\beta, \tag{2}$$

where we use the kinematic variables

$$z=\frac{(q_1+q_2)^2}{M_K^2},\quad y=\frac{p_K\cdot(q_1-q_2)}{M_K^2}, \tag{3}$$

and $\Lambda_\chi\approx 4\pi f_\pi\approx 1.17$ GeV.

This form for the two amplitudes does not correspond to a complete calculation in chiral perturbation theory at order p^6. It contains the complete one-loop calculation of order p^4 [1] and two types of terms of order p^6. The first type consists of the non-analytic terms in Eq. (2) that multiply the factors a_2 and $a_1(z)$. The inclusion of these terms is inspired by dispersion relations, and they originate in p^4 corrections to the $K\to 3\pi$ amplitudes [14,15]. The relevant constants which enter a_1 and a_2 are extracted from an analysis of $K\to 3\pi$ data. The second type of term consists of the analytic terms that arise from tree-level contributions from order p^6 chiral Lagrangians. These contributions can be grouped into three unknown constants: α_1, α_2 and β corresponding to the three possible Lorentz invariant forms which occur at order p^6 for the $K_L\pi^0\gamma\gamma$ vertex [11]. From the analysis of $K\to 3\pi$ in Ref. [14], we have

$$a_1(z)=0.38+0.13Y_0-0.0059Y_0^2,$$
$$Y_0=\frac{\left(z-r_\pi^2-\frac{1}{3}\right)}{r_\pi^2},$$
$$a_2=6.5, \tag{4}$$

with $r_\pi=m_\pi/M_K$. The loop form factors are given by [11]

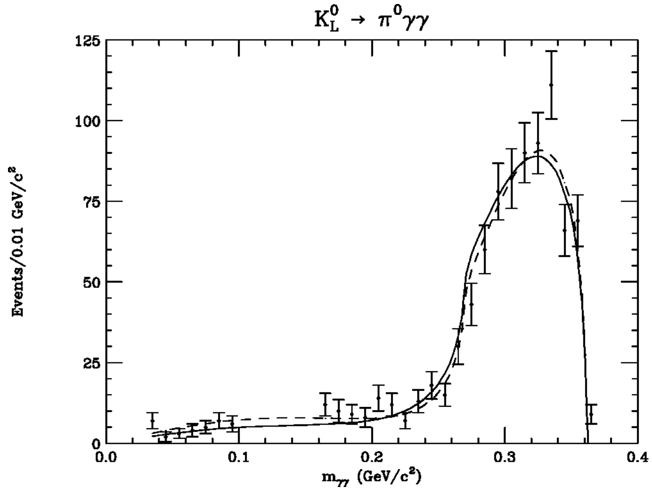

FIG. 1. Two different fits to the data from Ref. [16], as explained in the text. The solid line is a one-parameter fit corresponding to Eq. (6), the dashed line is the three-parameter fit shown in Eq. (7).

$$F(z)=1-\frac{4}{z}\left[\arcsin\left(\frac{1}{2}\sqrt{z}\right)\right]^2,\quad z\leqslant 4,$$
$$=1+\frac{1}{z}\left(\log\frac{1-\sqrt{1-4/z}}{1+\sqrt{1-4/z}}+i\pi\right)^2,\quad z\geqslant 4,$$
$$R(z)=-\frac{1}{6}+\frac{2}{z}\left[1-\sqrt{4/z-1}\arcsin\left(\frac{1}{2}\sqrt{z}\right)\right],\quad z\leqslant 4,$$
$$-\frac{1}{6}+\frac{2}{z}+\frac{\sqrt{1-4/z}}{z}\left(\log\frac{1-\sqrt{1-4/z}}{1+\sqrt{1-4/z}}+i\pi\right),\quad z\geqslant 4.$$

In the analysis of Ref. [11], which has become standard, the three unknown constants were fixed in terms of the contribution they receive from vector-meson exchange, supplemented with a minimal subtraction ansatz:

$$\alpha_1=-4a_V,$$
$$\alpha_2=12a_V-0.65,$$
$$\beta=-8a_V-0.13, \tag{5}$$

and this form has been used, for example, by KTeV [16] to fit their data with $a_V=-0.72\pm0.05\pm0.06$. In Eq. (5) β is no longer independent of $\alpha_{1,2}$; therefore it is clear that this ansatz introduces model-dependent correlations between the B amplitude (the one responsible for a large CP-conserving $K_L\to\pi^0e^-e^-$), and the A amplitude which dominates the $K_L\to\pi^0\gamma\gamma$ mode, but which does not contribute significantly to $K_L\to\pi^0e^+e^-$.

In Fig. 1 we reproduce the data from Ref. [16] as can be read from their published paper. We superimpose on the data the best fit we obtain in terms of the parameter a_V as a solid line. Our fit gives $a_V=-0.95$ with a χ^2/d.o.f.$=46/27$, which corresponds to

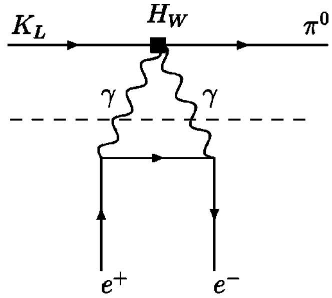

FIG. 2. Contribution from the on-shell two-photon intermediate state to $B_{CP}(K_L\to\pi^0e^+e^-)$.

$$\alpha_1=3.8,\quad \alpha_2=-12.0,\quad \beta=7.5. \tag{6}$$

Notice that our value for a_V is not the same value quoted by Ref. [16] because we do not have access to the raw data and hence we have not taken into consideration any background or detector issues. Nevertheless, we feel that it is fair to compare this fit to our best three-parameter fit obtained in the same way. This one is presented in Fig. 1 as the dashed line, and corresponds to

$$\alpha_1=0,\quad \alpha_2=1.7,\quad \beta=-5. \tag{7}$$

For this fit we obtain a χ^2/d.o.f.=37/25, slightly better than Eq. (6). Clearly it is up to the experimentalists to present a complete best fit to the data using the general form, Eqs. (1), (2), and taking into consideration all the experimental issues.[1] However, it should be clear from Fig. 1 that even though the current data are consistent with the vector dominance assumption, they cannot rule out other scenarios. In fact, our three parameter best fit is not consistent with the vector meson dominance assumption. In the Appendix we explore the significance of our fit showing the range allowed for its three parameters within one sigma from our best χ^2. This provides an estimate of the errors involved.

Although the two types of fit are indistinguishable as far as describing the $K_L\to\pi^0\gamma\gamma$ spectrum, they result in completely different predictions for the unitarity bound on the CP-conserving contribution to $K_L\to\pi^0e^+e^-$. To evaluate it we need to calculate the absorptive contribution from the on-shell two-photon intermediate state to $K_L\to\pi^0e^+e^-$, as depicted in Fig. 2. This yields the following bounds on the CP-conserving part of $B_{CP}(K_L\to\pi^0e^+e^-)$:

$$B_{CP}(K_L\to\pi^0e^+e^-) \geqslant \begin{cases} 2.3\times10^{-12} & \text{vector meson dominance (VMD)},\\ 3.4\times10^{-12} & \text{three-parameter fit.}\end{cases} \tag{8}$$

The above contribution is not the full absorptive part since there is a further cut due to on-shell pions. Moreover, the full CP-conserving amplitude includes a contribution from the dispersive part of the amplitude, with off-shell photons (and pions). The general form of the amplitude is

$$\mathcal{M}_{CP}(K_L\to\pi^0e^+e^-)=G_8\alpha_{EM}^2Kp_K\cdot(k_{e^+}-k_{e^-})\times(p_K+p_\pi)^\mu\bar{u}\gamma_\mu v, \tag{9}$$

where K is the result of the loop calculation and the extra antisymmetry under $k_{e^+}\leftrightarrow k_{e^-}$ is a reflection of the properties under a CP transformation. Introducing a form factor to regularize the virtual photon couplings, an expression for K [7] is obtained:

$$K=\frac{B(x)}{16\pi^2m_K^2}\left[\frac{2}{3}\log\left(\frac{m_\rho^2}{-s}\right)-\frac{1}{4}\log\left(\frac{-s}{m_e^2}\right)+\frac{7}{18}\right], \tag{10}$$

where $s=(k_{e^+}+k_{e^-})^2$. The log factor is of course expected, since the photon absorptive part comes from the expansion $\log(-s)=\log s+i\pi$. This representation of the amplitude leads to CP-conserving branching ratios:

$$B_{CP}(K_L\to\pi^0e^+e^-)=\begin{cases} 4.8\times10^{-12} & \text{VMD}\\ 7.3\times10^{-12} & \text{three-parameter fit.}\end{cases} \tag{11}$$

III. RESONANCE MODELS FOR α_1,α_2 AND β

In this section we present the contributions of scalar and tensor mesons to the parameters α_1, α_2 and β. We will be able to show that the tensor meson $f_2(1270)$, in particular, can contribute at a level comparable to that of vector mesons and yet produce a different pattern for the three constants.

We have chosen to follow the notation of [11], where the parameters α_1, α_2 and β are defined by the expression for the invariant amplitudes A and B as in Eq. (2). It is convenient to relate these parameters to the three Lorentz invariant couplings that can be derived from a chiral Lagrangian at order p^6. Writing these couplings as

$$\mathcal{L}=\frac{G_8\alpha_{EM}}{4\pi}\left(c_1K_L\pi^0F^{\mu\nu}F_{\mu\nu}+\frac{c_2}{M_K^2}\partial^\alpha K_L\partial_\alpha\pi^0F^{\mu\nu}F_{\mu\nu}+\frac{c_3}{M_K^2}\partial_\alpha K_L\partial^\beta\pi^0F^{\alpha\mu}F_{\mu\beta}\right), \tag{12}$$

where $F_{\mu\nu}$ is the usual electromagnetic field strength tensor, one finds that

[1] We proceed keeping the branching ratio fixed to the one measured by KTeV [16] $(1.68\pm0.07\pm0.08)\times10^{-6}$, in the normalization of our fits. Eventually the parameters extracted from all our fits yield branching ratios very close to the experimental one and well within its errors.

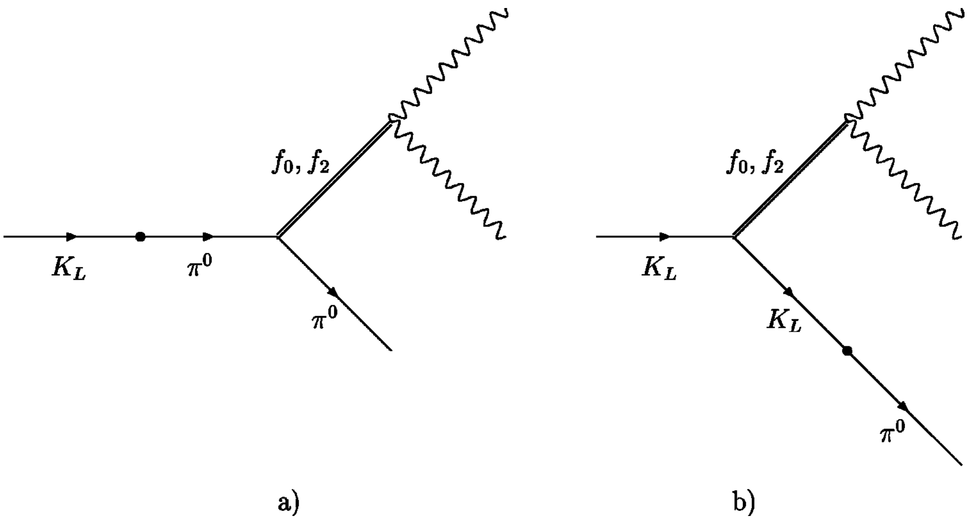

FIG. 3. Scalar- and tensor-meson resonance Feynman diagrams contributing to $K_L \to \pi^0 \gamma\gamma$. The dots in (a) and (b) represent flavor-changing mass-insertions in the incoming and outcomimg particles, respectively [1,3,24].

$$\alpha_1 = -2c_2 + \frac{c_3}{2},$$

$$\alpha_2 = 4c_1 + 2c_2 + \frac{c_3}{2},$$

$$\beta = -c_3. \tag{13}$$

The couplings that occur at order p^6 in a vector meson dominance model have been obtained in [12]. They are of the form

$$\mathcal{L}_V = \frac{G_8 \alpha_{EM}}{4\pi} \frac{4a_V}{M_K^2} (\partial^\alpha K_L \partial_\alpha \pi^0 F^{\mu\nu} F_{\mu\nu} + 2\partial_\alpha K_L \partial^\beta \pi^0 F^{\alpha\mu} F_{\mu\beta}) \tag{14}$$

and, therefore, the prediction of vector meson dominance is that

$$\alpha_1 = -4a_V,$$

$$\alpha_2 = 12a_V,$$

$$\beta = -8a_V. \tag{15}$$

This prediction is at the heart of Eq. (5), and differs from it only by small additional constants which appear in a particular regularization scheme for the loop amplitudes [11]. Although this pattern is a firm prediction of vector meson dominance models, a specific value for a_V is not. For example, in Ref. [12] the values $a_V = 0.32$ or $a_V = -0.32$ can be obtained depending on whether or not one uses the so called "weak deformation model." This is just another way of saying that the concept of "vector meson dominance" is not uniquely defined for the weak interactions. In addition, phenomenological treatments of vector mesons such as those of Ref. [17] include effects from $\eta - \eta'$ mixing, which are formally of higher order, but which result in significantly different "vector meson" contributions to $K_L \to \pi^0 \gamma\gamma$. It is worth mentioning that a quark model estimate of the parameters α_1, α_2 and β [18] yields the same pattern as in Eq. (15) with $a_V = (N_c/27) g_A^2 (M_K^2/m^2)$ in the notation of [18].

In addition to the vector meson exchange contributions, the parameters α_1, α_2 and β may receive contributions from the exchange of scalar and tensor resonances. The effect of scalar resonances near 1 GeV has been found to be small [19], and we include it here for completeness. Moreover, we sidestep the issue of a possible scalar resonance in the vicinity of 500 MeV because the physics of this broad enhancement in the $J = I = 0$ $\pi\pi$ scattering amplitude is, to a large extent, already included in the treatment of the pion loops. We concentrate instead in resonances near 1 GeV such as the $f_0(980)$, and take the simplest form for the scalar-pion and scalar-photon interactions [20] (we use U as in the notation of Gasser and Leutwyler [21]):

$$\mathcal{L}_S = g_\pi S \, \mathrm{Tr}(D^\mu U D_\mu U^\dagger) + \frac{\alpha_{EM}}{4\pi} g_\gamma S F^{\mu\nu} F_{\mu\nu}. \tag{16}$$

We have not included a coupling of the scalar field proportional to light quark masses because it does not contribute to $K_L \to \pi^0 \gamma\gamma$, and because there is not enough experimental information on scalar-meson decays to extract it.

The coupling g_π can be determined from the decay width of the scalar into two pions. Adding the charged and neutral modes we obtain

$$\Gamma(S \to \pi\pi) = \frac{3}{8\pi f_\pi^4} \sqrt{1 - 4r_{\pi S}^2} \, g_\pi^2 M_S^3 (1 - 2r_{\pi S}^2 + 4r_{\pi S}^4), \tag{17}$$

with $r_{\pi S} = M_\pi / M_S$. If we identify the scalar meson with the $f_0(980)$, and use the particle data book figures $B(f_0 \to \pi^+ \pi^-) = 2/3$, $B(f_0 \to \pi^0 \pi^0) = 1/3$ [22] and the NOMAD result $\Gamma(f_0) = 35 \pm 12$ MeV [23] we find $g_\pi \sim \pm 5$ MeV (we cannot decide the sign ambiguity from the experimental rates).

The width for the scalar-meson decay into two photons allows us to determine g_γ. We find for the width

TABLE I. A comparison of parameters for $K_L \to \pi^0\gamma\gamma$ for various contributions discussed in the text. We contrast these contributions with our best three-parameter fit, as well as with our best fit within the VMD ansatz.

	Vector ($a_V = \pm 0.32$)	Scalar	Tensor	Our Best Fit	Best Fit a_V
α_1	∓ 1.2	∓ 0.08	± 0.25	0	3.8
α_2	± 3.6	± 0.08	∓ 1.7	1.7	-12
β	∓ 2.4	0	± 1.5	-5	7.5

$$\Gamma(S\to\gamma\gamma) = \left(\frac{\alpha_{EM}}{4\pi}\right)^2 \frac{g_\gamma^2 M_S^3}{4\pi}. \tag{18}$$

If again we identify the scalar with the $f_0(980)$ and use the particle data book value $\Gamma(f_0\to\gamma\gamma) = 0.39^{+0.10}_{-0.13} \times 10^{-3}$ MeV [22], we find $g_\gamma \sim \pm 3.9\times 10^{-3}$ MeV^{-1}.

Collecting these results we finally obtain for the contribution of the scalar $f_0(980)$ to $K_L\to\pi^0\gamma\gamma$ (see Fig. 3):

$$\alpha_1 = -\alpha_2 = -16 g_\pi g_\gamma \frac{M_K^2}{M_S^2} \sim \mp 0.08, \quad \beta = 0. \tag{19}$$

In a similar manner we can determine the contribution from a tensor meson. A simple look at the low energy data for the reaction $\gamma\gamma\to\pi^0\pi^0$ [13] suffices to motivate the potential importance of the $f_2(1270)$ for our amplitudes through diagrams such as those in Fig. 3. Following Ref. [20] we write the lowest order couplings of a tensor meson $T_{\mu\nu}$ to pions and photons as

$$\mathcal{L}_T = h_\pi T^{\mu\nu}\mathrm{Tr}(D_\mu U D_\nu U^\dagger) + \frac{\alpha_{EM}}{4\pi} h_\gamma T^{\mu\nu} F_{\mu\alpha} F^{\alpha}_{\nu}. \tag{20}$$

For the inclusive width of the tensor meson into two pions, and following Ref. [25] for the description of the spin 2 states, we obtain

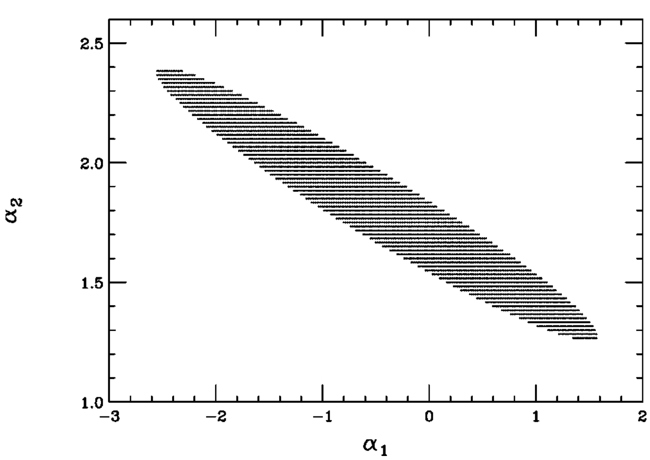

FIG. 4. Scatter plot of the parameter space allowed for α_1, α_2 with a fixed $\beta=-5$, within one sigma from our $\chi^2_{\min}$.

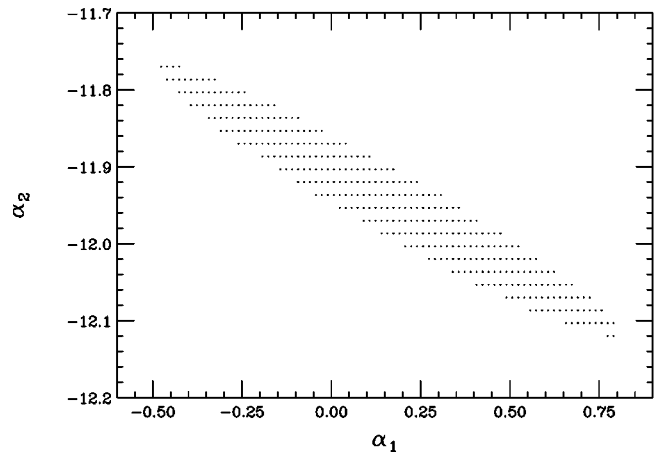

FIG. 5. Same as Fig. 4 with $\beta = 8.55$.

$$\Gamma(T\to\pi\pi) = \frac{3 h_\pi^2 M_T^3}{240\pi f_\pi^4}(1-4m_\pi^2/M_T^2)^{5/2}. \tag{21}$$

For the decay width of the tensor meson into two photons we find

$$\Gamma(T\to\gamma\gamma) = \left(\frac{\alpha_{EM}}{4\pi}\right)^2 \frac{h_\gamma^2 M_T^3}{80\pi}. \tag{22}$$

Identifying the tensor meson with the $f_2(1270)$ and using the particle data book values for mass and partial widths [22], we obtain $h_\pi \sim \pm 40$ MeV and $h_\gamma \sim \pm 0.03$ MeV^{-1}.

The tensor (f_2) contribution to the parameters α_1, α_2 and β can be read from the interaction that results after the tensor meson has been integrated out

$$\mathcal{L}_T = \frac{G_8\alpha_{EM}}{4\pi}\frac{4h_\pi h_\gamma}{M_T^2}\left(\frac{2}{3}\partial^\alpha K_L \partial_\alpha \pi^0 F^{\mu\nu}F_{\mu\nu} + 2\partial_\alpha K_L \partial^\beta \pi^0 F^{\alpha\mu}F_{\mu\beta}\right). \tag{23}$$

The resulting contributions are

$$\alpha_1 = \frac{4}{3} h_\pi h_\gamma \frac{M_K^2}{M_T^2} \sim \pm 0.25,$$

$$\alpha_2 = -\frac{28}{3} h_\pi h_\gamma \frac{M_K^2}{M_T^2} \sim \mp 1.7,$$

$$\beta = 8 h_\pi h_\gamma \frac{M_K^2}{M_T^2} \sim \pm 1.5. \tag{24}$$

We summarize our results in Table I.

IV. CONCLUSION

We expect new data for $K_L\to\pi^0\gamma\gamma$ from KTeV and NA48 in the near future, and this makes a reanalysis of this mode timely. We have argued that the new results should not be analyzed in terms of the vector meson dominance ansatz, but rather in a model independent way, and that this entails the use of three parameters. These three parameters are related to the three *a priori* undetermined counterterms entering the amplitude, as shown in Eqs. (12),(13).

To illustrate the previous point we have re-examined the fit to the 1999 KTeV data. We find that the general, three-parameter fit is slightly better than the old fit in terms of a_V, and we show our results in Fig. 1. The difference between the two procedures appears to be small in the $K_L \to \pi^0 \gamma\gamma$ spectrum. Nevertheless, it leads to significantly different predictions for the CP-conserving component of $K_L \to \pi^0 e^+ e^-$, which can be seen in Eqs. (8),(11). New data, with higher statistics, should be able to better distinguish the two cases.

As a further motivation for abandoning the usual parametrization, we have also shown that the $f_2(1270)$ tensor meson can yield an important contribution to the counterterms, and that this contribution cannot be cast in the one-parameter framework of vector meson dominance.

ACKNOWLEDGMENTS

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APPENDIX

To compute an estimate of the errors involved in our fits we calculate the range of variation of the three parameters from our "best fit" $\chi^2_{\rm min}=37$ to $\chi^2_{\rm min}+1$ (corresponding to one standard deviation), obtaining:

$$-2.0<\alpha_1<1.9,$$
$$0.8<\alpha_2<2.5,$$
$$-5.3<\beta<-4.5. \qquad \text{(A1)}$$

The "central" values give roughly a linear equation:

$$\alpha_2+0.29\alpha_1=1.65. \qquad \text{(A2)}$$

This is consistent with $z-r_\pi^2\sim 0.3$ dominating any z dependence.

The above values are to be compared with the ones given in Eq. (7):

$$\alpha_1=0,$$
$$\alpha_2=1.7,$$
$$\beta=-5. \qquad \text{(A3)}$$

In Fig. 4 we present a one-sigma plot of the parameter space for α_1 and α_2 with a fixed $\beta=-5.0$.

It is possible to redo the calculation keeping each time one parameter variable and the other two fixed at the values of Eq. (7). In this case they are much more constrained:

$$-0.5<\alpha_1<0.6,$$
$$1.5<\alpha_2<1.8,$$
$$-5.1<\beta<-4.9. \qquad \text{(A4)}$$

There exists another region in the parameter space where χ^2 is within one sigma from $\chi^2_{\rm min}$:

$$-0.6<\alpha_1<0.8,$$
$$-12.2<\alpha_2<-11.8,$$
$$8.4<\beta<8.7. \qquad \text{(A5)}$$

Note that the one-parameter fit in terms of a_V lies closer to this second region.

Figure 5 is analogous to Fig. 4 assuming $\beta=8.55$, and is consistent with the "central values" linear equation

$$\alpha_2+0.28\alpha_1=-11.9. \qquad \text{(A6)}$$

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