Radiative transitions in heavy mesons in a relativistic quark model

J. L. Goity
Department of Physics, Hampton University, Hampton, Virginia 23668
and Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606

W. Roberts*
National Science Foundation, 4201 Wilson Boulevard, Arlington, Virginia 22230
(Received 22 December 2000; published 3 October 2001)

The radiative decays of D^*, B^*, and other excited heavy mesons are analyzed in a relativistic quark model for the light degrees of freedom and in the limit of heavy quark spin-flavor symmetry. The analysis of strong decays carried out in the corresponding chiral quark model is used to calculate the strong decays and determine the branching ratios of the radiative D^* decays. Consistency with the observed branching ratios requires the inclusion of the heavy quark component of the electromagnetic current and the introduction of an anomalous magnetic moment for the light quark. It is observed that not only D, but also B meson transitions within a heavy quark spin multiplet are affected by the presence of the heavy quark current.

DOI: 10.1103/PhysRevD.64.094007 PACS number(s): 13.25.−k, 12.39.Fe, 12.39.Hg, 12.39.Ki

I. INTRODUCTION

In recent articles, we examined the spectra [1] and strong decay widths [2] of a number of heavy-light mesons in a relativistic chiral-quark model. The strong decays were assumed to take place through pion or kaon emission from the brown muck of the heavy meson, with the heavy quark being essentially a spectator in the decay. We found that relativistic effects in the decays were large, as many results we obtained were quite different from those obtained in analogous nonrelativistic calculations [3].

It is well known that the meson emission decays of the ground-state vector mesons are suppressed or forbidden by phase space. For the D^* mesons, the measured radiative partial widths are of the same order of magnitude as their partial widths for pion emission as a result of this suppression. The branching ratios (BR) reported are [4]

$$\mathrm{BR}(D^{*0}\to D^0\pi^0)=61.9\pm 2.9\%,$$

$$\mathrm{BR}(D^{*0}\to D^0\gamma)=38.1\pm 2.9\%,$$

$$\mathrm{BR}(D^{*+}\to D^+\pi^0)=30.6\pm 2.5\%,$$

$$\mathrm{BR}(D^{*+}\to D^0\pi^+)=68.3\pm 1.4\%,$$

$$\mathrm{BR}(D^{*+}\to D^+\gamma)=1.1^{+2.1}_{-0.7}\%,$$

$$\mathrm{BR}(D_s^*\to D_s\pi^0)=5.8\pm 2.5\%,$$

$$\mathrm{BR}(D_s^*\to D_s\gamma)=94.2\pm 2.5\%.$$

Of the three total widths, that of the $D^{*\pm}$ has only recently been measured by the CLEO Collaboration [5], with the result $\Gamma_{D^{*\pm}}=96\pm 4\pm 22$ keV. For the D^{*0}, only an upper limit is known [4], namely $\Gamma_{D^{*0}}<2$ MeV, and for the D_s^*, no information is currently available. The use of isospin symmetry arguments for the strong decay amplitudes, together with the reported branching ratios of the D^{*0}, suggest the rough upper limit $\Gamma_{D^{*0}}<75$ keV. Finally, no information is currently available on other radiative transitions, except for the transition $D_{s1}(1^+)\to D_s^*\gamma$ quoted as "seen" by the Particle Data Group [4].

In addition to the states mentioned above, the electromagnetic decays of a number of the lower-lying states, including orbitally excited states, are expected to be significant or even dominant. One example are the B^* mesons, where the B^*-B mass differences are of the order of 50 MeV, leaving the radiative mode as the only possible one. Note that no experimental information on the B^* radiative decays is available. For some of the orbitally excited D_s and B_s states, K meson emission is forbidden or suppressed by phase space, leaving the electromagnetic decays and isospin violating and/or Okubo-Zweig-Iizuka (OZI) violating pion emission as the only possible decay modes. Examples of such states are the D_s^* and the $D_s\,(0^+,1^+)$ and $B_s\,(0^+,1^+)$ doublets. A calculation of the radiative decays is thus crucial for understanding these states.

The radiative decays of heavy mesons have been examined by a number of authors, in a variety of different frameworks. Kamal and Xu [7] have treated the decays of the ground state vector mesons in a simple quark model. Fayyazuddin and Mobarek [8] have also used a simple quark model to study these decays, but have also looked at the radiative decays of the $(1^+,2^+)$ multiplet. Oda, Ishida and Morikawa [9] have used a covariant oscillator model, while Avila [10] uses a covariant model to describe the mesons, and both sets of authors deal only with the M_1 decays of the ground state vector mesons. Cho and Georgi [11], Cheng *et al.* [12], and Amundson *et al.* [13] use the heavy hadron chiral symmetry to treat these decays, while Körner *et al.* [14] use the heavy quark effective theory in a hybrid ap-

*Present address: Department of Physics, Old Dominion University, Norfolk, VA 23529 and Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606.

proach, supplemented with quark model input, to treat the decays of the $(1^+,2^+)$ multiplet. Lähde, Nyfält and Riska [15] treat the decays within the formalism of the Blankenbecler-Sugar equation. Prior to the work mentioned above, Pham [16], Rosner [17], Miller and Singer [18], and Eichten *et al.* [19] have all examined these decays in different scenarios.

The model that we present has features in common with many of those discussed by the authors mentioned in the previous paragraph, but it also has aspects that are different from all of them. Perhaps the most significant difference is the inclusion of an anomalous magnetic moment for the light quark in the meson. Miller and Singer [18] also include an anomalous magnetic moment, but for the heavy quark. In the context of the model we present, we believe that the expected non-perturbative dressing of the light quark argues for endowing the light quark with an anomalous magnetic moment. Our model treats the light quark in a fully relativistic manner, incorporates the symmetries of the chiral and heavy quark limits, includes contributions from both the heavy and light quark currents and, as discussed below, is gauge invariant to the order in the heavy quark limit to which we work.

II. MODEL

The strong decays of excited heavy mesons were analyzed in the framework of the relativistic chiral quark model [2], where the quasi-Goldstone pseudoscalars couple directly to the constituent light quark according to the strictures of chiral symmetry. This interaction is

$$\mathcal{L}_{\Pi q}=\frac{g_A^q}{2F_\pi}\ \overline{\mathbf{q}}\gamma^\mu\gamma_5\partial_\mu\Pi\ \mathbf{q}+\cdots$$

$$\Pi=\begin{bmatrix} \pi^0+\eta/\sqrt{3} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0+\eta/\sqrt{3} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\overline{K^0} & -\dfrac{2}{\sqrt{3}}\eta \end{bmatrix}, \qquad (1)$$

where $\mathbf{q}$ is the flavor-triplet of light quark spinors, and $F_\pi \simeq 93$ MeV is the pion decay constant [SU(3) breaking in the decay constants was taken into account in the calculations]. The axial vector coupling of the constituent quarks g_A^q was determined in [2] from the known strong decays turning out to be approximately equal to 0.8. As mentioned in the Introduction, in [2] it was found that relativistic effects in the strong decays are in some cases dramatic. In view of this, and for the sake of consistency, it is important to carry out the analysis of the electromagnetic (EM) decays within the same relativistic quark model and the $1/m_Q$ expansion where m_Q is the heavy quark mass.

In order to analyze the EM decays the different matrix elements of the EM current must be calculated. For a given heavy meson, the EM current is

$$J^\mu=J_q^\mu+J_Q^\mu=e\,\mathcal{Q}_q\overline{q}\left(\gamma^\mu+\frac{1}{\Lambda}\sigma^{\mu\nu}\overleftrightarrow{\partial}_\nu\right)q+e\,\mathcal{Q}_Q\overline{Q}\,\gamma^\mu Q, \qquad (2)$$

where $\mathcal{Q}_q$ and $\mathcal{Q}_Q$ are, respectively, the charges of the light and heavy quarks. An anomalous magnetic moment for the light quark is introduced, as this is found to be crucial for a consistent description of the D^* radiative decays. Here $\Lambda = 2\tilde{m}_q/\kappa^q$, where κ^q is the anomalous magnetic moment. Throughout this work, the constituent quark masses are taken to be $\tilde{m}_u=\tilde{m}_d=253$ MeV, $\tilde{m}_s=450$ MeV, $m_c=1.53$ GeV, and $m_b=4.87$ GeV.

In the limit of infinite mass of the heavy quark the light quark component of the current is the only one contributing to transitions, its matrix elements being of order unity in the $1/m_Q$ expansion. The suppression of the heavy quark current can be easily verified by considering its Gordon decomposition into the convection and spin currents and by performing an expansion in $1/m_Q$. The matrix elements between heavy quark states are

$$\langle s_Q',p'|J^0|s_Q,p\rangle=e\frac{\mathcal{Q}_Q}{2m_Q}\left\{(E+E')\,\delta_{s_Q s_Q'}+\frac{1}{2m_Q}\langle s_Q'|\vec{k}\cdot\vec{\sigma}\vec{p}\cdot\vec{\sigma}-\vec{p}\,'\cdot\vec{\sigma}\vec{k}\cdot\vec{\sigma}|s_Q\rangle\right\}+\cdots$$

$$\langle s_Q',p'|\vec{J}|s_Q,p\rangle=e\frac{\mathcal{Q}_Q}{2m_Q}\{(\vec{p}+\vec{p}\,')\,\delta_{s_Q s_Q'}-i\langle s_Q'|\vec{k}\times\vec{\sigma}|s_Q\rangle\}+\cdots, \qquad (3)$$

where $\vec{k}$ is the photon momentum and the $\cdots$ denote terms of order $1/m_Q^2$ and higher. The first term in each curly brackets comes from the convection current, and the second arises from the spin current. As made clear below, at order $1/m_Q$ the heavy quark current will only affect those transitions within a heavy-quark spin multiplet, which means that the heavy quark current must affect the spin state of the heavy quark. The convection current affects the spin state of the heavy quark first at order $1/m_Q^2$ and is, therefore, of higher order than the terms considered in this work. Thus, only the heavy-quark spin current has to be considered in what follows. The spatial components of the spin current are proportional to the heavy quark's magnetic moment, i.e., of order $1/m_Q$, while the time component is of order $1/m_Q^2$ and, as the convection current, irrelevant to our analysis.

In order to further understand the role of the heavy quark current, it is instructive to examine its contributions within the framework of the heavy quark effective theory (HQET). Consider for this the transition between D mesons $D_a \to D_b\gamma$. Let us define

$$M_a=m_c+\Lambda_a,\quad M_b=m_c+\Lambda_b, \qquad (4)$$

and let the velocities of the hadrons be v_a and v_b, respectively, with the four-momentum of the photon being k_μ. In the rest frame of the parent hadron, and to order $1/m_c^2$,

TABLE I. Matrix elements of the heavy quark vector current for a few selected decays, and the $1/m_c$ suppression factors associated with them. The polarization vector (tensor) of the initial state meson is denoted by ϵ.

Multiplet	Example decay	V_μ	Suppression
$(0^-,1^-)$	$1^-\to 0^-$	$\xi(v_a\cdot v_b)\epsilon_{\mu\nu\alpha\beta}\epsilon^\nu v_a^\alpha v_b^\beta$	$\frac{1}{m_c}$
$(0^+,1^+)$	$1^+\to 0^-$	$\tau_{1/2^+}(v_a\cdot v_b)[(v_a\cdot v_b-1)\epsilon_\mu-\epsilon\cdot v_b v_{a\mu}]$	$\frac{1}{m_c^2}$
$(1^+,2^+)$	$2^+\to 0^-$	$\tau_{3/2^+}(v_a\cdot v_b)\epsilon^{\rho\nu}v_{b\nu}\epsilon_{\mu\rho\alpha\beta}v_a^\alpha v_b^\beta$	$\frac{1}{m_c^2}$
$(1^-,2^-)$	$2^-\to 0^-$	$\tau_{3/2^-}(v_a\cdot v_b)\epsilon^{\rho\nu}[(v_a\cdot v_b-1)g_{\rho\mu}v_{b\nu}-v_{b\rho}v_{b\nu}v_{a\mu}]$	$\frac{1}{m_c^3}$
$(0^-,1^-)'$	$1^{-'}\to 0^-$	$\tau_{1/2^-}(v_a\cdot v_b)(v_a\cdot v_b-1)\epsilon_{\mu\nu\alpha\beta}\epsilon^\nu v_a^\alpha v_b^\beta$	$\frac{1}{m_c^3}$

$$v_a\cdot K=(\Lambda_a-\Lambda_b)\left[1-\frac{\Lambda_a-\Lambda_b}{2m_c}+\frac{\Lambda_a(\Lambda_a-\Lambda_b)}{2m_c^2}+\cdots\right],$$

$$v_a\cdot v_b=1+\frac{(\Lambda_a-\Lambda_b)^2}{2m_c^2}+\cdots. \tag{5}$$

For transitions within a multiplet, such as for $D^*\to D\gamma$, the mass difference $\Lambda_a-\Lambda_b$ is of order $1/m_c$.

Assuming that the initial meson is at rest, and that the emerging photon defines the z axis, we can write

$$v_a=(1,0,0,0),$$

$$v_b=\left(1+\frac{(\Lambda_a-\Lambda_b)^2}{2m_c^2},0,0,-\frac{(\Lambda_a-\Lambda_b)}{m_c}\left(1-\frac{\Lambda_a+\Lambda_b}{2m_c}\right)\right). \tag{6}$$

With all this we can now obtain the suppressions in $1/m_c$ of the different matrix elements of the heavy quark current with respect to the light quark one. Since we are interested in the emission of a real photon, only those pieces of the current matrix elements that can couple to a real photon are of interest to us. There is a generic $1/m_c$ suppression due to the magnetic moment of the heavy quark. For transitions within a heavy-quark spin multiplet this is the only suppression, while for transitions between states in two different multiplets the suppression is instead $1/m_c^n$, where $n=\max\{3,1+|l-l'|\}$ if the states have the same parity, and $n=1+|l-l'|$ if the states have opposite parity. Here, one power of $1/m_c$ arises as discussed in the text after Eq. (3), and the further $1/m_c$ suppressions stem from the overlap of the initial state light quark wave function with the boosted one of the final state. Table I shows the forms of the leading order (in HQET) matrix elements for a few selected transitions. Note that with the form of the matrix elements used here the Isgur-Wise form factors are of zeroth order in the $1/m_c$ expansion. Also shown in the table is the suppression with respect to the corresponding light quark current matrix element. Note that for the decays between any given pair of heavy-quark spin multiplets, all of the matrix elements of the heavy-quark current appear at the same order in the heavy quark expansion.

For transverse photons, current conservation implies that terms in the matrix elements of the currents that are proportional to $v_{a,b}^\mu$ do not contribute to the amplitude. As discussed previously, the recoil factor $(v_a\cdot v_b-1)$ is of order $1/m_c^2$, while terms like $\epsilon(v_a)\cdot v_b$ and $\epsilon(v_b)\cdot v_a$ either vanish, or are of order $1/m_c$. Moreover, for decays within a multiplet, the mass difference is also of order $1/m_c$, providing a further suppression for such decays. In the case of the decays from the radially excited $(0^-,1^-)'$ multiplet to the ground state multiplet the matrix element has a form analogous to that shown in the first row of Table I, except for the extra suppression factor $1/m_c^2$ that results from the overlap of the wave functions. In summary, for the transitions $D^*\to D\gamma$ as well as all other transitions we consider, the contributions of the heavy quark to the current are subleading in the heavy quark expansion, being suppressed by one power of $1/m_c$ in the case of transitions within a heavy-quark spin multiplet, and at least two powers of $1/m_c$ in the case of transitions between different multiplets.

Since in the case of D mesons the charge of the heavy quark is 2/3 and m_c is not very large, one has to keep the contributions suppressed by $1/m_c$. Indeed, if one would disregard these contributions, one immediately finds an inconsistency in the radiative branching ratios for the non-strange D^*-mesons. From isospin symmetry in the pion emission amplitudes and the relation $\Gamma(D^{*+}\to D^+\gamma)=\Gamma(D^{*0}\to D^0\gamma)/4$, that results when the heavy-quark electromagnetic current is disregarded, along with the corresponding neutral pion emission branching ratios, one obtains $\mathrm{BR}(D^{*+}\to D^{+}\gamma)/\mathrm{BR}(D^{*0}\to D^{0}\gamma)\simeq 0.18$, which is much larger than the experimental ratio $0.03^{+0.05}_{-0.02}$. As we see later, the inclusion of the heavy quark contributions largely remedies this discrepancy. We also find that even the B meson transitions within a heavy-quark spin multiplet are noticeably affected by the presence of the heavy-quark current. Thus, throughout we will keep the contributions of order $1/m_Q$ to the intra-multiplet transitions due to the heavy-quark current. In this case, and as already mentioned before, only the spatial components of the spin current are relevant.

In the following we work in the same framework as our previous paper [2]. We write the wave function of the light valence quark as

$$\psi_{jlm}=\begin{pmatrix} iF(r)\Omega_{jlm} \\ G(r)\Omega_{j\tilde{l}m}\end{pmatrix},\qquad \tilde{l}=2j-l, \tag{7}$$

where the radial wave functions are real, and the spinor harmonics are given by

$$\Omega_{(j=l+1/2)\;lm}=\begin{pmatrix} \sqrt{\frac{j+m}{2j}}\;Y_{l\,m-1/2} \\ \sqrt{\frac{j-m}{2j}}\;Y_{l\,m+1/2}\end{pmatrix},$$
$$\Omega_{(j=l-1/2)\;lm}=\begin{pmatrix} \sqrt{\frac{j+1-m}{2(j+1)}}\;Y_{l\,m-1/2} \\ -\sqrt{\frac{j+m+1}{2(j+1)}}\;Y_{l\,m+1/2}\end{pmatrix}. \tag{8}$$

We follow here the conventions of Bjorken and Drell [6].

Straightforward evaluation of the matrix elements of the electromagnetic current in the rest frame of the heavy meson gives

$$\langle J',M',j',l'|J_q^0|J,M,j,l\rangle =\sum_{l}(-i)^l Y^*_{l(M'-M)}(\hat{k})\langle l,M'-M;J,M|J',M'\rangle \times T_0^q(k,l,J,j,l,J',j',l',\Lambda), \tag{9}$$

$$\langle J',M',j',l'|J_q^m|J,M,j,l\rangle =\sum_{l,l'}(-i)^l Y^*_{l(M'-M-m)}(\hat{k})\langle l',M'-M;J,M|J',M'\rangle \times\langle 1,m;l,M'-M-m|l',M'-M\rangle \times T_1^q(k,k^0,l,l',J,j,l,J',j',l',\Lambda), \tag{10}$$

and

$$\langle J',M',j',l'|J_Q^m|J,M,j,l\rangle = iY^*_{1(M'-M-m)}(\hat{k})\langle 1,m;1,M'-M-m|1,M'-M\rangle \times\langle 1,M'-M,J,M|J',M'\rangle T_1^Q(J,j,l,J',j',l'), \tag{11}$$

where $k=|\vec{k}|$. Here, we use the angular momentum projection basis, so that $m=\pm 1,0$. Using the standard notations for the $3j$, $6j$ and $9j$ symbols, we define two reduced matrix elements R_0 and R_1 as

$$R_0(j,l,j',l',l)=(-1)^{j+l+1/2}\frac{1}{\sqrt{4\pi}}\sqrt{(2l+1)(2l+1)(2l'+1)(2j+1)(2j'+1)} \times\begin{pmatrix} l & l & l' \\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} l & l & l' \\ 1/2 & j' & j\end{Bmatrix} \tag{12}$$

and

$$R_1(j,l,j',l',l,l')=(-1)^{j+j'+l}\sqrt{\frac{3}{2\pi}}\sqrt{(2l+1)(2l+1)(2l'+1)(2j+1)(2j'+1)} \times\begin{pmatrix} l & l & l' \\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} 1/2 & l & j \\ 1/2 & l' & j' \\ 1 & l & l'\end{Bmatrix}. \tag{13}$$

With this, the expressions for the reduced amplitudes T_0 and T_1 are

$$
\begin{aligned}
&T_0^q(k,l,J,j,l,J',j',l',\Lambda)\\
&=4\pi e\mathcal{Q}_q\sqrt{2J+1}(-1)^{J'}\begin{Bmatrix} l & j & j'\\ 1/2 & J' & J\end{Bmatrix}\Bigg[(-1)^{j+1/2}\int dr\ \ r^2 j_l(kr)[(-1)^{l+l'}F'(r)F(r)R_0(j,l,j',l',l)\\
&\quad+(-1)^{\tilde{l}+\tilde{l}'}G'(r)G(r)R_0(j,\tilde{l},j',\tilde{l}',l)]+i\frac{k}{\Lambda}\sum_{l'}(-i)^{l'-l}(-1)^{j'+1/2}\sqrt{(2l+1)(2l'+1)}\begin{pmatrix} l' & 1 & l\\ 0 & 0 & 0\end{pmatrix}\\
&\quad\times\int dr\, r^2 j_{l'}(kr)[F'(r)G(r)(-1)^{\tilde{l}+l'}R_1(j,\tilde{l},j',l',l',l)+G'(r)F(r)(-1)^{\tilde{l}'+l}R_1(j,l,j',\tilde{l}',l',l)]\Bigg], \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
&T_1^q(k,k_0,l,l',J,j,l,J',j',l',\Lambda)\\
&=4\pi e\mathcal{Q}_q\sqrt{2J+1}\sqrt{2l'+1}(-1)^{J'}\begin{Bmatrix} l' & j & j'\\ 1/2 & J' & J\end{Bmatrix}\Bigg[(-1)^{l'+j'+1/2}\int dr\, r^2 j_l(kr)[-iF'(r)G(r)\\
&\times(-1)^{\tilde{l}+l'}R_1(j,\tilde{l},j',l',l,l')+iG'(r)F(r)(-1)^{\tilde{l}'+l}R_1(j,l,j',\tilde{l}',l,l')]+i\frac{k_0}{\Lambda}(-1)^{l'+j'+1/2}\int dr\, r^2 j_l(kr)\\
&\times[F'(r)G(r)(-1)^{\tilde{l}+l'}R_1(j,\tilde{l},j',l',l,l')+G'(r)F(r)(-1)^{\tilde{l}'+l}R_1(j,l,j',\tilde{l}',l,l')]\\
&-\sqrt{2}\,\frac{k}{\Lambda}\sum_L(-i)^{L-l}(-1)^{j'+1/2+L+l}\sqrt{3(2l+1)(2L+1)}\begin{pmatrix} L & 1 & l\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} 1 & 1 & 1\\ L & l' & l\end{Bmatrix}\\
&\times\int dr\, r^2 j_L(kr)[(-1)^{l+l'}F'(r)F(r)R_1(j,l,j',l',L,l')-(-1)^{\tilde{l}+\tilde{l}'}G'(r)G(r)R_1(j,\tilde{l},j',\tilde{l}',L,l')]\Bigg] \qquad (15)
\end{aligned}
$$

and

$$
\begin{aligned}
&T_1^Q(J,j,l,J',j',l')\\
&=i\,k\,(-1)^{J+j-1/2}\delta_{ll'}\delta_{jj'}\,\frac{e\mathcal{Q}_Q}{m_Q}\,\sqrt{4\pi}\sqrt{2J+1}\\
&\quad\times\begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2}\\ j & J' & J\end{Bmatrix}. \qquad (16)
\end{aligned}
$$

Note that we have kept only the leading contribution to T_1^Q in the $1/m_Q$ expansion. In particular, this means that T_1^Q must vanish unless the initial and final states belong to the same heavy-quark spin multiplet. The radial wave functions F and G are those used in Ref. [2], which were obtained using the potential and parameters in Ref. [1]. We have explicitly checked that the matrix elements of the electromagnetic current given above do satisfy the constraints imposed by current conservation and the relations implied by heavy quark spin-flavor symmetry. Notice in particular that the piece of the heavy quark current kept here, namely the spatial components of the spin current, is obviously conserved because it contains the factor $\vec{k}\times\vec{\sigma}$.

In terms of the reduced amplitudes T_0 and T_1, the radiative decay widths are written as

$$
\begin{aligned}
\Gamma_\gamma(Jjl\to J'j'l')=&\frac{2J'+1}{2J+1}\frac{E'}{E}\frac{k}{8\pi^2}\\
&\times\sum_{l=|J-J'|}^{J+J'}\Bigg[-|T_0^q(k,l,J,j,l,J',j',l',\Lambda)|^2\\
&+\sum_{l'=l-1}^{l+1}|T_1^q(k,k_0,l,l',J,j,l,J',j',l',\Lambda)\\
&+\delta_{l1}\delta_{l'1}T_1^Q(J,j,l,J',j',l')|^2\Bigg]. \qquad (17)
\end{aligned}
$$

III. RESULTS

In three tables we show the results obtained using the model. The uncertainties shown in the tables are theoretical kinematic uncertainties due to the uncertainties in the masses of the observed states. For states that have not yet been observed, the uncertainties in the masses are taken as ± 20 MeV. In each table, we present results for three values of κ^q. The two non-zero values are chosen to reproduce as well as possible the experimentally reported branching fractions. Using $g_A^q=0.8$ to determine the strong decay widths, a fit to those branching fractions leads to $\kappa^q=0.55$. The value of $\kappa^q=0.45$ is chosen to illustrate the sensitivity to this quantity. We emphasize here that the value of κ^q is the same for

TABLE II. Radiative decay widths of non-strange heavy mesons: only transitions within heavy quark spin multiplets are shown.

Decay	k	$\bar{c}d$ states			k	$\bar{b}d$ states		
	(MeV)	$\Gamma(\kappa^q=0)$	$\Gamma(\kappa^q=0.45)$	$\Gamma(\kappa^q=0.55)$	(MeV)	$\Gamma(\kappa^q=0)$	$\Gamma(\kappa^q=0.45)$	$\Gamma(\kappa^q=0.55)$
$1^-\to0^-$	136	50 ± 2 eV	904^{+25}_{-24} eV	1.5 ± 0.0 keV	45	37^{+5}_{-4} eV	182^{+22}_{-21} eV	228^{+28}_{-26} eV
$1^+\to0^+$	127	510^{+257}_{-194} eV	$1.6^{+0.8}_{-0.6}$ keV	$1.9^{+0.9}_{-0.7}$ keV	40	$4.5^{+10.7}_{-4.0}$ eV	$1.0^{+2.3}_{-0.9}$ eV	$2.9^{+6.7}_{-2.6}$ eV
$2^+\to1^+$	32	$7.0^{+22.7}_{-6.7}$ eV	$0.6^{+1.8}_{-0.5}$ eV	$0.1^{+0.4}_{-0.1}$ eV	20	$1.9^{+13.3}_{-1.9}$ eV	$5.5^{+37.9}_{-5.5}$ eV	$6.5^{+45.1}_{-6.5}$ eV
$2^-\to1^-$	30	11^{+38}_{-10} eV	20^{+71}_{-19} eV	23^{+80}_{-22} eV	0	$0.0^{+0.1}_{-0.0}$ eV	0.0 ± 0.0 eV	0.0 ± 0.0 eV
$1^{-\prime}\to0^{-\prime}$	117	153^{+91}_{-65} eV	311^{+149}_{-119} eV	590^{+290}_{-229} eV	40	15^{+34}_{-13} eV	101^{+234}_{-88} eV	131^{+303}_{-114} eV
		$\bar{c}u$ states				$\bar{b}u$ states		
$1^-\to0^-$	137	7.3 ± 0.2 keV	26 ± 1 keV	32 ± 1 keV	45	84 ± 10 eV	572^{+71}_{-65} eV	740^{+92}_{-85} eV
$1^+\to0^+$	127	$1.9^{+1.0}_{-0.7}$ keV	102^{+62}_{-43} eV	$6.6^{+6.6}_{-3.5}$ eV	40	$3.0^{+7.0}_{-2.6}$ eV	21^{+48}_{-18} eV	36^{+83}_{-31} eV
$2^+\to1^+$	32	77^{+245}_{-73} eV	158^{+505}_{-150} eV	180^{+576}_{-171} eV	20	$2.8^{+19.2}_{-2.8}$ eV	$12.8^{+88.5}_{-12.8}$ eV	$16.0^{+110.8}_{-16.0}$ eV
$2^-\to1^-$	30	40^{+141}_{-38} eV	15^{+52}_{-14} eV	11^{+38}_{-10} eV	0	0.0 ± 0.1 eV	0.0 ± 0.0 eV	$0.0^{+0.1}_{-0.0}$ eV
$1^{-\prime}\to0^{-\prime}$	117	$3.1^{+1.7}_{-1.2}$ keV	$13.3^{+7.1}_{-5.3}$ keV	$16.5^{+8.8}_{-6.6}$ keV	40	26^{+60}_{-23} eV	308^{+713}_{-269} eV	412^{+955}_{-360} eV

all light flavors. Since κ^q is determined solely by the strong interaction, it is an SU(3) singlet, except for SU(3) breaking corrections of order $(m_s-m_{u,d})/\Lambda_{\rm QCD}$, which we disregard here.

Table II shows the results obtained for the non-strange heavy mesons. Since all these mesons decay primarily through pion emission, most of the radiative decays are not likely to be measurable. We show only the intra-multiplet decays, but these are also unlikely to be measured. Among these, the ones of real interest are the decays within the ground state multiplet. As mentioned above, if we choose $g_A^q=0.8$, and use the reported ratios of widths for the D^* mesons, the fit value is $\kappa^q=0.55$, while $\kappa^q=0.45$ also gives a reasonable description of the data. The implied total widths of the D^{*0} are then 74 keV and 68 keV, respectively, for the two values of κ^q, very close to the estimated upper bound of about 70 keV mentioned earlier. For the D^{*+}, we obtain a total pion emission width of 93 keV, and for the non-zero values of κ^q the radiative branching ratio is either 1.5% or 1.0%, both close to the experimentally measured value. The corresponding total width is about 95 keV.

The heavy quark current plays an important role in the radiative decays of the D^* mesons, providing a large reduction of the radiative widths of the D^{*+} and D_s^*, and an enhancement of the D^{*0} width. Without the contribution of the heavy quark the radiative widths of the D^{*+}, D^{*0} and D_s^* mesons are 5.1 keV, 20.8 keV and 2.7 keV, respectively, evaluated with $\kappa=0.55$, while the corresponding widths with the heavy quark contribution are 1.5 keV, 32 keV, and 321 eV, respectively. In the B mesons the heavy quark electromagnetic current has a much smaller effect, as expected. Without the contribution of the heavy quark the radiative widths of the B^{*+}, B^{*0} and B_s^* mesons are 796 eV, 199 eV and 112 eV, respectively, while the inclusion of the heavy quark electromagnetic current changes these values to 740 eV, 228 eV, and 136 eV, respectively. We observe that the light quark contribution receives a suppression because the photon momentum is not that small (140 MeV). The D^* and B^* decays are found to be very sensitive to the anomalous magnetic moment of the light quark. While in a nonrelativistic approximation the anomalous magnetic moment contribution to the width manifests itself in the factor $(1+\kappa^q)^2$, relativistic effects turn out to give an enhancement of the anomalous magnetic moment piece by roughly a factor of four. For the other decay widths that are predominantly of M_1 type, which includes all the intra-multiplet decays, there is a similar sensitivity to the anomalous magnetic moment.

The widths obtained for the B^* mesons are quite small, as one would expect from the fact that the available phase space is smaller than in the D^* mesons. It is evident from the fact that the ratio of the B^+ to the B^0 width is not approximately equal to four, that the heavy quark current has a substantial effect. If the light quark has no anomalous magnetic moment, the B^{*0} width is found to be 40 eV, and it rises to 244 eV when $\kappa^q=0.55$. The B^{*+} width is about a factor of two to three larger than the B^{*0} width. The intra-multiplet partial widths of the excited mesons are negligible with respect to their radiative widths for decay into the ground state mesons. We display them only for the sake of completeness.

Table III shows the branching ratios and widths obtained in this model. Also shown in that table are some representative results presented in the literature. All of the models predict similar branching ratios, but there is some spread in the predicted widths, especially for the D^{*0}.

The results that we obtain for the strange heavy mesons D_s and B_s are shown in Table IV. There, we list only those decays for states whose kaon emission decays are forbidden or suppressed by phase space. We also show the intra-multiplet decays. Here, as in the non-strange D mesons, the results are very sensitive to the value of κ^q. In addition, states like the $(0^+,1^+)$ doublet, which would be broad if kaon emission could take place, are predicted to be a few tens of keV wide. Other states, like the radially excited $(0^-,1^-)$ doublet, are of the order of 10 keV in width. The branching ratios reported for the D_s^* would give total widths for this state of about 107 eV, 175 keV or 341 keV, depend-

TABLE III. Total widths and branching ratios for charmed vector mesons. The results shown are from Miller and Singer (MS) [18]; Eichten *et al.* (Eichten) [19]; Pham [16]; Rosner [17]; Kamal and Xu (Kamal) [7]; Cheng *et al.* (Cheng) [12]; and the present work. The numbers in the columns for the present work correspond to $g_A^q=0.8$, and $\kappa^q=0.45$, $\kappa^q=0.55$, respectively, and are calculated using the appropriate ''central'' values shown in Tables II and IV. The last column shows the current experimental values.

Quantity	MS	Eichten	Pham	Rosner	Kamal	Cheng	This work $\kappa^q=0.45$	This work $\kappa^q=0.55$	Experiment
BR($D^{*+}\to D^+\pi^0$)	31.2	28.5	29.4	30.9	30.0	31.2	30.5	30.3	30.6±2.5
BR($D^{*+}\to D^0\pi^+$)	67.5	68.5	64.7	67.8	68.0	67.3	68.5	68.1	68.3±1.4
BR($D^{*+}\to D^+\gamma$)	1.3	3.0	5.9	1.3	2.0	1.5	1.0	1.5	$1.1^{+2.1}_{-0.7}$
Γ($D^{*+}\to$all) (keV)	79.0	78.0	142.8	83.9	86.4	141.0	94.3	94.9	96±4±22 keV [5]
BR($D^{*0}\to D^0\pi^0$)	64.3	55.2	71.4	70.6	66.0	66.7	61.5	56.5	61.9±2.9
BR($D^{*0}\to D^0\gamma$)	35.7	44.8	28.6	29.4	34.0	33.3	38.5	43.5	38.1±2.9
Γ($D^{*0}\to$all) (keV)	59.4	78.6	120.4	56.2	64.2	102.0	67.6	73.6	<2 MeV
Γ($D_s^{*+}\to D_s^+\gamma$) (keV)					0.21	0.3	0.2	0.3	

TABLE IV. Radiative decay widths of strange heavy mesons.

Decay	k (MeV)	$\bar{c}s$ states $\Gamma(\kappa^q=0)$	$\bar{c}s$ states $\Gamma(\kappa^q=0.45)$	$\bar{c}s$ states $\Gamma(\kappa^q=0.55)$	k (MeV)	$\bar{b}s$ states $\Gamma(\kappa^q=0)$	$\bar{b}s$ states $\Gamma(\kappa^q=0.45)$	$\bar{b}s$ states $\Gamma(\kappa^q=0.55)$
$1^-\to 0^-$	139	101±3 eV	165±4 eV	321^{+9}_{-8} eV	47	35±3 eV	113±10 eV	136±12 eV
$1^+\to 0^+$	127	547^{+276}_{-209} eV	$1.1^{+0.5}_{-0.4}$ keV	$1.2^{+0.6}_{-0.5}$ keV	40	$4.0^{+9.4}_{-3.5}$ eV	0.0±0.1 eV	0.0±0.1 eV
$2^+\to 1^+$	38	13^{+33}_{-12} eV	$4.8^{+12.2}_{-4.3}$ eV	$3.5^{+8.9}_{-3.1}$ eV	30	$5.7^{+20.3}_{-5.5}$ eV	$11.7^{+41.7}_{-11.2}$ eV	$13.3^{+47.5}_{-12.8}$ eV
$2^-\to 1^-$	40	$26.1^{+60.5}_{-22.8}$ eV	$38.5^{+89.1}_{-33.6}$ eV	$41.6^{+96.2}_{-36.3}$ eV	0	$0.0^{+0.8}_{-0.0}$ eV	$0.0^{+0.2}_{-0.0}$ eV	$0.0^{+0.2}_{-0.0}$ eV
$1^{-\prime}\to 0^{-\prime}$	117	177^{+104}_{-74} eV	21^{+8}_{-7} eV	72^{+32}_{-28} eV	40	13^{+31}_{-12} eV	53^{+124}_{-47} eV	66^{+153}_{-58} eV
$0^+\to 1^-$	253	24.9±1.9 keV	14.5±0.9 keV	16.2±1.0 keV	324	35.9±2.6 keV	20.1±1.4 keV	22.6±1.6 keV
$1^+\to 0^-$	483	17.2±0.7 keV	10.3±0.6 keV	11.4±0.6 keV	406	15.9±1.0 keV	9.0±0.6 keV	10.1±0.7 keV
$1^+\to 1^-$	366	25.1±1.4 keV	14.0±0.8 keV	15.8±0.9 keV	362	27.5±1.8 keV	15.4±1.1 keV	17.3±1.2 keV
$1^+\to 0^-$	503	25.2±0.5 keV	31.1±0.8 keV	30.0±0.7 keV	406	$30.5^{+0.1}_{-0.2}$ keV	$38.3^{+0.0}_{-0.2}$ keV	$36.8^{+0.0}_{-0.2}$ keV
$1^+\to 1^-$	387	14.6±0.2 keV	22.8±1.2 keV	21.0±1.0 keV	362	15.8±0.4 keV	23.4±1.5 keV	21.8±1.2 keV
$2^+\to 0^-$	534	1.6±0.2 keV	8.6±1.0 keV	6.9±0.8 keV	434	795^{+158}_{-139} eV	$4.2^{+0.8}_{-0.7}$ keV	3.4±0.7 keV
$2^+\to 1^-$	419	41.5±0.0 keV	$55.9^{+0.5}_{-0.6}$ keV	$53.0^{+0.4}_{-0.5}$ keV	390	$46.2^{+0.4}_{-0.6}$ keV	$61.1^{+1.2}_{-1.4}$ keV	$58.2^{+1.0}_{-1.2}$ keV
$1^+\to 0^+$	150	$103^{+39.7}_{-32.4}$ eV	1.4±0.5 keV	1.0±0.4 keV	40	$2.2^{+5.0}_{-1.9}$ eV	$30.4^{+69.9}_{-26.6}$ eV	$23.0^{+52.9}_{-20.1}$ eV
$1^+\to 1^+$	25	$0.3^{+1.2}_{-0.3}$ eV	$3.7^{+17.3}_{-3.7}$ eV	$2.8^{+13.1}_{-2.8}$ eV	0	$0.0^{+0.1}_{-0.0}$ eV	$0.0^{+1.9}_{-0.0}$ eV	$0.0^{+1.5}_{-0.0}$ eV
$2^+\to 0^+$	186	0.1±0.1 eV	$3.8^{+3.1}_{-1.9}$ eV	$2.7^{+2.2}_{-1.3}$ eV	70	0.0±0.0 eV	0.0±0.0 eV	0.0±0.0 eV
$2^+\to 1^+$	62	$12.0^{+14.7}_{-8.1}$ eV	168^{+203}_{-113} eV	$127^{+153}_{-85.1}$ eV	30	$1.4^{+4.9}_{-1.3}$ eV	$19.5^{+68.7}_{-18.7}$ eV	$14.7^{+51.9}_{-14.1}$ eV
$0^{-\prime}\to 1^-$	450	$157^{+24.7}_{-22.1}$ eV	1.4±0.1 keV	1.1±0.1 keV	492	$106^{+33.3}_{-33.5}$ eV	1.4±0.1 keV	1.0±0.1 keV
$1^{-\prime}\to 0^-$	655	$59.2^{+29.7}_{-22.8}$ eV	$0.1^{+8.7}_{-0.1}$ eV	$1.3^{+13.2}_{-0.5}$ eV	571	$0.2^{+3.7}_{-0.2}$ eV	$200^{+62.3}_{-64.4}$ eV	$137^{+48.0}_{-51.0}$ eV
$1^{-\prime}\to 1^-$	548	$9.4^{+11.9}_{-7.5}$ eV	$482^{+98.6}_{-96.4}$ eV	$346^{+78.5}_{-78.2}$ eV	528	$28.2^{+16.3}_{-20.2}$ eV	696^{+123}_{-112} eV	$511^{+101}_{-93.3}$ eV
$0^{-\prime}\to 1^+$	98	3.3±0.6 keV	4.0±0.7 keV	3.8±0.7 keV	138	4.6±0.5 keV	5.6±0.7 keV	5.4±0.7 keV
$1^{-\prime}\to 0^+$	328	2.4±0.0 keV	2.6±0.1 keV	2.5±0.1 keV	216	2.1±0.1 keV	2.6±0.1 keV	2.5±0.1 keV
$1^{-\prime}\to 1^+$	211	4.0±0.2 keV	4.9±0.2 keV	4.8±0.2 keV	177	3.7±0.3 keV	4.6±0.3 keV	4.5±0.3 keV
$0^{-\prime}\to 1^+$	74	4.6±1.2 keV	4.3±1.1 keV	4.3±1.1 keV	138	9.0±1.4 keV	8.2±1.3 keV	8.4±1.3 keV
$0^{-\prime}\to 2^+$	37	$0.0^{+0.2}_{-0.0}$ eV	$0.1^{+1.1}_{-0.1}$ eV	$0.1^{+0.9}_{-0.1}$ eV	109	$5.3^{+6.6}_{-3.3}$ eV	$33.1^{+41.8}_{-20.8}$ eV	$26.4^{+33.3}_{-16.5}$ eV
$1'\to 1^+$	188	2.1±0.3 keV	2.1±0.4 keV	2.1±0.3 keV	177	2.0±0.3 keV	2.0±0.4 keV	2.0±0.3 keV
$1^{-\prime}\to 2^+$	153	8.1±1.1 keV	7.5±1.1 keV	7.6±1.1 keV	148	8.1±1.2 keV	7.5±1.2 keV	7.6±1.2 keV

ing on whether we use $\kappa^q=0$, $\kappa^q=0.45$ or $\kappa^q=0.55$, respectively. These numbers imply $\Gamma(D_s^*\to D_s\pi^0)=6.2\pm2.2$ eV, 10.1 ± 3.5 eV or 19.7 ± 7.0 eV. We should observe here that the prediction of our model for the D^{*+} radiative branching ratio and the observed strong decay branching ratio of the D_s^* give that $\mathrm{BR}(D_s^*\to D_s\pi^0)\cdot\mathrm{BR}(D^{*+}\to D^+\gamma)$ is about 2×10^{-3}, a result that is much larger than the value proposed in Ref. [20], where this product is estimated in a model of the isospin violating decay $D_s^*\to D_s\pi^0$ and found to be about 8×10^{-5}. From the experimental branching ratios one obtains $6.4\pm4.9\times10^{-4}$, which falls between those two numbers.

The strong decays of many of the excited strange heavy mesons will proceed either through the emission of one or two pions; only the $(1^-,2^-)$, $(0^{-\prime},1^{-\prime})$, and the $(1^+,2^+)$ states can decay emitting a kaon. The $(1^+,2^+)$ states have masses that lie very close to the threshold for kaon emission. However, these decays are expected to be predominantly D wave, so that the centrifugal suppression will lead to very small decay widths [2]. Similarly, in the $(0^{-\prime},1^{-\prime})$ multiplet, the $0^{-\prime}$ may actually lie below the threshold for kaon production, meaning that its electromagnetic decays could provide a significant portion of its total decay rate. The pionic decay widths of these states have not been studied to the best of our knowledge. We expect them to be in the range from a few tens to a hundred keV. Thus, several of the excited states will decay radiatively with an important branching fraction.

We note that heavy-quark spin symmetry would require that both partners in a heavy quark spin multiplet should have the same partial width for transitions to other multiplets. In the table this holds only approximately because of phase space corrections due to the subleading intra-multiplet mass splittings. In a few cases the effect is dramatic because the phase space is small to start with.

IV. CONCLUSIONS

In summary, we have obtained results for a variety of radiative heavy meson transitions in the relativistic quark model. To agree with observed decays it is found that the light quark must have an anomalous magnetic moment of about 0.5. Important corrections subleading in the expansion in the inverse of the heavy quark mass are observed. These corrections are very important in the D-meson sector for transitions within the same heavy quark spin multiplet, while in B mesons those contributions are much smaller but not quite negligible. The decays of excited D_s mesons are particularly interesting as their strong decays are suppressed. The experimental study of some of the radiative and strong decays would clearly impact on our understanding of the structure of heavy mesons and their excited states.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation through Grants No. PHY-9733343 (J.L.G.), No. PHY-9457892 and No. PHY 9820458 (W.R.), by the Department of Energy through Contracts No. DE-AC05-84ER40150 (J.L.G. and W.R.) and No. DE-FG05-94ER40832 (W.R.), and by sabbatical leave support from the Southeastern Universities Research Association (J.L.G.).

[1] J. Zeng, J. W. Van Orden, and W. Roberts, Phys. Rev. D **52**, 5229 (1995).
[2] J. L. Goity and W. Roberts, Phys. Rev. D **60**, 034001 (1999).
[3] J. L. Goity and W. Roberts, Phys. Rev. D **51**, 3459 (1995).
[4] Particle Data Group, D. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[5] CLEO Collaboration, M. Dubrovin, hep-ex/0105030.
[6] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
[7] A. N. Kamal and Q. P. Xu, Phys. Lett. B **284**, 421 (1992).
[8] Fayyazuddin and O. H. Mobarek, Phys. Rev. D **48**, 1220 (1993); **50**, 2329 (1994).
[9] M. Oda, S. Ishida, and A. Morikawa, presented at HADRON 95, 6th International Conference on Hadron Spectroscopy, Manchester, England, 1995.
[10] M. Avila, Phys. Rev. D **49**, 309 (1994).
[11] P. Cho and H. Georgi, Phys. Lett. B **296**, 408 (1992); **300**, 410 (1993).
[12] H.-Y. Cheng, C.-Y. Cheung, G.-L. Lin, Y. C. Lin, T.-M. Yan, and H.-L. Yu, Phys. Rev. D **47**, 1030 (1993).
[13] J. F. Amundson *et al.*, Phys. Lett. B **296**, 415 (1992).
[14] J. G. Körner, D. Pirjol, and K. Schilcher, Phys. Rev. D **47**, 3955 (1993).
[15] T. A. Lähde, C. J. Nyfält, and D. O. Riska, Nucl. Phys. **A674**, 141 (2000).
[16] T. N. Pham, Phys. Rev. D **25**, 2955 (1982).
[17] J. L. Rosner, in *Particles and Fields 3, Proceedings of the Banff Summer Institute*, Banff, Canada, 1988, edited by A. N. Kamal and F. C. Khanna (World Scientific, Singapore, 1989), p. 395.
[18] G. A. Miller and P. Singer, Phys. Rev. D **37**, 2564 (1985).
[19] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Phys. Rev. D **21**, 203 (1980).
[20] P. Cho and M. B. Wise, Phys. Rev. D **49**, 6228 (1994).