How can a heavy Higgs boson be consistent with the precision electroweak measurements?

Michael E. Peskin
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

James D. Wells
Department of Physics, University of California, Davis, California 95616
and Lawrence Berkeley National Laboratory, Berkeley, California 94720

(Received 2 February 2001; published 28 September 2001)

The fit of precision electroweak data to the minimal standard model currently gives an upper limit on the Higgs boson mass of 170 GeV at 95% confidence. Nevertheless, it is often said that the Higgs boson could be much heavier in more general models. In this paper, we critically review models that have been proposed in the literature that allow a heavy Higgs boson consistent with the precision electroweak constraints. All have unusual features, and all can be distinguished from the minimal standard model either by improved precision measurements or by other signatures accessible to next-generation colliders.

DOI: 10.1103/PhysRevD.64.093003 PACS number(s): 12.15.Lk, 12.15.Ji

I. INTRODUCTION

Is there a Higgs boson? What is its mass? These are among the most pressing questions of contemporary elementary particle physics. The final months of experiments at the CERN e^+e^- collider LEP showed tantalizing hints of the appearance of the Higgs boson. But with the LEP run now ended, we will not see further experimental evidence to confirm or refute these suggestions for many years. Thus, it is important to reexamine the indirect constraints on the Higgs boson mass and to understand their power as well as possible.

The most important indirect information on the Higgs boson comes from precision measurements of the weak interactions [1]. The minimal standard model (MSM)—defined as the SU(3)×SU(2)×U(1) gauge theory of quarks and leptons with a single elementary Higgs field to break the electroweak symmetry—provides a good fit to the corpus of precision electroweak data. The fit presented at the most recent International Conference on High-Energy Physics predicts the mass of the Higgs boson to be less than 170 GeV at 95% confidence [2]. Although this limit may be weakened slightly by improved measurements of the renormalization of α [3], it remains true that the MSM with a Higgs boson of mass above 250 GeV is strongly inconsistent with the current data.

On the other hand, it is likely that the correct picture of electroweak symmetry breaking requires ingredients beyond the MSM. In principle, these new ingredients could affect the electroweak fit and weaken the upper limit on the Higgs boson mass. Specific models have been presented in which there is no significant upper limit. In this paper, we will review and catalog models with new physics beyond the MSM which allow a heavy Higgs boson to be consistent with the precision electroweak measurements. We will give strategies for producing such models, and we will investigate what properties these models share.

The predictions of the MSM depend on the Higgs boson mass through loop diagrams which contain the Higgs boson as a virtual particle. A general model of electroweak symmetry breaking might not contain a Higgs boson as a light, narrow resonance. However, any such model must contain an SU(2)×U(1) gauge theory and some new particles or fields which spontaneously break its symmetry. These new fields must couple to the W and Z bosons and thus contribute to electroweak radiative corrections. The constraint of the precision electroweak fit is that these corrections should be of the same size as those produced by a light elementary Higgs boson. Models in which the Higgs boson is composite or the symmetry-breaking sector is strongly interacting typically contain larger corrections, comparable to those of a heavy elementary Higgs boson. The precision electroweak constraint then requires that other radiative corrections in these models cancel this contribution down to the small value produced by a light Higgs boson.

About 10 years ago, at the beginning of the era of precision electroweak measurements, several groups studied these corrections in the simplest technicolor models of a strongly interacting Higgs sector. They found that the new contributions typically add to the heavy Higgs effect rather than canceling it, giving an even stronger disagreement with the precision data [4–6]. To build models with a heavy Higgs boson that are compatible with the precision data, we need to find counterexamples to this general trend.

One way to address this question is to represent the Higgs sector by the most general possible effective Lagrangian. Recently, a number of groups have shown that, by adding high-dimension operators to this effective Lagrangian, it is possible to compensate the effect of a heavy Higgs boson and relax the upper bound on the Higgs boson mass [7–10]. We believe that this line of argument, though correct, is incomplete. The effective Lagrangian description of the Higgs sector is obtained by starting with a complete theory of electroweak symmetry breaking and integrating out the high-energy degrees of freedom. The full theory predicts not only the particular operator coefficients relevant to the Higgs mass bound but also other effects, which might include interesting low-energy signatures [11]. Also, it could happen that a particular set of operator coefficients cannot be produced from any complete theory, or may require a full theory so contrived as to be unacceptable. To investigate these issues, we must go beyond the effective Lagrangian description and ask what ingredients are needed in the full theory to compensate the effect of a heavy Higgs boson.

In this paper, we will attempt to make general statements about the Higgs mass bound based on explicit models. It is not so easy to make statements that cover all possible models. However, as we will review in Sec. II, the question of how to relax the constraint on the Higgs boson mass is related to other questions about the electroweak constraints that were raised just shortly after the inception of the precision electroweak program 10 years ago. Considerable ingenuity has been applied to these questions, and a substantial literature has been generated. In this paper, we will review this literature and extract lessons from it.

We have noticed that all explicit models proposed in the literature to relax the bound on the Higgs boson mass use one of three specific mechanisms, which we will attempt to describe transparently. In Sec. II, we will briefly review the present status of constraints on the Higgs boson mass from precision electroweak interactions, using the language of S and T variables [6]. Then, in Secs. III, IV, and V, we will discuss the three mechanisms in turn, showing that each has a simple explanation in terms of the S,T formalism. For definiteness, we will focus on models of new physics that would make a Higgs boson of 500 GeV with standard model couplings consistent with the precision electroweak bounds. This is a less severe criterion than that of allowing a model with no narrow Higgs resonances and true Higgs-sector strong interactions.

Before we begin, we have one more important introductory comment. In models in which electroweak symmetry breaking arises from an elementary Higgs boson, theoretical consistency often places a stringent upper bound on the mass of the Higgs boson which is independent of any requirement from the data. In particular, the postulate that all interactions in nature are weakly coupled up to a grand unification scale at 10^{16} GeV implies by itself a very strong constraint on the Higgs boson mass [12]. The general class of supersymmetric grand unified theories has been studied exhaustively and found to give an upper bound of 205 GeV [13]. In addition, because of decoupling, models which contain an elementary Higgs boson—even those which, like supersymmetry, contain a huge number of new particles—typically give only small additional contributions to electroweak radiative corrections beyond the effects present in the standard model. Thus, the most familiar examples of physics beyond the standard model are compatible only with a light Higgs boson. To allow the Higgs boson to be heavy, we must go further afield.

II. *S-T* ANALYSIS

The precision electroweak constraints are conveniently represented by fitting the data to the MSM augmented by two "oblique" parameters, S and T, which represent the effects of new physics on the W and Z vacuum polarization amplitudes [6,14]. We describe the method briefly. The parameter S describes weak-isospin symmetric and T describes weak isotriplet contributions to W and Z loop diagrams. Precision electroweak observables are linear functions of S and T. Thus, each measurement picks out an allowed band in the S-T plane, and measurements of several processes restrict one to a bounded region in this plane.

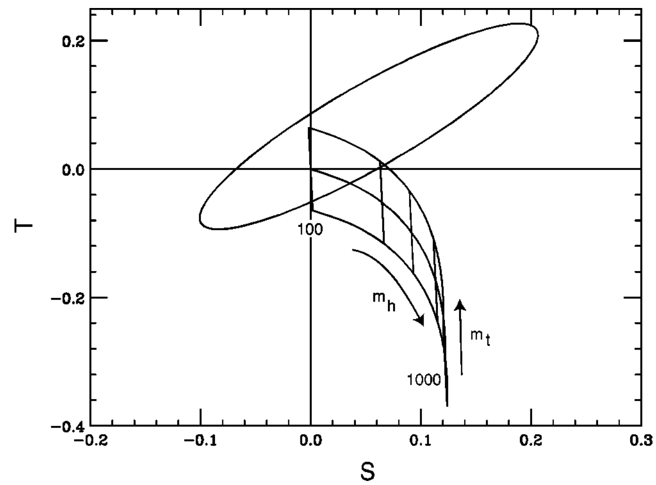

FIG. 1. Fit of the precision electroweak data to the MSM plus the S,T parameters described in the text. The fit is based on the values of m_W, $\sin^2\theta_w^{\rm eff}$, and Γ_l shown in Table I. The ellipse shows the 68% two-dimensional confidence region (1.5σ). The banana-shaped figure shows the central value of a fit to the MSM for $m_t = 174.3 \pm 5.1$ GeV and m_h varying from 100 to 1000 GeV, with $m_h = 200$, 300, and 500 GeV marked with vertical bands. An active version of this figure can be obtained by downloading the additional files deposited with the eprint.

By convention, the point $S=T=0$ corresponds to the prediction of the MSM for fixed "reference" values of the top quark and Higgs boson mass. In this paper, we will take the reference values to be $m_t = 174.3$ GeV (the current central value from the Tevatron experiments [15]) and $m_h = 100$ GeV. Shifts in these reference values can be compensated by shifts in S and T. In Fig. 1, we show the 68% confidence contour (1.51σ) for a current S-T fit.

A fit with different reference values of m_t and m_h has confidence contours of the same shape but with a different center. The differences from the S and T values at the original reference masses indicate the shifts in S and T that best compensate the change in the contributions from the top quark and Higgs boson masses. Following Takeuchi, we translate the ellipse for a given (m_t, m_h) back to the position for a light Higgs boson, and then consider the translation to represent the (S,T) position associated with the new top quark and Higgs bosen masses. With this definition, we obtain the (S,T) values shown in the figure as a banana-shaped grid to represent the MSM with $m_t = 174.3 \pm 5.1$ GeV [15] and m_h running from 100 to 1000 GeV. The condition $S=T=0$ means "no new physics." From the figure, the fit strongly favors a light Higgs boson and excludes a 500 GeV Higgs boson at the 5σ level.

The fit in Fig. 1 uses only the values of the three best measured electroweak observables, m_W, $\sin^2\theta_w^{\rm eff}$ (the value of the weak mixing angle which appears in Z^0 decay asymmetries), and Γ_l (the leptonic width of the Z^0). Accurate analytic expressions for the standard model predictions for these quantities have been given in [16,17]. The current values of

TABLE I. Current values of the three best measured electroweak parameters, from [2]. In the fit describe din the text, the uncertainty in the last column [17], due to the uncertainty in $\sigma(m_z^2)$, is added in quadrature.

Parameter	Current value	α Effect
m_W (GeV)	80.434 ± 0.037	± 0.003
$\sin^2\theta_w^{\rm eff}$	0.23147 ± 0.00017	± 0.00006
Γ_ℓ (MeV)	83.984 ± 0.086	± 0.028

these quantities are displayed in Table I. A fit with this data only gives

$$S=0.05\pm 0.10, \quad T=0.07\pm 0.11. \tag{1}$$

This restricted data set actually carries most of the information in a complete fit to the corpus of weak interaction data. A rather sophisticated fit presented by Swartz at the 1999 Lepton-Photon Conference [18] gave the same errors as in Eq. (1) with a central value $(S,T)=(-0.04,-0.06)$. Our fit to three data points with the values used by Swartz again gives the same errors and a central value $(S,T)=(0.02,-0.02)$. The compact procedure used here makes little difference for most of the paper, but it will considerably simplify the analysis of Sec. V.

The earliest fits to S and T had central values which were substantially negative compared to the standard model prediction. It was pointed out in [6] that a negative value of S is especially problematic; since S is the zeroth moment of a distribution whose first moment is positive and whose second moment is zero, S will be positive in any simple model. This led to a number of papers on mechanisms which generated negative S. These mechanisms are directly relevant to our present concern. If there is a heavy Higgs boson, it is clear from Fig. 1 that we must add additional ingredients to the theory to compensate the effect of this Higgs boson on S and T. Since it is difficult to generate a negative shift in S, the list of helpful additions is severely restricted.

III. METHOD A: NEGATIVE *S*

As we have indicated in Sec. I, we have exhaustively surveyed explicit models of electroweak symmetry breaking which produce shifts in the S and T parameters. It turns out that all such models use one of three mechanisms to move S and T from the region predicted by a heavy Higgs boson to the region preferred by the S-T fit to data. In this section and Secs. III and IV we will discuss these mechanisms in turn. For definiteness, we will consider models which contain a 500 GeV Higgs particle and additional content associated with dynamical electroweak symmetry breaking. We refer to the contributions from this additional content as $\Delta S,\Delta T$.

The first method for reconciling a heavy Higgs boson with the precision electroweak fits is to add particles whose vacuum polarization integral shifts S in the negative direction. Typically, new heavy particles give a positive shift in S. However, several specific multiplets have been found which can give negative contributions to S. In this section, we will review models of this type.

Georgi [19] and Dugan and Randall [20] considered a scalar field which transforms according to a definite representation of SU(2)×SU(2), where the first factor is the weak interaction gauge group and the second factor is the additional symmetry required to preserve the small value of the ρ parameter [21]. We denote the representation by (j_L,j_R) according to the spin under each SU(2) group. When electroweak symmetry is spontaneously broken, the diagonal SU(2) (“custodial SU(2)”) is preserved, and the large multiplet breaks up into smaller multiplets of definite spin J under this symmetry. The smallest possible value of J is $j_-=|j_L-j_R|$. It turns out that, if the particles with smallest J are the lightest, the multiplet produces negative ΔS. As long as the SU(2) symmetry is exact, the contribution to ΔT is zero.

It is interesting to ask how large a value of ΔS can be produced in this model. To make a simple estimate, assume that the particle with $J=j_-$ has the lowest mass m, and all other particles in the multiplet have a common mass M. Then ΔS contains a logarithm of the mass ratio

$$\Delta S\sim\frac{1}{3\pi}X\log\frac{M^2}{m^2} \tag{2}$$

with

$$X=-\left[\left(\frac{(j_++1)}{(j_-+1)}\right)^2-1\right]\frac{j_-(j_-+1)(2j_-+1)}{12}, \tag{3}$$

and $j_+=(j_L+j_R)$. We give a more complete expression for ΔS in the Appendix. The simplest example, $(j_L,j_R)=(1,\frac{1}{2})$, yields a puny coefficient $X/3\pi=-0.024$. Larger values can be obtained by using multiplets with larger weak isospin. It is important to note that the logarithm cannot be large. Since the mass splitting between M and m violates weak isospin, this splitting must be generated by electroweak symmetry breaking and so cannot be greater than about 100 GeV.

In Fig. 2, we plot the contributions to ΔS from some representative multiplets as a function of the light mass m, assuming a mass splitting of 100 GeV. The values shown should be compared to the contribution $\Delta S=+0.11$ from a 500 GeV Higgs boson. Since the scalars involved in this mechanism couple to the weak interactions, they will certainly be found at an e^+e^- collider that can reach their pair-production threshold. From the figure, we see that it is possible that the required particles might escape detection at 500 GeV e^+e^- collider, but that this requires large multiplets of new particles and isospins $J>2$.

We are aware of only one paper that makes use of this mechanism within a fully developed model of electroweak symmetry breaking. Luty and Sundrum [22] devised a set of technicolor models in which the pseudo-Goldstone bosons contribute a negative ΔS. However, to obtain $\Delta S\simeq -0.1$ from this source, they needed technifermions with $j_L=2$ and pseudo-Goldstone bosons as light as 200 GeV. Larger values of $|\Delta S|$ could be obtained from larger isospin multiplets.

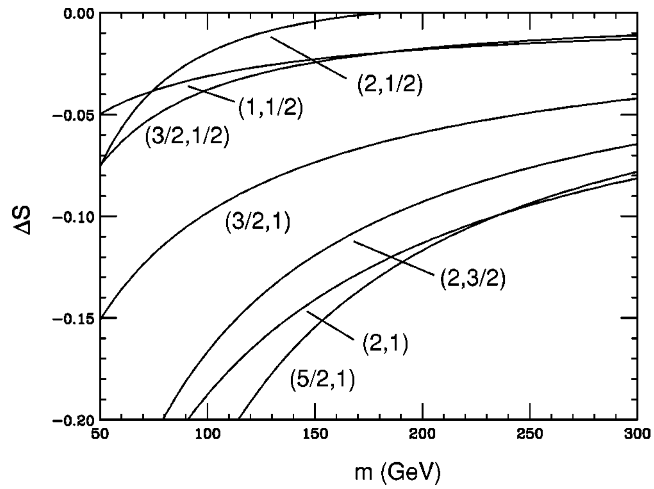

FIG. 2. Shift in S induced by the vacuum polarization of various multiplets in the Dugan-Randall scenerio described in the text. The curves assume that the lightest state in the multiplet is split in mass from the other states by 100 GeV. The various Dugan-Randall multiplets are labeled by (j_L, j_R), and the corresponding shifts in S are plotted against the mass of the lightest state.

However, these large multiplets gave compensatory positive contributions to ΔS from the technicolor dynamics. Other problematical aspects of these models are also pointed out in [22].

Gates and Terning [23] noticed that one can obtain negative S by introducing electroweak-singlet Majorana fermions which also have Dirac mass terms with isodoublet particles. In this mechanism, the contribution to S again contains a logarithm of the mass ratio of the particles split in mass by electroweak symmetry breaking. The largest negative contributions to ΔS that can be obtained by this method are of the size

$$\Delta S \sim -\frac{1}{6\pi}\log\frac{M_1^2}{M_2^2}, \tag{4}$$

where M_1 and M_2 are mass eigenvalues split by electroweak symmetry breaking. This formula briefly led to excitement that the negative S values found in electroweak fits could be explained by a chargino of mass 60–80 GeV [24]. Unfortunately, for charginos heavier than m_Z, the contribution decouples and vanishes as m_Z^2/M_2^2.

Thus, to explicitly obtain negative ΔS by introducing new particles along the lines of Dugan and Randall, one must either introduce very large multiplets or require that some of these particles remain light. Light particles associated with this mechanism necessarily have electroweak charge and can be found at an e^+e^- collider.

IV. METHOD B: NEW VECTORS

The second method for reconciling a heavy Higgs boson with the precision electroweak fits is to change the weak-interaction gauge group, adding heavy $Z^{0\prime}$ vector bosons. The effects of such new bosons on the precision electroweak fits were studied by a number of groups in the early 1990's [25–28]. Rizzo explicitly studied their effect as a method for obtaining negative ΔS [29]. More recently, Casalbuoni *et al.* have studied the compensation of the heavy Higgs effect by new vectors in a model with an added gauge group SU(2)×SU(2) [30].

In all of these papers, the effects of the $Z^{0\prime}$ is studied by mapping it to a shift of three variables representing the oblique electroweak corrections, S, T, and U. We find it more instructive to use a slightly different strategy that ignores U. We will compute the shifts in our three well-measured electroweak parameters due to the $Z^{0\prime}$, fit the data to S and T taking these shifts into account, and see if the resulting effect on S and T can compensate the effect of a heavy Higgs boson.

For the pattern of shifts induced by a $Z^{0\prime}$ boson, we find the following: Consider a $Z^{0\prime}$ boson whose mixing with the standard Z^0 is represented by the mass matrix

$$m^2 = \begin{pmatrix} m^2 & \gamma m_Z^2 \\ \gamma m_Z^2 & M^2 \end{pmatrix}, \tag{5}$$

where γ is a parameter of order 1. It is natural that the off-diagonal terms are of the same order of magnitude as m_Z and much less than M^2, since in typical models the heavy mass M^2 results from an SU(2)×U(1) singlet expectation value, while both the Z^0 mass and the off-diagonal terms result from the expectation values of standard Higgs fields. The observed Z^0 mass is given by the lower eigenvalue of this matrix, $m_Z^2 = m^2(1-\delta)$, and the physical Z^0 contains an admixture ξ of the original $Z^{0\prime}$, where

$$\delta = \gamma^2\frac{m_Z^2}{M^2}, \quad \xi = \gamma\frac{m_Z^2}{M^2}, \tag{6}$$

to leading order in (m_Z^2/M^2). Let the current coupling the $Z^{0\prime}$ to leptons l^- have the form

$$\Delta\mathcal{L} = g' Z_\mu^{0\prime}\{\overline{l_L}\gamma^\mu q_L l_L + \overline{l_R}\gamma^\mu q_R l_R\}. \tag{7}$$

Then the $Z^{0\prime}$ induces the shifts

$$\begin{aligned} \Delta m_W &= 57.\delta \ \ (\text{GeV}), \\ \Delta \sin^2\theta_w^{\text{eff}} &= -0.33\delta + 0.22 q_L\xi + 0.26 q_R\xi, \\ \Delta\Gamma_l &= 100\delta - 170 q_L\xi + 150 q_R\xi \ \ (\text{MeV}). \end{aligned} \tag{8}$$

Symbolic versions of these expressions are given in the Appendix.

To demonstrate the effect of these shifts, consider first the simple case $q_L = q_R = 0$, and take $\gamma = 1$. (This last choice is conservative, since typically γ is of order $\sin^2\theta_w$.) Now set the Higgs boson mass to 500 GeV, add to the MSM prediction the shifts shown in Eq. (8), and fit for S and T. The result is the set of contours shown in Fig. 3. We see almost complete compensation of the heavy Higgs boson effect for $M \sim 2$ TeV. The figure shows how we would plot this compen-

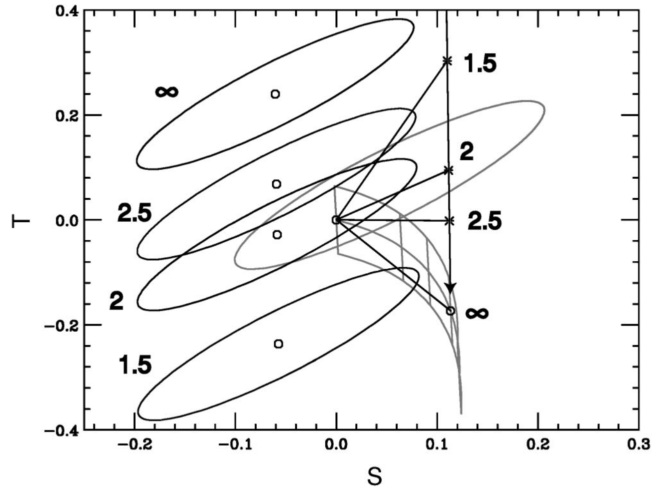

FIG. 3. Fit of the precision electroweak data to the MSM with $m_h = 500$ GeV and shifts of the electroweak parameters due to a $Z^{0\prime}$, plus the effects of the S,T parameters. The four darker ellipses correspond to fits with $M = 1.5$, 2.0, and 2.5 TeV and ∞. The lighter ellipse and the grid are those plotted in Fig. 1. This diagram shows how the centers of the various fits with different values of M (symbolized by ○) can be plotted as shifts of (S,T) with respect the standard model ellipse (symbolized by *). These shifts represent the combined contribution of the $Z^{0\prime}$ and the heavy Higgs boson, and fall on a line which tends to the heavy Higgs boson prediction for $M \to \infty$. We see almost complete compensation of the heavy Higgs boson effect for $M \sim 2$ TeV.

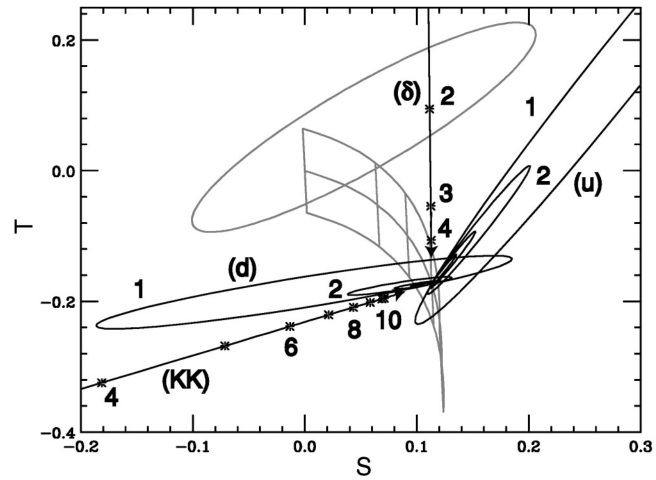

FIG. 4. Contributions to S and T from a Higgs boson with $m_h = 500$ GeV, plus a heavy $Z^{0\prime}$. The contributions are computed and displayed as indicated in Fig. 3. Four different models are considered: (δ) model of Fig. 3, with $\gamma = 1$, $q_{L,R} = 0$; (u) rank-1 E_6 models with mixing due to a Higgs field H_u; (d) rank-1 E_6 models with mixing due to a Higgs field H_d; (KK) extra-dimension model of Ref. [31]. The numbers indicate the values of the $Z^{0\prime}$ mass M, always in TeV, and the asterisks represent the (S,T) shifts, as in Fig. 3, for the variously labeled values of M. For the E_6 models, there are two parameters to vary: the mass M and the mixing angle θ. For these models, we have plotted the contours swept out as one changes the mixing angle for fixed values of M. All of the $Z^{0\prime}$ predictions tend to the 500 GeV MSM point as $M \to \infty$.

sation as a translation of the center of the fit, in the same way that we plotted the (S,T) contributions from shifts in m_t and m_h within the MSM.

In principle, the fit might have become worse as the $Z^{0\prime}$ pulls the three variables in directions that are not possible within the MSM. To show that this does not happen significantly, we have plotted the various new ellipses at the same χ^2 value as the reference ellipse copied from Fig. 1. If a third variable U were required, the contours would become smaller for low $Z^{0\prime}$ masses, but this clearly does not happen. In this special case with only Z-Z' mixing, the effect of the $Z^{0\prime}$ is actually completely described by a shift of T, $\Delta T = \delta/\alpha$. However, it is true in the other examples we have studied that the main effect of the $Z^{0\prime}$ is to shift the center of the (S,T) fit while maintaining a fit with reasonable χ^2.

Using this formalism, we can investigate the region of parameters for any $Z^{0\prime}$ model in which the shifts due to the $Z^{0\prime}$ compensate those of a heavy Higgs boson. In Fig. 4, we show the results for the fit centers as a function of the parameters of the $Z^{0\prime}$, for the model described in the previous paragraph and for several other models from the literature. A commonly discussed class of $Z^{0\prime}$ models are the rank-1 E_6 models, obtained by considering the $Z^{0\prime}$ to be an arbitrary linear combination of the two U(1) bosons in E_6 that are orthogonal to the bosons of SU(3)×SU(2)×U(1). The SO(10)$Z^{0\prime}$ and the "superstring-inspired" $Z^{0\prime}$ are particular cases of these models. The predictions of these models for precision electroweak parameters depend through ξ on the quantum numbers of the Higgs field responsible for Z-Z' mixing. This Higgs field could be either an H_u, with $I = \frac{1}{2}$, $Y = \frac{1}{2}$, or an H_d, with $I = \frac{1}{2}$, $Y = -\frac{1}{2}$, or the mixing could receive contributions from Higgs fields of both types. Explicit formulas for the various contributions from these models are given in the Appendix. In Fig. 4, we plot the compensation as a function of the E_6 mixing angle for each of the two extreme cases, for fixed values of M. In these models, partial compensation is possible only for relatively low values of M, below 1.5 TeV.

Figure 4 contains a substantial amount of detailed information, but it also contains two simple messages. First, it is possible within the space of Z' models to arrange shifts $(\Delta S, \Delta T)$ that move the precision electroweak fit in almost any direction. Second, in any given model, these shifts can be large enough to influence the conclusion about a heavy Higgs boson only if the mass of the Z' is small enough.

The final model shown in Fig. 4 is a model involving extra space dimensions [31]. This is an example of a number of models presented recently in which gauge fields live in a higher-dimensional space that is compactified to our ordinary 3+1 dimensions [32–35]. In these models, the new vectors are the Kaluza-Klein excitations of the MSM gauge bosons.

These models with extra dimensions are interesting to this study because one of the simplest of these theories—with gauge fields in five dimensions and the Higgs boson and fermions in four dimensions—exhibits compensation between the effects of the new vector bosons and the heavy Higgs boson. If we denote the mass of the first KK excited state of the gauge bosons by $M_{\rm KK}$, the electroweak observables are altered in this model by

$$\Delta m_W = 87.0 \frac{m_Z^2}{M_{\rm KK}^2} \ \text{(GeV)}$$

$$\Delta \sin^2 \theta_w^{\rm eff} = -1.09 \frac{m_Z^2}{M_{\rm KK}^2} \tag{9}$$

$$\Delta \Gamma_l = -220 \frac{m_Z^2}{M_{\rm KK}^2} \ \text{(MeV)}.$$

The effect of these changes is to move the (S,T) value to a region of the plane where it is less constrained. Though the 68% C.L. ellipse does not intersect the KK line in Fig. 4, the expanded ellipse corresponding to the 99% C.L. reaches into the lower left-hand corner of the plot and intersects the KK line for $M_{\rm KK}\sim 3$ TeV. A detailed global fit done in the summer of 1998 [31] indicated that the 95% C.L. upper bound on the Higgs boson mass could reach as high as 300–500 GeV for $M_{\rm KK}$ in the range 3–5 TeV. With the latest experimental numbers, we find that the upper limit on the Higgs boson mass is still relaxed in this model, though it does not extend beyond 300 GeV.

Another interesting property of this model is that the coupling of the new sector is large. By this we mean both that a large number of new states participate and that the couplings of individual states are larger by a factor of $\sqrt{2}$ than those of the corresponding MSM bosons. These features allow compensation for a mass of the lightest new vector about twice as high as the mass of the single $Z^{0\prime}$ boson in the E_6 models considered above.

A variation on this scheme is suggested in [36], which considers a higher-dimensional model with a low quantum gravity scale. In this case, the precision electroweak corrections are distorted by the effect of the radion, a scalar degree of freedom from the gravity sector. The corrections involve the value of the underlying Planck scale M and the Higgs field coupling to curvature through a term $\frac{1}{2}\xi R h^2$. The model allows compensation of the electroweak corrections and a heavy Higgs boson, but only when M or M/ξ is close to 1 TeV.

The common feature of these four models, and of other $Z^{0\prime}$ models we have studied, is that the values of the $Z^{0\prime}$ mass needed to compensate the effect of a heavy Higgs boson is well within the reach of next-generation colliders. The CERN Large Hadron Collider (LHC) should be able to find a $Z^{0\prime}$ as a narrow resonance for masses up to 4 TeV. A 500 GeV e^+e^- linear collider can see the effect of the $Z^{0\prime}$ as a perturbation of the cross section for $e^+e^- \to f\bar{f}$, with similar sensitivity in the $Z^{0\prime}$ mass [37]. The information from proton and electron colliders is complementary, and a complete picture of the $Z^{0\prime}$ is obtained by combining the two sets of measurements. In the case of the extra-dimension model, the mass of the first new vector excitation is predicted to be higher. However, in this model, the larger couplings give enhanced sensitivity, up to 6 TeV for the LHC and above 10 TeV for a 500 GeV e^+e^- linear collider, so the general conclusion applies to this model as well.

V. METHOD C: POSITIVE T

In both methods of compensating a heavy Higgs boson that we have discussed so far, the compensation leads to new physics signatures that should be observed at next-generation pp and e^+e^- colliders. However, there is one further compensation strategy that can evade this requirement. Looking again at Fig. 1, we see that it is possible to bring a model with a heavy Higgs boson back into reasonable agreement with the precision electroweak fit without changing S at all, by adding new particles that lead to positive ΔT. For example, the shift $\Delta S = 0$, $\Delta T = 0.3$ due to new physics brings a model with a 500 GeV Higgs boson within 1 sigma of the central value.

Most models with new physics produce a nonzero, positive ΔT [38]. In fact, the contribution to ΔT can easily be of order 1. Particles with mass much larger than 1 TeV can contribute to ΔT if their masses have an up-down flavor asymmetry. The contribution is on the order of

$$\Delta T \sim \frac{m_U^2 - m_D^2}{m_U^2 + m_D^2}. \tag{10}$$

Even though $m_U - m_D$ can be at most of order 100 GeV because it must arise from electroweak symmetry breaking, this contribution can easily be large enough to compensate for the effect of a heavy Higgs boson for values of the U and D masses that are inaccessible to any collider.

Several recently proposed models allow a heavy Higgs boson to make use of this mechanism. The first is the "top-color seesaw" of Dobrescu and Hill [39]. In this model, the new physics needed to break electroweak symmetry arises from a heavy, weak-SU(2)-singlet fermion χ. In the simplest top-color seesaw model, one finds [40]

$$\alpha \Delta T = \frac{3}{16\pi^2} \frac{g_{\rm tc}^4}{4} \frac{v^2}{m_\chi^2} \left[1 + 2 \frac{\lambda_t^2}{g_{\rm tc}^2} \log \frac{m_\chi^2}{m_t^2} \right], \tag{11}$$

where $g_{\rm tc}\sim 3$ is the top-color coupling, $\lambda_t = 1$ is the top quark Yukawa coupling, $v = 246$ GeV is the weak interaction scale, and m_χ is the mass of a new heavy fermion. For $m_\chi = 1$ TeV, this expression gives $\Delta T = 7.2$. However, it is permissible in this model to raise m_χ arbitrarily, although very high m_χ requires fine tuning of the underlying parameters. For $m_\chi = 5$ TeV, we find $\Delta T = 0.3$, which gives a reasonable fit to the precision electroweak data with heavy Higgs boson. It is argued in [39] that this choice does not yet require fine tuning. A mapping of the ellipse in Fig. 1 into the m_χ, m_h plane gives the interesting contour seen in Fig. 7 of [40]. Similar behavior is seen in the "top flavor" model of [41].

A recent paper by Chankowski *et al.* [42] argues that the two-Higgs–doublet model can be made consistent with the electroweak fits for a Higgs boson mass of 500 GeV. The strategy of this paper is to adjust the Higgs spectrum to give the required positive contribution to the ρ parameter ($\Delta\rho = \alpha \Delta T$); the model gives only a tiny shift in S. A recent paper by He *et al.* [43] suggests adding a fourth generation of quarks and leptons. The additional fermions increase S, but even $S \sim 0.2$ may be accomodated by choosing the mass spectrum to give an appropriate value of T.

Technicolor models can also be made consistent with the precision electroweak fits through this strategy. Most technicolor models lead to values of S larger than the value for a 1000 GeV Higgs boson, $S > 0.12$ [6]. In typical cases, the values of S and T are positive and of order 1. Models have been proposed in which the technicolor enhancements to S and T are of order 0.1 or smaller [44–46]. But in all models that have been studied, except for [22] cited above, the lower bound for S still applies. Still, it is possible to construct a technicolor model that is consistent with the electroweak data in spite of this bound, by including enough weak isospin breaking to give a small positive correction to T. Such a model would, for example, have $S \sim 0.15$, $T \sim 0.2$. Models of this type would not have a visible Higgs boson and might not contain any new particles below the first techni-rho resonance at about 2 TeV [47].

It is important to note that the models we have discussed in the last few paragraphs are minimal ones that represent the worst-case scenarios for the colliders of the next generation. More typical and realistic models of top color and technicolor contain additional ingredients that form the basis for further experimental signatures. These include additional gauge groups [48] or extra space dimensions [49] in the case of top color and light techni-pions and techni-rho states [50] in the case of technicolor.

However, even those models using this strategy which predict little or no new physics at the next generation of colliders will be clearly distinguishable from the MSM by improved precision electroweak measurements. Foreseeable improvements in the precision electroweak fit are shown in Fig. 5. The larger contour shows the effect of a measurement of m_W to 15 MeV, as might be expected from the LHC [51]. The smaller contour shows the effect of the precision measurements expected from a high-luminosity e^+e^- linear collider run at the Z^0 and at the W^+W^- threshold: m_W to 6 MeV, $\sin^2\theta_w^{\rm eff}$ to 0.000 02, Γ_l to 0.04 MeV [52–55]. With this latter set of measurements, the point in the (S,T) space favored by this strategy is separated from prediction of the MSM with a light Higgs boson by more than 5 sigma. Thus, these measurements would clearly prove the presence of new physics and would indicate the route by which today's precision electroweak constraint is evaded.

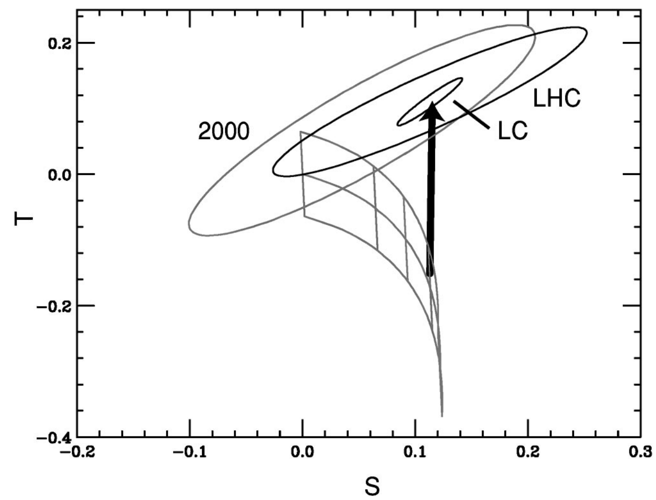

FIG. 5. Future improvements in the determination of precision electroweak parameters. The lighter ellipse and grid are those plotted in Fig. 1. The heavier ellipses, both centered at $(S,T) = (0.11,0.11)$, correspond to an improved W mass measurement with an error of 15 MeV, as would be expected from the LHC, and measurements of m_W, $\sin^2\theta_w^{\rm eff}$, and Γ_l with errors of 6 MeV, 0.000 02, and 0.04 MeV, respectively, as would be expected from the precision electroweak program at an e^+e^- linear collider [52–55].

VI. CONCLUSIONS

The precision electroweak data are consistent with the minimal standard model only if the Higgs boson mass is very low, $m_h < 170$ GeV at 95% C.L. This result would predict that the colliders of the next generation, possibly including the upgraded Tevatron, will be able to discover and study the Higgs boson. However, if the minimal standard model is not correct, there are scenarios in which new physics contributions conspire with a heavier Higgs boson to allow agreement with the precision electroweak data.

In this paper, we have argued that, despite this, one cannot freely assume that the Higgs boson is heavy in the face of the precision electroweak constraint. The particular new physics that compensates the effect of a heavy Higgs boson has a price and, to evade the MSM constraint on the Higgs boson mass, one must be prepared to pay it.

Many popular models of physics beyond the standard model do not allow a heavy Higgs boson at any price. Supersymmetric grand unified theories are an example. Among models that allow a heavy Higgs boson in principle, any successful model must introduce new physics that can perturb the precision electroweak observables in the correct direction by a sufficiently large amount. In this paper, we have made that statement precise, using the S,T formalism, and we have reviewed the various strategies suggested in the literature. In fact, the entire literature to date is exhausted by only three strategies, which we have described in detail.

Two of these strategies, method A, which gives new contributions of negative ΔS, and method B, which introduces new vector bosons, have distinctive signatures that should be observed at the next e^+e^- linear collider. Method A requires new light bosons or fermions with electroweak charge. Method B requires that the new vector bosons be sufficiently light to create large perturbations of the cross sections for e^+e^- annihilation to fermion pairs. In fact, models using this strategy create a very interesting physics scenario, in which the new vector particles are also directly observable as resonances at the LHC.

The third strategy, method C, is less dramatic, and specific models exist in which no new particles beyond the MSM are observable either at an e^+e^- linear collider or at the LHC. However, this strategy leads to a prediction for the precision electroweak parameters that is distinctive, and that can be distinguished from the MSM prediction with a high level of confidence by the improved level of precision in the electroweak parameters that a next-generation e^+e^- linear collider should achieve.

We cannot close off the idea that the Higgs boson is very heavy on purely theoretical grounds. But we have emphasized in this paper that models in which the Higgs boson is heavy have specific properties which must be taken into account in any discussion of future experimental prospects. In particular, these models generally lead in their own way to an interesting experimental program for the next-generation colliders.

ACKNOWLEDGMENTS

We are grateful to Gordy Kane and Ken Lane for inspiration, to Bogdan Dobrescu, Andreas Kronfeld, and the participants of the Berkeley 2000 Linear Collider Workshop for valuable discussions, and to Jonathan Bagger and Morris Swartz for useful correspondence. The work of M.E.P. was supported by the Department of Energy, Contract No. DE-AC03-76SF00515. The work of J.D.W. was supported by the Department of Energy and the Alfred P. Sloan Foundation.

APPENDIX

In this appendix, we present various explicit formulas that are used in the text.

In Sec. III, we analyzed the Dugan-Randall models [20] in the most favorable case in which the scalar particle with smallest J has a low mass m while the other particles in the multiplet have a large mass M. In Eq. (2), we quoted only the leading logarithm in the formula for ΔS. The complete formula for this case is

$$\Delta S=\frac{1}{3\pi}\left[X\log\frac{M^2}{m^2}+2X'B(m,M)\right], \tag{A1}$$

where

$$\begin{aligned}X&=\left[1-\left(\frac{(j_++1)}{(j_-+1)}\right)^2\right]\frac{j_-(j_-+1)(2j_-+1)}{12},\\ X'&=\left[1+\left(\frac{(j_++1)}{(j_-+1)}\right)^2\right.\\ &\quad\left.-\frac{j_+(j_++2)+j_-^2}{j_-(j_-+1)}\right]\frac{j_-(j_-+1)(2j_-+1)}{12},\end{aligned} \tag{A2}$$

$$\begin{aligned}B(m,M)&=-\frac{m^4(m^2-3M^2)}{(M^2-m^2)^3}\log\frac{M^2}{m^2}\\ &\quad+\frac{5M^4-22M^2m^2+5m^4}{6(M^2-m^2)^2}.\end{aligned}$$

In Sec. IV, we analyzed the effect of a $Z^{0\prime}$ boson on the best-measured precision electroweak observables. In Eq. (8), we quoted numerical formulas for the shifts in m_W, $\sin^2\theta_w^{\rm eff}$, and Γ_l induced by a $Z^{0\prime}$ in terms of the parameters δ, ξ, $q_{L,R}$ defined in Eqs. (6),(7). The corresponding analytic formulas are

$$\begin{aligned}\Delta m_W&=\frac{1}{2}\frac{c^2}{c^2-s^2}m_W\delta,\\ \Delta\sin^2\theta_w^{\rm eff}&=-\frac{s^2c^2}{c^2-s^2}\delta+s\xi(q_R(1-2s^2)+2s^2q_L),\\ \Gamma L_l&=\Gamma_l\left\{\left(\frac{1-2s^2}{s^2c^2}+\frac{4(1-4s^2)}{1-4s^2+8s^4}\right)\frac{s^2c^2}{c^2-s^2}\delta\right.\\ &\quad\left.-\frac{4}{1-4s^2+8s^4}s\xi(q_L(1-2s^2)-2s^2q_R)\right\},\end{aligned} \tag{A3}$$

where $s=\sin\theta_w$, $c=\cos\theta_w$. In the rank-1 E_6 models [25],

$$\begin{aligned}q_L&=\cos\theta\frac{3}{2\sqrt{6}}+\sin\theta\frac{1}{6}\sqrt{\frac{5}{2}}\\ q_R&=\cos\theta\frac{1}{2\sqrt{6}}-\sin\theta\frac{1}{6}\sqrt{\frac{5}{2}},\end{aligned} \tag{A4}$$

where θ is the mixing angle between the two U(1) bosons, defined so that $\theta=0$ corresponds to the SO(10) boson χ and $\theta=\pi/2$ to the E_6 boson ψ. The expressions for δ and ξ require a parameter γ, which depends on the quantum numbers of the Higgs boson responsible for SU(2)×U(1) breaking and Z^0-$Z^{0\prime}$ mixing. In general, we would expect both Higgs fields H_u and H_d to obtain vacuum expectation values, which are conventionally written

$$\langle H_u^0\rangle=\frac{v}{\sqrt{2}}\sin\beta\quad\langle H_d^0\rangle=\frac{v}{\sqrt{2}}\cos\beta, \tag{A5}$$

with $v=246$ GeV. Then

$$\begin{aligned}\gamma&=2s\sin^2\beta\left(\cos\theta\frac{1}{\sqrt{6}}-\sin\theta\sqrt{\frac{5}{18}}\right)\\ &\quad+2s\cos^2\beta\left(\cos\theta\frac{1}{\sqrt{6}}+\sin\theta\sqrt{\frac{5}{18}}\right).\end{aligned} \tag{A6}$$

Cases (d) and (u) shown in Fig. 4 correspond to the cases $\beta=0$ and $\beta=\pi/2$, respectively.

[1] LEP Electroweak Working Group Report, "A combination of preliminary electroweak measurements and constraints on the standard model," CERN-EP-2000-016, 2000.

[2] A. Gurtu, talk presented at XXXth International Conference on High Energy Physics (ICHEP 2000), Osaka, Japan, 2000.

[3] For a summary of developments, see D. Strom, talk presented at 5th International Symposium on Radiative Corrections (RADCOR 2000), Carmel, California, 2000.

[4] B. Holdom and J. Terning, Phys. Lett. B **247**, 88 (1990).
[5] M. Golden and L. Randall, Nucl. Phys. **B361**, 3 (1991).
[6] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. D **46**, 381 (1992).
[7] R. Barbieri and A. Strumia, Phys. Lett. B **462**, 144 (1999).
[8] J. A. Bagger, A. F. Falk, and M. Swartz, Phys. Rev. Lett. **84**, 1385 (2000).
[9] C. Kolda and H. Murayama, J. High Energy Phys. **07**, 035 (2000).
[10] R. S. Chivukula, C. Holbling, and N. Evans, Phys. Rev. Lett. **85**, 511 (2000).
[11] G. L. Kane and J. D. Wells, ''Pursuing the origin of electroweak symmetry breaking: A 'Bayesian physics' argument for a $\sqrt{s} \leq 600$ GeV e^+e^- collider,'' hep-ph/0003249.
[12] N. Cabibbo, L. Maiani, G. Parisi, and R. Petronzio, Nucl. Phys. **B158**, 295 (1979).
[13] G. L. Kane, C. Kolda, and J. D. Wells, Phys. Rev. Lett. **70**, 2686 (1993); J. R. Espinosa and M. Quiros, Phys. Lett. B **302**, 51 (1993); Phys. Rev. Lett. **81**, 516 (1998).
[14] For other, almost equivalent, parametrizations, see W. J. Marciano and J. L. Rosner, Phys. Rev. Lett. **65**, 2963 (1990); D. C. Kennedy and P. Langacker, *ibid.* **65**, 2967 (1990); G. Altarelli and R. Barbieri, Phys. Lett. B **253**, 161 (1991).
[15] R. Partridge, in *Proceedings of the XXIX International Conference on High-Energy Physics (ICHEP 98)*, edited by A. Astbury, D. Axen, and J. Robinson (World Scientific, Singapore, 1999), hep-ex/9811035.
[16] G. Degrassi, P. Gambino, M. Passera, and A. Sirlin, Phys. Lett. B **418**, 209 (1998); G. Degrassi and P. Gambino, Nucl. Phys. **B567**, 3 (2000).
[17] W. J. Marciano, Proceedings of the Fifth International Conference on Physics Potential and Development of Muon Colliders (MUMU99) (to be published), hep-ph/0003181.
[18] M. L. Swartz, in Proceedings of the XIX International Symposium on Photon and Lepton Interactions at High Energy (LP99), edited by J. A. Jaros and M. E. Peskin, 2000, p. 307, hep-ex/9912026.
[19] H. Georgi, Nucl. Phys. **B363**, 301 (1991).
[20] M. J. Dugan and L. Randall, Phys. Lett. B **264**, 154 (1991).
[21] P. Sikivie, L. Susskind, M. Voloshin, and V. Zakharov, Nucl. Phys. **B173**, 189 (1980).
[22] M. A. Luty and R. Sundrum, Phys. Rev. Lett. **70**, 529 (1993).
[23] E. Gates and J. Terning, Phys. Rev. Lett. **67**, 1840 (1991).
[24] G. Altarelli, R. Barbieri, and F. Caravaglios, Phys. Lett. B **314**, 357 (1993).
[25] P. Langacker and M. Luo, Phys. Rev. D **45**, 278 (1992).
[26] B. Holdom, Phys. Lett. B **259**, 329 (1991).
[27] G. Altarelli, R. Barbieri, and S. Jadach, Nucl. Phys. **B369**, 3 (1992).
[28] G. Altarelli, R. Casalbuoni, S. De Curtis, N. Di Bartolomeo, R. Gatto, and F. Feruglio, Phys. Lett. B **318**, 139 (1993).
[29] T. G. Rizzo, Phys. Rev. D **50**, 2256 (1994).
[30] R. Casalbuoni, S. De Curtis, D. Dominici, R. Gatto, and M. Grazzini, Phys. Lett. B **435**, 396 (1998).
[31] T. G. Rizzo and J. D. Wells, Phys. Rev. D **61**, 016007 (2000).
[32] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).
[33] A. Pomarol and M. Quiros, Phys. Lett. B **438**, 255 (1998).
[34] A. Delgado, A. Pomarol, and M. Quiros, J. High Energy Phys. **01**, 030 (2000).
[35] A. Strumia, Phys. Lett. B **466**, 107 (1999).
[36] C. Csaki, M. L. Graesser, and G. D. Kribs, Phys. Rev. D **63**, 065002 (2001).
[37] T. G. Rizzo, Int. J. Mod. Phys. A **13**, 2245 (1998).
[38] M. B. Einhorn, D. R. Jones, and M. Veltman, Nucl. Phys. **B191**, 146 (1981).
[39] B. A. Dobrescu and C. T. Hill, Phys. Rev. Lett. **81**, 2634 (1998).
[40] H. Collins, A. Grant, and H. Georgi, Phys. Rev. D **61**, 055002 (2000).
[41] H. He, T. Tait, and C. P. Yuan, Phys. Rev. D **62**, 011702(R) (2000).
[42] P. Chankowski, T. Farris, B. Grzadkowski, J. F. Gunion, J. Kalinowski, and M. Krawczyk, Phys. Lett. B **496**, 195 (2000).
[43] H. He, N. Polonsky, and S. Su, Phys. Rev. D **64**, 053004 (2001). .
[44] R. Sundrum, Nucl. Phys. **B395**, 60 (1993).
[45] T. Appelquist and J. Terning, Phys. Lett. B **315**, 139 (1993).
[46] T. Appelquist, J. Terning, and L. C. Wijewardhana, Phys. Rev. Lett. **79**, 2767 (1997).
[47] It should be noted that such a heavy techni-rho can be observed by experiments at the LHC and by precision studies at a 500 GeV e^+e^- collider with high luminosity; see M. S. Chanowitz, talk presented at Zuoz Summer School on Hidden Symmetries and Higgs Phenomena, Zuoz, Switzerland, 1998, hep-ph/9812215; T. Barklow, talk presented at Linear Collider Workshop 2000 (LCWS 2000), Fermilab, 2000.
[48] R. S. Chivukula, B. A. Dobrescu, H. Georgi, and C. T. Hill, Phys. Rev. D **59**, 075003 (1999).
[49] H. Cheng, B. A. Dobrescu, and C. T. Hill, Nucl. Phys. **B589**, 249 (2000).
[50] K. Lane, Phys. Rev. D **60**, 075007 (1999); ''Technicolor 2000,'' hep-ph/0007304.
[51] F. Gianotti and M. Pepe-Altarelli, Nucl. Phys. B (Proc. Suppl.) **89**, 177 (2000).
[52] R. Hawkings and K. Monig, Eur. Phys. J. C **8**, 1 (1999).
[53] G. W. Wilson, in *Physics and Experiments with Future e^+e^- Linear Colliders* (LCWS99), edited by E. Fernández and A. Pacheco (University Auton. de Barcelona, Bellaterra, 2000), LC-PHSM-2001-009.
[54] J. Erler, S. Heinemeyer, W. Hollik, G. Weiglein, and P. M. Zerwas, Phys. Lett. B **486**, 125 (2000).
[55] P. C. Rowson and M. Woods, ''Experimental issues for precision electroweak physics at a high-luminosity Z factory,'' hep-ex/0012055.