Factorization in color-favored *B*-meson decays to charm

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Improved *B* meson decay data have permitted more incisive tests of factorization predictions. A concurrent benefit is the ability to constrain the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$. Using a simultaneous fit to differential distributions $d\Gamma(\overline{B}^0 \to D^{(*)+} l^- \overline{\nu}_l)/dq^2$ and the rates for the color-favored decays $\overline{B}^0 \to D^{(*)+}(\pi^-, \rho^-, a_1^-)$, we find $|V_{cb}| = 0.0415 \pm 0.0022$. The slope of the universal Isgur-Wise form factor is described by a parameter found to be $\rho^2 = 1.52 \pm 0.11$. Taking the D_s meson decay constant from the world average of direct measurements, we predict satisfactorily the branching ratios for $\overline{B}^0 \to D^{(*)+} D_s^{(*)-}$. Ratios of helicity amplitudes for color-favored processes are also found to be in accord with predictions.

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I. INTRODUCTION

Semileptonic weak hadron decays provide useful information on form factors of the weak current. The lepton pair can then be replaced with a hadron, permitting the calculation of nonleptonic decays. Although this hadron can reinteract with the rest of the system, the effects of this reinteraction sometimes can be neglected or evaluated. In such cases one is employing the *factorization hypothesis*. An early version of this hypothesis [1] was recently justified for certain decays of hadrons containing heavy quarks [2].

In the present paper we update and test some factorization predictions first made a number of years ago [3]. We compare the values of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ and form factor shapes obtained from (1) the differential distribution $d\Gamma(\overline{B}^0 \to D^{(*)+} l^- \overline{\nu}_l)/dq^2$ [4–9] and (2) the color-favored two-body nonleptonic decays $\overline{B}^0 \to D^{(*)+}(\pi^-, \rho^-, a_1^-)$. We find that the consistency between nonleptonic and semileptonic determinations is at least as good as that among the semileptonic determinations themselves.

Using a combined fit to semileptonic and nonleptonic decays and the measured value of the D_s meson decay constant f_{D_s}, we then predict the rates for $\overline{B}^0 \to D^{(*)+} D_s^{(*)-}$ and find that they are in accord with experiment. We thus find that factorization holds not only in color-favored cases in which the current produces a light meson, where it has been justified [2], but also when the current produces a heavy meson, where no such justification has been presented. The importance of such processes has recently been stressed by Lipkin [10]. We also find that new experimental ratios of helicity amplitudes for color-favored processes agree with predictions. We shall ignore small nonfactorizable contributions to color-favored $\overline{B}^0$ decays as discussed, for example, in Ref. [11].

In Sec. II we review the factorization predictions for the decays $\overline{B}^0 \to W^{*-} D^{(*)+}$, where the virtual W^{*-} produces either a lepton pair $l^- \overline{\nu}_l$ or a hadron π^-, ρ^-, a_1^-, D_s^-, or D_s^{*-}. These processes are purely color favored. We do not consider the corresponding B^- decays, for which the nonleptonic processes receive both color-favored and color-suppressed contributions. We then (Sec. III) discuss the differential distributions $d\Gamma(\overline{B}^0 \to D^{(*)+} l^- \overline{\nu}_l)/dq^2$ and the information they can provide regarding the values of $|V_{cb}|$ and the form factor slope at the normalization point. Results of fits to $\overline{B}^0$ two-body decays to charmed final states are presented and compared with those from semileptonic decays in Sec. IV. We discuss the predictions of the factorization hypothesis for decays in which the weak current produces a $D_s^{(*)}$ in Sec. V and for ratios of helicity amplitudes in Sec. VI. Section VII concludes. An Appendix summarizes parameters of error ellipses used in combining data.

II. NOTATION AND PREDICTIONS

We review the notation which is described in more detail in Ref. [3]. We consider processes in which a semileptonic $\overline{B}^0$ decay of the form shown in Fig. 1(a) can be related to the corresponding decay with the lepton pair replaced by a quark pair, illustrated in Fig. 1(b). The matrix element for production of a pseudoscalar meson $P(q)$ of four-momentum q from the vacuum by the axial vector current is

$$\langle P(q)|A_\mu|0\rangle = i f_P q_\mu, \tag{1}$$

while that for production of a vector meson by the vector current is

$$\langle V(q)|V_\mu|0\rangle = \epsilon_\mu^* M_V f_V, \tag{2}$$

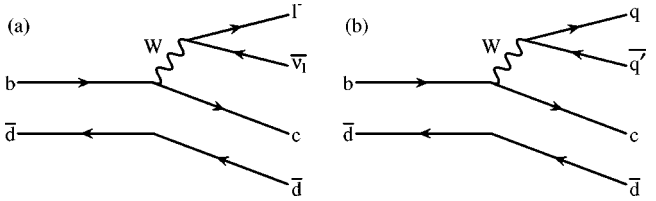

FIG. 1. Feynman diagrams for $\overline{B}^0$ decay illustrating the factorization hypothesis. A semileptonic decay (a) is related to a hadronic decay (b) by the replacement of the lepton pair of four-momentum q with a $q\overline{q}'$ pair of effective mass q^2.

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and that for production of an axial vector meson by the axial vector current is

$$\langle A(q)|A_\mu|0\rangle = \epsilon_\mu^* M_A f_A . \tag{3}$$

The form factors for the $\overline{B}^0(v) \to D^{(*)}(v')$ transitions are described in the heavy-quark limit by one universal function of the Lorentz-invariant variable $w \equiv v\cdot v'$, where v and v' are invariant four-velocities: $v \equiv p_{\overline{B}^0}/m_B$, $v' \equiv p_{D^{(*)}}/m_{D^{(*)}}$. We take $c=1$ and note that $q=p-p'$. Another expression for w is then

$$w = \frac{p_B \cdot p_{D^{(*)}}}{m_B m_{D^{(*)}}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}. \tag{4}$$

[A variable $(v-v')^2 = 2(1-w)$ was called w in Ref. [3].] If ϵ denotes the polarization vector of the final D^*, we may write [1]

$$\langle D(v')|V_\mu|B(v)\rangle = \sqrt{m_D m_B} F_V(w)(v+v')_\mu , \tag{5}$$

$$\langle D^*(v',\epsilon)|A_\mu|B(v)\rangle = \sqrt{m_D m_B} F_A(w)[\epsilon_\mu^*(1+v\cdot v') - \epsilon^*\cdot v\, v'_\mu], \tag{6}$$

$$\langle D^*(v',\epsilon)|V_\mu|B(v)\rangle = -i\sqrt{m_D m_B} F_V(w)\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu} v^\alpha v'^\beta . \tag{7}$$

We take $m_B = 5.28$ GeV and $m_D = m_{D^{*+}} = 2.01$ GeV or $m_{D^+} = 1.87$ GeV depending on the final-state charmed meson. The maximum momentum transfer occurs when the recoiling $D^{(*)}$ is at rest in the B rest frame, so $q^2_{\rm max} = (m_B - m_{D^{(*)}})^2$ and hence $w \geqslant 1$. Another useful relation is

$$y \equiv \frac{q^2}{m_B^2} = 1 - 2w\sqrt{\zeta^{(*)}} + \zeta^{(*)}, \quad \zeta^{(*)} \equiv \frac{m_{D^{(*)}}^2}{m_B^2}. \tag{8}$$

The differential decay width as a function of w for $\overline{B}^0 \to D^{(*)+} l^- \overline{\nu_l}$ can then be written as

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_B^2 m_{D^{(*)}}^3 \sqrt{w^2-1} f^{(*)}(w)|F_{V,A}(w)|^2, \tag{9}$$

where, for $B \to D l \overline{\nu_l}$,

$$f(w) = (w^2-1)(1+\sqrt{\zeta})^2,$$

for $B \to D_T^* l \overline{\nu_l}$,

$$f_T^*(w) = 4wy(w+1),$$

and for $B \to D_L^* l \overline{\nu_l}$,

$$f_L^*(w) = (1-\sqrt{\zeta^*})^2 (w+1)^2.$$

We shall take the form factors $F_{V,A}(w)$ to be parametrized as in Ref. [12]. The form factor $F_V(w)$ governing the process $\overline{B}^0 \to D^+ l^- \overline{\nu_l}$ can be expressed as

$$F_V(w) = F_V(1) \times [1 - 8\rho_{F_V}^2 z + (51\rho_{F_V}^2 - 10)z^2 - (252\rho_{F_V}^2 - 84)z^3], \tag{10}$$

where $z \equiv (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$ (the corresponding variable for elastic $B \to B$ transitions, a natural one for discussing analyticity in dispersion relations, was introduced in [13]), while the form factor $F_A(w)$ governing $B \to D^* l \overline{\nu_l}$ is related to the axial-vector form factor $A_1(w)$ by

$$\left[1 + \frac{4w}{w+1}\frac{1 - 2w\sqrt{\zeta^*} + \zeta^*}{(1-\sqrt{\zeta^*})^2}\right]|F_A(w)|^2 = \left\{2\frac{1-2w\sqrt{\zeta^*}+\zeta^*}{(1-\sqrt{\zeta^*})^2}\left[1 + \frac{w-1}{w+1}R_1(w)^2\right] + \left[1 + \frac{w-1}{1-\sqrt{\zeta^*}}(1 - R_2(w))\right]^2\right\}|A_1(w)|^2. \tag{11}$$

$A_1(w)$ can similarly be parametrized as

$$A_1(w) = A_1(1) \times [1 - 8\rho_{A_1}^2 z + (53\rho_{A_1}^2 - 15)z^2 - (231\rho_{A_1}^2 - 91)z^3]. \tag{12}$$

These forms are motivated by dispersion relations [12–14]. $R_1(w)$ and $R_2(w)$ are given by

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2,$$
$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2. \tag{13}$$

In this paper we use the CLEO experimental results for $R_1(1)$ and $R_2(1)$ [4]:

$$R_1(1) = 1.18 \pm 0.30 \pm 0.12, \tag{14}$$

$$R_2(1) = 0.71 \pm 0.22 \pm 0.07. \tag{15}$$

As we know, $\rho_{F_V}^2$ and $\rho_{A_1}^2$ are the slope parameters for the form factors $F_V(w)$ and $A_1(w)$ at zero recoil, respectively. The difference between $\rho_{A_1}^2$ and $\rho_{F_A}^2$, the slope parameter for $F_A(w)$ at $w=1$, is predicted to be $\rho_{F_A}^2 - \rho_{A_1}^2 = 0.21$ [12]. Recall that $\rho_{F_V}^2$ and $\rho_{F_A}^2$ are actually the same in the single pole model. To make a connection between $F_{V,A}(w)$ and the single pole form factor [3]

$$\mathcal{F}_{V,A}(w) = \mathcal{F}_{V,A}(1)/[1 - 2(1-w)/w_{0(V,A)}^2], \tag{16}$$

we assume $\rho_{F_V}^2 = \rho_{A_1}^2 - 0.21$. From now on we will simplify $\rho_{A_1}^2$ as ρ^2. This parameter describes the slope of the Isgur-Wise [15] form factor at the zero-recoil point: $\rho^2 = [dF_{A_1}(w)/dw]|_{w=1}$ (see, e.g., [16]).

At $w=1$, the vector and axial vector form factors are expected to behave as $F_V(1) = \eta_V(1 + \delta_{1/m_b})$, $F_A(1) = \eta_A(1 + \delta_{1/m_b^2})$. Here $\eta_V = 1.022 \pm 0.004$ and $\eta_A = 0.960$

± 0.007 are QCD corrections [17]. The terms δ_{1/m_b} and δ_{1/m_b^2} are nonperturbative in origin, and correspond physically to the excitation of states other than D and D^*. Lacking a reliable method for estimating the term δ_{1/m_b} in $F_V(1)$ [18], we set it equal to zero. The absence of a δ_{1/m_b} term in $F_A(1)$ is the subject of Luke's theorem [19]. We take $\delta_{1/m_b^2}=-0.05 \pm 0.035$ [20,21], with the product $F_A(1)=0.913\pm 0.042$ as used in Ref. [4]. Equation (9) can then be integrated with respect to w to yield predicted decay rates as functions of the two parameters ρ^2 and $|V_{cb}|$.

The decay widths of some nonleptonic modes may be obtained under the assumption of factorization. For simplicity we assume all $\overline{B}^0\to D^+M^-$ transitions to involve $F_V(w_M)$ and all $\overline{B}^0\to D^{*+}M^-$ transitions to involve $F_A(w_M^*)$, where

$$w_M\equiv(m_B^2+m_D^2-m_M^2)/(2m_Bm_D),$$

$$w_M^*\equiv(m_B^2+m_{D^*}^2-m_M^2)/(2m_Bm_{D^*}). \tag{17}$$

We then find

$$\Gamma(\overline{B}^0\to D^+\pi^-)=\frac{G_F^2}{32\pi}|V_{cb}|^2|V_{ud}|^2m_B^3f_\pi^2|a_1|^2|F_V(w_\pi)|^2(1-\sqrt{\zeta})^2\lambda^{1/2}(1,\zeta,\zeta_\pi)\frac{[(1+\sqrt{\zeta})^2-\zeta_\pi]^2}{4\sqrt{\zeta}}, \tag{18}$$

$$\Gamma(\overline{B}^0\to D^{*+}\pi^-)=\frac{G_F^2}{32\pi}|V_{cb}|^2|V_{ud}|^2m_B^3f_\pi^2|a_1|^2|F_A(w_\pi^*)|^2(1+\sqrt{\zeta^*})^2\lambda^{1/2}(1,\zeta^*,\zeta_\pi)\frac{\lambda(1,\zeta^*,\zeta_\pi)}{4\sqrt{\zeta^*}}, \tag{19}$$

$$\Gamma(\overline{B}^0\to D^+\rho^-)=\frac{G_F^2}{32\pi}|V_{cb}|^2|V_{ud}|^2m_B^3f_\rho^2|a_1|^2|F_V(w_\rho)|^2(1+\sqrt{\zeta})^2\lambda^{1/2}(1,\zeta,\zeta_\rho)\frac{\lambda(1,\zeta,\zeta_\rho)}{4\sqrt{\zeta}}, \tag{20}$$

$$\Gamma(\overline{B}^0\to D^{*+}\rho^-)=\frac{G_F^2}{32\pi}|V_{cb}|^2|V_{ud}|^2m_B^3f_\rho^2|a_1|^2|F_A(w_\rho^*)|^2N(\zeta^*,\zeta_\rho)\lambda^{1/2}(1,\zeta^*,\zeta_\rho)\frac{(1+\sqrt{\zeta^*})^2-\zeta_\rho}{4\sqrt{\zeta^*}}, \tag{21}$$

$$\Gamma(\overline{B}^0\to D^+a_1^-)=\frac{G_F^2}{32\pi}|V_{cb}|^2|V_{ud}|^2m_B^3f_{a_1}^2|a_1|^2|F_V(w_{a_1})|^2(1+\sqrt{\zeta})^2\lambda^{1/2}(1,\zeta,\zeta_{a_1})\frac{\lambda(1,\zeta,\zeta_{a_1})}{4\sqrt{\zeta}}, \tag{22}$$

$$\Gamma(\overline{B}^0\to D^{*+}a_1^-)=\frac{G_F^2}{32\pi}|V_{cb}|^2|V_{ud}|^2m_B^3f_{a_1}^2|a_1|^2|F_A(w_{a_1}^*)|^2N(\zeta^*,\zeta_{a_1})\lambda^{1/2}(1,\zeta^*,\zeta_{a_1})\frac{(1+\sqrt{\zeta^*})^2-\zeta_{a_1}}{4\sqrt{\zeta^*}}, \tag{23}$$

where $\zeta_M=m_M^2/m_B^2$ while

$$N(\zeta^*,\zeta_M)\equiv(1-\sqrt{\zeta^*})^2[(1+\sqrt{\zeta^*})^2-\zeta_M]+4\zeta_M(1+\zeta^*-\zeta_M), \tag{24}$$

$$\lambda(a,b,c)\equiv a^2+b^2+c^2-2ab-2ac-2bc. \tag{25}$$

The QCD correction $|a_1|$ is taken to be 1.05 for all processes; this is a sufficiently good approximation to the actual situation, in which values differ by less than a percent from process to process [2]. In the limit of small m_π, the results (18) and (19) are special cases of the simple Bjorken relation [1]

$$\Gamma(\overline{B}^0\to D^{(*)+}\pi^-)=6\pi^2f_\pi^2|V_{ud}|^2|a_1|^2\left.\frac{d\Gamma(\overline{B}^0\to D^{(*)+}l^-\overline{\nu}_l)}{dq^2}\right|_{q^2=m_\pi^2}. \tag{26}$$

III. SEMILEPTONIC DECAYS

The CLEO Collaboration [4] at the Cornell Electron Storage Ring (CESR) and the ALEPH, DELPHI, and OPAL Collaborations [5–7] at LEP [8] have measured the spectra in lepton-pair squared effective mass q^2 (equivalently, in the Isgur-Wise variable w) for the decay $\overline{B}^0\to D^{*+}l^-\overline{\nu}_l$. The spectra may then be fitted for $F_A(1)|V_{cb}|$ and ρ^2. There is a strong correlation between the two parameters. The results are shown in Fig. 2 and Table I, where we have taken $F_A(1)=0.913\pm 0.042$ as in Ref. [4]. Our fitted parameters for the CLEO data differ slightly from those presented by the collaboration itself, since we wished to generate an error ellipse and therefore fitted the spectral points directly without taking account of point-to-point correlations. For com-

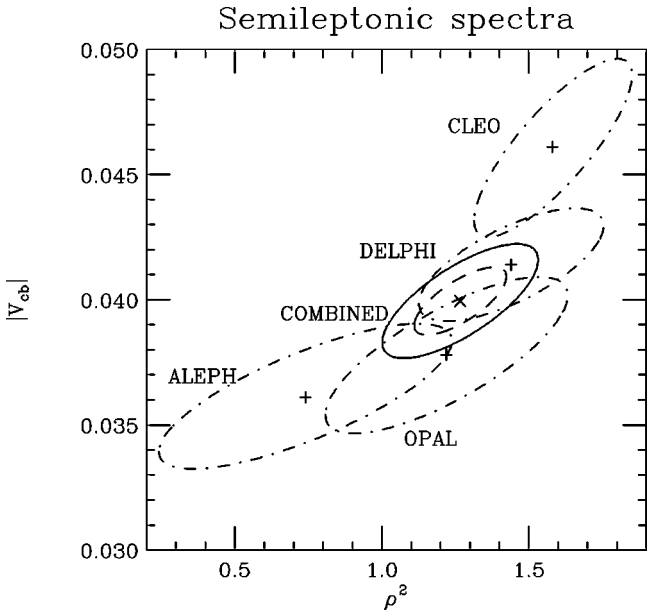

FIG. 2. Error ellipses corresponding to $\Delta\chi^2=1$ for fits to $\overline{B}^0 \to D^{*+} l^- \overline{\nu}_l$ spectra. Dot-dashed lines correspond to fits to individual experiments, not including theoretical error in $F_A(1)$. Fit to combined semileptonic data without (with) error in $F_A(1)$ is shown by the dashed (solid) ellipse. The errors we quote on each variable in Table I correspond to $\pm 1\sigma$ extremes in which the other variable is also permitted to vary. The plotted points (+ for individual experiments, $\times$ for combined data) correspond to χ^2 minima.

parison, CLEO quotes $|V_{cb}|=0.0464\pm0.0020(\text{stat.})\pm0.0021(\text{syst.})\pm0.0021(\text{theor.})$ and $\rho^2=1.67\pm0.11\pm0.22$. The combined fit implies $|V_{cb}|=0.0399\pm0.0023$ and $\rho^2=1.27\pm0.26$.

The CLEO Collaboration has also measured the spectrum for the decay $\overline{B}^0 \to D^+ l^- \overline{\nu}_l$ [9]. A fit to this spectrum with the form factor (10) and with $F_V(1)=1.022$ yields $|V_{cb}|=0.0459^{+0.0053}_{-0.0044}$ and $\rho^2=1.33^{+0.21}_{-0.25}$.

IV. NONLEPTONIC TWO-BODY DECAYS AND COMBINED FIT

We fit rates for nonleptonic two-body decays [Eqs. (18)–(23)] to experimental averages, allowing (as in the fit to semileptonic spectra) for variation of $|V_{cb}|$ and ρ^2. We use the averages of Ref. [22] except for modes for which new values have been presented [23–25]; these are summarized and averaged in Table II. For the decays involving $D_s^{(*)}$ (to be discussed in the Sec. V) the last errors in the second and third columns of Table II refer to a common systematic error in D_s branching ratios, which are based on $\mathcal{B}(D_s^+ \to \phi\pi^+)=(3.6\pm0.9)\%$, and are combined accordingly. In our fits we use $|V_{ud}|=0.974$, $\tau_B=1.548$ ps, and $f_\pi=131$ MeV. f_ρ and f_{a_1} are determined to be 209 MeV and 229 MeV [2], respectively, from the branching ratios for $\tau \to \rho\nu$ and $\tau \to a_1\nu$.

TABLE I. Values of $|V_{cb}|$ and ρ^2 obtained from fits to individual spectra in $\overline{B}^0 \to D^{*+} l^- \overline{\nu}_l$ decays.

Experiment	$\lvert V_{cb}\rvert$	ρ^2
CLEO [4]	0.0461 ± 0.0036	1.58 ± 0.27
ALEPH [5]	0.0361 ± 0.0029	0.74 ± 0.50
DELPHI [6]	0.0378 ± 0.0031	1.22 ± 0.41
OPAL [7]	0.0414 ± 0.0023	1.44 ± 0.32
Combined[a]	0.0399 ± 0.0023	1.27 ± 0.26

[a]Errors include common theoretical error on $F_A(1)$.

The fit to two-body nonleptonic decays alone gives rise to a different correlation between $|V_{cb}|$ and ρ^2 than that to the $\overline{B}^0 \to D^{(*)+} l^- \overline{\nu}_l$ spectra, since the decay rates are dominated by low q^2 and hence high w. Contours of $\Delta\chi^2=1$ (1σ) for nonleptonic decays are shown along with the $\Delta\chi^2=1$ contours for $\overline{B}^0 \to D^{(*)+} l^- \overline{\nu}_l$ spectra in Fig. 3. Also shown are the $\Delta\chi^2=1$ contours for the combined fit without and with common theoretical errors. We find $|V_{cb}|=0.0415\pm0.0022$ and $\rho^2=1.52\pm0.11$. The results are summarized in Table III. The error on $|V_{cb}|$ is dominated by the theoretical uncertainty on the form factors at $w=1$, which we take to have the same fractional value (0.042/0.913) for vector and axial form factors.

The fitted branching ratios are compared with experimental data in Table IV. We also show the predictions of a recent investigation based on a more detailed application of the factorization hypothesis [2]. The quality of the fit is acceptable except for a slight excess in the predicted branching ratio for $\overline{B}^0 \to D^{*+}\pi^-$.

It is interesting to compare the form factors based on Eqs. (10) and (11) with the simple pole model (16) [3], where $m_B w_{0(V,A)}$ has the interpretation of the mass of a pole in the weak $b \to c$ (V,A) current. The axial form factor for $\rho^2_{A_1}=1.52$ is compared with a pole model with $w_{0A}=1.17$ in Fig. 4. Also shown are CLEO [4] and DELPHI [6] data points. The pole-model form factor is almost indistinguishable from that [12] motivated by dispersion relations. The value $w_{0A}=1.12\pm0.17$, found in Ref. [3], is consistent with the present determination $w_{0A}=1.17\pm0.08$.

The vector form factor (10) is characterized by a slope

TABLE II. Branching ratios for $\overline{B}^0$ decays averaged for our fits, in units of 10^{-3}.

Mode	Value	Value	Average
$D^{*+}\pi^-$	2.76 ± 0.21 [22]	$2.9\pm0.3\pm0.3$ [23]	2.79 ± 0.19
$D^{*+}\rho^-$	6.8 ± 3.4 [22]	$11.2\pm1.1\pm2.5$ [23]	9.5 ± 2.1
$D^{*+}D_s^-$	$11.0\pm1.8\pm1.0\pm2.8$ [24]	$7.1\pm2.4\pm2.5\pm1.8$ [25]	10.0 ± 3.1
$D^{*+}D_s^{*-}$	$18.2\pm3.7\pm2.4\pm4.6$ [24]	$25.4\pm3.8\pm5.3\pm6.4$ [25]	20.5 ± 6.3

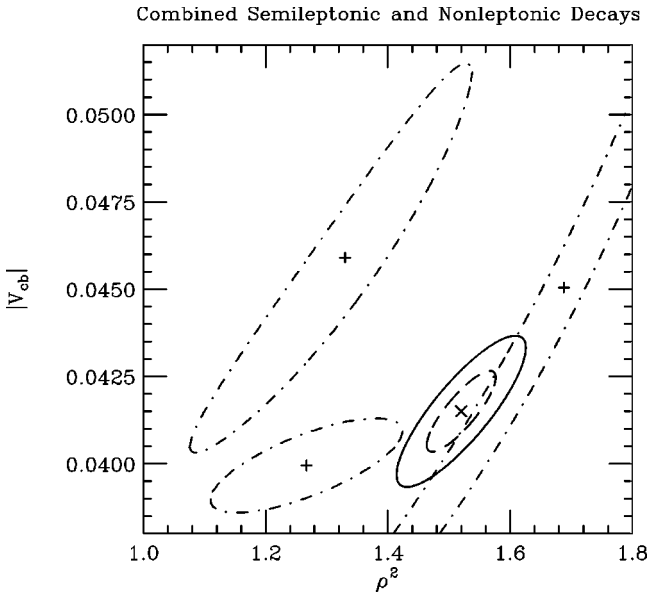

FIG. 3. Right-hand pair of dash-dotted curves: Contours of $\Delta\chi^2=1$ (1σ) for fit to nonleptonic two-body decays $\overline{B}^0 \to D^{(*)+}(\pi^-,\rho^-,a_1^-)$. Upper dash-dotted ellipse: Contour of $\Delta\chi^2=1$ (1σ) for fit to $\overline{B}^0 \to D^+ l^- \overline{\nu}_l$ spectrum. Lower dash-dotted ellipse: Combined fit to semileptonic $\overline{B}^0 \to D^{*+} l^- \overline{\nu}_l$ decays. All these fits are performed assuming $F_A(1)=0.913$ and $F_V(1)=1.022$. Contour of $\Delta\chi^2=1$ (1σ) for fit to combined semileptonic and nonleptonic data without (with) common theoretical error in form factor normalization is shown by the dashed (solid) ellipse. χ^2 minima are indicated by $+$ for nonleptonic and $\overline{B}^0 \to D^{(*)+} l^- \overline{\nu}_l$ decays and by $\times$ for combination of all data.

parameter $\rho^2_{F_V}=\rho^2_{A_1}-0.21$ [12] and hence $\rho^2_{F_V}=1.31$. It is compared to a pole form factor with $w_{0V}=1.14$ and to the CLEO data [9] in Fig. 5. Thus, a nearly universal pole position characterizes the vector and axial form factors, as in Ref. [3]. The CLEO data lie slightly above the predicted form factor but have the same w dependence, as one can also see in Fig. 3. It should be recalled that, in contrast to the case of the axial-vector form factor, theoretical estimates of the $\mathcal{O}(1/m_b)$ correction to the vector form factor are lacking [18]. The normalization of the CLEO data may reflect our ignorance of this correction.

TABLE III. Values of $|V_{cb}|$ and ρ^2 obtained from fits to nonleptonic two-body $\overline{B}^0 \to D^{(*)+}(\pi^-,\rho^-,a_1^-)$ decays and $\overline{B}^0 \to D^{(*)+} l^- \overline{\nu}_l$ decays.

Decays	$\lvert V_{cb}\rvert$	ρ^2
Nonleptonic	0.0450 [a]	1.69 [a]
$\overline{B}^0 \to D^{*+} l^- \overline{\nu}_l$ spectra	0.0399 ± 0.0023	1.27 ± 0.26
$\overline{B}^0 \to D^+ l^- \overline{\nu}_l$ spectrum	$0.0459^{+0.0053}_{-0.0044}$ [a]	$1.33^{+0.21}_{-0.25}$ [a]
Combined [b]	0.0415 ± 0.0022	1.52 ± 0.11

[a]Large correlated errors; see Fig. 3.

[b]Errors include common theoretical error of $\delta F_A(1)/F_A(1)=4.6\%$.

TABLE IV. Branching ratios in units of 10^{-3}: comparison between data and predictions.

Decay mode	Data	Ref. [2][a]	Our fit	χ^2 contribution
$\overline{B}^0 \to D^+ \pi^-$	3.0 ± 0.4	3.27	3.19	0.22
$\overline{B}^0 \to D^{*+} \pi^-$	2.79 ± 0.19	3.05	3.10	2.70
$\overline{B}^0 \to D^+ \rho^-$	7.9 ± 1.4	7.64	7.92	0.00
$\overline{B}^0 \to D^{*+} \rho^-$	9.5 ± 2.1	7.59	8.78	0.12
$\overline{B}^0 \to D^+ a_1^-$	6.0 ± 3.3	7.76	9.10	0.88
$\overline{B}^0 \to D^{*+} a_1^-$	13.0 ± 2.7	8.53	12.2	0.09

[a]For preferred values of form factors and $|a_1|=1.05$.

V. $D_s^{(*)}$ PRODUCTION BY THE WEAK CURRENT

When the $q\overline{q'}$ meson in Fig. 1(b) is a D_s^- or D_s^{*-}, Eqs. (18)–(23) can be used for the respective predictions for $\Gamma(\overline{B}^0 \to D^+ D_s^-)$, $\Gamma(\overline{B}^0 \to D^{*+} D_s^-)$, $\Gamma(\overline{B}^0 \to D^+ D_s^{*-})$, and $\Gamma(\overline{B}^0 \to D^{*+} D_s^{*-})$ by replacing $f_\pi \to f_{D_s}$, $f_\rho \to f_{D_s^*}$, $\zeta_\pi \to \zeta_{D_s} \equiv m^2_{D_s}/m^2_B$, $\zeta_\rho \to \zeta_{D_s^*} \equiv m^2_{D_s^*}/m^2_B$, and other corresponding substitutions of kinematic variables w and N. We shall assume $f_{D_s^*}=f_{D_s}=270 \pm 16 \pm 34$ MeV based on an experimental average of rates for $D_s \to \mu\nu$ and $D_s \to \tau\nu$ [26]; the latter error is the common systematic error associated with a 25% uncertainty in the branching ratio for $D_s^+ \to \phi\pi^+$. (For comparison, the value of 259 ± 74 MeV was found in Ref. [3] by utilizing the observed branching ratios of processes in which the weak current produced a D_s or D_s^*.)

We then predict the branching ratios for $D_s^{(*)}$ production by the weak current shown in Table V. Experimental values are from Ref. [22] $(D^+ D_s^{(*)-})$ or Table II $(D^{*+} D_s^{(*)-})$. The

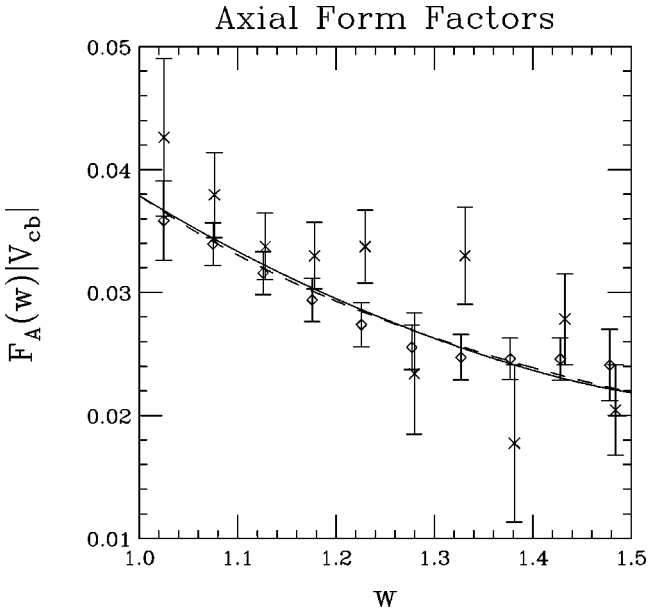

FIG. 4. Form factors (11) (solid curve) for $\rho^2_{A_1}=1.52$ and pole-model (16) (dashed curve) for $w_{0A}=1.17$. Data points are from CLEO (crosses) and DELPHI (diamonds).

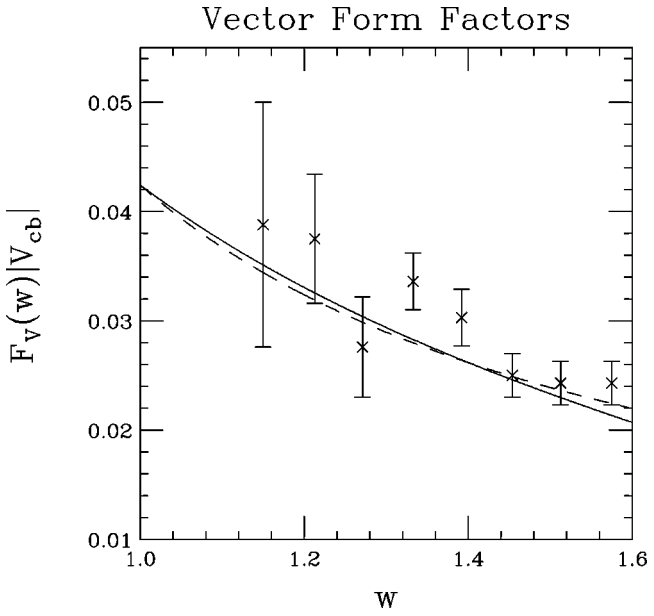

FIG. 5. Form factors (10) (solid curve) for $\rho^2_{F_V}=1.31$ and pole-model (16) (dashed curve) for $w_{0V}=1.14$ compared with CLEO data (plotted points).

predictions have an overall 25% uncertainty associated with all D_s branching ratios. They are as well obeyed as those for the light mesons. An additional prediction involving heavy meson production by the weak current [3] is that $\mathcal{B}(\overline{B}^0 \to D^{*+}D^{*-}) / \mathcal{B}(\overline{B}^0 \to D^{*+}D_s^-) = 0.13(f_D/f_{D_s})^2 \simeq 0.09$, where f_D and f_{D_s} are the decay constants for the nonstrange and strange D mesons. The experimental value for this ratio [27] is $0.06^{+0.04}_{-0.03}$.

VI. RATIOS OF HELICITY AMPLITUDES

The decays of spinless particles to two-vector mesons are describable [28] by amplitudes A_0 (longitudinal polarization), $A_\parallel$ (linear parallel polarization), and $A_\perp$ (linear perpendicular polarization), normalized such that $|A_0|^2+|A_\parallel|^2+|A_\perp|^2=1$. Factorization predicts $(|A_0|^2,|A_\parallel|^2,|A_\perp|^2)=(88,10,2)\%$ for $\overline{B}^0 \to D^{*+}D_s^{*-}$ and $(55,39,6)\%$ for $\overline{B}^0 \to D^{*+}\rho^-$. Experimental values are only quoted for $|A_0|^2$: $(87.8\pm3.4\pm3.0)\%$ for $\overline{B}^0 \to D^{*+}D_s^{*-}$ [27] and $(50.6\pm13.9\pm3.6)\%$ for $\overline{B}^0 \to D^{*+}\rho^-$ [29]. These agree with the predictions, as does the intermediate case of $\rho'(1418)$ production [30].

TABLE V. Comparison of predictions for branching ratios (in units of 10^{-3}) involving $D_s^{(*)}$ production by the weak current in $\overline{B}^0$ decays.

Decay mode	Data	Prediction
$D^+D_s^-$	8.0 ± 3.0	14.9 ± 4.1
$D^{*+}D_s^-$	10.0 ± 3.1	8.6 ± 2.4
$D^+D_s^{*-}$	10.0 ± 5.0	10.0 ± 2.8
$D^{*+}D_s^{*-}$	20.5 ± 6.3	24.0 ± 6.7

VII. CONCLUSIONS

New data on B meson decays have improved the precision of tests of some early factorization predictions [3], and yield a value $|V_{cb}|=0.0415\pm0.0022$ when CLEO and CERN e^+e^- collider LEP data on $\overline{B}^0 \to D^{(*)+}l^-\overline{\nu}_l$ spectra are combined with two-body nonleptonic decays $\overline{B}^0 \to D^{(*)+}(\pi^-,\rho^-,a_1^-)$. The slope of the universal Isgur-Wise form factor at the normalization point $w=1$ is found to be described by the parameter $\rho^2=1.52\pm0.11$. These values are only slightly different from those based on $\overline{B}^0 \to D^{(*)+}l^-\overline{\nu}_l$ spectra alone, indicating that factorization for color-favored decays and universality of $\overline{B} \to D^{(*)}$ form factors are reasonable approximations. Consistency between nonleptonic and semileptonic determinations is at least as good as that among the semileptonic processes themselves. Our neglect of $\mathcal{O}(1/m_b)$ corrections to the vector form factor may underestimate the rate for $\overline{B}^0 \to D^+l^-\overline{\nu}_l$ slightly.

Satisfactory rates for processes involving $D_s^{(*)}$ production by the weak current are obtained when the world average of direct measurements for f_{D_s} is used, and when it is assumed that $f_{D_s^*}=f_{D_s}$. Ratios of helicity amplitudes for color-favored processes are also found to be in accord with predictions.

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APPENDIX: PARAMETRIZATION OF ERROR ELLIPSES

In order to combine results of fits to semileptonic decays in the absence of the ALEPH and DELPHI raw spectra, we have parametrized their fits in terms of error ellipses corresponding to $\Delta\chi^2=1$, and generated corresponding ellipses for our own fits to CLEO and DELPHI data. The equations describing these ellipses are given below. Also shown are the contributions to $\Delta\chi^2$ for each set of $\overline{B}^0 \to D^{*+}l^-\overline{\nu}_l$ data when $x\equiv\rho^2$ and $y\equiv|V_{cb}|$ are taken to equal their values $x=1.52$ and $y=0.0415$ in the global fit. The corresponding $\Delta\chi^2$ value is 4.02 for the sum of the nonleptonic modes listed in Table IV, which, when combined with the $\Delta\chi^2$ values for the semileptonic spectra, leads to a total of $\Delta\chi^2=23.2$ for the fit to five semileptonic spectra and six nonleptonic decay rates. The largest source of this $\Delta\chi^2$ (10.3) is the higher overall scale of the $\overline{B}^0 \to D^+l^-\overline{\nu}_l$ spectrum measured by CLEO, with some contribution also from the disparity between the CLEO and ALEPH fits. In comparison with these disagreements among purely semileptonic processes,

the fits to nonleptonic decays do not fare badly at all.

CLEO:

$$41.8814(x-x_c)^2-5174.3(x-x_c)(y-y_c)+238793(y-y_c)^2=1,$$

$$x_c=1.58,\quad y_c=0.0461,\quad \Delta\chi^2=3.78;$$

ALEPH:

$$9.6816(x-x_a)^2-2554.0(x-x_a)(y-y_a)+287560(y-y_a)^2=1,$$

$$x_a=0.74,\quad y_a=0.0361,\quad \Delta\chi^2=3.52;$$

OPAL:

$$19.4601(x-x_o)^2-3771.3(x-x_o)(y-y_o)+379980(y-y_o)^2=1,$$

$$x_o=1.44,\quad y_o=0.0414,\quad \Delta\chi^2=0.10;$$

DELPHI:

$$14.1777(x-x_d)^2-2856.4(x-x_d)(y-y_d)+246120(y-y_d)^2=1,$$

$$x_d=1.22,\quad y_d=0.0378,\quad \Delta\chi^2=1.47;$$

CLEO+ALEPH+OPAL+DELPHI (without common theoretical normalization error; divide all coefficients by 2.850 for ellipse with this error):

$$85.2008(x-\overline{x})^2-14356(x-\overline{x})(y-\overline{y})+1152453(y-\overline{y})^2=1,$$

$$\overline{x}=1.27,\quad \overline{y}=0.0399;$$

CLEO+ALEPH+OPAL+DELPHI+DECAYS (without common theoretical normalization error; divide all coefficients by 3.514 for ellipse with this error):

$$1059.7(x-\tilde{x})^2-86781(x-\tilde{x})(y-\tilde{y})+2524400(y-\tilde{y})^2=1,$$

$$\tilde{x}=1.52,\quad \tilde{y}=0.0415.$$

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