Almost maximal lepton mixing with large T violation in neutrino oscillations and neutrinoless double beta decay

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We point out two simple but instructive possibilities to construct the charged lepton and neutrino mass matrices, from which the nearly bimaximal neutrino mixing with large T violation can naturally emerge. The two lepton mixing scenarios are very compatible with current experimental data on solar and atmospheric neutrino oscillations, and one of them may lead to an observable T-violating asymmetry between $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ transitions in the long-baseline neutrino oscillation experiments. Their implications for the neutrinoless double beta decay are also discussed.

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I. INTRODUCTION

Recent observation of atmospheric and solar neutrino anomalies, particularly that in the super-Kamiokande experiment [1], has provided robust evidence that neutrinos are massive and lepton flavors are mixed. Analyses of the atmospheric neutrino deficit favor $\nu_\mu \rightarrow \nu_\tau$ as the dominant oscillation mode with the mass-squared difference $\Delta m^2_{\rm atm} \sim 10^{-3}$ eV^2 and the mixing factor $\sin^2 2\theta_{\rm atm} > 0.88$ at the 90% confidence level. As for the solar neutrino anomaly, there are four possible solutions belonging to two categories: (a) solar ν_e neutrinos changing to active ν_μ or sterile ν_s neutrinos due to the long-wavelength vacuum oscillation with the parameters $\Delta m^2_{\rm sun} \sim 10^{-10}$ eV^2 and $\sin^2 2\theta_{\rm sun} \approx 1$ [2]; (b) the matter-enhanced $\nu_e \rightarrow \nu_\mu$ or $\nu_e \rightarrow \nu_s$ oscillations via the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism with $\Delta m^2_{\rm sun} \sim 10^{-5}$ eV^2 and $\sin^2 2\theta_{\rm sun} \sim 1$ (large-angle solution), with $\Delta m^2_{\rm sun} \sim 10^{-6}$ eV^2 and $\sin^2 2\theta_{\rm sun} \sim 10^{-2}$ (small-angle solution), or with $\Delta m^2_{\rm sun} \sim 10^{-7}$ eV^2 and $\sin^2 2\theta_{\rm sun} \sim 1$ (low solution) [3]. Although the large-angle MSW solution seems to be somehow favored by the present super-Kamiokande and SNO data [1,4], the other three solutions have not been convincingly ruled out. To pin down the true solution to the solar neutrino problem remains a challenging task of the next round of solar neutrino experiments.

The strong hierarchy between $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sun}$, together with the small ν_3 component in the ν_e configuration [5], implies that atmospheric and solar neutrino oscillations decouple approximately from each other. Each of them is dominated by a single mass scale,[1] which can be set as

$$\Delta m^2_{21} \equiv m^2_2 - m^2_1 \approx \pm \Delta m^2_{\rm sun}, \quad (1)$$

$$\Delta m^2_{32} \equiv m^2_3 - m^2_2 \approx \pm \Delta m^2_{\rm atm}.$$

Of course $\Delta m^2_{31} \approx \Delta m^2_{32}$ holds in this approximation. As a consequence, the mixing factors of solar and atmospheric neutrino oscillations in the disappearance-type experiments (i.e., $\nu_e \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_\mu$) are simply given by

$$\sin^2 2\theta_{\rm sun} = 4|V_{e1}|^2 |V_{e2}|^2,$$
$$\sin^2 2\theta_{\rm atm} = 4|V_{\mu 3}|^2 (1 - |V_{\mu 3}|^2), \quad (2)$$

where V is the 3×3 lepton flavor-mixing matrix linking the neutrino mass eigenstates (ν_1, ν_2, ν_3) to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$. The present experimental data seem to favor the large-angle MSW solution to the solar neutrino problem. In this case, $\sin^2 2\theta_{\rm atm} \sim \sin^2 2\theta_{\rm sun} \sim O(1)$. Then two large mixing angles can be drawn from Eq. (2): one between the second and third lepton families and the other between the first and second lepton families.[2]

A particularly interesting limit is $\sin^2 2\theta_{\rm atm} = \sin^2 2\theta_{\rm sun} = 1$, corresponding uniquely (up to a trivial sign or phase rearrangement) to

$$V_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (3)$$

the so-called "bimaximal" flavor-mixing pattern [9]. There have been a lot of discussions about the bimaximal and nearly bimaximal neutrino mixing scenarios [10]. While the latter could straightforwardly be obtained from slight modi-

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[1]Throughout this paper we do not take the Liquid Scintillation Neutrino Detector (LSND) evidence for neutrino oscillations [6], which has not been independently confirmed by other experiments [7], into account.

[2]The conjecture that two of the three lepton flavor mixing angles could be extraordinarily large (i.e., equal or close to 45°) had been made by several authors [8] before the first-round super-Kamiokande data appeared in 1998.

fications of the former, the arbitrariness in doing so has to be resolved by imposing simple flavor symmetries or dynamical constraints on the charged lepton and neutrino mass matrices. In Ref. [11], for example, it has been shown that a nearly bimaximal neutrino mixing pattern can naturally arise from the explicit breaking of the lepton flavor democracy.

The present paper aims to discuss two simple but instructive possibilities to construct the lepton mass matrices, from which two almost bimaximal neutrino mixing patterns can directly be derived. We find that these two scenarios have practically indistinguishable consequences on solar and atmospheric neutrino oscillations, but their predictions for leptonic CP or T violation are quite different. To be specific, we calculate the deviation of solar neutrino mixing from maximal mixing in each scenario. We illustrate that one of the two lepton mixing patterns may lead to an observable T-violating asymmetry between $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ transitions in the long-baseline neutrino oscillation experiments. The implications of our phenomenological models on the neutrinoless double beta decay are also discussed in some detail.

II. NEARLY BIMAXIMAL MIXING

The fact that the masses of three active neutrinos are extremely small is presumably attributed to the Majorana feature of the neutrino fields [12]. In this picture, the light (left-handed) neutrino mass matrix M_ν must be symmetric and can be diagonalized by a single unitary transformation:

$$U_\nu^\dagger M_\nu U_\nu^* = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{4}$$

The charged lepton mass matrix M_l is in general non-Hermitian, hence the diagonalization of M_l needs a biunitary transformation:

$$U_l^\dagger M_l \widetilde{U}_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}. \tag{5}$$

The lepton flavor-mixing matrix V, defined to link the neutrino mass eigenstates (ν_1, ν_2, ν_3) to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$, measures the mismatch between the diagonalization of M_l and that of M_ν:

$$V = U_l^\dagger U_\nu. \tag{6}$$

Note that (m_1, m_2, m_3) in Eq. (4) and (m_e, m_μ, m_τ) in Eq. (5) are physical (positive) masses of light neutrinos and charged leptons, respectively.

In the flavor basis where M_l is diagonal (i.e., $U_l = \mathbf{1}$ being a unity matrix), the flavor-mixing matrix is simplified to $V = U_\nu$. The bimaximal neutrino mixing pattern $U_\nu = V_0$ can then be constructed from the product of the Euler rotation matrices

$$R_{12}(\theta_x) = \begin{pmatrix} \cos\theta_x & \sin\theta_x & 0 \\ -\sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

and

$$R_{23}(\theta_y) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_y & \sin\theta_y \\ 0 & -\sin\theta_y & \cos\theta_y \end{pmatrix} \tag{8}$$

with special rotation angles $\theta_x = \theta_y = 45°$:

$$V_0 = R_{23}(45°) \otimes R_{12}(45°) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{9}$$

Obviously the vanishing of the (1,3) element in V_0 assures an exact decoupling between solar $(\nu_e \rightarrow \nu_\mu)$ and atmospheric $(\nu_\mu \rightarrow \nu_\tau)$ neutrino oscillations. The corresponding neutrino mass matrix M_ν turns out to be

$$M_\nu = V_0 \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V_0^T = \begin{pmatrix} A_\nu - B_\nu & C_\nu & -C_\nu \\ C_\nu & A_\nu & B_\nu \\ -C_\nu & B_\nu & A_\nu \end{pmatrix}, \tag{10}$$

where

$$A_\nu = \frac{m_3}{2} + \frac{m_1 + m_2}{4}, \quad B_\nu = \frac{m_3}{2} - \frac{m_1 + m_2}{4}, \quad C_\nu = \frac{m_2 - m_1}{2\sqrt{2}}. \tag{11}$$

A trivial sign or phase rearrangement for $U_\nu = V_0$ may lead to a slightly different form of M_ν [9,13], but the relevant physical consequences on neutrino oscillations are essentially unchanged. If the masses of ν_1 and ν_2 neutrinos are nearly degenerate (i.e., $m_1 \approx m_2$), one can arrive at a simpler texture of M_ν, in which $A_\nu \approx (m_3 + m_1)/2$, $B_\nu \approx (m_3 - m_1)/2$, and $C_\nu \approx 0$ hold.

It is worthwhile at this point to give a brief comment on the mathematical structure of M_ν obtained in Eq. (10). Indeed M_ν can be decomposed as follows:

$$M_\nu = A_\nu \mathbf{I}_A + B_\nu \mathbf{I}_B + C_\nu \mathbf{I}_C, \tag{12}$$

where

$$\mathbf{I}_A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$\mathbf{I}_B = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{13}$$
$$\mathbf{I}_C = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}.$$

It becomes clear that the diagonalization of M_ν requires an *algebraic* unitary matrix which is able to diagonalize $\mathbf{I}_B$ and $\mathbf{I}_C$ simultaneously. This unitary matrix is just V_0 given in Eq. (9). Although the aforementioned decomposition is by no means unique, it might have a meaningful interpretation in an underlying model of neutrino masses with specific flavor symmetries.

We observe that the bimaximal neutrino mixing pattern will be modified, if U_l deviates somehow from the unity matrix. This can certainly happen, provided that the charged lepton mass matrix M_l is not diagonal in the flavor basis where the neutrino mass matrix M_ν takes the form given in Eq. (10). As $U_\nu = V_0$ describes a product of two special Euler rotations in the real (2,3) and (1,2) planes, the simplest form of U_l which allows $V = U_l^\dagger U_\nu$ to cover the whole 3×3 space should be $U_l = R_{12}(\theta_x)$ or $U_l = R_{31}(\theta_z)$ (see Ref. [14] for a detailed discussion). To incorporate T violation in neutrino oscillations, however, the complex rotation matrices

$$R_{12}(\theta_x, \phi_x) = \begin{pmatrix} \cos\theta_x & \sin\theta_x e^{i\phi_x} & 0 \\ -\sin\theta_x e^{-i\phi_x} & \cos\theta_x & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{14}$$

and

$$R_{31}(\theta_z, \phi_z) = \begin{pmatrix} \cos\theta_z & 0 & \sin\theta_z e^{i\phi_z} \\ 0 & 1 & 0 \\ -\sin\theta_z e^{-i\phi_z} & 0 & \cos\theta_z \end{pmatrix} \tag{15}$$

should be used [15]. In this case, we arrive at lepton flavor mixing of the pattern

$$V_{(x)} = \begin{pmatrix} c_x & -s_x e^{i\phi_x} & 0 \\ s_x e^{-i\phi_x} & c_x & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{c_x}{\sqrt{2}} + \frac{s_x}{2} e^{i\phi_x} & \frac{c_x}{\sqrt{2}} - \frac{s_x}{2} e^{i\phi_x} & -\frac{s_x}{\sqrt{2}} e^{i\phi_x} \\ -\frac{c_x}{2} + \frac{s_x}{\sqrt{2}} e^{-i\phi_x} & \frac{c_x}{2} + \frac{s_x}{\sqrt{2}} e^{-i\phi_x} & \frac{c_x}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{16}$$

or of the pattern

$$V_{(z)} = \begin{pmatrix} c_z & 0 & -s_z e^{i\phi_z} \\ 0 & 1 & 0 \\ s_z e^{-i\phi_z} & 0 & c_z \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{c_z}{\sqrt{2}} - \frac{s_z}{2} e^{i\phi_z} & \frac{c_z}{\sqrt{2}} + \frac{s_z}{2} e^{i\phi_z} & -\frac{s_z}{\sqrt{2}} e^{i\phi_z} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{c_z}{2} + \frac{s_z}{\sqrt{2}} e^{-i\phi_z} & -\frac{c_z}{2} + \frac{s_z}{\sqrt{2}} e^{-i\phi_z} & \frac{c_z}{\sqrt{2}} \end{pmatrix}, \tag{17}$$

where $s_x \equiv \sin\theta_x$, $c_z \equiv \cos\theta_z$, and so on. It is obvious that $V_{(x)}$ and $V_{(z)}$ represent two nearly bimaximal flavor-mixing scenarios, if the rotation angles θ_x and θ_z are small in magnitude.

As the mixing angle θ_x or θ_z arises from the diagonalization of M_l, it is expected to be a simple function of the ratios of charged lepton masses. Then the strong mass hierarchy of charged leptons naturally assures the smallness of θ_x or θ_z, as one can see later on.

III. CONSTRAINTS ON $\sin^2 2\theta_{\rm sun}$ and $\sin^2 2\theta_{\rm atm}$

Indeed the proper texture of M_l which leads to the flavor-mixing pattern $V_{(x)}$ is

$$M_l^{(x)} = \begin{pmatrix} 0 & C_l & 0 \\ C_l^* & B_l & 0 \\ 0 & 0 & A_l \end{pmatrix}, \tag{18}$$

where $A_l = m_\tau$, $B_l = m_\mu - m_e$, and $C_l = \sqrt{m_e m_\mu}\, e^{i\phi_x}$. The mixing angle θ_x in $V_{(x)}$ is then given by

$$\tan(2\theta_x) = 2\frac{\sqrt{m_e m_\mu}}{m_\mu - m_e}. \tag{19}$$

On the other hand, the proper texture of M_l which gives rise to the mixing pattern $V_{(z)}$ reads as follows:

$$M_l^{(z)} = \begin{pmatrix} 0 & 0 & C_l \\ 0 & B_l & 0 \\ C_l^* & 0 & A_l \end{pmatrix}, \tag{20}$$

where $A_l = m_\tau - m_e$, $B_l = m_\mu$, and $C_l = \sqrt{m_e m_\tau}\, e^{i\phi_z}$. The mixing angle θ_z in $V_{(z)}$ turns out to be

$$\tan(2\theta_z) = 2\frac{\sqrt{m_e m_\tau}}{m_\tau - m_e}. \tag{21}$$

Taking the hierarchy of charged lepton masses (i.e., $m_e \ll m_\mu \ll m_\tau$) into account, one obtains

$$s_x \approx \sqrt{\frac{m_e}{m_\mu}}, \qquad s_z \approx \sqrt{\frac{m_e}{m_\tau}}, \tag{22}$$

to a good degree of accuracy. Numerically, we find $\theta_x \approx 3.978°$ and $\theta_z \approx 0.972°$ with the inputs $m_e = 0.511$ MeV, $m_\mu = 105.658$ MeV, and $m_\tau = 1.777$ GeV [16].

Now let us calculate the mixing factors of solar and atmospheric neutrino oscillations in the disappearance-type experiments. Using Eq. (2), we arrive straightforwardly at

$$\sin^2 2\theta_{\rm sun} = 1 - s_x^2(1 + 2\cos^2\phi_x), \qquad \sin^2 2\theta_{\rm atm} = 1 - s_x^4 \tag{23}$$

for $V_{(x)}$; and

$$\sin^2 2\theta_{\rm sun} = 1 - s_z^2(1 + 2\cos^2\phi_z), \qquad \sin^2 2\theta_{\rm atm} = 1 \tag{24}$$

for $V_{(z)}$. Allowing ϕ_x and ϕ_z to take arbitrary values, we find that the magnitude of $\sin^2 2\theta_{\rm sun}$ lies in the following range:

$$1 - 3s_i^2 \leq \sin^2 2\theta_{\rm sun} \leq 1 - s_i^2, \tag{25}$$

where $i = x$ or z. Numerically, we obtain $0.986 \leq \sin^2 2\theta_{\rm sun} \leq 0.995$ for $V_{(x)}$ and $0.999 \leq \sin^2 2\theta_{\rm sun} \leq 1.000$ for $V_{(z)}$. Note that $\sin^2 2\theta_{\rm atm} = 1.000$ holds in both cases. Therefore the two nearly bimaximal neutrino mixing patterns are practically indistinguishable in the experiments of solar and atmospheric neutrino oscillations. They may be distinguished from each other with the measurements of $|V_{e3}|$ and CP or T violation in the long-baseline neutrino oscillation experiments.

It is worth mentioning that Gonzalez-Garcia, Peña-Garay, Nir, and Smirnov have recently defined a small real parameter ϵ to describe the deviation of solar neutrino mixing from maximal mixing [17]:

$$\sin^2\theta_{\rm sun} \equiv \frac{1-\epsilon}{2} \tag{26}$$

with $|\epsilon| \ll 1$. This parameter proves very useful for phenomenological studies of the solar neutrino problem [17]: the probabilities of solar neutrino oscillations depend quadratically on ϵ in vacuum, and linearly on ϵ if matter effects dominate. It is then our interest to calculate ϵ in the nearly bimaximal neutrino mixing scenarios under discussion. We notice that

$$\sin^2 2\theta_{\rm sun} = 1 - \epsilon^2 \tag{27}$$

results from Eq. (24) exactly. Comparing Eq. (27) with Eqs. (23) and (24), we obtain

$$|\epsilon| = s_x\sqrt{1 + 2\cos^2\phi_x} \tag{28}$$

for $V_{(x)}$ and

$$|\epsilon| = s_z\sqrt{1 + 2\cos^2\phi_z} \tag{29}$$

for $V_{(z)}$. Given ϕ_x and ϕ_z of arbitrary values, the allowed region of $|\epsilon|$ turns out to be $0.069 \leq |\epsilon| \leq 0.120$ in the scenario of $V_{(x)}$ and $0.017 \leq |\epsilon| \leq 0.029$ in the scenario of $V_{(z)}$. Both ranges of $|\epsilon|$ are phenomenologically interesting for solar neutrino oscillations, as comprehensively discussed in Ref. [17].

IV. LEPTONIC *T* VIOLATION

The strength of CP or T violation in neutrino oscillations, regardless of whether neutrinos are Dirac or Majorana particles, is measured by a universal and rephasing-invariant parameter $\mathcal{J}$ [14], defined through the following equation:

$$\mathrm{Im}(V_{\alpha i}V_{\beta j}V_{\alpha j}^* V_{\beta i}^*) = \mathcal{J}\sum_{\gamma,k}(\varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk}), \tag{30}$$

in which the Greek subscripts run over (e,μ,τ), and the Latin subscripts run over $(1,2,3)$. Considering the two lepton mixing scenarios proposed in Sec. II, we obtain

$$\mathcal{J} = \begin{cases} \dfrac{c_x s_x}{4\sqrt{2}}\sin\phi_x & \text{for } V_{(x)}, \\[2ex] \dfrac{c_z s_z}{4\sqrt{2}}\sin\phi_z & \text{for } V_{(z)}. \end{cases} \tag{31}$$

For illustration, we typically take $\phi_x = \phi_z = 90°$. Then we arrive at $\mathcal{J} \approx 0.012$ and $\mathcal{J} \approx 0.003$, respectively, for $V_{(x)}$ and

$V_{(z)}$. The former could be determined from the probability asymmetry between $\nu_\mu \rightarrow \nu_e$ and $\overline{\nu_\mu} \rightarrow \overline{\nu_e}$ transitions (CP-violating asymmetry), or that between $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ transitions (T-violating asymmetry) in a long-baseline neutrino oscillation experiment [18], if the Earth-induced matter effects were assumed to be absent or negligible:

$$\begin{aligned}\Delta P &= P(\nu_\mu \rightarrow \nu_e) - P(\overline{\nu_\mu} \rightarrow \overline{\nu_e}) \\ &= P(\nu_\mu \rightarrow \nu_e) - P(\nu_e \rightarrow \nu_\mu) \\ &= 16\mathcal{J} \sin F_{12} \sin F_{23} \sin F_{31} \\ &\approx 16\mathcal{J} \sin F_{21} \sin^2 F_{32}, \end{aligned} \tag{32}$$

where $F_{ij} = 1.27\Delta m_{ij}^2 L/E$ with L being the distance between the neutrino source and the detector (in unit of km) and E being the neutrino beam energy (in unit of GeV). In realistic long-baseline neutrino oscillation experiments, however, the terrestrial matter effects are by no means small and must be taken into account.

It is generally expected that the T-violating asymmetry between $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ transitions is less sensitive to matter effects than the CP-violating asymmetry between $\nu_\mu \rightarrow \nu_e$ and $\overline{\nu_\mu} \rightarrow \overline{\nu_e}$ transitions [19]. For simplicity, we concentrate only on T violation in the following. In analogy to Eq. (32), the matter-corrected T-violating asymmetry can be expressed as

$$\begin{aligned}\Delta \tilde{P} &= \tilde{P}(\nu_\mu \rightarrow \nu_e) - \tilde{P}(\nu_e \rightarrow \nu_\mu) \\ &= 16\tilde{\mathcal{J}} \sin \tilde{F}_{12} \sin \tilde{F}_{23} \sin \tilde{F}_{31} \\ &\approx 16\tilde{\mathcal{J}} \sin \tilde{F}_{21} \sin^2 \tilde{F}_{32}, \end{aligned} \tag{33}$$

where $\tilde{F}_{ij} = 1.27\Delta \tilde{m}_{ij}^2 L/E$ and $\Delta \tilde{m}_{ij}^2 \equiv \tilde{m}_i^2 - \tilde{m}_j^2$ with $\tilde{m}_i$ being the effective neutrino masses in matter. The relation between $\tilde{\mathcal{J}}$ and $\mathcal{J}$ reads [20]

$$\tilde{\mathcal{J}} = \mathcal{J} \frac{\Delta m_{21}^2}{\Delta \tilde{m}_{21}^2} \frac{\Delta m_{31}^2}{\Delta \tilde{m}_{31}^2} \frac{\Delta m_{32}^2}{\Delta \tilde{m}_{32}^2}, \tag{34}$$

where

$$\begin{aligned}\Delta \tilde{m}_{21}^2 &= \frac{2}{3}\sqrt{x^2 - 3y}\sqrt{3(1-z^2)}, \\ \Delta \tilde{m}_{31}^2 &= \frac{1}{3}\sqrt{x^2 - 3y}[3z + \sqrt{3(1-z^2)}], \\ \Delta \tilde{m}_{32}^2 &= \frac{1}{3}\sqrt{x^2 - 3y}[3z - \sqrt{3(1-z^2)}], \end{aligned} \tag{35}$$

and

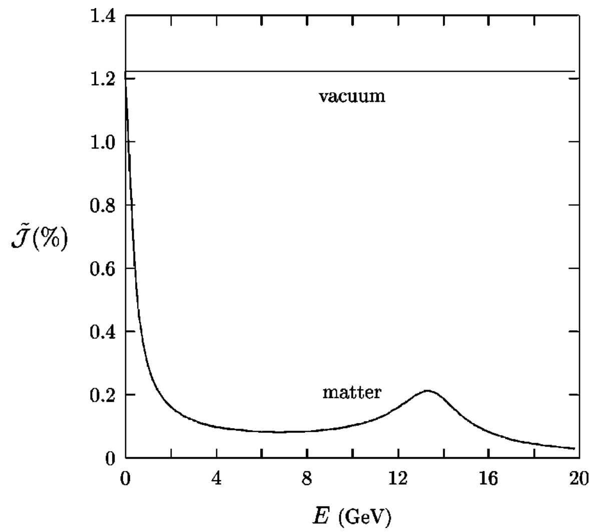

FIG. 1. Illustrative plot for matter effects on the universal T-violating parameter $\tilde{\mathcal{J}}$, where $\Delta m_{21}^2 \approx 5 \times 10^{-5}$ eV^2, $\Delta m_{31}^2 \approx 3 \times 10^{-3}$ eV^2, and $\phi_x = 90°$ have typically been input.

$$\begin{aligned} x &= \Delta m_{21}^2 + \Delta m_{31}^2 + A, \\ y &= \Delta m_{21}^2 \Delta m_{31}^2 + A[\Delta m_{21}^2(1 - |V_{e2}|^2) \\ &\quad + \Delta m_{31}^2(1 - |V_{e3}|^2)], \\ z &= \cos\left[\frac{1}{3}\arccos\frac{2x^3 - 9xy - 27A\Delta m_{21}^2 \Delta m_{31}^2 |V_{e1}|^2}{2(x^2 - 3y)^{3/2}}\right]. \end{aligned} \tag{36}$$

The terrestrial matter effects are described by the parameter $A = 2\sqrt{2}G_F N_e E$ [21], with N_e being the background density of electrons and E being the neutrino beam energy. Assuming the matter density of the Earth's crust to be constant, one may get $A \approx 2.2 \times 10^{-4}$ eV$^2 E/$[GeV] as a good approximation [22].

To illustrate, let us calculate $\tilde{J}$ and $\Delta \tilde{P}$ for two scenarios of the long-baseline neutrino oscillation experiments: $L = 730$ km and $L = 2100$ km. The former baseline corresponds to a neutrino source at Fermilab pointing toward the Soudan mine or that at CERN toward the Gran Sasso underground laboratory, and the latter corresponds to a possible high-intensity neutrino beam from the High Energy Proton Accelerator in Tokaimura to a detector located in Beijing [23]. We typically take $\Delta m_{21}^2 \approx 5 \times 10^{-5}$ eV^2 (the large-angle MSW solution to the solar neutrino problem) and $\Delta m_{32}^2 \approx 3 \times 10^{-3}$ eV^2, as well as $\phi_x = 90°$ based on the almost bimaximal lepton mixing pattern $V_{(x)}$. The numerical results of $\tilde{J}$ and $\Delta \tilde{P}$ as functions of the neutrino beam energy E are shown in Figs. 1 and 2, respectively. We observe that the magnitude of $\tilde{\mathcal{J}}$ can significantly be suppressed due to matter effects. This feature of $\tilde{\mathcal{J}}$ makes the measurement of leptonic CP- and T-violating asymmetries more difficult in practice. Indeed the T-violating asymmetry $\Delta \tilde{P}$ is quite small in the chosen range of the neutrino beam energy (1 GeV $\leqslant E \leqslant 20$ GeV), at most at the percent level. The terrestrial

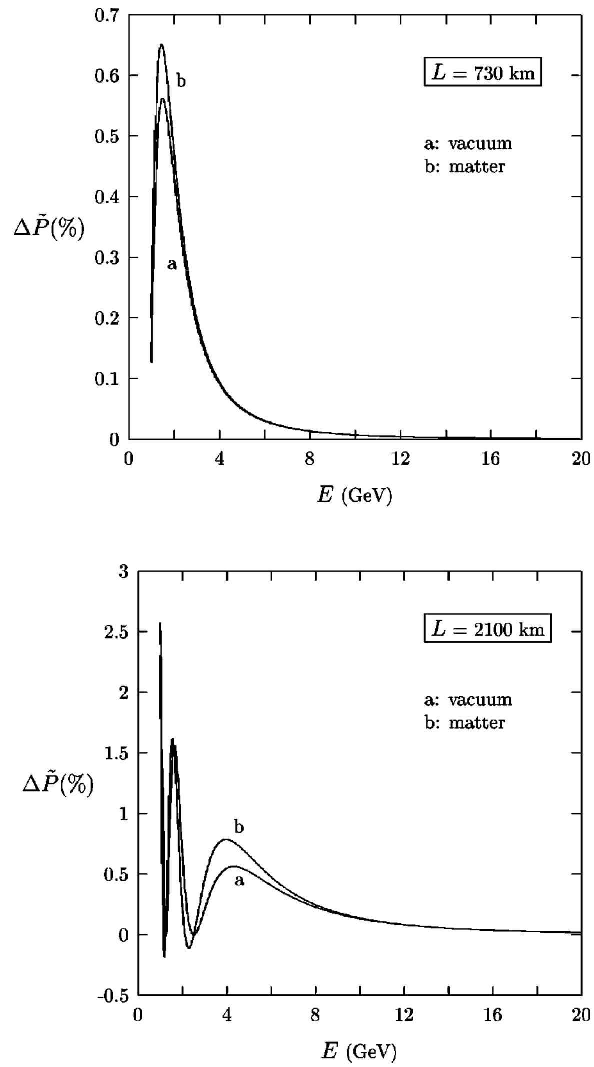

FIG. 2. Illustrative plot for matter effects on the T-violating asymmetry $\Delta\tilde{P}$ between $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_\mu$ transitions, where $\Delta m^2_{21} \approx 5\times10^{-5}$ eV^2, $\Delta m^2_{31} \approx 3\times10^{-3}$ eV^2, and $\phi_x = 90°$ have typically been input.

matter effects on $\Delta\tilde{P}$ are in general insignificant and negligible, except the case of the resonance enhancement at $E \sim 1.5$ GeV for $L = 730$ km or at $E \sim 4$ GeV for $L = 2100$ km. It should be noted that $\Delta\tilde{P} \approx \Delta P$ has no way to lead to $\tilde{\mathcal{J}} \approx \mathcal{J}$. Therefore a relatively clean signal of T violation, even measured in the future long-baseline neutrino experiments, does not mean that the fundamental T-violating parameter ($\mathcal{J}$ or ϕ_x) can *directly* be determined. To pin down those genuine parameters of flavor mixing and T violation, we must first of all understand the terrestrial matter effects to a high degree of accuracy. More reliable knowledge of the Earth's matter density profile is unavoidably required for our long-baseline neutrino oscillation experiments.

V. NEUTRINOLESS DOUBLE BETA DECAY

So far we have only introduced a Dirac-type T-violating phase into the lepton flavor-mixing matrix V. The latter may in general consist of two additional T-violating phases of the Majorana type, i.e.,

$$V \Rightarrow \hat{V} = V P_\nu , \tag{37}$$

where $P_\nu = \mathrm{diag}\{1, e^{i\rho}, e^{i\sigma}\}$ is a diagonal Majorana phase matrix. Although ρ and σ have no effect on CP or T violation in normal neutrino-neutrino and antineutrino-antineutrino oscillations, they are expected to play an important role in the neutrinoless double beta decay, whose effective mass term is given as

$$\langle m_{\nu_e} \rangle = \left| \sum_{i=1}^{3} (m_i \hat{V}^2_{ei}) \right| . \tag{38}$$

The current experimental bound is $\langle m_{\nu_e} \rangle < 0.34$ eV, obtained by the Heidelberg-Moscow Collaboration at the 90% confidence level [24]. For the two nearly bimaximal lepton mixing scenarios under discussion, $\langle m_{\nu_e} \rangle$ reads as follows:

$$\begin{aligned} \langle m_{\nu_e} \rangle_{(x)} &= \left| \frac{\alpha}{2} c_x^2 + \frac{\beta}{\sqrt{2}} s_x c_x e^{i\phi_x} + \frac{\gamma}{4} s_x^2 e^{i2\phi_x} \right| , \\ \langle m_{\nu_e} \rangle_{(z)} &= \left| \frac{\alpha}{2} c_z^2 - \frac{\beta}{\sqrt{2}} s_z c_z e^{i\phi_z} + \frac{\gamma}{4} s_z^2 e^{i2\phi_z} \right| , \end{aligned} \tag{39}$$

where

$$\begin{aligned} \alpha &= m_1 + m_2 e^{i2\rho}, \\ \beta &= m_1 - m_2 e^{i2\rho}, \\ \gamma &= m_1 + m_2 e^{i2\rho} + 2 m_3 e^{i2\sigma}. \end{aligned} \tag{40}$$

Note that $s_x \approx \sqrt{m_e/m_\mu} \approx 0.069$ and $s_z \approx \sqrt{m_e/m_\tau} \approx 0.017$, therefore $c_x \approx c_z \approx 1$ is an excellent approximation. If the spectrum of neutrino masses were known, one would be able to simplify the expression of $\langle m_{\nu_e} \rangle_{(x)}$ or $\langle m_{\nu_e} \rangle_{(z)}$ and confront it with the present experimental bound. Subsequently let us take four specific but interesting cases of the neutrino mass spectrum for example.

(a) $m_1 \approx m_2 \approx m_3$. In this case, the third term of $\langle m_{\nu_e} \rangle_{(x)}$ or $\langle m_{\nu_e} \rangle_{(z)}$ is negligible. We then arrive at

$$\begin{aligned} \langle m_{\nu_e} \rangle_{(x)} &\approx m_1 \left| \frac{1}{2}(1 + e^{i2\rho}) + \frac{s_x}{\sqrt{2}} (1 - e^{i2\rho}) e^{i\phi_x} \right| , \\ \langle m_{\nu_e} \rangle_{(z)} &\approx m_1 \left| \frac{1}{2}(1 + e^{i2\rho}) - \frac{s_z}{\sqrt{2}} (1 - e^{i2\rho}) e^{i\phi_z} \right| . \end{aligned} \tag{41}$$

If $\rho \approx \pm 90°$ holds, we obtain $\langle m_{\nu_e} \rangle_{(x)} \approx \sqrt{2} s_x m_1$ and $\langle m_{\nu_e} \rangle_{(z)} \approx \sqrt{2} s_z m_1$. The experimental bound $\langle m_{\nu_e} \rangle < 0.34$ eV is then assured for $m_1 \leqslant 3$ eV of pattern $\hat{V}_{(x)}$ or for $m_1 \leqslant 15$ eV of pattern $\hat{V}_{(z)}$. If the value of ρ is not close to $\pm 90°$, one may obtain $\langle m_{\nu_e} \rangle \approx m_1 |\cos\rho|$ for both lepton mixing patterns. Such a constraint could provide some information on the Majorana phase ρ, provided that the magnitude of m_1 were already known.

(b) $m_1 \approx m_2 \gg m_3$. One may easily check that the results of $\langle m_{\nu_e}\rangle_{(x)}$ and $\langle m_{\nu_e}\rangle_{(z)}$ in this case are essentially the same as those in case (a).

(c) $m_1 \approx m_2 \ll m_3$. In this case, we obtain $m_3 \approx \sqrt{\Delta m^2_{32}} \approx \sqrt{\Delta m^2_{\rm atm}} \leqslant 0.1$ eV. Then m_1 and m_2 should be of $O(10^{-2})$ eV or smaller. Note that the contribution of m_3 to $\langle m_{\nu_e}\rangle$ is always suppressed by s_x or s_z. Therefore the magnitude of $\langle m_{\nu_e}\rangle$ is at most of $O(10^{-2})$ eV for either $\hat{V}_{(x)}$ or $\hat{V}_{(z)}$, much smaller than the present experimental bound.

(d) $m_1 \ll m_2 \ll m_3$. This "normal" neutrino mass hierarchy[3] leads to $m_3 \approx \sqrt{\Delta m^2_{32}} \approx \sqrt{\Delta m^2_{\rm atm}} \leqslant 0.1$ eV as well as $m_2 \approx \sqrt{\Delta m^2_{21}} \approx \sqrt{\Delta m^2_{\rm sun}} \leqslant 0.01$ eV, where the upper limit of m_2 corresponds to the large-angle MSW solution to the solar neutrino problem. In this case, Eq. (39) can be simplified as

$$\langle m_{\nu_e}\rangle_{(x)} \approx \frac{1}{2}\left|m_2 e^{i2(\rho-\sigma)} + m_3 s_x^2 e^{i2\phi_x}\right|,$$
$$\langle m_{\nu_e}\rangle_{(z)} \approx \frac{1}{2}\left|m_2 e^{i2(\rho-\sigma)} + m_3 s_z^2 e^{i2\phi_x}\right|. \tag{42}$$

We see that $\langle m_{\nu_e}\rangle \leqslant O(10^{-2})$ eV must hold for both nearly bimaximal lepton mixing patterns.

The neutrinoless double beta decay itself is certainly not enough to determine the two Majorana T-violating phases ρ and σ. One may in principle study some other possible lepton-number–nonconserving processes, in which the Majorana phases can show up, to get more constraints on ρ and σ. However, all such processes are suppressed in magnitude by an extremely small factor compared to normal weak interactions [15,25]. Hence it seems practically impossible to measure or constrain ρ and σ in any experiment other than the one associated with the neutrinoless double beta decay.

[3]The "inverse" neutrino mass hierarchy $m_1 \gg m_2 \gg m_3$, which is apparently in conflict with our choices $\Delta m^2_{21} \approx \pm \Delta m^2_{\rm sun}$ and $\Delta m^2_{32} \approx \pm \Delta m^2_{\rm atm}$ in Eq. (1), will not be taken into account in this paper.

VI. SUMMARY

We have discussed two simple possibilities to construct the charged lepton and neutrino mass matrices, from which two almost bimaximal neutrino mixing patterns can naturally emerge. Both scenarios are favored by the atmospheric neutrino oscillation data, and are compatible with either the large-angle (or low) MSW solution or the vacuum oscillation solution to the solar neutrino problem. While the two lepton mixing patterns have practically indistinguishable consequences on solar and atmospheric neutrino oscillations, their predictions for leptonic CP or T violation are different and distinguishable. Only one of them is likely to yield an observable T-violating asymmetry between $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_\mu$ transitions in the long-baseline neutrino oscillation experiments. To be specific, we have taken two typical baselines ($L=730$ km and $L=2100$ km) to illustrate the magnitude of T violation and its dependence on the terrestrial matter effects. The implications of our nearly bimaximal neutrino mixing scenarios on the neutrinoless double beta decay have also been discussed in some detail. We expect that a variety of neutrino experiments in the near future could provide crucial tests of the existing lepton mixing models and give useful hints towards the symmetry or dynamics of lepton mass generation.

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