Normalization of the neutrino-deuteron cross section

John F. Beacom*

NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500

Stephen J. Parke[†]

Theoretical Physics Department, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500
(Received 12 June 2001; published 4 October 2001)

As is well known, the comparison of the solar neutrino fluxes measured in SuperKamiokande (SK) by $\nu + e^- \rightarrow \nu + e^-$ and in the Sudbury Neutrino Observatory (SNO) by $\nu_e + d \rightarrow e^- + p + p$ can provide a "smoking gun" signature for neutrino oscillations as the solution to the solar neutrino puzzle. This occurs because SK has some sensitivity to all active neutrino flavors whereas SNO can isolate electron neutrinos. This comparison depends crucially on the normalization and uncertainty of the theoretical charged-current neutrino-deuteron cross section. We address a number of effects which are significant enough to change the interpretation of the SK-SNO comparison.

Both SuperKamiokande (SK) and Sudbury Neutrino Observatory (SNO) are sensitive to solar neutrinos with energies above about 5 MeV. In SK, these are detected by $\nu + e^- \rightarrow \nu + e^-$, with possible (indistinguishable) contributions from all active flavors. In particular, if there are $\nu_e \rightarrow \nu_\mu, \nu_\tau$ oscillations, then the latter contribute to the measured flux with a cross section 6 to 7 times smaller than for ν_e. In SNO, on the other hand, the detection reaction $\nu_e + d \rightarrow e^- + p + p$ can isolate the ν_e flux.

The measured flux of solar neutrinos in SK, in units of the expected electron neutrino flux from the standard solar model (SSM) [1], is 0.45 ± 0.02, with the systematic uncertainty dominating [2]. This measured flux may or may not include a contribution from ν_μ, ν_τ (in any linear combination, since they interact via the neutral current). In an energy-independent (as suggested by the absence of any distortion in the SK recoil electron spectrum [2]) two-flavor oscillation scenario, there are two extreme cases [3]. First, for $\nu_e \rightarrow \nu_s$, the ν_e flux in these units is 0.45, and the undetectable ν_s (sterile neutrino) flux is 0.55. Second, for $\nu_e \rightarrow \nu_\mu, \nu_\tau$, the ν_e flux is 0.34, and the ν_μ, ν_τ flux 0.66, so that the measured flux in SK is $0.34 + 0.66/6 = 0.45$. In the first case, SNO will measure 0.45, and in the second case, 0.34. More generally, these arguments can be rephrased as a ratio to eliminate the SSM flux normalization and its $\simeq 20\%$ uncertainty, since the total incident flux is the same for both SK and SNO. Also, a small correction is necessary for the $^3\text{He} + p$ (hep) neutrinos [1,4] that add to the dominant ^8B neutrinos.

This possible difference of 0.11 is small enough that the uncertainties must be scrutinized closely. Below, we closely follow the analogous results for the percent-level corrections to the theoretical $\bar{\nu}_e + p \rightarrow e^+ + n$ cross section [5,6] that are necessary to achieve agreement with experiment [7].

While results from SNO [8,9] have not yet been reported, they have said at conferences [10] that they expect their flux

───────────
*Email address: beacom@fnal.gov
[†]Email address: parke@fnal.gov

measurement uncertainty to be dominated by the $\sim 3\%$ theoretical uncertainty [11,12] on the neutrino-deuteron cross section, at least eventually.

In the SNO proposal [8], the uncertainty on the neutrino-deuteron cross sections was assumed to be about 10% (for comparison, the experimental measurements of neutrino-deuteron cross sections have uncertainties of 10–40 % [13]). Since that time, the calculations have been redone by a number of authors, with decreasing quoted uncertainties. The most refined recent calculations are those of Butler, Chen, and Kong (BCK) [11] and Nakamura, Sato, Gudkov, and Kubodera (NSGK) [12]. Each claims an uncertainty of about 3% in the range of energies relevant for solar neutrinos, and they agree to about 1% [11]. It is now important to consider issues that affect the normalization of the total cross section at a comparable level. Three such effects have been overlooked by BCK and NSGK; these effects are comparable and add constructively.

First, at low energies, the neutrino-deuteron cross section is dominated by the Gamow-Teller transition, so that the cross section scales as g_A^2, where g_A is the weak axial coupling to nucleons, and the angular distribution [6] of the outgoing electron is nearly of the form $1 - \frac{1}{3}\cos\theta$. The present value of g_A is -1.267 ± 0.004 [14]. BCK use -1.26, which makes their cross section about 1% too small, and NSGK use -1.254, which makes their cross section about 2% too small. This effect is trivial in nature but must be included.

Second, a more subtle effect occurs because both BCK and NSGK use the Fermi constant G_F as determined from muon decay [14]. The radiative corrections to low-energy (much less than the W mass) weak processes are frequently divided into "inner" (which are energy-independent) and "outer" (which are generally energy-dependent) corrections. The inner radiative correction is universal to a given reaction and those related to it by crossing symmetry, and can thus be considered to renormalize G_F for each set of diagrams. This renormalization is different for purely leptonic and semileptonic processes, e.g., muon decay and neutron beta decay (it is also different for neutral-current weak processes involving

a nucleon). Since the inner radiative correction arises from diagrams with internal γ and Z exchanges with high momenta, the quark structure of the nucleon is resolved and the fact that the nucleon is bound in a deuteron is irrelevant. Thus the inner radiative correction increases the $\nu_e + d \rightarrow e^- + p + p$ cross section by 2.4%, just as for $\bar{\nu}_e + p \rightarrow e^+ + n$ [6].

Third, the outer radiative correction must be considered. This arises from bremsstrahlung diagrams with a single external photon and from diagrams with low-momentum internal γ exchange. The only calculation of the radiative corrections for $\nu_e + d \rightarrow e^- + p + p$ is given by Towner [15]. If the bremsstrahlung photon energy is included in the deposited energy, the total radiative corrections are only mildly energy-dependent, and are given in Towner's Table II as about 4.4%. Taking this at face value, and subtracting the above inner radiative correction of 2.4%, Towner's result for the outer radiative correction to $\nu_e + d \rightarrow e^- + p + p$ is nearly constant at about 2%. This correction is somewhat larger than the corresponding corrections for $\bar{\nu}_e + p \rightarrow e^+ + n$ [5], and $\nu_e + n \rightarrow e^- + p$ [16], each about 1% and decreasing with increasing energy. In all cases, the total cross section is increased.

Thus, BCK and NSGK each underestimate the total cross section by about 6%. If the overlooked normalization effects discussed above are taken into account, it does seem reasonable to use the quoted nuclear-physics uncertainty of 3%. It should be noted that these very sophisticated calculations differ from the simplest treatment only by about 10% (while a number of corrections contribute, none is as large as their sum) [11,12]. Thus, roughly speaking, in order for the cross section to be known to 3%, the corrections only have to be known to 30%, which seems reasonable.

These simple considerations are backed up by the detailed results of BCK and NSGK. BCK have shown that up to next-to-next-to-leading order in their effective field theory treatment, only one unmeasured parameter $L_{1,A}$ (it appears at next-to-leading order) appreciably affects their result. They have also shown that their effective field theory series is convergent, with the contribution from each order about ten times smaller than the previous order. NSGK state that Ref. [17] did not include all of the known exchange-current contributions; when also neglected in NSGK, NSGK agree with Ref. [17] to about 1%. Similarly, BCK can reproduce the results of Ref. [17] by adjusting the value of $L_{1,A}$. BCK and NSGK agree with each other and Ref. [18] at the 1–2% level.

Bahcall, Krastev, and Smirnov [19] quote a theoretical uncertainty of 6% on the neutrino-deuteron total cross section based on nuclear-physics difference in the calculated cross sections of Refs. [17,18,20]. Since then, almost all of the differences between Refs. [17,18] has been explained by NSGK, similarly for the difference between Ref. [18] and the effective range calculation of Ref. [20], where no exchange current effects are included. Thus, a theoretical uncertainty of 6% seems too conservative.

So far, we have only considered the corrections to the normalization of the total cross section. We now turn to the
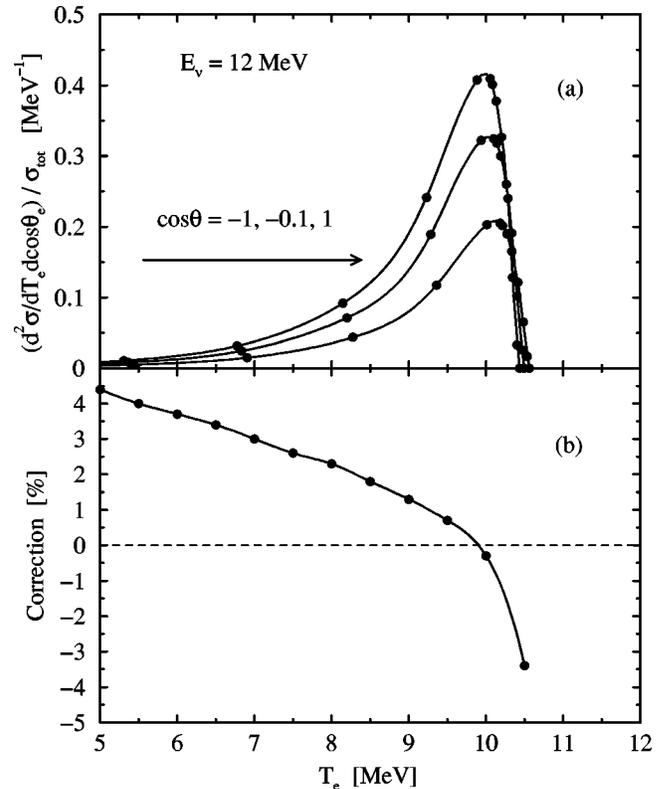

FIG. 1. (a) The differential cross section without radiative corrections for $\nu_e + d \rightarrow e^- + p + p$ as a function of the electron kinetic energy T_e, for selected values of the electron cosine $\cos\theta_e$, for $E_\nu = 12$ MeV. The points are from a table provided by Kubodera (private communication), based on Ref. [18], and the lines are spline fits. The height differences for different $\cos\theta_e$ are mostly accounted for by the angular distribution $d\sigma/d\cos\theta_e \simeq 1 - \frac{1}{3}\cos\theta_e$ (note $\langle\cos\theta\rangle \simeq -0.1$). (b) Total radiative corrections $(\alpha/\pi)g_I(E_\nu, T_e)$ to the differential cross section as a function of T_e, using Towner's Table I (points) [15], also for $E_\nu = 12$ MeV. The line is a spline fit. For the curves in both panels, other neutrino energies yield similar shapes when considered as a function of T_e/E_ν.

radiative corrections to the differential cross section. The differential cross section without radiative corrections for a representative neutrino energy is shown in Fig. 1(a). SNO may initially operate with a high electron energy threshold, above the peak in the electron spectrum expected from the ^{8}B neutrino spectrum. If so, a slight downward shift (of order 100 keV) in the electron energies can cause a few-percent decrease in the number of events above threshold (when integrated over the ^{8}B spectrum). For photons below about 1 MeV, the Compton-scattered electrons will not produce detectable Čerenkov light [21]. Thus, one of the effects of bremsstrahlung will be to lower the electron energies from the case with no bremsstrahlung. In Towner's Table I, the bremsstrahlung energy is considered undetected, and the correction to $d\sigma/dT_e$ is as large as $+5\%$ to -4% from low to high scattered electron energies, and nearly vanishing at electron kinetic energy $T_e \simeq E_\nu - 2$ MeV. This is shown in Fig. 1(b). The qualitative features of Towner's total radiative corrections can be reproduced by a small downward shift in the electron energies.

Using Towner's Table I, where the bremsstrahlung energy is considered undetected, we can calculate the total cross section by integrating

$$\frac{d\sigma}{dT_e} = \left(\frac{d\sigma}{dT_e}\right)^0 \times \left[1 + \frac{\alpha}{\pi} g_I(E_\nu, T_e)\right], \qquad (1)$$

where the superscript 0 indicates the cross section without radiative corrections. Using Towner's Table II, where the bremsstrahlung energy is considered to add to the detected electron energy, we can calculate the total cross section by integrating

$$\frac{d\sigma}{dX} = \left(\frac{d\sigma}{dX}\right)^0 \times \left[1 + \frac{\alpha}{\pi} g_{II}(E_\nu, X)\right], \qquad (2)$$

where X is the sum of electron and bremsstrahlung energies (without radiative corrections, this is just the electron energy). In the latter case, the total radiative corrections are nearly constant at $(\alpha/\pi)g_{II}(E_\nu, X) \simeq 4.4\%$. If there are no cuts on the kinematic variables, then these two integrals must be identical. We explicitly made this test (using spline fits) for $E_\nu = 12$ MeV, and find it not to be the case. In the first case, we find an average correction of 0.7% above $T_e = 5$ MeV (this is also obvious by inspection of Fig. 1, if the correction is evaluated at the average energy for the differential cross section), much less than the 4.4% for the second case. The neglected fraction of the $d\sigma/dT_e$ integral below 5 MeV is about 3%, so in order to reproduce the integrated total corrections of 4.4%, the correction below 5 MeV (not given in Towner's Table I) would have to be of order 100 times larger than that above 5 MeV. Thus, we are unable to

see how the results of Towner's Table I can be consistent with the seemingly reasonable results of his Table II.

Given the importance of the radiative corrections for the SK-SNO comparison, additional work is needed, in particular on the bremsstrahlung spectrum. For a sufficiently soft bremsstrahlung spectrum, the corrections to the total cross section will be applicable. Otherwise, corrections to the differential cross section will also be necessary.

In conclusion, three overlooked effects conspire to increase the normalization of the total cross section for $\nu_e + d \rightarrow e^- + p + p$ by about 6%. As noted, the uncertainty in the measured neutrino flux in SNO is expected to be eventually dominated by the uncertainty in the theoretical cross section. In addition, if SNO operates with a high threshold, the effects of the radiative corrections on the differential cross section must be considered. These effects, if not taken into account, could qualitatively change the outcome of the SK-SNO comparison, which is a SSM-independent test for the appearance of the active flavors ν_μ, ν_τ resulting from neutrino oscillations (e.g., in a $\phi_{\nu_\mu, \nu_\tau}/\phi_{SSM}$ versus ϕ_{ν_e}/ϕ_{SSM} plot, where ϕ is the neutrino flux). If the effects discussed above, in particular the QED radiative corrections, are correctly taken into account, then the 3% theoretical uncertainty indicated by BCK and NSGK for the neutrino-deuteron cross section in the energy range appropriate for solar neutrinos is attainable.

---

[1] J.N. Bahcall, M.H. Pinsonneault, and S. Basu, Astrophys. J. **555**, 990 (2001).

[2] S. Fukuda *et al.*, Phys. Rev. Lett. **86**, 5651 (2001).

[3] W. Kwong and S.P. Rosen, Phys. Rev. D **54**, 2043 (1996); F.L. Villante, G. Fiorentini, and E. Lisi, *ibid.* **59**, 013006 (1999); G.L. Fogli, E. Lisi, A. Palazzo, and F.L. Villante, *ibid.* **63**, 113016 (2001); J.N. Bahcall, P.I. Krastev, and A.Y. Smirnov, J. High Energy Phys. **05**, 015 (2001); V. Barger, D. Marfatia, and K. Whisnant, Phys. Lett. B **504**, 14 (2001).

[4] L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky, and S. Rosati, Phys. Rev. Lett. **84**, 5959 (2000); L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky, S. Rosati, and J.F. Beacom, Phys. Rev. C **63**, 015801 (2001).

[5] P. Vogel, Phys. Rev. D **29**, 1918 (1984).

[6] P. Vogel and J.F. Beacom, Phys. Rev. D **60**, 053003 (1999).

[7] Y. Declais *et al.*, Phys. Lett. B **338**, 383 (1994).

[8] G.T. Ewan *et al.*, "Sudbury Neutrino Observatory Proposal," Report No. SNO-87-12, 1987.

[9] J. Boger *et al.*, Nucl. Instrum. Methods Phys. Res. A **449**, 172 (2000).

[10] S.M. Oser, at "Pheno 2001," http://pheno.physics.wisc.edu/pheno01/; R.G.H. Robertson, at "Neutrino Telescopes 2001," http://axpd24.pd.infn.it/conference2001/venice01.html

[11] M. Butler, J. Chen, and X. Kong, Phys. Rev. C **63**, 035501 (2001).

[12] S. Nakamura, T. Sato, V. Gudkov, and K. Kubodera, Phys. Rev. C **63**, 034617 (2001).

[13] S.E. Willis *et al.*, Phys. Rev. Lett. **44**, 522 (1980); S.P. Riley *et al.*, Phys. Rev. C **59**, 1780 (1999); Y.V. Kozlov *et al.*, Phys. At. Nucl. **63**, 1016 (2000).

[14] D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[15] I.S. Towner, Phys. Rev. C **58**, 1288 (1998).

[16] D.A. Dicus, E.W. Kolb, A.M. Gleeson, E.C. Sudarshan, V.L. Teplitz, and M.S. Turner, Phys. Rev. D **26**, 2694 (1982).

[17] S. Ying, W.C. Haxton, and E.M. Henley, Phys. Rev. C **45**, 1982 (1992).

[18] K. Kubodera and S. Nozawa, Int. J. Mod. Phys. E **3**, 101 (1994).

[19] J.N. Bahcall, P.I. Krastev, and A.Y. Smirnov, Phys. Rev. D **62**, 093004 (2000).

[20] J.N. Bahcall and E. Lisi, Phys. Rev. D **54**, 5417 (1996).

[21] H. Robertson (private communication).