Classification of conformality models based on non-Abelian orbifolds

Paul H. Frampton
Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27599

Thomas W. Kephart
Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37325
(Received 6 February 2001; published 27 September 2001)

A systematic analysis is presented of compactifications of the type IIB superstring on $\mathrm{AdS}_5 \times \mathrm{S}^5/\Gamma$ where Γ is a non-Abelian discrete group. Every possible Γ with order $g \leqslant 31$ is considered. 45 such groups exist but a majority cannot yield chiral fermions due to a certain theorem that is proved. The lowest order to embrace the non-SUSY standard $SU(3)\times SU(2)\times U(1)$ model with three chiral families is $\Gamma = D_4 \times Z_3$, with $g=24$; this is the only successful model found in the search. The consequent uniqueness of the successful model arises primarily from the scalar sector, prescribed by the construction, being sufficient to allow the correct symmetry breakdown.

DOI: 10.1103/PhysRevD.64.086007 PACS number(s): 11.25.Mj, 11.15.Ex, 11.15.Pg, 11.30.Pb

I. INTRODUCTION

In particle phenomenology, the impressive success of the standard theory based on $SU(3)\times SU(2)\times U(1)$ has naturally led to the question of how to extend the theory to higher energies. One is necessarily led by weaknesses and incompleteness in the standard theory. If one extrapolates the standard theory as it stands one finds (approximate) unification of the gauge couplings at $\sim 10^{16}$ GeV. But then there is the *hierarchy* problem of how to explain the occurrence of the tiny dimensionless ratio $\sim 10^{-14}$ of the weak scale to the unification scale. Inclusion of gravity leads to a *superhierarchy* problem of the ratio of the weak scale to the Planck scale, $\sim 10^{19}$ GeV, to even tinier $\sim 10^{-17}$ dimensionless ratios. Although this is obviously a very important problem about which conformality in itself is not informative, we shall first discuss the hierarchy rather than the superhierarchy.

There are four well-defined approaches to the hierarchy problem: (1) supersymmetry, (2) technicolor, (3) extra dimensions, and (4) conformality.

Supersymmetry has the advantage of rendering the hierarchy technically natural: once the hierarchy is put into the Lagrangian it need not be retuned in perturbation theory. Supersymmetry predicts superpartners of all the known particles, and these are predicted to be at or below a TeV scale if supersymmetry is related to the electroweak breaking. Inclusion of such hypothetical states improves the gauge coupling unification. On the negative side, supersymmetry does not explain the origin of the hierarchy.

Technicolor postulates that the Higgs boson is a composite of a fermion-antifermion pair bound by a new (technicolor) strong dynamics at or below the TeV scale. This obviates the hierarchy problem. On the minus side, no convincing simple model of technicolor has been found.

Extra dimensions can have a range as large as $1(\mathrm{TeV})^{-1}$ and the gauge coupling unification can occur quite differently than in only four spacetime dimensions. This replaces the hierarchy problem with a different fine-tuning question of why the extra dimension is restricted to a distance corresponding to the weak interaction scale. There is also a potentially serious problem with the proton lifetime.

Conformality is inspired by superstring duality and assumes that the particle spectrum of the standard model is enriched such that there is a conformal fixed point of the renormalization group at the TeV scale. Above this scale the coupling do not run so the hierarchy is nullified.

Conformality is the approach followed in this paper. We shall systematically analyze the compactification of a type IIB superstring on $\mathrm{AdS}_5 \times \mathrm{S}^5/\Gamma$ where Γ is a discrete non-Abelian group.

The duality between weak and strong coupling field theories, and then between all the different superstring theories has led to a revolution in our understanding of strings. Equally profound, is the AdS conformal field theory (CFT) duality which is the subject of the present article. This AdS CFT duality is between string theory compactified on anti–de-Sitter space and conformal field theory.

Until very recently, the possibility of testing string theory seemed at best remote. The advent of AdS/CFT's and large-scale string compactification suggest that this point of view may be too pessimistic, since both could lead to $\sim$100-TeV evidence of strings. With this thought in mind, we are encouraged to build AdS/CFT models with realistic fermionic structure, and reduce to the standard model below ~ 1 TeV.

Using AdS/CFT duality, one arrives at a class of gauge field theories of special recent interest. The simplest compactification of a ten-dimensional superstring on a product of an AdS space with a five-dimensional spherical manifold leads to an $\mathcal{N}=4$ $SU(N)$ supersymmetric gauge theory, well known to be conformally invariant [1]. By replacing the manifold S^5 by an orbifold S^5/Γ, one arrives at less supersymmetries corresponding to $\mathcal{N}=2$, 1 or 0 depending [2] on whether (i) $\Gamma \subset SU(2)$, (ii) $\Gamma \subset SU(3)$ but $\Gamma \not\subset SU(2)$, or (iii) $\Gamma \subset SU(4)$ but $\Gamma \not\subset SU(3)$ respectively, where Γ is in all cases a subgroup of $SU(4)\sim SO(6)$, the isometry of the S^5 manifold.

It was conjectured in Ref. [3] that such $SU(N)$ gauge theories are conformal in the $N\rightarrow\infty$ limit. In Ref. [4] it was conjectured that at least a subset of the resultant nonsupersymmetric $\mathcal{N}=0$ theories are conformal even for finite N, and that one of this subsets provides the right extension of the standard model. Some first steps to check this idea were made in Ref. [5]. Model building based on an Abelian Γ was studied further in Refs. [6–8], arriving in Ref. [8] at an $SU(3)^7$ model based on $\Gamma=Z_7$ which has three families of chiral fermions, a correct value for $\sin^2\theta$ and a conformal scale ~ 10 TeV.

The case of non-Abelian orbifolds bases on non-Abelian Γ has not previously been extensively studied [21], partially due to the fact that it is apparently somewhat more mathematically sophisticated. However, we shall show here that it can be handled systematically as in the Abelian case, and leads to richer structures and interesting results.

In such constructions, the cancellation of chiral anomalies in the four-dimensional theory, as is necessary in extension of the standard model (e.g. Refs. [9,10]), follows from the fact that the progenitor ten-dimensional superstring theory has a canceling hexagon anomaly [11].

We consider all non-Abelian discrete groups of order $g<32$. These were described in detail in Refs. [12,15]. There are exactly 45 such non-Abelian groups. Because the gauge group arrived at in this construction [6] is $\otimes_i SU(Nd_i)$ where d_i are the dimensions of the irreducible representations of Γ, and one can expect to arrive at models such as the Pati-Salam $SU(4)\times SU(2)\times SU(2)$ model [16] by choosing $N=2$ and combining two singlets and a doublet in the **4** of $SU(4)$. Indeed we shall show that such an accommodation of the standard model is possible by using a non-Abelian Γ.

The procedures for building a model within such a conformality approach are the following: (1) Choose Γ. (2) Choose a proper embedding $\Gamma\subset SU(4)$ by assigning the components of the **4** of $SU(4)$ to irreps of Γ, while at the same time ensuring that the **6** of $SU(4)$ is real. (3) Choose N, in the gauge group $\otimes_i SU(Nd_i)$. (4) Analyze the patterns of spontaneous symmetry breaking.

In the present study we shall most often choose $N=2$ and aim at the gauge group $SU(4)\times SU(2)\times SU(2)$. To obtain chiral fermions, it is necessary [6] that the **4** of $SU(4)$ be complex: $\mathbf{4}\neq\mathbf{4}^*$. Actually this condition is not quite sufficient to ensure chirality in the present case because of the pseudoreality of $SU(2)$. We must ensure that the **4** is not just pseudoreal.

This last condition means that many of our 45 candidates for Γ do not lead to chiral fermions. For example, $\Gamma=Q_{2n}\subset SU(2)$ has irreps of appropriate dimensionalities for our purpose, but with $N=2$ it will not sustain chiral fermions under $SU(4)\times SU(2)\times SU(2)$ because these irreps are all, like $SU(2)$, pseudoreal.[1] Applying the rule that **4** must be neither real nor pseudoreal leaves a total of only 19 possible non-Abelian discrete groups of order $g\leqslant 31$. The smallest group which avoids pseudoreality has order $g=16$ but gives only two families. The technical details of our systematic search will be given in Secs. V and VI. The simplest interesting non-Abelian case has $g=24$, and gives three chiral families in a Pati-Salam-type model [16].

Before proceeding to details, it is worth reminding the reader that the conformal field theory that it exemplifies should be free of all divergences, even logarithmic ones, if the conformality conjecture is correct, and be completely finite. Further the theory is originating from a superstring theory in a higher dimension (10) and contains gravity [17–19] by compactification of the higher-dimensional graviton already contained in that superstring theory. In the CFT as we derive it, gravity is absent because we have not kept these graviton modes (of course, their influence on high-energy physics experiments is generally completely negligible unless the compactification scale is ''large'' [20]); here we shall neglect the effects of gravity.

It is worthwhile noting the degree of constraint imposed on the symmetry and particle content of a model as the number of irreps N_R of the discrete group Γ associated with the choice of orbifold changes. The number of gauge groups grows linearly in N_R, the number of scalar irreps grows roughly quadratically with N_R, and the chiral fermion content is highly Γ dependent. If we require a minimal Γ that is large enough for the model generated to contain the fermions of the standard model and have sufficient scalars to break the symmetry to the level of that of the standard model, then $\Gamma=Q\times Z_3$ appears to be that minimal choice [21].

Although a decade ago the chances of testing string theory seemed at best remote, recent progress has given us hope that such tests may indeed be possible in AdS/CFT's. The model provided here demonstrates that the standard model can be accommodated in these theories, and suggests the possibility of a rich spectrum of new physics just around the TeV corner.

II. NON-ABELIAN GROUPS WITH ORDER $g\leqslant 31$

From any good textbook on finite groups [12] we may find a tabulation of the number of finite groups as a function of the order g, the number of elements in the group. Up to order 31 there is a total of 93 different finite groups of which slightly over one half (48) are Abelian.

Among finite groups, the non-Abelian examples have the advantage of nonsinglet irreducible representations which can be used to interrelate families. Which such group to select is based on simplicity: the minimum order and most economical use of representations [13–15].

Let us first dispense with the Abelian groups. These are all made up from the basic unit Z_p, the order p group formed from the pth roots of unity. It is important to note that the product Z_pZ_q is identical to Z_{pq} if and only if p and q have no common prime factor.

If we write the prime factorization of g as

$$g=\prod_i p_i^{k_i}, \tag{1}$$

where the product is over primes, it follows that the number $N_a(g)$ of inequivalent Abelian groups of order g is given by

[1]Note that were we using $N\geqslant 3$, then a pseudoreal **4** would give chiral fermions.

$$N_a(g)=\prod_{k_i} P(k_i), \tag{2}$$

where $P(x)$ is the number of unordered partitions of x. For example, for order $g=144=2^4 3^2$ the value would be $N_a(144)=P(4)P(2)=5\times 2=10$. For $g\leqslant 31$ it is simple to evaluate $N_a(g)$ by inspection. $N_a(g)=1$ unless g contains a nontrivial power ($k_i\geqslant 2$) of a prime. These exceptions are $N_a(g=4,9,12,18,20,25,28)=2$, $N_a(8,24,27)=3$, and $N_a(16)=5$. This confirms that

$$\sum_{g=1}^{31} N_a(g)=48. \tag{3}$$

We do not consider the Abelian cases further in this paper.

Of the non-Abelian finite groups, the best known are perhaps the permutation groups S_N (with $N\geqslant 3$) of order N. The smallest non-Abelian finite group is S_3 ($\equiv D_3$), the symmetry of an equilateral triangle with respect to all rotations in a three dimensional sense. This group initiates two infinite series S_N and D_N. Both have elementary geometrical significance since the symmetric permutation group S_N is the symmetry of the N-plex in N dimensions while the dihedral group D_N is the symmetry of the planar N-agon in three dimensions. As a family symmetry, the S_N series becomes uninteresting rapidly as the order and the dimensions of the represenetions increase. Only S_3 and S_4 are of any interest as symmetries associated with the particle spectrum [14], also the order (number of elements) of the S_N groups grow factorially with N. The order of the dihedral groups D_N are $2N$ and so increase only linearly with N, and their irreducible representations are all one and two dimensional. This is reminiscent of the representations of the electroweak $SU(2)_L$ used in nature. Each D_N is a subgroup of $O(3)$ and has a counterpart double dihedral (also known as dicyclic) group Q_{2N} of order $4N$, which is a subgroup of the double covering $SU(2)$ of $O(3)$.

With only the use of D_N, Q_{2N}, S_N, and the tetrahedral group T (of order 12, the even permutations subgroup of S_4) we find (see Table I) 32 of the 45 non-Abelian groups up to order 31, either as simple groups or as products of simple non-Abelian groups with Abelian groups. (Note that $D_6\simeq Z_2\times D_3$, $D_{10}\simeq Z_2\times D_5$ and $D_{14}\simeq Z_2\times D_7$.) Some of these groups are familiar from crystallography and chemistry, but the non-Abelian groups that do not embed in $SU(2)$ are less likely to have seen wide usage.

There remain 13 other groups formed by twisted products of Abelian factors. Only certain such twistings are permissible, namely (completing all $g\leqslant 31$) those given in Table II. It can be shown that these 13 groups exhaust the classification of *all* inequivalent finite groups up to order 31 [12].

Of the 45 non-Abelian groups, the dihedrals (D_N) and double dihedrals (Q_{2N}), of order $2N$ and $4N$ respectively, form the simplest sequences. In particular, they fall into subgroups of $O(3)$ and $SU(2)$ respectively, the two simplest non-Abelian continuous groups.

For D_N and Q_{2N}, the multiplication tables, as derivable from the character tables, are simple to express in general [15]. D_N, for odd N, has two singlet representations $1,1'$ and $m=(N-1)/2$ doublets $2_{(j)}$ $(1\leqslant j\leqslant m)$. The multiplication rules are

TABLE I. The non-Abelian finite groups of order <32 constructed from direct products of Z_N, D_N, Q_{2N}, S_N, and T.

g	
6	$D_3\equiv S_3$
8	$D_4, Q=Q_4$
10	D_5
12	D_6, Q_6, T
14	D_7
16	$D_8, Q_8, Z_2\times D_4, Z_2\times Q$
18	$D_9, Z_3\times D_3$
20	D_{10}, Q_{10}
22	D_{11}
24	$D_{12}, Q_{12}, Z_2\times D_6, Z_2\times Q_6, Z_2\times T$, $Z_3\times D_4, Z_3\times Q, Z_4\times D_3, S_4$
26	D_{13}
28	D_{14}, Q_{14}
30	$D_{15}, D_5\times Z_3, D_3\times Z_5$

$$1'\times 1'=1, \quad 1'\times 2_{(j)}=2_{(j)}, \tag{4}$$

$$2_{(i)}\times 2_{(j)}=\delta_{ij}(1+1')+2_{(min[i+j,N-i-j])} + (1-\delta_{ij})2_{(|i-j|)}. \tag{5}$$

For even N, D_N has four singlets $1,1',1'',1'''$ and $(m-1)$ doublets $2_{(j)}$ $(1\leqslant j\leqslant m-1)$, where $m=N/2$ with multiplication rules

$$1'\times 1'=1''\times 1''=1'''\times 1'''=1, \tag{6}$$

$$1'\times 1''=1''';1''\times 1'''=1';1'''\times 1'=1'', \tag{7}$$

$$1'\times 2_{(j)}=2_{(j)}, \tag{8}$$

$$1''\times 2_{(j)}=1'''\times 2_{(j)}=2_{(m-j)}, \tag{9}$$

$$2_{(j)}\times 2_{(k)}=2_{|j-k|}+2_{(min[j+k,N-j-k])} \tag{10}$$

if $k\neq j$, $(m-j)$,

$$2_{(j)}\times 2_{(j)}=2_{(min[2j,N-2j])}+1+1' \tag{11}$$

TABLE II. All non-Abelian finite groups of order <32 containing twisted products of Abelian factors.

g	
16	$Z_2\tilde{\times}Z_8$ (two, excluding D_8), $Z_4\tilde{\times}Z_4, Z_2\tilde{\times}(Z_2\times Z_4)$ (two)
18	$Z_2\tilde{\times}(Z_3\times Z_3)$
20	$Z_4\tilde{\times}Z_5$
21	$Z_3\tilde{\times}Z_7$
24	$Z_3\tilde{\times}Q, Z_3\tilde{\times}Z_8, Z_3\tilde{\times}D_4$
27	$Z_9\tilde{\times}Z_3, Z_3\tilde{\times}(Z_3\times Z_3)$

if $j \neq m/2$,

$$2_{(j)} \times 2_{(m-j)} = 2_{|m-2j|} + 1'' + 1''' \tag{12}$$

and

$$2_{m/2} \times 2_{m/2} = 1 + 1' + 1'' + 1'''. \tag{13}$$

This last is possible only if m is even and hence if N is divisible by 4.

For Q_{2N}, there are four singlets (1, $1'$, $1''$, and $1'''$) and $(N-1)$ doublets $2_{(j)}$ $[1 \leqslant j \leqslant (N-1)]$. The singlets have multiplication rules

$$1 \times 1 = 1' \times 1' = 1, \tag{14}$$

$$1'' \times 1'' = 1''' \times 1''' = 1', \tag{15}$$

$$1' \times 1'' = 1''', \quad 1''' \times 1' = 1'' \tag{16}$$

for $N = (2k+1)$, but are identical to those for D_N when $N = 2k$. The products involving the $2_{(j)}$ are identical to those given for D_N (N even) above.

This completes the multiplication rules for 19 of the 45 groups. As they are not available in the literature, and somewhat tedious to work out, we have provided complete multiplication tables for all the non-Abelian groups with order $g \leqslant 31$ in the Appendix.

III. MATHEMATICAL THEOREM

Theorem: A pseudoreal 4 of $SU(4)$ *cannot yield chiral fermions*. In Ref. [6] it was proved that if the embedding in $SU(4)$ is such that **4** is real, $\mathbf{4} = \mathbf{4}^*$, then the resultant fermions are always nonchiral. It was implied there that the converse holds, that if **4** is complex, $\mathbf{4} \neq \mathbf{4}^*$, then the resulting fermions are necessarily chiral. Actually for $\Gamma \subset SU(2)$ one encounters the intermediate possibility that the **4** is *pseudoreal*. In the present section we shall show that if **4** is pseudoreal then the resultant fermions are necessarily nonchiral. The converse now holds: if the **4** is neither real nor pseudoreal then the resultant fermions are chiral.

For $\Gamma \subset SU(2)$ it is important that the embedding be consistent with the chain $\Gamma \subset SU(2) \subset SU(4)$; otherwise the embedding is not a consistent one. One way to see the inconsistency is to check the reality of $\mathbf{6} = (\mathbf{4} \otimes \mathbf{4})_{antisymmetric}$. If $\mathbf{6} \neq \mathbf{6}^*$ then the embedding is clearly improper. To avoid this inconsistency, it is sufficient to include only complete irreducible representations of $SU(2)$ in the **4** of $SU(4)$.

An explicit example will best illustrate this propriety constraint on embeddings. Let us consider $\Gamma = Q_6$, the dicyclic group of order $g = 12$. This group has six inequivalent irreducible representations 1, $1'$, $1''$, $1'''$, 2_1, and 2_2. 1, $1'$, and 2_1 are real. $1''$ and $1'''$ are a complex conjugate pair, and 2_2 is pseudoreal. To embed $\Gamma = Q_6 \subset SU(4)$ we must choose from the special combinations which are complete irreducible representations of $SU(2)$ namely 1, $2 = 2_2$, $3 = 1' + 2_1$ and $4 = 1'' + 1''' + 2_2$. In this way the embedding either makes the **4** of $SU(4)$ real, e.g., $4 = 1 + 1' + 2_1$, in which case the theorem of Ref. [6] applies, and non-chirality results; or else **4** is pseudoreal, e.g., $4 = 2_2 + 2_2$. In this case one can check that the embedding is consistent because $(\mathbf{4} \otimes \mathbf{4})_{antisymmetric}$ is real. But it is equally easy to check that the product of this pseudoreal **4** with the complete set of irreducible representations of Q_6 is again real and that the resultant fermions are nonchiral.

The lesson is contained in the following theorem: *To obtain chiral fermions from compactification on* $\mathrm{AdS}_5 \times \mathrm{S}_5/\Gamma$, *the embedding of* Γ *in* $SU(4)$ *must be such that the 4 of* $SU(4)$ *is neither real nor pseudoreal.*

IV. CHIRAL FERMIONS FOR ALL NON-ABELIAN $g \leqslant 31$

Looking at the full list of non-Abelian discrete groups of order $g \leqslant 31$ as given explicitly in Ref. [15] we see that of the 45 such groups 32 are simple groups or semidirect products thereof; these 32 are listed in the table on page 4691 of Ref. [15], and reproduced in Sec. II above. The remaining 13 are formed as semidirect product groups (SDPG's) and are listed in the Table on page 4692 of Ref. [15] and in Sec. II. We shall follow this classification closely.

Using the pseudoreality considerations of Sec. III, we can pare down the full list of 45 to only 19 which include 13 SDPG's. The lowest order non-Abelian group Γ which can lead to chiral fermions is $g = 16$. The only possible orders $g \leqslant 31$ are the seven values $g = 16(5[5SDPGs])$, $18(2[1SDPG])$, $20(1[1SDPG])$, $21(1[1SDPG])$, $24(6[3SDPGs])$, $27(2[2SDPGs])$, and $30(2[0SDPG])$. In parentheses we show the number of groups at order g, and the number of these that are SDPG's is in square brackets; they add to (19[13 SDPG's]). We shall proceed with the analysis systematically, in progressively increasing magnitudes of g.

$g = 16$. The nonpseudoreal groups number five, and all are SDPG's. In the notation of Thomas and Wood [12], which we shall follow for definiteness both here and in the Appendix, they are 16/8, 9, 10, 11, and 13. So we now treat these in the order they are enumerated by Thomas and Wood. Again, the relevant multiplication tables are collected in the Appendix.

Group 16/8; also designated $(Z_4 \times Z_2) \tilde{\rtimes} Z_2$. This group has eight singlets $1_1, 1_2, \ldots, 1_8$ and two doublets 2_1 and 2_2. In the embedding of 16/8 in $SU(4)$ we must avoid the sin-

TABLE III. Chiral fermions for 16/8 with $\mathbf{4} = (2_1, 2_1)$.

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2
1_1									$\times\times$	
1_2									$\times\times$	
1_3									$\times\times$	
1_4									$\times\times$	
1_5										$\times\times$
1_6										$\times\times$
1_7										$\times\times$
1_8										$\times\times$
2_1					$\times\times$	$\times\times$	$\times\times$	$\times\times$		
2_2	$\times\times$	$\times\times$	$\times\times$	$\times\times$						

TABLE IV. Chiral fermions for 16/8 with $\mathbf{4}=(1_2,1_5,2_1)$.

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2
1_1		×			×				×	
1_2	×					×			×	
1_3				×			×		×	
1_4			×					×	×	
1_5	×						×			×
1_6		×				×				×
1_7			×					×		×
1_8				×			×			×
2_1					×	×	×	×	×	×
2_2	×	×	×	×					×	×

glet 1_1; otherwise there will be a residual supersymmetry with $\mathcal{N}\geqslant 1$. Consider the embedding defined by $\mathbf{4}=(2_1,2_1)$. To find the surviving chiral fermions we need to product $\mathbf{4}$ with all ten of the irreps of 16/8. The results are given in Table III.

If we choose $N=2$, the gauge group is $SU(2)^8 \times SU(4)^2$, and the entries in Table III correspond to bifundamental representations of this group [e.g., the entry nearest the top right corner at the position $(1_1,2_1)$ is the representation $2(2,1,1,1,1,1,1,1;\bar{4},1)$]. If we identify the diagonal subgroup of the first four $SU(2)$'s as $SU(2)_L$, that of the second four as $SU(2)_R$, and that of the two $SU(4)$'s as color $SU(4)$, the result is nonchiral due to the symmetry about the main diagonal of the above table.

On the other hand, if we identify $\mathbf{4}_1$ with $\overline{\mathbf{4}}_2$ there are potentially eight chiral families

$$8[(2,1,4)+(1,2,\bar{4})] \tag{17}$$

under $SU(2)_L \times SU(2)_R \times SU(4)$. This is the maximum total chirality for this orbifold, but, as we will see in Sec. V, the allowed chirality at any stage is as usual determined by spontaneous symmetry breaking (SSB) generated in the scalar sector. In this section we give the maximum chirality for each orbifold; in Sec. V we study SSB for those models with sufficient chirality to accommodate at least three families.

Because $2_1=2_2^*$ form a complex conjugate pair, the embedding $\mathbf{4}=(2_1,2_2)$ is pseudoreal, $\mathbf{4}\equiv\mathbf{4}^*$ and the fermions are nonchiral as is easily confirmed. For this embedding, the result is nonchiral for either of the cases $\mathbf{4}_1\equiv\mathbf{4}_2$ or $\mathbf{4}_1\equiv\overline{\mathbf{4}}_2$. (In the future, we shall not even consider such trivially real nonchiral embeddings.)

Finally, for 16/8, consider the embedding $\mathbf{4}=(1_2,1_5,2_1)$; see Table IV. [In general there will be many equivalent embeddings. We will give one member of each equivalence class. Cases that are obviously nonchiral (vectorlike) will, in general, be ignored, except for a few instructive examples at order 16 and 18.] These examples of embeddings for $\Gamma=16/8$ clearly show how the number of chiral families depends critically on the choice of embedding $\Gamma\subset SU(4)$. To actually achieve a model that is phenomenologically viable, we must study the possible routes through SSB for each chiral model. We postpone this until we find all models of potential interest.

Group 16/9; also designated $[(Z_4\times Z_2)\tilde{\times}Z_2]'$. This group has irreps which comprise eight singlets $1_1,\ldots,1_8$ and two doublets $2_1,2_2$. With the embedding $\mathbf{4}=(2_1,2_2)$ and using the multiplication table from the Appendix, we arrive at the fermion bilinears. These are nonchiral and the model has no families. This was the only potentially chiral embedding. In what follows, nonchiral models will not be displayed, however, as the unification scale can be rather low in AdS/CFT models, it would also be interesting to investigate vectorlike models of this class.

Group 16/10; also designated $Z_4\tilde{\times}Z_4$. The multiplication table is identical to that for 16/9, as mentioned in the Appendix; thus the model building for 16/10 is also identical to 16/9 and merits no additional discussion.

Group 16/11; also designated $Z_8\tilde{\times}Z_2$. Again there are eight singlets and two doublets. The singlets $1_{1,3,5,7}$ are real while the other singlets fall into two conjugate pairs $1_2=1_4^*$ and $1_6=1_8^*$. The doublets are complex: $2_1=2_2^*$.

The multiplication table in the Appendix includes the products $1_{1,3,5,7}\times 2_{1,2}=2_{1,2}$ and $1_{2,4,6,8}\times 2_{1,2}=2_{2,1}$. Also $2_1\times 2_1=2_2\times 2_2=1_2+1_4+1_6+1_8$, while $2_1\times 2_2=1_1+1_3+1_5+1_7$. This means that there are no interesting (legitimate and chiral) embeddings of the type $1+1+2$ or $2+2$.

TABLE V. Chiral fermions for 16/11 with $\mathbf{4}=(1_2,1_2,1_2,1_2)$.

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2
1_1		$(\times)^4$								
1_2			$(\times)^4$							
1_3				$(\times)^4$						
1_4	$(\times)^4$									
1_5						$(\times)^4$				
1_6							$(\times)^4$			
1_7								$(\times)^4$		
1_8					$(\times)^4$					
2_1										$(\times)^4$
2_2									$(\times)^4$	

TABLE VI. Chiral fermions for 16/11 with $\mathbf{4}=(1_2,1_2,1_2,1_4)$.

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2
1_1		×××		×						
1_2	×		×××							
1_3		×		×××						
1_4	×××		×							
1_5						×××		×		
1_6					×		×××			
1_7						×		×××		
1_8					×××		×			
2_1										××××
2_2									××××	

The most chiral possibility is the embedding $\mathbf{4}=(1_2,1_2,1_2,1_2)$ which leads to the fermions in Table V. [In this table, $(\times)^4 \equiv (\times\times\times\times)$.] This gives rise to twelve chiral families if we set $N=3$ and identify and $3_1=3_4=3_5=3_8$, $3_2=3_6$ and $3_3=3_7$. Under $SU(3)^3$ the chiral fermions are

$$8[(3,\bar{3},1)+(1,3,\bar{3})+(\bar{3},3,1)] \tag{18}$$

together with real nonchiral representations. In Sec. VI where we discuss spontaneous symmetry breaking, we will see if this type of unification is possible.

With the different embedding $\mathbf{4}=(1_2,1_2,1_2,1_4)$ the model changes to a less chiral but still interesting fermion configuration given in Table VI. If we can identify $SU(3)$'s as $3_1\equiv 3_4 \equiv 3_5 \equiv 3_8$, $3_2 \equiv 3_6$ and $3_3\equiv 3_7$ this embedding gives just four chiral families:

$$4[(3,\bar{3},1)+(1,3,\bar{3})+(\bar{3},3,1)] \tag{19}$$

under $SU(3)^3$ together with real representations. To check consistency, we have verified that real and legitimate embeddings for 16/11 like $\mathbf{4}=(1_3,1_3,1_3,1_3)$ and $\mathbf{4}=(2_1,2_2)$ give no chiral fermions.

Group 16/13; also designated $[Z_8 \tilde{\times} Z_2]''$. Of the five non-pseudoreal $g=16$ non-Abelian Γ's, 16/13 is unique in having only four inequivalent singlets 1_1, 1_2, 1_3, and 1_4 but three doublets 2_1, 2_2, and 2_3.

All four singlets are real: $1_i=1_i^*$. The three doublets comprise a conjugate complex pair $2_1=2_3^* \neq 2_1^*$ and the real $2_2=2_2^*$.

With the embedding $\mathbf{4}=(1_3,1_4,2_1)$ the resultant model has the chiral fermion of Table VII. For $N=2$ if we identify $SU(2)_L$ with the diagonal subgroup of the first and fourth $SU(2)$'s, and $SU(2)_R$ with the diagonal subgroup of the second and third $SU(2)$'s, then there are four chiral families if we embed $\mathbf{4}_1 \equiv \bar{\mathbf{4}}_3$ and break $SU(4)_2$ completely.

For 16/13 with $\mathbf{4}=(2_1,2_1)$ the chiral fermions are given in Table VIII. With $\mathbf{4}_1\equiv\bar{\mathbf{4}}_3$ there are potentially eight chiral families. A similar result occurs, of course, for $\mathbf{4}=(2_3,2_3)$. But the embedding $\mathbf{4}=(2_1,2_3)$ is manifestly nonchiral because of the symmetry of the table, as can be shown from the embedding as follows: Even though 2_1 and 2_3 are complex, $2_1=2_3^*$, so $4^*=(2_1,2_3)^*=(2_3,2_1)$. We can rotate this within $SU(4)$ to $4=(2_1,2_3)$. Therefore, the 4 is pseudoreal and the fermions are vectorlike as expected. Also the embedding $\mathbf{4}=(2_2,2_2)$ in 16/13 gives rise to no chirality and hence to zero families. Finally, the embedding $\mathbf{4}=(2_1,2_2)$ of 16/13 leads to the intermediate situation shown in Table IX. This gives rise to four potential chiral families with the identification $\mathbf{4}_1\equiv\bar{\mathbf{4}}_3$.

To summarize the "double doublet" embeddings $\mathbf{4}=(2_i,2_j)$ of 16/13: for equivalent embeddings $(i,j)=(1,1)$ or $(3,3)$, there are up to eight chiral families; for the other mutually equivalent cases $(i,j)=(1,2)$, $(3,2)$, $(2,3)$, or $(2,1)$ there are up to four chiral families; finally for the pseudoreal cases $(i,j)=(1,3)$, $(3,1)$ and the real case $(2,2)$ there

TABLE VII. Chiral fermions for 16/13 with $\mathbf{4}=(1_3,1_4,2_1)$.

	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1			×	×	×		
1_2			×	×			×
1_3	×	×					×
1_4	×	×			×		
2_1		×	×		×	×	×
2_2					×	××	×
2_3	×			×	×	×	×

TABLE VIII. Chiral fermions for 16/13 with $\mathbf{4}=(2_1,2_1)$.

	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1					××		
1_2							××
1_3							××
1_4					××		
2_1		××	××			××	
2_2					××		××
2_3	××			××		××	

TABLE IX. Chiral fermions for 16/13 with $\mathbf{4}=(2_1,2_2)$.

	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1					$\times$	$\times$	
1_2						$\times$	$\times$
1_3						$\times$	$\times$
1_4					$\times$	$\times$	
2_1		$\times$	$\times$		$\times$	$\times$	$\times$
2_2	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$
2_3	$\times$			$\times$	$\times$	$\times$	$\times$

are, because of the mathematical theorem (and as we have now verified by direct calculation), no chiral fermions.

g=18. The nonpseudoreal groups number two, and one is a SDPG. In the notation of Thomas and Wood [12] they are 18/3 and 18/5. We now treat these in the order they were enumerated by Thomas and Wood.

Group 18/3; also designated $D_3\times Z_3$. This group has irreps which fall into six singlets 1, $1'$, 1α, $1'\alpha$, $1\alpha^2$, and $1'\alpha^2$ and three doublets 2, 2α, and $2\alpha^2$. Using the D_3 multiplication table from the Appendix, the embedding $\mathbf{4}=(1\alpha,1',2\alpha)$ yields the fermions of Table X. When $N=2$ there is sufficient chirality to provide three families but we will find the spontaneous symmetry breaking difficult to carry out.

Group 18/5; also designated $(Z_3\times Z_3)\tilde{\times}Z_2$. This group has two singlets 1 and $1'$ and four doublets 2_1, 2_2, 2_3, and 2_4. Using the multiplication table from the Appendix we compute the models corresponding to the three inequivalent embeddings $\mathbf{4}=(1',1',2_1)$, $\mathbf{4}=(2_1,2_1)$ and $\mathbf{4}=(2_1,2_2)$; however, all three turn out to be vectorlike. This is easy to understand when one realizes that all of the irreducible representations of 18/5 are individually either real or pseudoreal [12], making a complex embedding of **4** impossible.

g=20. There is one nonpseudoreal group, a SDPG. In the notation of Thomas and Wood [12] it is 20/5.

Group 20/5; also designated $Z_5\tilde{\times}Z_4$. The group has four singlets 1_1, 1_2, 1_3, and 1_4 and a 4. The singlets 1_1 and 1_3 are real, and the other two form a complex conjugate pair $1_2=1_4^*$. **6**, which is the antisymmetric product $\mathbf{6}=(4\times 4)_a$, must be real for a legitimate embedding. The two inequivalent choices, bearing in mind the multiplication table provided in the Appendix are $\mathbf{4}=(1_2,1_2,1_2,1_2)$ and $\mathbf{4}=(1_2,1_2,1_2,1_4)$.

TABLE X. Chiral fermions for 18/3 with $\mathbf{4}=(1\alpha,1',2\alpha)$.

	1	$1'$	2	1α	$1'\alpha$	2α	$1\alpha^2$	$1'\alpha^2$	$2\alpha^2$
1		$\times$		$\times$		$\times$			
$1'$	$\times$				$\times$	$\times$			
2			$\times$	$\times$	$\times$	$\times\times$			
1α					$\times$		$\times$		$\times$
$1'\alpha$				$\times$				$\times$	$\times$
2α						$\times$	$\times$	$\times$	$\times\times$
$1\alpha^2$	$\times$		$\times$					$\times$	
$1'\alpha^2$		$\times$	$\times$				$\times$		
$2\alpha^2$	$\times$	$\times$	$\times\times$						$\times$

TABLE XI. Chiral fermions for 20/5 with $\mathbf{4}=(1_2,1_2,1_2,1_2)$.

	1_1	1_2	1_3	1_4	4
1_1		$\times\times\times\times$			
1_2			$\times\times\times\times$		
1_3				$\times\times\times\times$	
1_4	$\times\times\times\times$				
4					$\times\times\times\times\times$

The first $\mathbf{4}=(1_2,1_2,1_2,1_2)$ yields the chiral fermions in Table XI. Putting $N=3$ this embedding gives four chiral families when we identify $SU(3)_3\equiv SU(3)_4$ and drop real representations, giving

$$4[(3,\bar{3},1)+(1,3,\bar{3})+(\bar{3},1,3)] \tag{20}$$

under $SU(3)\times SU(3)\times SU(3)$. This possibility for the 20/5 non-Abelian orbifold certainly merits further study. The symmetry breaking for this model will be investigated in Sec. V.

The second inequivalent embedding $\mathbf{4}=(1_2,1_2,1_2,1_4)$ gives rise to the fermions in Table XII. Identifying $SU(3)_3\equiv SU(3)_4$ as before for $N=3$ this is less chiral and gives rise to just two chiral families

$$2[(3,\bar{3},1)+(1,3,\bar{3})+(\bar{3},1,3)] \tag{21}$$

under $SU(3)\times SU(3)\times SU(3)$.

g=21. One nonpseudoreal group exists in this case: a SDPG. In the notation of Thomas and Wood [12], it is 21/2.

Group 21/2; also designated $Z_7\tilde{\times}Z_3$. This group has irreps which comprise three singlets 1_1, 1_2, and 1_3 and two triplets 3_1 and 3_2. With the embedding $\mathbf{4}=(1_2,3_1)$ (recall that 1_1 must be avoided to obtain $\mathcal{N}=0$), the resultant fermions are given in Table XIII.

Putting $N=2$, the gauge group is $SU(2)^3\times SU(6)^2$. Clearly the model is chiral, as seen in the asymmetry of the table. For example, put $SU(2)_L\equiv SU(2)_1$ and $SU(2)_R\equiv SU(2)_2$, break $SU(2)_3$ entirely, and use $\mathbf{6}_1\to\mathbf{4},\mathbf{6}_2\to\bar{\mathbf{4}}$ to find two chiral families.

g=24. The nonpseudoreal groups number six, and three are SDPG's. In the notation of Thomas and Wood [12] they are 24/7, 8, 9, 13, 14, and 15. So we now treat these in the order they were enumerated by Thomas and Wood.

TABLE XII. Chiral fermions for 20/5 with $\mathbf{4}=(1_2,1_2,1_2,1_4)$.

	1_1	1_2	1_3	1_4	4
1_1		$\times\times\times$		$\times$	
1_2	$\times$		$\times\times\times$		
1_3		$\times$		$\times\times\times$	
1_4	$\times\times\times$		$\times$		
4					$\times\times\times\times\times$

TABLE XIII. Chiral fermions for 21/2 with $\mathbf{4}=(1_2,3_1)$.

	1_1	1_2	1_3	3_1	3_2
1_1		$\times$		$\times$	
1_2			$\times$	$\times$	
1_3	$\times$			$\times$	
3_1				$\times\times$	$\times\times$
3_2	$\times$	$\times$	$\times$	$\times$	$\times\times$

Group 24/7; also designated $D_4\times Z_3$. This group has 12 singlets $1_1\alpha^i$, $1_2\alpha^i$, $1_3\alpha^i$, and $1_4\alpha^i$ $(i=0–2)$ and three doublets $2\alpha^i$ $(i=0–2)$; here $\alpha=\exp(i\pi/3)$. The embedding $\mathbf{4}=(1_1\alpha,1_2,2\alpha)$ was studied in detail in our previous paper [21] where it was shown how it can lead to precisely three chiral families in the standard model. For completeness we include the chiral fermions in Table XIV (it was presented in a different equivalent way in Ref. [21]).

By identifying $SU(4)$ with the diagonal subgroup of $SU(4)_{2,3}$, breaking $SU(4)_1$ to $SU(2)'_L\times SU(2)'_R$, then identifying $SU(2)_L$ with the diagonal subgroup of $SU(2)_{6,7,8}$ and $SU(2)'_L$ and $SU(2)_R$ with the diagonal subgroup of $SU(2)_{10,11,12}$ and $SU(2)'_R$, we are led to a three-family model as explained already in Ref. [21].

It is convenient to represent the chiral fermions in a quiver diagram [22] as shown in Fig. 1. This model is especially interesting because, uniquely among the large number of models examined in this study, the prescribed scalars are sufficient to break the gauge symmetry to that of the standard model.

Group 24/8; also designated $Q\times Z_3$. The multiplication tables of D_4 and Q and hence the multiplication tables of 24/7 and 24/8 are identical. Model building for 24/8 is therefore the same as 24/7 and merits no additional discussion.

Group 24/9; also designated $D_3\times Z_4$. This group generates one of the richest sets of chiral models in the class of

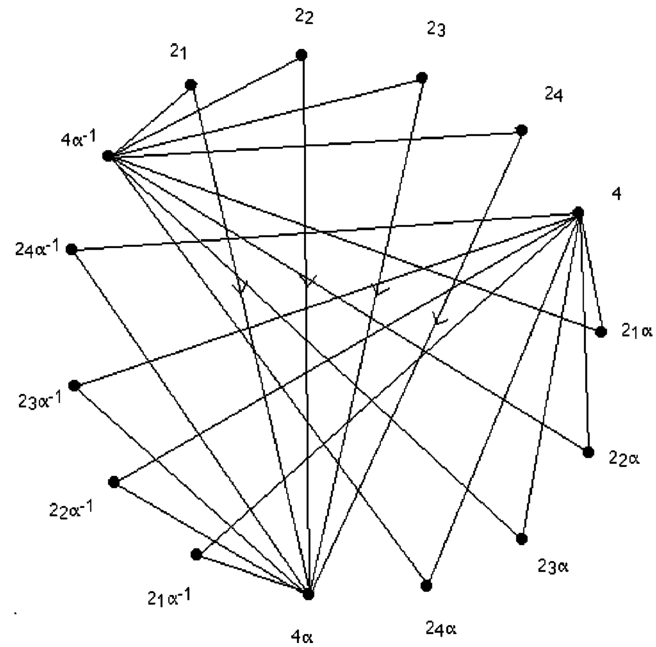

FIG. 1. Quiver diagram for chiral fermions in the 24/7 model.

models discussed in this paper. The group has, as irreps, eight singlets $(1_1\alpha^j,1_2\alpha^j)$ and four doublets $2\alpha^j$ $(j=0,1,2,3)$, where $\alpha=\exp(i\pi/4)$.

The embedding $\mathbf{4}=(1_1\alpha^{a_1},1_2\alpha^{a_2},2\alpha^{a_3})$ must satisfy $a_1\neq 0$ (for $\mathcal{N}=0$) and $a_1+a_2=-2a_3$ (mod 4) [to ensure the reality of $\mathbf{6}=(\mathbf{4}\times\mathbf{4})_a$]. There are several interesting possibilities including $(1_1\alpha,1_2\alpha,2\alpha)$, $(1_1\alpha,1_2\alpha^3,2\alpha^2)$, $(1_1\alpha^2,1_2,2\alpha^3)$, $(1_1\alpha^2,1_2,2\alpha)$, and $(1_1\alpha^2,1_2\alpha^2,2)$. The third and fourth cases are equivalent, as can be seen by letting α go to α^{-1}, and the last case has only real fermions since $\alpha^2=-1$, i.e., the fermions for $\mathbf{4}=(\mathbf{1_1}\alpha^{\mathbf{2}},\mathbf{1_2}\alpha^{\mathbf{2}},\mathbf{2})$ are vectorlike.

For 24/9 with $\mathbf{4}=(\mathbf{1_1}\alpha,\mathbf{1_2}\alpha^{\mathbf{3}},\mathbf{2}\alpha^{\mathbf{2}})$ we find the fermions are chiral and fall into the irreps as displayed in Table XV.

TABLE XIV. Chiral fermions for 24/7 with $\mathbf{4}=(1_1\alpha,1_2,2\alpha)$.

	1_1	1_2	1_3	1_4	2	$1_1\alpha$	$1_2\alpha$	$1_3\alpha$	$1_4\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$1_3\alpha^2$	$1_4\alpha^2$	$2\alpha^2$
1_1		$\times$				$\times$				$\times$					
1_2	$\times$						$\times$			$\times$					
1_3				$\times$				$\times$		$\times$					
1_4			$\times$						$\times$	$\times$					
2					$\times$	$\times$	$\times$	$\times$	$\times$	$\times$					
$1_1\alpha$							$\times$				$\times$				$\times$
$1_2\alpha$						$\times$						$\times$			$\times$
$1_3\alpha$									$\times$				$\times$		$\times$
$1_4\alpha$								$\times$						$\times$	$\times$
2α										$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$1_1\alpha^2$	$\times$				$\times$							$\times$			
$1_2\alpha^2$		$\times$			$\times$						$\times$				
$1_3\alpha^2$			$\times$		$\times$									$\times$	
$1_4\alpha^2$				$\times$	$\times$								$\times$		
$2\alpha^2$	$\times$	$\times$	$\times$	$\times$	$\times$										$\times$

TABLE XV. Chiral fermions for 24/9 with $\mathbf{4}=(\mathbf{1_1}\alpha,\mathbf{1_2}\alpha^3,\mathbf{2}\alpha^2)$.

	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1				×					×		×	
1_2					×				×	×		
2						×	×	×	×			×
$1_1\alpha$		×					×					×
$1_2\alpha$	×							×				×
2α			×						×	×	×	×
$1_1\alpha^2$			×		×					×		
$1_2\alpha^2$			×	×							×	
$2\alpha^2$	×	×	×			×						×
$1_1\alpha^3$	×					×		×				
$1_1\alpha^3$		×				×	×					
$2\alpha^3$			×	×	×	×			×			

TABLE XVI. Chiral fermions for 24/9 with $\mathbf{4}=(1_1\alpha,1_2\alpha,2\alpha)$.

	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1				×	×	×						
1_2				×	×	×						
2				×	×	×××						
$1_1\alpha$							×	×	×			
$1_2\alpha$							×	×	×			
2α							×	×	×××			
$1_1\alpha^2$										×	×	×
$1_2\alpha^2$										×	×	×
$2\alpha^2$										×	×	×××
$1_1\alpha^3$	×	×	×									
$1_2\alpha^3$	×	×	×									
$2\alpha^3$	×	×	×××									

TABLE XVII. Chiral fermions for 24/9 with $\mathbf{4}=(1_1\alpha^2,1_2,2\alpha)$.

	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1		×				×	×					
1_2	×					×		×				
2			×	×	×	×			×			
$1_1\alpha$					×				×	×		
$1_2\alpha$				×					×		×	
2α						×	×	×	×			×
$1_1\alpha^2$	×							×				×
$1_2\alpha^2$		×					×					×
$2\alpha^2$			×						×	×	×	×
$1_1\alpha^3$			×	×							×	
$1_2\alpha^3$			×		×					×		
$2\alpha^3$	×	×	×			×						×

TABLE XVIII. Chiral fermions for 24/9 with $\mathbf{4}=(2\alpha,2\alpha)$.

	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1						$\times\times$						
1_2						$\times\times$						
2				$\times\times$	$\times\times$	$\times\times$						
$1_1\alpha$									$\times\times$			
$1_2\alpha$									$\times\times$			
2α							$\times\times$	$\times\times$	$\times\times$			
$1_1\alpha^2$												$\times\times$
$1_2\alpha^2$												$\times\times$
$2\alpha^2$										$\times\times$	$\times\times$	$\times\times$
$1_1\alpha^3$			$\times\times$									
$1_2\alpha^3$			$\times\times$									
$2\alpha^3$	$\times\times$	$\times\times$	$\times\times$									

With the embedding $\mathbf{4}=(1_1\alpha,1_2\alpha,2\alpha)$, the chiral fermions are given in Table XVI. Identifying $SU(2)_L$ with the diagonal subgroup of $SU(2)_{1,2,3,4}$, $SU(2)_R$ with the diagonal subgroup of $SU(2)_{5,6,7,8}$ and the $\mathbf{4}$ of $SU(4)$ with the $\mathbf{4}$ of $SU(4)_{2,3}$ and the $\bar{\mathbf{4}}$ of $SU(4)_{1,4}$ leads to eight chiral families.

Taking the embedding $\mathbf{4}=(1_1\alpha^2,1_2,2\alpha)$ gives as chiral fermions of Table XVII. We identify $SU(2)_L$ and $SU(2)_R$ with the diagonal subgroups of $SU(2)_{1,2}$ and $SU(2)_{3,4}$, respectively, and completely break $SU(2)_{5,6,7,8}$. The generalized color embedding $\mathbf{4}\equiv\mathbf{4_1}\equiv\mathbf{4_2}\equiv\bar{\mathbf{4}}_\mathbf{3}\equiv\bar{\mathbf{4}}_\mathbf{4}$ leads to four chiral families. This can be reduced to three families by further symmetry breaking using the same idea as in Ref. [21]. An even more interesting embedding for 24/9 is to set $\mathbf{4}=(2\alpha,2\alpha)$ which gives a real $\mathbf{6}$ as required (since $\alpha^2=-1$ is real). See Table XVIII for the corresponding fermions. Identifying $SU(2)_L$ with the diagonal subgroup of $SU(2)_{1,3,5,7}$, $SU(2)_R$ with the diagonal subgroup of $SU(2)_{2,4,6,8}$, breaking $SU(4)_{1,3}$, and keeping the unbroken $SU(4)$ which is the diagonal subgroup of $SU(4)_{2,4}$ gives rise to eight chiral families:

$$8[(2,1,\bar{4})+(1,2,4)]. \tag{22}$$

The possibility of achieving the relevant symmetry breaking will be examined below in Sec. V.

Group 24/13; also designated $Q\tilde{\times}Z_3$. This group has three singlets 1_1, 1_2, and 1_3, three doublets 2_1, 2_2, and 2_3, and one triplet 3. For $N=2$ the gauge group is therefore $SU(2)^3\times SU(4)^3\times SU(6)$.

With the embedding $\mathbf{4}=(2_1,2_2)$ the chiral fermions are those of Table XIX. If we identify $SU(2)_L\equiv SU(2)_3$, $SU(2)_R\equiv SU(2)_2$, and break $SU(2)_1$ there are two chiral families for $\mathbf{4}\equiv\mathbf{4_1}\equiv\bar{\mathbf{4}}_\mathbf{2}\equiv\bar{\mathbf{4}}_\mathbf{3}$. However, for $N=3$ or larger there is more chirality (the total number of chiral states increases) when we increase N, and therefore the opportunities for model building increase. While we ignore these possibilities here, they are straightforward to investigate since the tables of fermions we display are independent on N. This is also why we display the fermions when the $N=2$ case leads to less than three families. If, instead, we embed $\mathbf{4}=(2_2,2_3)$ the fermions are manifestly nonchiral.

Group 24/14; also designated $Z_8\tilde{\times}Z_3$. There are eight singlets and four doublets, with multiplication table as in the Appendix. With the embedding $\mathbf{4}=(2_2,2_4)$ one arrives at an arrangement with no chiral families.

A chiral embedding occurs at $\mathbf{4}=(2_1,2_2)$ giving rise to the fermions of Table XX. For $N=2$, if we identify $SU(2)_L$ as the diagonal subgroup of $SU(2)_{1,2,5,6}$ and $SU(2)_R$ as the diagonal subgroup of $SU(2)_{3,4,7,8}$, then identify the $\mathbf{4}$ of $SU(4)$ with the $\mathbf{4}$ of $SU(4)_{2,3}$ and the $\bar{\mathbf{4}}$ of $SU(4)_{1,4}$, this model has eight chiral families under $SU(2)_L\times SU(2)_R\times SU(4)$.

Group 24/15; also designated $D_4\tilde{\times}Z_3$. The group 24/15 has nine inequivalent irreducible representations, four singlets and five doublets. With the embedding $\mathbf{4}=(2_3,2_5)$, the fermions are shown in Table XXI.

Identifying $SU(2)_L\equiv SU(2)_{1,3}$ and $SU(2)_R\equiv SU(2)_{2,4}$ gives rise to two chiral families for $N=2$. Another chiral embedding is $\mathbf{4}=(1_2,1_3,2_3)$ which gives the chiral fermions of Table XXII.

Identifying $SU(2)_L$ with the diagonal subgroup of 1_1 and 1_3, $SU(2)_R$ with 1_2 and 1_4, then identifying $2_3=\mathbf{4}$ and $2_4=\bar{\mathbf{4}}$, and finally breaking the other three $SU(4)$'s gives rise to six chiral families for $N=2$.

As an alternative 24/15 model we can embed $\mathbf{4}=(2_3,2_3)$ and obtain the fermions of Table XXIII. With

TABLE XIX. Chiral fermions for 24/13 with $\mathbf{4}=(2_1,2_2)$.

	1_1	1_2	1_3	2_1	2_2	2_3	3
1_1				$\times$	$\times$		
1_2					$\times$	$\times$	
1_3				$\times$		$\times$	
2_1	$\times$	$\times$					$\times\times$
2_2		$\times$	$\times$				$\times\times$
2_3	$\times$		$\times$				$\times\times$
3				$\times\times$	$\times\times$	$\times\times$	

TABLE XX. Chiral fermions for 24/14 with $\mathbf{4}=(2_1,2_2)$.

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2	2_3	2_4
1_1									×	×		
1_2										×	×	
1_3											×	×
1_4									×			×
1_5									×	×		
1_6										×	×	
1_7											×	×
1_8									×			×
2_1	×	×			×	×			×	×		
2_2		×	×			×	×			×	×	
2_3			×	×			×	×			×	×
2_4	×			×	×			×	×			×

TABLE XXI. Chiral fermions for 24/15 with $\mathbf{4}=(2_3,2_5)$.

	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1							×		×
1_2								×	×
1_3							×		×
1_4								×	×
2_1							×	××	×
2_2							××	×	×
2_3		×		×	××	×			
2_4	×		×		×	××			
2_5	×	×	×	×	×	×			

TABLE XXII. Chiral fermions for 24/15 with $\mathbf{4}=(1_2,1_3,2_3)$.

	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1		×	×				×		
1_2	×			×				×	
1_3	×			×			×		
1_4		×	×					×	
2_1					×	×		×	×
2_2					×	×	×		×
2_3		×		×	×		×	×	
2_4	×		×			×	×	×	
2_5					×	×			××

TABLE XXIII. Chiral fermions for 24/15 with $\mathbf{4}=(2_3,2_3)$.

	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1							××		
1_2								××	
1_3							××		
1_4								××	
2_1								××	××
2_2							××		××
2_3		××		××	××				
2_4	××		××			××			
2_5					××	××			

$SU(2)_L$ and $SU(2)_R$ as diagonal subgroups of $SU(2)_1 \times SU(2)_3$ and $SU(2)_2 \times SU(2)_4$ respectively, and breaking $SU(4)_4$ completely, this leads to four chiral families when $N=2$.

g=27. The non-pseudoreal groups number two and both are SDPG's. In the notation of Thomas and Wood [12] they are 27/4 and 27/5, and we treat them in that order.

Group 27/4; also designated $Z_9 \tilde{\times} Z_3$. 27/4 has nine singlet $1_1, \ldots, 1_9$ and two triplet 3_1 and 3_2 irreducible representations. We may choose the embedding $4=(1_2,3_1)$ for which the chiral fermions become those listed in Table XXIV.

Putting $N=2$, the gauge group is $SU(2)^9 \times SU(6)_1 \times SU(6)_2$ and the chiral fermions are

$$\left(\sum_{i=1}^{i=9} 2_i, \overline{6_1}\right) + (6_1, \overline{6_1} + 3(\overline{6_2})) + \left(6_2, \sum_{i=1}^{i=9} 2_i\right) + (6_2, \overline{6_2}). \tag{23}$$

Though asymmetric in representations, this result is anomaly free with respect to both $SU(6)_1$ and $SU(6)_2$.

Group 27/5; also designated $(Z_3 \times Z_3) \tilde{\times} Z_3$. The multiplication tables, and hence the model building, are identical for 27/4 and 27/5. The group 27/5 merits no separate further discussion.

TABLE XXIV. Chiral fermions for 27/4 with $\mathbf{4}=(1_2,3_1)$.

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_1		×								×	
1_2			×							×	
1_3	×									×	
1_4					×					×	
1_5						×				×	
1_6				×						×	
1_7								×		×	
1_8									×	×	
1_9							×			×	
3_1										×	×××
3_2	×	×	×	×	×	×	×	×	×		×

TABLE XXV. Chiral fermions for 30/2 with $\mathbf{4}=(1\alpha,1',2\alpha)$.

	1	$1'$	2	$2'\alpha$	1α	$1'\alpha$	2α	$2'\alpha$	$1\alpha^2$	$1'\alpha^2$	$2\alpha^2$	$2'\alpha^2$
1		$\times$			$\times$		$\times$					
$1'$	$\times$					$\times$	$\times$					
2			$\times$		$\times$	$\times$	$\times$	$\times$				
$2'$				$\times$			$\times$	$\times\times$				
1α						$\times$			$\times$		$\times$	
$1'\alpha$					$\times$					$\times$	$\times$	
2α							$\times$		$\times$	$\times$	$\times$	$\times$
$2'\alpha$								$\times$			$\times$	$\times\times$
$1\alpha^2$	$\times$		$\times$							$\times$		
$1'\alpha^2$		$\times$	$\times$						$\times$			
$2\alpha^2$	$\times$	$\times$	$\times$	$\times$							$\times$	
$2'\alpha^2$			$\times$	$\times\times$								$\times$

g=30. The nonpseudoreal groups number two, and neither is a SDPG. In the notation of [12], they are 30/2 and 30/3. We now treat these in the order they are enumerated by Thomas and Wood.

Group 30/2; also designated $D_5\times Z_3$. 30/2 has six singlets $1\alpha^i, 1'\alpha^i$ and six doublets $2\alpha^i$, $2'\alpha^i$ with $\alpha=\exp(i\pi/3)$ and $i=0,1,2$. Choosing $\mathbf{4}=(1\alpha,1',2\alpha)$ yields the fermions of Table XXV.

Identifying $SU(2)_L$ with the diagonal subgroup of $SU(2)_1\times SU(2)_2$ (associated with $1,1'$) and $SU(2)_R$ with the diagonal subgroup of $SU(2)_5\times SU(2)_6$ (associated with $1\alpha^2,1'\alpha^2$), we break the $SU(4)$'s associated with 2 and $2\alpha^2$ to arrive at two chiral families when $N=2$.

Group 30/3; also designated $D_3\times Z_5$. This group has irreps which comprise ten singlets and five doublets and yields, for $N=2$, the gauge group $SU(2)^{10}\times SU(4)^5$. As we have encountered for groups $D_3\times Z_p$ (with $g=6p$), the embedding $\mathbf{4}=(1\alpha^{a_1},1'\alpha^{a_2},2\alpha^{a_3})$ must satisfy $a_1+a_2=-2a_3 \pmod p$ for consistency, as well as $a_1\neq 0$ to ensure $\mathcal{N}=0$. There are several interesting such examples, one of which is $\mathbf{4}=(1\alpha,1',2\alpha^2)$ where Table XXVI displays the fermions.

In an obvious notation, the chiral fermions are

$$
\begin{aligned}
&(2_1+2_2,\overline{4}_3+4_4)+(2_3+2_4,\overline{4}_4+4_5)+(2_5+2_6,\overline{4}_5+4_1)\\
&\quad+(2_7+2_8,\overline{4}_1+4_2)+(2_9+2_{10},\overline{4}_2+4_3).
\end{aligned}
\tag{24}
$$

Identifying, for example (there are equivalent cyclic permutations), $SU(2)_L$ as the diagonal subgroup of $SU(2)_1\times SU(2)_2\times SU(2)_7\times SU(2)_8$, $SU(2)_R$ as the diagonal subgroup of $SU(2)_5\times SU(2)_6\times SU(2)_9\times SU(2)_{10}$, and the generalized color $SU(4)$ as the diagonal subgroup of

TABLE XXVI. Chiral fermions for 30/3 with $\mathbf{4}=(1\alpha,1',2\alpha^2)$.

	1	$1'$	2	1α	$1'\alpha$	2α	$1\alpha^2$	$1'\alpha^2$	$2\alpha^2$	$1\alpha^3$	$1'\alpha^3$	$2\alpha^3$	$1\alpha^4$	$1'\alpha^4$	$2\alpha^4$
1		$\times$		$\times$					$\times$						
$1'$	$\times$				$\times$				$\times$						
2			$\times$			$\times$	$\times$	$\times$	$\times$						
1α					$\times$		$\times$					$\times$			
$1'\alpha$				$\times$				$\times$				$\times$			
2α						$\times$			$\times$	$\times$	$\times$	$\times$			
$1\alpha^2$								$\times$		$\times$					$\times$
$1'\alpha^2$							$\times$				$\times$				$\times$
$2\alpha^2$									$\times$			$\times$	$\times$	$\times$	$\times$
$1\alpha^3$			$\times$								$\times$		$\times$		
$1'\alpha^3$			$\times$							$\times$				$\times$	
$2\alpha^3$	$\times$	$\times$	$\times$									$\times$			$\times$
$1\alpha^4$	$\times$					$\times$								$\times$	
$1'\alpha^4$		$\times$				$\times$							$\times$		
$2\alpha^4$			$\times$	$\times$	$\times$	$\times$									$\times$

$SU(4)_1 \times SU(4)_3$, and completely breaking $SU(4)_{2,4,5}$, gives rise to four chiral families.

We can examine the infinite series $D_3 \times Z_p$ for $p \geqslant 3$ (as necessary for nonpseudoreality). The order is $g=6p$. By generalizing the above discussions of 18/3 ($D_3 \times Z_3$), 24/9 ($D_3 \times Z_4$) and 30/3 ($D_3 \times Z_5$) we find that with the same type of embedding one arrives at a maximal number of $2[p/2]$ chiral families where $[x]$ is the largest integer not greater than x. For example, with $p=3,4,5,6,7,8,9,10, \ldots$ one obtains 2,4,4,6,6,8,8,10 . . . chiral families respectively. This is an example of accessing the more difficult non-Abelian Γ with $g \geqslant 32$ at least for orders $g=6p \geqslant 36$.

That completes the analysis of the occurrence of chiral fermions for Γ with $g \leqslant 31$. For the cases where there are $\geqslant 3$ chiral families, it remains to check whether the spectrum of complex scalars is sufficient to allow spontaneous symmetry breaking to the standard model gauge group. This is the subject of Secs. V and VI.

V. SCALAR SECTOR

In order to carry out spontaneous symmetry breaking in the chiral models we found in Sec. IV, we must first extract the scalar sector from Eq. (5), where **6** is obtained from the embedding of $(\mathbf{4}\times\mathbf{4})_A$, which in turn follows from the embedding of **4**. We only consider models of phenomenological interest, i.e., those which potentially have three or more families, but preferably three. With this perspective in mind we first collect the following models:

16/8 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_1)$ and $\chi=2^8$ with $N=2$.

16/8 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_5,\mathbf{2}_1)$ and $\chi=2^7$ with $N=2$.

16/11 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2)$ and $\chi=432$ with $N=3$.

16/11 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_4)$ and $\chi=216$ with $N=3$.

16/13 with $\mathbf{4}=(\mathbf{1}_3,\mathbf{1}_4,\mathbf{2}_1)$ and $\chi=2^6$ with $N=2$.

16/13 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_2)$ and $\chi=2^6$ with $N=2$.

16/13 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_1)$ and $\chi=2^7$ with $N=2$.

18/3 with $\mathbf{4}=(\mathbf{1}\alpha,\mathbf{1}',\mathbf{2}\alpha)$ and $\chi=192$ with $N=2$.

20/5 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2)$ and $\chi=144$ with $N=3$.

20/5 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_4)$ and $\chi=72$ with $N=3$.

21/2 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{3}_1)$ and $\chi=108$ with $N=2$.

24/7 with $\mathbf{4}=(\mathbf{1}\alpha,\mathbf{1}',\mathbf{2}\alpha)$ and $\chi=240$ with $N=2$.

24/9 with $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2\alpha^3,\mathbf{2}\alpha^2)$ and $\chi=320$ with $N=2$.

24/9 with $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2\alpha,\mathbf{2}\alpha)$ and $\chi=320$ with $N=2$.

24/9 with $\mathbf{4}=(\mathbf{1}_1\alpha^2,\mathbf{1}_2,\mathbf{2}\alpha)$ and $\chi=192$ with $N=2$.

24/9 with $\mathbf{4}=(\mathbf{2}\alpha,\mathbf{2}\alpha)$ and $\chi=384$ with $N=2$.

24/13 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_2)$ and $\chi=48$ with $N=2$.

24/14 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_2)$ and $\chi=192$ with $N=2$.

24/15 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_3,\mathbf{2}_3)$ and $\chi=2^7$ with $N=2$.

24/15 with $\mathbf{4}=(\mathbf{2}_3,\mathbf{2}_5)$ and $\chi=2^7$ with $N=2$.

24/15 with $\mathbf{4}=(\mathbf{2}_3,\mathbf{2}_3)$ and $\chi=2^8$ with $N=2$.

27/4 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{3}_1)$ and $\chi=324$ with $N=2$.

30/2 with $\mathbf{4}=(\mathbf{1}\alpha,\mathbf{1}',\mathbf{2}\alpha)$ and $\chi=336$ with $N=2$.

30/3 with $\mathbf{4}=(\mathbf{1}\alpha,\mathbf{1}',\mathbf{2}\alpha^2)$ and $\chi=320$ with $N=2$.

First we consider *16/8 with* $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{2}_1)$, where we have included this example to demonstrate improper embedding. This representation is complex and would be expected to lead to chiral fermions, but $\mathbf{6}=(\mathbf{4}\times\mathbf{4})_A=\mathbf{1}_1+2(\mathbf{2}_1+\mathbf{2}_1)+(\mathbf{1}_5+\mathbf{1}_6+\mathbf{1}_7+\mathbf{1}_8)_A$ is complex (for any choice of singlet in the last parenthetical expression), and therefore the embedding $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{2}_1)$ is improper and we need not consider this or other such models further.

Let us define the chirality measure χ of a model as the number of chiral fermion states. This variable applies to any irreps and provides a somewhat finer measure of chirality than the number of families. As spontaneous symmetry breaking proceeds, χ decreased (except under unusual circumstances). For instance, the standard model and minimal $SU(5)$ both initially have $\chi=45$. By the time the symmetry is broken to $SU(3)\times U_{EM}(1)$, $\chi=3$ since the neutrino's cannot acquire mass due to global *B-L* symmetry. On the other hand, three family $SO(10)$ and E_6 models start with $\chi=48$ and 81 respectively but both break to $\chi=0$.

In model building with AdS/CFT's we are faced with a number of choices. If we require the initial model be chiral before SSB, then we need $\chi \geqslant 45$ initially. However, since the scale of SSB M_{AdS} in these models can be relatively low (a few tens of TeV), vectorlike models are more appealing than usual, and we could allow an initial $\chi=0$ without resorting to incredibly detailed fine tunings. Our prejudice is to still require a chiral model with $\chi \geqslant 45$ initially in order to gain some control in model building, but we want to make it clear that, even though we have not displayed them explicitly, the entire class of vectorlike model based on the non-Abelian orbifold classification given here would be worthy of detailed study. There are also models (chiral or vectorlike) that break from G_{AdS} to $SU(3)\times U_{EM}(1)$ but without going through $SU(3)\times SU(2)\times U(1)$ directly. As M_{AdS} may be not far above M_Z, there may be models in this class that could be in agreement with current data, but again we restrict most of our discussion to chiral models that break through

TABLE XXVII. The scalars for 16/8 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_1)$, which fixes the $\mathbf{6}$ to be $\mathbf{6}=3(\mathbf{1}_5)+\mathbf{1}_6+\mathbf{1}_7+\mathbf{1}_8$. (Below we give only the embedding of the **4** in the table captions, as it fixes the **6** and hence the scalars.)

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1					$\times\times\times$	$\times$	$\times$	$\times$		
1_2					$\times$	$\times$	$\times$	$\times$		
1_3					$\times$	$\times$	$\times$	$\times$		
1_4					$\times$	$\times$	$\times$	$\times\times\times$		
1_5	$\times\times\times$	$\times$	$\times$	$\times$						
1_6	$\times$	$\times$	$\times$	$\times$						
1_7	$\times$	$\times$	$\times$	$\times$						
1_8	$\times$	$\times$	$\times$	$\times\times\times$						
2										$\times\times\times$
										$\times\times\times$
$2'$									$\times\times\times$	
									$\times\times\times$	

TABLE XXVIII. The scalars for 16/8 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_{4+i},\mathbf{2}_1)$.

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1					$\times(5)$	(6)			$\times$	$\times$
1_2					(6)	$\times(5)$			$\times$	$\times$
1_3							$\times(5)$	(6)	$\times$	$\times$
1_4							(6)	$\times(5)$	$\times$	$\times$
1_5	$\times(5)$	(6)							$\times$	$\times$
1_6	(6)	$\times(5)$							$\times$	$\times$
1_7			$\times(5)$	(6)					$\times$	$\times$
1_8			(6)	$\times(5)$					$\times$	$\times$
2	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times\times$
$2'$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times\times$	

TABLE XXIX. The scalars for 16/11 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2)$.

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1			$(\times)^6$							
1_2		$(\times)^6$								
1_3	$(\times)^6$									
1_4				$(\times)^6$						
1_5							$(\times)^6$			
1_6						$(\times)^6$				
1_7					$(\times)^6$					
1_8								$(\times)^6$		
2									$(\times)^6$	
$2'$										$(\times)^6$

TABLE XXX. The scalars for 16/11 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_4)$.

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1	$(\times)^3$		$(\times)^3$							
1_2		$(\times)^6$								
1_3	$(\times)^3$		$(\times)^3$							
1_4				$(\times)^6$						
1_5					$(\times)^3$		$(\times)^3$			
1_6						$(\times)^6$				
1_7					$(\times)^3$		$(\times)^3$			
1_8								$(\times)^6$		
2									$(\times)^6$	
$2'$										$(\times)^6$

the standard model. What is encouraging is the fact that orbifold AdS/CFT's provide such a wealth of potentially interesting models.

16/8 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_1)$. Here $\mathbf{6}=3(\mathbf{1}_5)+\mathbf{1}_6+\mathbf{1}_7+\mathbf{1}_8$ which is real so the embedding is proper and the scalar sector is given in Table XXVII.

16/8 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_{4+i},\mathbf{2}_1)$ and $\mathbf{6}=[\mathbf{1}_{x(i)},\mathbf{2},\mathbf{2}',(\mathbf{1}_5+\mathbf{1}_6+\mathbf{1}_7+\mathbf{1}_8)_{\mathbf{A}}]$, where $x=6$, 5, 8, or 7 for $i=1$, 2, 3, and 4. The fermionic sectors of these models are identical up to permutation, but there are two potential types of scalar sectors, depending on whether $\mathbf{1}_{x(i)}$ is the same as or different from the antisymmetric product $(\mathbf{2}_1\times\mathbf{2}_1)_A$. Let us relabel the singlets so $(\mathbf{2}_1\times\mathbf{2}_1)_A=\mathbf{1}_6$, and then choose $\mathbf{1}_{x(i)}$ to be either $\mathbf{1_5}$ or $\mathbf{1_6}$. Now the two inequivalent scalar sectors [in this instance, it is easier to analyze both models and show that neither phenomenology is interesting, rather than untangle the correct antisymmetric singlet in $(\mathbf{2}_1\times\mathbf{2}_1)_A$; see Sec. VI] are shown in Table XXVIII. Here (5) is replaced by an "$\times$" and (6) by a blank if $\mathbf{1}_{x(i)}=\mathbf{1_5}$, and vice versa if $\mathbf{1}_{x(i)}=\mathbf{1_6}$.

For 16/11 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2)$ and $\mathbf{6}=(\mathbf{1}_3,\mathbf{1}_3,\mathbf{1}_3,\mathbf{1}_3,\mathbf{1}_3,\mathbf{1}_3)$ we find the scalar of Table XXIX. The other interesting case for 16/11 is where $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_4)$ and $\mathbf{6}=(\mathbf{1}_1,\mathbf{1}_1,\mathbf{1}_1,\mathbf{1}_3,\mathbf{1}_3,\mathbf{1}_3)$ which leads to the scalars of Table XXX.

Moving on to 16/13 there are three cases of interest. The first is when $\mathbf{4}=(\mathbf{1}_3,\mathbf{1}_4,\mathbf{2}_1)$ and $\mathbf{6}=(\mathbf{1}_2,\mathbf{1}_c,\mathbf{2}_1,\mathbf{2}_3)$, where $\mathbf{1}_c=(\mathbf{2}_1\times\mathbf{2}_1)_{\mathbf{A}}$ and we have $\mathbf{1}_c$ is either $\mathbf{1}_2$ or $\mathbf{1}_3$ (unresolved here, but see Sec. VI) giving the scalars of Table XXXI.

Next, for 16/13, where $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_2)$ has $\mathbf{6}=(\mathbf{1}_a,\mathbf{1}_b,\mathbf{2}_1,\mathbf{2}_3)$, and where $\mathbf{1}_a=(\mathbf{2}_1\times\mathbf{2}_1)_{\mathbf{A}}=(1_2+1_3+2_2)_{\mathbf{A}}$ and $\mathbf{1}_b=(\mathbf{2}_2\times\mathbf{2}_2)_{\mathbf{A}}+(1_1+1_2+1_3+1_4)_{\mathbf{A}}$, we find the scalars in Table XXXII. Here we insert $\times$'s at the locations in parentheses when the singlets are chosen properly from the antisymmetric products of the doublets. There are three inequivalent choices: either (i) put $\times\times$ at location (2), (ii) put an $\times$ at (2) and one at (3), or (iii) put $\times$ at (2) and $\times$ at (1). All other choices lead to equivalent models. Thus without a detailed knowledge of the antisymmetric products, we can still reduce the analysis to the consideration of these three cases.

Finally for 16/13 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_1)$ and $\mathbf{6}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_3,\mathbf{2}_2)$ [which is equivalent to $\mathbf{6}=(\mathbf{1}_2,\mathbf{1}_3,\mathbf{1}_3,\mathbf{1}_3,\mathbf{2}_2)$ for SSB up to a relabeling of irreps] the scalar are given in Table XXXIII.

Moving on to 18/3 there is only one case of interest where $\mathbf{4}=(1'\alpha,1',2\alpha)$ and $\mathbf{6}=(1'\alpha,2\alpha,2\alpha^2,1'\alpha^2)$ and the scalars are in Table XXXIV.

Proceeding to 20/5 there are two interesting cases; the first has $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2)$ and $\mathbf{6}=(\mathbf{1}_3,\mathbf{1}_3,\mathbf{1}_3,\mathbf{1}_3,\mathbf{1}_3,\mathbf{1}_3)$ with the scalars shown in Table XXXV and is very much like the 16/11 model with similar embedding. Note that a VEV for any of these scalars renders the entire fermion sector vectorlike. The second example is 20/5 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_4)$ and $\mathbf{6}=(\mathbf{1}_1,\mathbf{1}_1,\mathbf{1}_1,\mathbf{1}_3,\mathbf{1}_3,\mathbf{1}_3)$ where we have the scalars of Table XXXVI.

Again at order 21 there is only one chiral set of models. It is 21/2 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{3}_1)$ and $\mathbf{6}=\mathbf{3}_1+\mathbf{3}_2$ which is real so the embedding is proper, and where the scalar sector is shown in Table XXXVII. (All other embeddings of the **4** with chiral fermions and $\mathcal{N}=0$ supersymmetry permutations are equivalent to this model.)

TABLE XXXI. The scalars for 16/13 with $\mathbf{4}=(\mathbf{1}_3,\mathbf{1}_4,\mathbf{2}_1)$.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1		$\times(2)$	(3)		$\times$		$\times$
1_2	$\times(2)$			(3)	$\times$		$\times$
1_3	(3)			$\times(2)$	$\times$		$\times$
1_4		(3)	$\times(2)$		$\times$		$\times$
2_1	$\times$	$\times$	$\times$	$\times$		$\times\times$	$\times\times$
2_2					$\times\times$	$\times\times$	$\times\times$
2_3	$\times$	$\times$	$\times$	$\times$	$\times\times$	$\times\times$	

TABLE XXXII. The scalars for 16/13 with $\mathbf{4}=(2_1,2_2)$.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1	(1)	(2)	(3)	(4)	$\times$		$\times$
1_2	(2)	(1)	(4)	(3)	$\times$		$\times$
1_3	(3)	(4)	(1)	(2)	$\times$		$\times$
1_4	(4)	(3)	(2)	(1)	$\times$		$\times$
2_1	$\times$	$\times$	$\times$	$\times$	(1)(4)	$\times\times$	(2)(3)
2_2					$\times\times$	(1)(2) (3)(4)	$\times\times$
2_3	$\times$	$\times$	$\times$	$\times$	(2)(3)	$\times\times$	(1)(4)

TABLE XXXIII. The scalars for 16/13 with $\mathbf{4}=(2_1,2_1)$.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1		$\times\times\times$	$\times$			$\times$	
1_2	$\times\times\times$					$\times$	
1_3	$\times$			$\times\times\times$		$\times$	
1_4			$\times\times\times$			$\times$	
2_1					$\times$		$\times\times$ $\times\times\times$
2_2	$\times$	$\times$	$\times$	$\times$		$\times\times$ $\times\times$	
2_3					$\times\times$ $\times\times\times$		$\times$

TABLE XXXIV. The scalars for 18/3 with $4=(1'\alpha,1',2\alpha)$.

$\otimes$	1	$1'$	2	1α	$1'\alpha$	2α	$1\alpha^2$	$1'\alpha^2$	$2\alpha^2$
1					$\times$	$\times$		$\times$	$\times$
$1'$				$\times$		$\times$	$\times$		$\times$
2				$\times$	$\times$	$\times\times$	$\times$	$\times$	$\times\times$
1α		$\times$	$\times$					$\times$	$\times$
$1'\alpha$	$\times$		$\times$				$\times$		$\times$
2α	$\times$	$\times$	$\times\times$				$\times$	$\times$	$\times\times$
$1\alpha^2$		$\times$	$\times$		$\times$	$\times$			
$1'\alpha^2$	$\times$		$\times$	$\times$		$\times$			
$2\alpha^2$	$\times$	$\times$	$\times\times$	$\times$	$\times$	$\times\times$			

TABLE XXXV. The scalars for 20/5 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2)$.

$\otimes$	1_1	1_2	1_3	1_4	4
1_1			$(\times)^6$		
1_2				$(\times)^6$	
1_3	$(\times)^6$				
1_4		$(\times)^6$			
4					

TABLE XXXVI. The scalars for 20/5 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_4)$.

$\otimes$	1_1	1_2	1_3	1_4	4
1_1	$\times\times\times$		$\times\times\times$		
1_2		$\times\times\times$		$\times\times\times$	
1_3	$\times\times\times$		$\times\times\times$		
1_4		$\times\times\times$		$\times\times\times$	
4					$(\times)^6$

TABLE XXXVII. The scalars for 21/2 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{3}_1)$.

$\otimes$	1_1	1_2	1_3	3_1	3_2
1_1				$\times$	$\times$
1_2				$\times$	$\times$
1_3				$\times$	$\times$
3_1	$\times$	$\times$	$\times$	$\times\times$	$\times\times\times$
3_2	$\times$	$\times$	$\times$	$\times\times\times$	$\times\times$

For 24/7 (or equivalently 24/8 since they have isomorphic irrep product tables) we have the model of Ref. [21] where $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2,\mathbf{2}\alpha)$ and $\mathbf{6}=(\mathbf{1}_2\alpha,\mathbf{1}_2\alpha^2,\mathbf{2}\alpha,\mathbf{2}\alpha^2)$, and the scalars are given by Table XXXVIII.

The next group of interest at order 24 is 24/9 where first we study the case with $\mathbf{4}=(\mathbf{1_1}\alpha,\mathbf{1_2}\alpha^3,\mathbf{2}\alpha^2)$ and $\mathbf{6}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{2}\alpha,\mathbf{2}\alpha^3)$ where the scalars are shown in Table XXXIX. Next the scalars for 24/9 are given in Table XL, when $\mathbf{4}=(\mathbf{1_1}\alpha,\mathbf{1_2}\alpha,\mathbf{2}\alpha)$ and $\mathbf{6}=(\mathbf{1}_2\alpha,\mathbf{2},\mathbf{1}_2\alpha,\mathbf{2}\alpha^2)$. Proceeding to 24/9 with $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2,\mathbf{2}\alpha)$ and $\mathbf{6}=(\mathbf{1}_2\alpha^2,\mathbf{2}\alpha,\mathbf{2}\alpha^{-1},\mathbf{1}_2\alpha^{-2})$ where $\alpha^4=1$, Table XLI provides the scalar sector. Finally there is the 24/9 case involving only doublets where $\mathbf{4}=(\mathbf{2}\alpha,\mathbf{2}\alpha)$ and $\mathbf{6}=3(\mathbf{1}_2\alpha^2)+\mathbf{1}_1\alpha^2+\mathbf{2}\alpha^2$, and the scalars are collected in Table XLII.

The next example of interest is 24/13 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_2)$ and $\mathbf{6}=\mathbf{1}_1+\mathbf{1}_2+\mathbf{1}_3+\mathbf{3}$ where Table XLIII lists the scalars.

There are two inequivalent models to investigate for the group *24/15:* they are $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_3,\mathbf{2}_3)$ where $\mathbf{6}=\mathbf{1}_4+\mathbf{1}_{2[4]}+\mathbf{2}_3+\mathbf{2}_4$ with the two choices of scalars. (For a discussion of the two possibilities, see the analysis in Sec. VI.) If $(\mathbf{2}_3\times\mathbf{2}_3)_A=\mathbf{1}_4$ the scalars are those of Table XLIV, but if $(\mathbf{2}_3\times\mathbf{2}_3)_A=\mathbf{1}_2$ then the top 4×4 changes in Table XLIV and is replaced with Table XLV. The other *24/15* case has $\mathbf{4}=(\mathbf{2}_3,\mathbf{2}_3)$ where $\mathbf{6}=3(\mathbf{1}_2)+\mathbf{1}_4+\mathbf{2}_1$ and the scalars are (this time swapping $\mathbf{1}_2$ and $\mathbf{1}_4$ gives equivalent models) given in Table XLVI.

The next model to evaluate is *27/4* with $\mathbf{4}=(\mathbf{1}_2,\mathbf{3}_1)$, where $\mathbf{6}=\mathbf{3}_1+\mathbf{3}_2$ is real, and the scalar sector is given in Table XLVII.

Finally, at order 30 we have *30/2* with $\mathbf{4}=(\mathbf{1}\alpha,\mathbf{1}',\mathbf{2}\alpha)$ and $\mathbf{6}=(\mathbf{1}'\alpha+\mathbf{2}\alpha+\mathbf{2}\alpha^{-1}+\mathbf{1}'\alpha^{-1})$ where $\alpha^3=1$. The scalar sector is shown in Table XLVIII.

The other possibility at order 30 is *30/3* with $\mathbf{4}=(\mathbf{1}\alpha,\mathbf{1}',\mathbf{2}\alpha^2)$ where $\mathbf{6}=\mathbf{1}'\alpha+\mathbf{2}\alpha^2+\mathbf{2}\alpha^3+\mathbf{1}'\alpha^4$ and $\alpha^5=1$, and where the scalars are provided by Table XLIX. The possible patterns of spontaneous symmetry breaking for all these models will be discussed in Sec. VI.

VI. SPONTANEOUS SYMMETRY BREAKING

We are now in a position to carry out spontaneous symmetry breaking for the models with fermions and scalars given in Secs. IV and V. We restrict ourselves to chiral models with the potential of at least three families ($\chi\geqslant45$) and for the most part consider only models with $N=2$, although we have included two $N=3$ models. Again, we move progressively through the models of increasing order of Γ. The model is completely fixed by Γ, the embedding of **4** in Γ, and the choice of N.

TABLE XXXVIII. The scalars for 24/7 with $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2,\mathbf{2}\alpha)$.

$\otimes$	1_1	1_2	1_3	1_4	2	$1_1\alpha$	$1_2\alpha$	$1_3\alpha$	$1_4\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$1_3\alpha^2$	$1_4\alpha^2$	$2\alpha^2$
1_1							×			×		×			×
1_2						×				×	×				×
1_3									×	×				×	×
1_4								×		×			×		×
2						×	×	×	×	×	×	×	×	×	×
$1_1\alpha$		×			×							×			×
$1_2\alpha$	×				×						×				×
$1_3\alpha$				×	×									×	×
$1_4\alpha$			×		×								×		×
2α	×	×	×	×	×						×	×	×	×	×
$1_1\alpha^2$		×			×		×			×					
$1_2\alpha^2$	×				×	×				×					
$1_3\alpha^2$				×	×				×	×					
$1_4\alpha^2$			×		×			×		×					
$2\alpha^2$	×	×	×	×	×	×	×	×	×	×					

TABLE XXXIX. The scalars for 24/9 with $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2\alpha^3,\mathbf{2}\alpha^2)$.

$\otimes$	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1		××				×						×
1_2	××					×						×
2			××	×	×	×				×	×	×
$1_1\alpha$			×		××				×			
$1_2\alpha$			×	××					×			
2α	×	×	×			××	×	×	×			
$1_1\alpha^2$						×		××				×
$1_2\alpha^2$						×	××					×
$2\alpha^2$				×	×	×			××	×	×	×
$1_1\alpha^3$			×						×		××	
$1_1\alpha^3$			×						×	××		
$2\alpha^3$	×	×	×				×	×	×			××

TABLE XL. The scalars for 24/9 with $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2\alpha,\mathbf{2}\alpha)$.

$\otimes$	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1								××	××			
1_2							××		××			
2							××	××	×× ××			
$1_1\alpha$											××	××
$1_2\alpha$										××		××
2α										××	××	×× ××
$1_1\alpha^2$		××	××									
$1_2\alpha^2$	××		××									
$2\alpha^2$	××	××	×× ××									
$1_1\alpha^3$					××	××						
$1_1\alpha^3$				××		××						
$2\alpha^3$				××	××	×× ××						

TABLE XLI. The scalars for 24/9 with $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2,\mathbf{2}\alpha)$.

$\otimes$	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1						×		××				×
1_2						×	××					×
2				×	×	×			××	×	×	×
$1_1\alpha$			×						×		××	
$1_2\alpha$			×						×	××		
2α	×	×	×				×	×	×			××
$1_1\alpha^2$		××				×						×
$1_2\alpha^2$	××					×						×
$2\alpha^2$			××	×	×	×				×	×	×
$1_1\alpha^3$			×		××				×			
$1_2\alpha^3$			×	××					×			
$2\alpha^3$	×	×	×			××	×	×	×			

The first relevant model is *16/8 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_1)$ and $N=2$*. The chiral fermions are

$$
\begin{aligned}
2[&(2,1,1,1,1,1,1,1;4,1)+(1,1,1,1,2,1,1,1;1,4)+(1,2,1,1,1,1,1,1;4,1)+(1,1,1,1,1,2,1,1;1,4)+(1,1,2,1,1,1,1,1;4,1)\\
&+(1,1,1,1,1,1,2,1;1,4)+(1,1,1,2,1,1,1,1;4,1)+(1,1,1,1,1,1,1,2;1,4)+(2,1,1,1,1,1,1,1;1,\bar{4})+(1,1,1,1,2,1,1,1;\bar{4},1)\\
&+(1,2,1,1,1,1,1,1;1,\bar{4})+(1,1,1,1,1,2,1,1;\bar{4},1)+(1,1,2,1,1,1,1,1;1,\bar{4})+(1,1,1,1,1,1,2,1;\bar{4},1)+(1,1,1,2,1,1,1,1;1,\bar{4})\\
&+(1,1,1,1,1,1,1,2;\bar{4},1)]
\end{aligned}
$$

and $\chi=2^8$. From the table of scalars for this model, we find that if we break $SU(4)\times SU(4)$ to the diagonal $SU_D(4)$, then the model becomes vectorlike.

All scalars that are nontrivial in the $SU(4)$'s are of the form $(1,1,1,1,1,1,1,1;4,\bar{4})$ + H.c., and a VEV for any one can be rotated such that the unbroken symmetry is $SU_D(4)$. All other scalars are $SU_i(2)\times SU_j(2)$ bilinears; hence we cannot break to a Pati-Salam (PS) model or any standard type chiral model.

16/8 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_{4+i},\mathbf{2}_1)$ and $N=2$, where $\mathbf{6}=[\mathbf{1}_{x(i)},\mathbf{2}_1,\mathbf{2}_2,(\mathbf{1}_5,\mathbf{1}_6,\mathbf{1}_7,\mathbf{1}_8)_A]$ with $x=6,5,8,7$ for $i=1,2,3,4$. These models have only half the initial chirality of the previous model ($\chi=2^7$), and the chiral fermions are given above if the overall factor of 2 is removed. As above, we need to break one $SU(4)$, either will do. We choose $SU_2(4)$. For the scalars shown, we can do this with, say, $(1,1,1,2,1,1,1,1;1,\bar{4})$ and $(1,1,1,1,1,1,1,2;1,4)$ VEV's. The remaining chiral fermion sector is

TABLE XLII. The scalars for 24/9 with $\mathbf{4}=(\mathbf{2}\alpha,\mathbf{2}\alpha)$.

$\otimes$	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1							×	×××	××			
1_2							×××	×	××			
2							××	××	××× ×××			
$1_1\alpha$										×	×××	××
$1_2\alpha$										×××	×	××
2α										××	××	××× ×××
$1_1\alpha^2$	×	×××	××									
$1_2\alpha^2$	×××	×	××									
$2\alpha^2$	××	××	××× ×××									
$1_1\alpha^3$				×	×××	××						
$1_2\alpha^3$				×××	×	××						
$2\alpha^3$				××	××	××× ×××						

TABLE XLIII. The scalars for 24/13 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_2)$.

$\otimes$	1_1	1_2	1_3	2_1	2_2	2_3	3
1_1	×	×	×				×
1_2	×	×	×				×
1_3	×	×	×				×
2_1				××	××	××	
2_2				××	××	××	
2_3				××	××	××	
3	×	×	×				××

$$(2,1,1,1,1,1;4)+(1,1,1,2,1,1;\overline{4})+(1,2,1,1,1,1;4)$$

$$+(1,1,1,1,2,1;\overline{4})+(1,1,2,1,1,1;4)+(1,1,1,1,1,2;\overline{4})$$

for $G=\Pi_k SU_k(2)\times SU(4)$, with $k=1,\ 2,\ 3,\ 5,\ 6$, and 7.

There are only $SU_i(2)\times SU_j(2)$ bilinear scalars of the form $(2_i,2_j)$ where $i=1,\ 2$, or 3 and $j=4,\ 5$, or 6, whose VEV's reduce chirality further, so we cannot reach a three-family PS model.

Note that what one would need is bilinears that would allow one to break $SU_1(2)\times SU_2(2)\times SU_3(2)$ to a diagonal subgroup $SU_L(2)$, and similarly for $SU_4(2)\times SU_5(2)\times SU_6(2)$ to $SU_R(2)$. This would then have been a three-family PS model.

16/11 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2)$ and $N=3$. This model is highly chiral, with $\chi=432$, and the chiral fermions are

$$6[(3,\overline{3},1,1,1,1,1,1;1,1)+(1,1,1,1,3,\overline{3},1,1;1,1)$$

$$+(1,3,\overline{3},1,1,1,1,1;1,1)+(1,1,1,1,1,3,\overline{3},1;1,1)$$

$$+(1,1,3,\overline{3},1,1,1,1;1,1)+(1,1,1,1,1,1,3,\overline{3};1,1)$$

$$+(\overline{3},1,1,3,1,1,1,1;1,1)+(1,1,1,1,\overline{3},1,1,3;1,1)].$$

We can ignore the $SU(6)\times SU(6)$ sector, since it can be broken completely without affecting the chirality. If we then give VEV's to $(1,1,1,8,1,1,1,1)$ and $(1,1,1,1,1,1,1,8)$ representations of $SU(3)^8$, we arrive at $6[(3,\overline{3},1)+(1,3,\overline{3})+(1,1,3)+(\overline{3},1,1)]$ in the $SU_{i+1}(3)\times SU_{i+2}(3)\times SU_{i+3}(3)$ sector for both $i=0$ and $i=1$. The $i=0$ sector can be broken completely with $(1,1,1,1,8,1)$-type VEV's plus $(1,1,1,3,1,\overline{3})$-type VEV's. The remaining fermions falling nearly into six $E_6\rightarrow SU(3)\times SU(3)\times SU(3)$-type families. While close, this model is still unsuccessful.

16/11 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{1}_4)$ and $N=3$. The chiral fermion sector is exactly half the previous case. Again we break $SU(6)\times SU(6)$ completely. Then breaking $\Pi_{j=4}^{8} SU_j(3)$ completely with $SU_j(3)$ octet VEV's finally gives us a chiral fermion sector $3[(3,\overline{3},1)+(1,3,\overline{3})+(1,1,3)+(\overline{3},1,1)]$. This is tantalizingly close to the three-family model we seek.

16/13: There are three potential models for this group. First consider the case with $\mathbf{4}=(2_1,2_1)$ *and* $N=2$. Here $\mathbf{6}=(1_2,1_2,1_2,1_3,2_2)$ and the chiral fermions are

TABLE XLIV. The scalars for 24/15 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_3,\mathbf{2}_3)$ if $(\mathbf{2}_3\times\mathbf{2}_3)_A=\mathbf{1}_4$.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1				××			×	×	
1_2			××				×	×	
1_3		××					×	×	
1_4	××						×	×	
2_1						××	×	×	××
2_2					××		×	×	××
2_3	×	×	×	×	×	×		××	
2_4	×	×	×	×	×	×	××		
2_5					××	××			××

$$2[(2,1,1,1;4,1,1)+(1,2,1,1;1,1,4)+(1,1,2,1;1,1,4)$$

$$+(1,1,1,2;4,1,1)+(2,1,1,1;1,1,\overline{4})+(1,2,1,1;\overline{4},1,1)$$

$$+(1,1,2,1;\overline{4},1,1)+(1,1,1,2;1,1,\overline{4})].$$

VEV's of the form $\langle 4_2,\overline{4}_2\rangle$ etc., can break $SU_2(4)$ completely [this group is irrelevant, since there are no chiral fermions with $SU_2(4)$ quantum numbers]. VEV's for $(4_1,\overline{4}_3)$ scalars then break $SU_1(4)\times SU_3(4)$ to $SU_D(4)$, such that the fermions become vectorlike. On the other hand, VEV's for $(2_4,4_2)+$H.c. reduce the chiral sector to

$$2[(2,1,1;1,4)+(1,2,1;4,1)+(1,1,2;4,1)+2(1,1,1;1,4)$$

$$+(2,1,1;\overline{4},1)+2(1,1,1;\overline{4},1)+(1,2,1;1,\overline{4})$$

$$+(1,1,2;1,\overline{4})],$$

and then a VEV for $(2_3,4_2)+$H.c. reduces this further to $2[(2,1;1,4)+(1,2;4,1)+(1,2;1,\overline{4})+(2,1;\overline{4},1)]$.

As above, a VEV for $(4_1,\overline{4}_3)$ scalars would render the model vectorlike, while just breaking $SU_3(4)$ would give a one-family model. However, this needs VEV's for $(2_1,2_4)$ and $(2_2,2_3)$, but no scalars of this type exist in the model. We conclude that the model has no Pati-Salam type phenomenology.

Next consider ***16/13*** *with* $\boldsymbol{4}=(\boldsymbol{2}_1,\boldsymbol{2}_2)$ *and* $N=2$. This time $\mathbf{6}$ is as given in Sec. V, but undetermined up to the identification of antisymmetric singlets in $(\mathbf{2}_i\times\mathbf{2}_i)_A$ with $i=1$ and 2. The chiral fermions are as in the $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_1)$ case, but with the overall factor of 2 deleted. A useful strategy is to perform a generic spontaneous symmetry breaking analysis to try to obtain a realistic Pati-Salam type phenomenology; then, if

TABLE XLV. The scalars in the top left 4×4 for 24/15 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_3,\mathbf{2}_3)$ if $(\mathbf{2}_3\times\mathbf{2}_3)_A=\mathbf{1}_2$.

	×		×
×		×	
	×		×
×		×	

TABLE XLVI. The scalars for 24/15 with $\mathbf{4}=(\mathbf{2}_3,\mathbf{2}_3)$.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1		$\times\times\times$		$\times$	$\times$				
1_2	$\times\times\times$		$\times$			$\times$			
1_3		$\times$		$\times\times\times$	$\times$				
1_4	$\times$		$\times\times\times$						
2_1	$\times$		$\times$		$\times$	$\times\times$			
2_2		$\times$			$\times\times$ $\times\times$	$\times\times$ $\times$			
2_3								$\times\times\times$ $\times\times$	$\times$
2_4							$\times\times\times$ $\times\times$		$\times$
2_5							$\times$	$\times$	$\times\times$ $\times\times$

successful, one asks whether the scalars required to carry out the breaking are included in the model. As above, $SU_2(4)$ is irrelevant and can be ignored. If we identify $SU_1(2)\times SU_4(2)$ with $SU_L(2)$ and $SU_2(2)\times SU_3(2)$ with $SU_R(2)$, we find $2[(2,1;1,4)+(1,2;4,1)+(1,2;1,\bar{4})+(2,1;\bar{4},1)]$. Now breaking one of the remaining $SU(4)$'s completely gives two families, and this is the best one can do. Hence independent of what scalars are available, there is no chance to obtain a model with three or more families.

The remaining *16/13* case is $\boldsymbol{4}=(\boldsymbol{1_3},\boldsymbol{1_4},\boldsymbol{2_1})$ *with* $N=2$. Now $\mathbf{6}=(\mathbf{1_2},\mathbf{2_1},\mathbf{2_3},\mathbf{1_c})$, but the chiral fermions are in the same representations as in the previous model, and so we can immediately conclude on general grounds that there is no viable phenomenology for this case.

18/3: Now consider *18/3 with* $\boldsymbol{4}=(\boldsymbol{1}\alpha,\boldsymbol{1}';\boldsymbol{2}\alpha)$ *and* $N=3$. This model has $\chi=192$ and chiral fermions

$$
\begin{aligned}
&(2,1,1,1,1,1;1,4,1)+(1,2,1,1,1,1;1,4,1)+(1,1,2,1,1,1;\bar{4},1,1)\\
&\quad+(1,1,1,2,1,1;\bar{4},1,1)+(1,1,2,1,1,1;1,1,4)\\
&\quad+(1,1,1,2,1,1;1,1,4)+(1,1,1,1,2,1;1,\bar{4},1)\\
&\quad+(1,1,1,1,1,2;1,\bar{4},1)+(1,1,1,1,2,1;4,1,1)\\
&\quad+(1,1,1,1,1,2;4,1,1)+(2,1,1,1,1,1;1,1,\bar{4})\\
&\quad+(1,2,1,1,1,1;1,1,\bar{4})+2[(1,1,1,1,1,1;\bar{4},4,1)\\
&\quad+(1,1,1,1,1,1;1,\bar{4},4)+(1,1,1,1,1,1;4,1,\bar{4})].
\end{aligned}
$$

Breaking $SU^6(2)$ to a single diagonal $SU(2)$ with all six $(2_i,2_j)$ type VEV's of $SU_i(2)\times SU_j(2)$, and then further VEV's of the types $(2;4,1,1)$, $(2;1,4,1)$, and $(2;1,1,4)$ to break the $SU(4)$'s to $SU(3)$'s leads to the set of remaining chiral fermions:

$$2[(3,\bar{3},1)+(1,3,\bar{3})+(\bar{3},1,3)].$$

Thus this route leads to two families.

If instead we seek a Pati-Salam model, there are several spontaneous symmetry breaking routes we need to investigate. If we break with $(1,1,1,1,1,1;\bar{4},4,1)$ scalars to $SU^6(2)\times SU_D(4)\times SU_3(4)$ we find the fermions remaining chiral are

$$
\begin{aligned}
&(2,1,1,1,1,1;4,1)+(1,2,1,1,1,1;4,1)+(1,1,2,1,1,1;1,4)\\
&\quad+(1,1,1,2,1,1;1,4)+(1,1,2,1,1,1;\bar{4},1)\\
&\quad+(1,1,1,2,1,1;\bar{4},1)+(2,1,1,1,1,1;1,\bar{4})\\
&\quad+(1,2,1,1,1,1;1,\bar{4}).
\end{aligned}
$$

Now breaking with a $(4_1,\bar{4}_3)$ or $(4_2,\bar{4}_3)$ VEV would render the model vectorlike, so we avoid this and instead give VEV's to $(2_5,4_1)$ and $(2_6,4_1)$ to break $SU_D(4)$ to $SU'(2)$. However, this yields at most two families.

We must try another route. If we avoid $(\bar{4},4)$ type VEV's and give VEV's only to (2,4) type scalars, we can proceed as follows: $\langle 2_1,4_2\rangle$, $\langle 2_2,4_2\rangle$, $\langle 2_3,\bar{4}_1\rangle$, and $\langle 2_4,\bar{4}_1\rangle$ VEV's break $SU^6(2)\times SU^3(4)$ down to $SU_5(2)\times SU_6(2)\times SU'(2)\times SU''(2)\times SU(4)$. Some fermions remain chiral but they are insufficient to construct families. We conclude that this model will not provide viable phenomenology.

TABLE XLVII. The scalars for 27/4 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{3}_1)$.

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_1										$\times$	$\times$
1_2										$\times$	$\times$
1_3										$\times$	$\times$
1_4										$\times$	$\times$
1_5										$\times$	$\times$
1_6										$\times$	$\times$
1_7										$\times$	$\times$
1_8										$\times$	$\times$
1_9										$\times$	$\times$
3_1	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times\times\times$
3_2	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times\times\times$	

TABLE XLVIII. The scalars for 30/2 with $\mathbf{4}=(\mathbf{1}\alpha,\mathbf{1}',\mathbf{2}\alpha)$.

$\otimes$	1	$1'$	2	$2'$	1α	$1'\alpha$	2α	$2'\alpha$	$1\alpha^2$	$1'\alpha^2$	$2\alpha^2$	$2'\alpha^2$
1						×	×			×	×	
$1'$					×		×		×		×	
2					×	×	×	×	×	×	×	×
$2'$							×	××			×	××
1α		×	×							×	×	
$1'\alpha$	×		×						×		×	
2α	×	×	×	×					×	×	×	×
$2'\alpha$			×	××							×	××
$1\alpha^2$		×	×			×	×					
$1'\alpha^2$	×		×		×		×					
$2\alpha^2$	×	×	×	×	×	×	×	×				
$2'\alpha^2$			×	××			×	××				

20/5 with $\mathbf{4}=(\mathbf{1_2},\mathbf{1_2},\mathbf{1_2},\mathbf{1_2})$ and *N=3*. The chiral $SU^4(3)$ fermions are $4[(3,\bar{3},1,1)+(1,3,\bar{3},1)+(1,1,3,\bar{3})+(\bar{3},1,1,3)]$. [The $SU(6)$ fermion does not participate, and will be ignored.] The only scalars are in representations $(3,1,\bar{3},1)$ + H.c. and $(1,3,1,\bar{3})$ + H.c. A VEV to, say, the first of these would break $SU_1(3)\times SU_3(3)$ to a diagonal $SU_D(3)$, and the fermions would become $4[(3,\bar{3},1)+(\bar{3},3,1)+(3,1,\bar{3})+(\bar{3},1,3)]$ under $SU_D(3)\times SU_2(3)\times SU_4(3)$, which is vectorlike. Hence any allowed VEV's immediately render the model vectorlike.

We get no farther with $\mathbf{4}=(\mathbf{1_2},\mathbf{1_2},\mathbf{1_2},\mathbf{1_4})$ and $N=3$, where $\mathbf{6}=(\mathbf{1_3},\mathbf{1_3},\mathbf{1_3},\mathbf{1_1},\mathbf{1_1},\mathbf{1_1})$, since this model has only half the chirality content of the previous case, and again VEV's will render it vectorlike.

21/2 with $\mathbf{4}=(\mathbf{1_2},\mathbf{3_1})$ and *N=2*. Now $\mathbf{6}=(\mathbf{3}_1,\mathbf{3}_2)$. (Other embeddings of **4** with $\mathcal{N}=0$ supersymmetry are permutation of the representations of this model and therefore all equivalent.) The fermions have $\chi=108$ and are

$$(2,1,1;6,1)+(1,2,1;6,1)+(1,1,2;6,1)+(2,1,1;1,\bar{6})$$

$$+(1,2,1;1,\bar{6})+(1,1,2;1,\bar{6})+(1,1,1;\bar{6},6).$$

A VEV for a $(\bar{6},6)$ scalar renders the model vectorlike. Our only other option is to give (2,6) type VEV's. $\langle 2,1,1;6,1\rangle$ breaks the gauge group to $SU_2(2)\times SU_3(2)\times SU(5)\times SU(6)$ with chiral fermions $2(1,1;5,1)+(1,2;5,1)+(2,1;5,1)+(1,1;1,\bar{6})+(2,1;1,\bar{6})+(1,2;1,\bar{6})+(1,1,1;\bar{5},6)$. There is insufficient fermion content for a three family Pati-Salam model if we identify $SU_2(2)\times SU_3(2)$ with $SU_L(2)\times SU_R(2)$. Our only other choice is to obtain one of these $SU(2)$'s from $SU(5)\times SU(6)$. For instance a $\langle 2_2,5\rangle$ VEV breaks the gauge group to $SU_3(2)\times SU(4)\times SU(6)$ but the remaining chiral fermions are $4(1,4,1)+(2,4,1)+3(1,1,\bar{6})+(2,1,\bar{6})+(1,2;1,\bar{6})+(1,1,6)+(1,\bar{4},6)$. We cannot identify $SU(4)$ with $SU_{PS}(4)$, so this group can only be in $SU(6)$. Breaking $SU(6)$ with an adjoint to $SU(2)\times SU(4)$ leaves

TABLE XLIX. The scalars for 30/3 with $\mathbf{4}=(\mathbf{1}\alpha,\mathbf{1}',\mathbf{2}\alpha^2)$.

$\otimes$	1	$1'$	2	1α	$1'\alpha$	2α	$1\alpha^2$	$1'\alpha^2$	$2\alpha^2$	$1\alpha^3$	$1'\alpha^3$	$2\alpha^3$	$1\alpha^4$	$1'\alpha^4$	$2\alpha^4$
1					×				×			×		×	
$1'$				×					×			×	×		
2						×	×	×	×	×	×	×			×
1α		×						×				×			×
$1'\alpha$	×						×					×			×
2α			×						×	×	×	×	×	×	×
$1\alpha^2$			×		×						×				×
$1'\alpha^2$			×	×						×					×
$2\alpha^2$	×	×	×			×						×	×	×	×
$1\alpha^3$			×			×		×						×	
$1'\alpha^3$			×			×	×						×		
$2\alpha^3$	×	×	×	×	×	×			×						×
$1\alpha^4$		×				×			×		×				
$1'\alpha^4$	×					×			×	×					
$2\alpha^4$			×	×	×	×	×	×	×			×			

us with $SU(2)\times SU(4)\times SU(2)\times SU(4)$ fermions that are again insufficient for a three family Pati-Salam model.

24/7 with $\mathbf{4}=(\mathbf{1}\alpha,\mathbf{1}',\mathbf{2}\alpha)$ for $N=2$. This model, the only successful one in the present broad search, was discussed in detail in Ref. [21], but for completeness we repeat the derivation here.

The original gauge group at the conformality scale is $SU(4)^3\times SU(2)^{12}$, with chiral fermions as given in Sec. IV and complex scalars as given in Sec. V above. If we break the three $SU(4)$'s to a single diagonal $SU(4)$ subgroup, chirality is lost. To avoid this we break $SU(4)_1$ completely and then break $SU(4)_\alpha\times SU(4)_{\alpha^2}$ to its diagonal subgroup $SU(4)_D$. The appropriate VEV's are available as $[(4_1,2_b\alpha^k)+\mathrm{H.c.}]$ with b (b runs from 1 to 4) arbitrary but $k=1$ or $k=2$. The second step requires an $SU(4)_D$ singlet VEV from $(\bar{4}_\alpha,4_{\alpha^2})$ and/or $(4_\alpha,\bar{4}_{\alpha^2})$. Once a choice is made for b (we take $b=4$), the remaining fermions are, in an intuitive notation,

$$\sum_{a=1}^{a=3}[(2_a\alpha,1,4_D)+(1,2_a\alpha^{-1},\bar{4}_D)], \tag{25}$$

which has the same content as a three family Pati-Salam model, though with a separate $SU(2)_L\times SU(2)_R$ per family.

To further reduce the symmetry we must arrange to break to a single $SU(2)_L$ and a single $SU(2)_R$. This is achieved by modifying step one where $SU(4)_1$ was broken. Consider the block diagonal decomposition of $SU(4)_1$ into $SU(2)_{1L}\times SU(2)_{1R}$. The representations $(2_a\alpha,4_1)$ and $(2_a\alpha^{-1},4_1)$ decompose as $(2_a\alpha,4_1)\rightarrow(2_a\alpha,2,1)+(2_a\alpha,1,2)$ and $(2_a\alpha^{-1},4_1)\rightarrow(2_a\alpha^{-1},2,1)+(2_a^{-1},1,2)$. Now if we give VEV's of equal magnitude to $(2_a\alpha,2,1)$, $a=1$, 2, and 3, and equal magnitude VEV's to $(2_a\alpha^{-1},1,2)$, $a=1$, 2, and 3, we break $SU(2)_{1L}\times\Pi_{a=1}^{a=3}SU(2_a\alpha)$ to a single $SU(2)_L$ and we break $SU(2)_{1R}\times\Pi_{a=1}^{a=3}SU(2_a\alpha^{-1})$ to a single $SU(2)_R$. Finally, VEV's for $(2_4\alpha,2,1)$ and $(2_4\alpha,1,2)$ as well as $(2_4\alpha^{-1},2,1)$ and $(2_4\alpha^{-1},1,2)$ ensure that both $SU(2_4\alpha)$ and $SU(2_4\alpha^{-1})$ are broken and that only three families remain chiral. The final set of chiral fermions is then $3[(2,1,4)+(1,2,\bar{4})]$ with gauge symmetry $SU(2)_L\times SU(2)_R\times SU(4)_D$. To achieve the final reduction to the standard model, an adjoint VEV from $(\bar{4}_\alpha,4_{\alpha^2})$ and/or $(4_\alpha,\bar{4}_{\alpha^2})$ is used to break $SU(4)_D$ to $SU(3)\times U(1)$, and a right-handed doublet is used to break $SU(2)_R$.

24/9 with $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2\alpha^3,\mathbf{2}\alpha^2)$ for $N=2$. The original gauge group at the conformality scale is $SU(4)^4\times SU(2)^8$, with chiral fermions as given in Sec. IV and complex scalars as given in Sec. V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4)\times SU(2)_L\times SU(2)_R$ requires the identifications $SU(2)_{1_1}=SU(2)_{1_2}=SU(2)_{1_1\alpha}=SU(2)_{1_2\alpha}=SU(2)_L$ and $SU(2)_{1_1\alpha^3}=SU(2)_{1_2\alpha^2}=SU(2)_{1_1\alpha^3}=SU(2)_{1_2\alpha^3}=SU(2)_R$, while, for example, $SU(4)_2=SU(4)_{2\alpha}=\bar{\mathbf{4}}$ of $SU(4)$ and $SU(4)_{2\alpha^2}=SU(4)_{2\alpha^3}=\mathbf{4}$ of $SU(4)$, where here and below this simplified notation implies diagonal subgroups.

However, the scalars tabulated for this case in Sec. V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/9 with $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2\alpha,\mathbf{2}\alpha)$ for $N=2$. The original gauge group at the conformality scale is $SU(4)^4\times SU(2)^8$ with chiral fermions as given in Sec. IV and complex scalars as given in Sec. V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4)\times SU(2)_L\times SU(2)_R$ requires the identifications $SU(2)_{1_1}=SU(2)_{1_2}=SU(2)_{1_1\alpha}=SU(2)_{1_2\alpha}=SU(2)_L$ and $SU(2)_{1_1\alpha^3}=SU(2)_{1_2\alpha^2}=SU(2)_{1_1\alpha^3}=SU(2)_{1_2\alpha^3}=SU(2)_R$; while, for example, $SU(4)_2=SU(4)_{2\alpha^3}=\bar{\mathbf{4}}$ of $SU(4)$ and $SU(4)_{2\alpha}=SU(4)_{2\alpha^2}=\mathbf{4}$ of $SU(4)$. However, the scalars tabulated for this case in Sec. V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/9 with $\mathbf{4}=(\mathbf{1}_1\alpha^2,\mathbf{1}_2,\mathbf{2}\alpha)$ for $N=2$. The original gauge group at the conformality scale is $SU(4)^4\times SU(2)^8$, with chiral fermions as given in Sec. IV and complex scalars as given in Sec. V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4)\times SU(2)_L\times SU(2)_R$ requires the identifications $SU(2)_{1_1}=SU(2)_{1_2}=SU(2)_{1_1\alpha}=SU(2)_{1_2\alpha}=SU(2)_L$ and $SU(2)_{1_1\alpha^3}=SU(2)_{1_2\alpha^2}=SU(2)_{1_1\alpha^3}=SU(2)_{1_2\alpha^3}=SU(2)_R$; while, for example, $SU(4)_2=SU(4)_{2\alpha^3}=\bar{\mathbf{4}}$ of $SU(4)$ and $SU(4)_{2\alpha}=SU(4)_{2\alpha^2}=\mathbf{4}$ of $SU(4)$. But again the scalars tabulated for this case in Sec. V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/9 with $\mathbf{4}=(\mathbf{2}\alpha,\mathbf{2}\alpha)$ *for* $N=2$. The original gauge group at the conformality scale is $SU(4)^4\times SU(2)^8$, with chiral fermions as given in Sec. IV and complex scalars as given in Sec. V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4)\times SU(2)_L\times SU(2)_R$ requires the identifications $SU(2)_{1_1}=SU(2)_{1_1\alpha}=SU(2)_{1_1\alpha^2}=SU(2)_{1_1\alpha^3}=SU(2)_L$ and $SU(2)_{1_2\alpha}=SU(2)_{1_2\alpha}=SU(2)_{1_2\alpha^2}=SU(2)_{1_2\alpha^3}=SU(2)_R$; while, for example, $SU(4)_{2\alpha}=SU(4)_{2\alpha^3}=\mathbf{4}$ of $SU(4)$, and $SU(4)_2$ and $SU(4)_{2\alpha^2}$ are broken. However, the scalars tabulated for this case in Sec. V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/13 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_2)$ for $N=2$. The original gauge group at the conformality scale is $SU(6)\times SU(4)^3\times SU(2)^3$ with chiral fermions as given in Sec. IV and complex scalars as given in Sec. V above. According to the analysis in Sec. IV this orbifold permits only two chiral families and is therefore not of phenomenological interest.

24/14 with $\mathbf{4}=(\mathbf{2}_1,\mathbf{2}_2)$ for $N=2$. The original gauge group at the conformality scale is $SU(4)^4\times SU(2)^8$ with chiral fermions as given in Sec. IV and complex scalars as given in Sec. V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4)\times SU(2)_L\times SU(2)_R$ requires the identifications $SU(2)_{1_1}=SU(2)_{1_2}=SU(2)_{1_5}=SU(2)_{1_6}=SU(2)_L$

and $SU(2)_{1_3}=SU(2)_{1_4}=SU(2)_{1_5}=SU(2)_{1_6}=SU(2)_R$; while, for example, $SU(4)_{2_2}=SU(4)_{2_3}=\mathbf{4}$ of $SU(4)$ and $SU(4)_{2_1}=SU(4)_{2_4}=\overline{\mathbf{4}}$ of $SU(4)$ where by this simplified notation we again imply diagonal subgroups. However, the scalars tabulated for this case in Sec. V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/15 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{1}_3,\mathbf{2}_3)$ *for* $N=2$. The original gauge group at the conformality scale is $SU(4)^5\times SU(2)^4$ with chiral fermions as given in Sec. IV and complex scalars as given in Sec. V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4)\times SU(2)_L\times SU(2)_R$ requires the identifications $SU(2)_{1_1}=SU(2)_{1_3}=SU(2)_L$ and $SU(2)_{1_2}=SU(2)_{1_4}=SU(2)_R$; while, for example, $SU(4)_{2_3}=SU(4)_{2_4}=\mathbf{4}$ of $SU(4)$. However, the scalars tabulated for this case in Sec. V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/15 with $\mathbf{4}=(\mathbf{2}_3,\mathbf{2}_5)$ *for* $N=2$. The original gauge group at the conformality scale is $SU(4)^5\times SU(2)^4$, with chiral fermions as given in Sec. IV and complex scalars as given in Sec. V above. According to the analysis in Sec. IV this orbifold permits only two chiral families and is hence not phenomenologically interesting.

24/15 with $\mathbf{4}=(\mathbf{2}_3,\mathbf{2}_3)$ *for* $N=2$. The original gauge group at the conformality scale is $SU(4)^5\times SU(2)^4$ with chiral fermions as given in Sec. IV and complex scalars as given in Sec. V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4)\times SU(2)_L\times SU(2)_R$ requires the identifications $SU(2)_{1_1}=SU(2)_{1_3}=SU(2)_L$ and $SU(2)_{1_2}=SU(2)_{1_4}=SU(2)_R$; while, for example, $SU(4)_{2_3}=SU(4)_{2_4}=\mathbf{4}$ of $SU(4)$. However, the scalars tabulated for this case in Sec. V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

27/4 with $\mathbf{4}=(\mathbf{1}_2,\mathbf{3}_1)$ *with* $N=2$. Here $\mathbf{6}=(\mathbf{3}_1,\mathbf{3}_2)$ and the chiral fermions are given in Sec. IV and all scalars are of type $(2_i,\overline{6}_1),(2_i,6_2)$ or $(6_1,\overline{6}_2)$ for $i=1,2,\ldots,9$. A VEV for the $(6_1,\overline{6}_2)+\text{H.c.}$ scalar breaks $SU_1(6)\times SU_2(6)$ to $SU_D(6)$, and the model becomes vectorlike. Hence we must break only with (2,6) type scalars if there is any hope of a viable model. We give VEV's to $(2_i,6_1)$ scalars for $i=1,2,\ldots,5$ to break $SU_1(6)$ completely, and VEV's to $(2_j,6_2)$ for $j=6,7$ to break $SU_2(6)$ to $SU(4)$. Then the remaining unbroken gauge group is $SU_8(2)\times SU_9(2)\times SU(4)$ with fermions $(2,1,4)+(1,2,4)+4(1,1,\overline{4})$, which are chiral but not of the correct form.

A more successful variation is obtained with $(2_i,6_1)$ scalar VEV's for $i=1$, 2, 3 and 4 to break the gauge group to $SU_5(2)\times SU_6(2)\times SU_7(2)\times SU_8(2)\times SU_9(2)\times SU'(2)\times SU(6)$ and then VEV's for $(2_5,6_2)$ and $(2_6,6_2)$ to break to $SU_7(2)\times SU_8(2)\times SU_9(2)\times SU'(2)\times SU(4)$ which has chiral fermions $(2,1,1,1,4)+(1,2,1,1,4)+(1,1,2,1,4)+3(1,1,1,2,\overline{4})$. If we could break $SU_7(2)\times SU_8(2)\times SU_9(2)$ to a diagonal $SU(2)$ subgroup, we would have a three-family Pati-Salam model. However, the scalars to accomplish this are not in the spectrum. If we could give VEV's to $(2_i,6_1)$ scalars for $i=7,8,9$ to break $SU_7(2)\times SU_8(2)\times SU_9(2)$ to a $U_Y(1)$ without disturbing the $SU'(2)$ subgroup of $SU_1(6)$, and a further $(2_j,6_2)$ VEV, say $(2_1,6_2)$, to break $SU(4)$ to $SU_C(3)$, then we would have a true three family standard [i.e., $U_Y(1)\times SU_{EW}(2)\times SU_C(3)$] model upon identifying $SU'(2)$ with $SU_{EW}(2)$.

30/2 with $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2,\mathbf{2}\alpha)$ and $N=2$. Here $\mathbf{6}=(\mathbf{1}_2\alpha,\mathbf{1}_2\alpha^2,\mathbf{2}\alpha,\mathbf{2}\alpha^2)$, and the gauge group is $SU^6(2)\times SU^6(4)$. This group has chiral fermions

$$
\begin{aligned}
&(2,1,1,1,1,1;1,1,4,1,1,1)+(1,2,1,1,1,1;1,1,4,1,1,1)+(1,1,2,1,1,1;\overline{4},1,1,1,1,1)+(1,1,1,2,1,1;\overline{4},1,1,1,1,1)\\
&+(1,1,1,1,1,1;1,\overline{4},4,1,1,1)+(1,1,1,1,1,1;\overline{4},1,4,1,1,1)+2(1,1,1,1,1,1;1,\overline{4},1,4,1,1)+(1,1,1,1,1,1;\overline{4},1,1,4,1,1)\\
&+(1,1,2,1,1,1;1,1,1,1,4,1)+(1,1,1,2,1,1;1,1,1,1,4,1)+(1,1,1,1,2,1;1,1,\overline{4},1,1,1)+(1,1,1,1,1,2;1,1,\overline{4},1,1,1)\\
&+(1,1,1,1,1,1;1,1,1,\overline{4},4,1)+(1,1,1,1,1,1;1,1,\overline{4},1,4,1)+2(1,1,1,1,1,1;1,1,1,\overline{4},1,4)+(1,1,1,1,1,1;1,1,\overline{4},1,1,4)\\
&+(1,1,1,1,2,1;4,1,1,1,1,1)+(1,1,1,1,1,2;4,1,1,1,1,1)+(2,1,1,1,1,1;1,1,1,1,\overline{4},1)+(1,2,1,1,1,1;1,1,1,1,\overline{4},1)\\
&+(1,1,1,1,1,1;4,1,1,1,\overline{4},1)+(1,1,1,1,1,1;4,1,1,1,1,\overline{4})+2(1,1,1,1,1,1;1,4,1,1,1,\overline{4})+(1,1,1,1,1,1;1,4,1,1,\overline{4},1).
\end{aligned}
$$

The spontaneous symmetry breaking analysis for this model is quite unwieldy, but for the most part can be carried out systematically. For example, breaking with $(1,1,1,1,1,1;1,\overline{4},1,4,1,1)$, $(1,1,1,1,1,1;\overline{4},1,1,4,1,1)$, $(1,1,1,1,1,1;4,1,1,1,\overline{4},1)$ and $(1,1,1,1,1,1;1,1,\overline{4},1,1,4)$ VEV's reduces $SU^6(4)$ to $SU_1(4)\times SU_D(4)$, with fermions remaining chiral in representations:

$$(2,1,1,1,1,1;1,4)+(1,2,1,1,1,1;1,4)+(1,1,2,1,1,1;\bar{4},1)$$
$$+(1,1,1,2,1,1;\bar{4},1)+(1,1,2,1,1,1;1,4)+(1,1,1,2,1,1;1,4)$$
$$+(1,1,1,1,2,1;1,\bar{4})+(1,1,1,1,1,2;1,\bar{4})$$
$$+(1,1,1,1,2,1;4,1)+(1,1,1,1,1,2;4,1)+(2,1,1,1,1,1;1,\bar{4})$$
$$+(1,2,1,1,1,1;1,\bar{4}).$$

Now (1,1,1,1,2,1;4,1) and (1,1,1,1,1,2;4,1) VEV's break $SU_5(2)\times SU_6(2)\times SU_1(4)$ to $SU'(2)$ with fermions remaining chiral in the representations

$$(2,1,1,1;4)+(1,2,1,1;4)+(1,1,2,1;4)+(1,1,1,2;4)$$
$$+2(1,1,1,1;\bar{4})+2(1,1,1,1;\bar{4})+(2,1,1,1;\bar{4})$$
$$+(1,2,1,1;\bar{4}),$$

which is already insufficient to provide three normal families. Other analyses of spontaneous symmetry breaking toward constructing a Pati-Salam model starting with this 30/2 model are similarly unsuccessful.

An alternative is to seek a trinification model. To this end, consider only the $SU^6(4)$ fermion sector

$$(1,\bar{4},4,1,1,1)+(\bar{4},1,4,1,1,1)+2(1,\bar{4},1,4,1,1)$$
$$+(\bar{4},1,1,4,1,1)+(1,1,1,\bar{4},4,1)+(1,1,\bar{4},1,4,1)$$
$$+2(1,1,1,\bar{4},1,4)+(1,1,\bar{4},1,1,4)+(4,1,1,1,\bar{4},1)$$
$$+2(4,1,1,1,1,\bar{4})+2(1,4,1,1,1,\bar{4})+(1,4,1,1,\bar{4},1).$$

Identifying $SU_1(4)$ with $SU_2(4)$, $SU_3(4)$ with $SU_4(4)$ and $SU_5(4)$ with $SU_6(4)$ would lead to five families of the form $5[(\bar{4},4,1)+(1,\bar{4},4)+(4,1,\bar{4})]$; however, there are no scalars of the type needed to carry this out.

This analysis is not exhaustive and there may be models where $SU_L(2)$ or $SU_R(2)$ or both are contained in $SU^6(4)$. Since we are starting with a group of rank 24, and seek the standard model of rank 4 or a unified model thereof of rank 5 or 6, and since there are 66 Higgs representations in the theory, the spontaneous symmetry breaking possibilities are rather complex. The $N=3$ case is obviously even more complicated, with initial rank 42, and one could try to automate the search for phenomenological models, although we have not attempted to do so.

30/3 with $\mathbf{4}=(\mathbf{1}_1\alpha,\mathbf{1}_2,\mathbf{2}\alpha^2)$ *and N=2.* We now have $\mathbf{6}=(\mathbf{1}_2\alpha,\mathbf{1}_2\alpha^4,\mathbf{2}\alpha^3,\mathbf{2}\alpha^2)$ where $\alpha^5=1$.

The chiral $SU^{10}(2)\times SU^5(4)$ fermions are

$$(1^{10};\bar{4},4,1,1,1)+(1^{10};\bar{4},1,4,1,1)+(1^4,2,1^5;\bar{4},1,1,1,1)+(1^5,2,1^4;\bar{4},1,1,1,1)+(1^{10};1,\bar{4},4,1,1)$$
$$+(1^{10};1,\bar{4},1,4,1)+(1^6,2,1^3;1,\bar{4},1,1,1)+(1^7,2,1^2;1,\bar{4},1,1,1)+(1^{10};1,1,\bar{4},4,1)+(1^{10};1,1,\bar{4},1,4)$$
$$+(1^8,2,1^1;1,1,\bar{4},1,1)+(1^9,2;1,1,\bar{4},1,1)+(1^{10};1,1,1,\bar{4},4)+(1^{10};4,1,1,\bar{4},1)$$
$$+(2,1^9;1,1,1,\bar{4},1)+(1^1,2,1^8;1,1,1,\bar{4},1)+(1^{10};4,1,1,1,\bar{4})+(1^{10};1,4,1,1,\bar{4})+(1^2,2,1^7;1,1,1,1,\bar{4})$$
$$+(1^3,2,1^6;1,1,1,1,\bar{4}).$$

Consider the bifundamentals only. VEV's for $(1,1,1,\bar{4},4)$ and $(1,\bar{4},4,1,1)$ scalars reduce the chiral fermion sector to $2[(\bar{4},4,1)+(1,\bar{4},4)+(4,1,\bar{4})]$, which provides at most a two family model.

If instead we try to construct a Pati-Salam model, and note that there are 20 (2;4) type fermions, and that we need six appropriate ones of these for three families, we must take care in the spontaneous symmetry breaking to preserve this much chirality. If we (i) break $SU_2(4)\times SU_4(4)\times SU_5(4)$ completely and (ii) break $SU_1(4)\times SU_3(4)$ to $SU_{PS}(4)$, while (iii) equating $SU_5(2)$, $SU_6(2)$, $SU_9(2)$ and (iv) equating $SU_{10}(2)$ with $SU_L(2)$, and $SU_1(2)$, $SU_2(2)$, $SU_7(2)$ and $SU_8(2)$ with $SU_R(2)$, and (v) breaking $SU_3(2)\times SU_4(2)$ completely, we would be left with a four family Pati-Salam model. Can we do this? (ii) is accomplished with (a) $(1^{10};\bar{4},1,4,1,1)$; then (i) requires (b) $(1^{10};1,\bar{4},1,4,1)$ and (c) $(1^{10};1,\bar{4},1,1,4)$ to obtain a $SU_D(4)$. Breaking this to nothing, we assume that VEV's (a) and (b) allow no freedom to rotate the (c) VEV to diagonal form. Now, at this point, we are stymied, as there are insufficient $(2_i,2_j)$ representations of $SU_i(2)\times SU_j(2)$ to accomplish (v).

Finally, one can imagine that there exist models with either $SU_L(2)$ or $SU_R(2)$ or both coming from $SU^5(4)$, but we see no obvious way to carry this out. On the other hand, since there are 60 Higgs representations we are unable to categorically eliminate this possibility.

VII. SUMMARY

We have shown how AdS/CFT duality leads to a large class of models which can provide interesting extensions of the standard model of particle phenomenology. The naturally occurring $\mathcal{N}=4$ extended supersymmetry was completely broken to $\mathcal{N}=0$ by choice of orbifolds S^5/Γ such that $\Gamma\not\subset SU(3)$.

In the present work, we systematically studied all such non-Abelian Γ's with order $g\leqslant 31$. We have seen how chiral fermions require that the embedding of Γ be neither real nor pseudoreal. This dramatically reduces the number of possibilities to obtain chiral fermions. Nevertheless, many candidates for models which contain the chiral fermions of the three-family standard model were found.

However, the requirement that the spontaneous symmetry breaking down to the correct gauge symmetry of the standard model be permitted by the prescribed scalar representations eliminates most of the surviving models. We found only one allowed model based on the $\Gamma=D_4\times Z_3$ orbifold. We initially expected to find more examples in our search. The moral for model building is interesting. Without the rigid framework of string duality the scalar sector would be arbitrarily chosen to permit the required spontaneous symmetry breaking. This is the normal practice in the standard model, in grand unification, in supersymmetry and so on. With string duality, the scalar sector is prescribed by the construction and only in one very special case does it permit the required symmetry breaking. This leads us to give more credence to the $\Gamma=D_4\times Z_3$ example that does work, and to encourage its further study to check whether it can have any connection to the real world.

ACKNOWLEDGMENTS

P.H.F. thanks the Department of Physics and Astronomy, Vanderbilt University for hospitality while this work was in progress. T.W.K. thanks the Department of Physics and Astronomy, at the University of North Carolina and the Aspen Center for Physics for hospitality while this work was in progress. This work was supported in part by the U. S. Department of Energy under Grant Nos. DE-FG02-97ER-41036 and DE-FG05-85ER-40226.

APPENDIX

The following Tables L–LXXIX are the irreducible representation muliplication tables for non-Abelian groups with $g\leqslant 31$:

TABLE L. The group $D_3=S_3$, 6/2.

$\otimes$	1	$1'$	2
1	1	$1'$	2
$1'$	$1'$	1	2
2	2	2	$1+1'+2$

TABLE LI. The group D_4, 8/4.

$\otimes$	1_1	1_2	1_3	1_4	2
1_1	1_1	1_2	1_3	1_4	2
1_2	1_2	1_1	1_4	1_3	2
1_3	1_3	1_4	1_1	1_2	2
1_4	1_4	1_3	1_2	1_1	2
2	2	2	2	2	$1_1+1_2+1_3+1_4$

TABLE LII. The group Q, 8/5.

$\otimes$	1_1	1_2	1_3	1_4	2
1_1	1_1	1_2	1_3	1_4	2
1_2	1_2	1_1	1_4	1_3	2
1_3	1_3	1_4	1_1	1_2	2
1_4	1_4	1_3	1_2	1_1	2
2	2	2	2	2	$1_1+1_2+1_3+1_4$

TABLE LIII. The group D_5, 10/2.

$\otimes$	1	$1'$	2	$2'$
1	1	$1'$	$1''$	$2'$
$1'$	$1'$	1	1	$2'$
2	2	2	$1+1'+2'$	$2+2'$
$2'$	$2'$	$2'$	$2+2'$	$1+1'+2$

TABLE LIV. The group T, 12/4.

$\otimes$	1	$1'$	$1''$	3
1	1	$1'$	$1''$	3
$1'$	$1'$	$1''$	1	3
$1''$	$1''$	1	$1'$	3
3	3	3	3	$1+1'+1''+3+3$

TABLE LV. The group D_6, 12/3.

$\otimes$	1_1	1_2	1_3	1_4	2	$2'$
1_1	1_1	1_2	1_3	1_4	2	$2'$
1_2	1_2	1_1	1_4	1_3	$2'$	2
1_3	1_3	1_4	1_1	1_2	$2'$	2
1_4	1_4	1_3	1_2	1_1	2	$2'$
2	2	$2'$	$2'$	2	$1_1+1_4+2'$	1_2+1_3+2
$2'$	$2'$	2	2	$2'$	1_2+1_3+2	$1_1+1_4+2'$

TABLE LVI. The group Q_6, 12/5.

$\otimes$	1_1	1_2	1_3	1_4	2	$2'$
1_1	1_1	1_2	1_3	1_4	2	$2'$
1_2	1_2	1_3	1_4	1_1	$2'$	2
1_3	1_3	1_4	1_1	1_2	2	$2'$
1_4	1_4	1_1	1_2	1_3	$2'$	2
2	2	$2'$	2	$2'$	$1_1+1_3+2'$	1_2+1_4+2
$2'$	$2'$	2	$2'$	2	1_2+1_4+2	$1_1+1_2+2'$

TABLE LVII. The group D_7, 14/2.

$\otimes$	1	$1'$	2_1	2_2	2_3
1	1_1	$1'$	2_1	2_2	2_3
$1'$	$1'$	1	2_1	2_2	2_3
2_1	2_1	2_1	$1+1'+2_2$	2_1+2_3	2_2+2_3
2_2	2_2	2_2	2_1+2_3	$1+1'+2_3$	2_1+2_2
2_3	2_3	2_3	2_2+2_3	2_1+2_2	$1+1'+2_1$

TABLE LVIII. The group $(Z_4\times Z_2)\tilde{\rtimes}Z_2$, 16/8.

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_2	1_2	1_1	1_4	1_3	1_6	1_5	1_8	1_7	2	$2'$
1_3	1_3	1_4	1_1	1_2	1_7	1_8	1_5	1_6	2	$2'$
1_4	1_4	1_3	1_2	1_1	1_8	1_7	1_6	1_5	2	$2'$
1_5	1_5	1_6	1_7	1_8	1_1	1_2	1_3	1_4	$2'$	2
1_6	1_6	1_5	1_8	1_7	1_2	1_1	1_4	1_3	$2'$	2
1_7	1_7	1_8	1_5	1_6	1_3	1_4	1_1	1_2	$2'$	2
1_8	1_8	1_7	1_6	1_5	1_4	1_3	1_2	1_1	$2'$	2
2	2	2	2	2	$2'$	$2'$	$2'$	$2'$	$1_5+1_6+1_7+1_8$	$1_1+1_2+1_3+1_4$
$2'$	$2'$	$2'$	$2'$	$2'$	2	2	2	2	$1_1+1_2+1_3+1_4$	$1_5+1_6+1_7+1_8$

TABLE LIX. The group $Z_4\tilde{\rtimes}Z_4$, 16/10.

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_2	1_2	1_3	1_4	1_1	1_6	1_7	1_8	1_5	$2'$	2
1_3	1_3	1_4	1_1	1_2	1_7	1_8	1_5	1_6	2	$2'$
1_4	1_4	1_1	1_2	1_3	1_8	1_5	1_6	1_7	$2'$	2
1_5	1_5	1_6	1_7	1_8	1_1	1_2	1_3	1_4	2	$2'$
1_6	1_6	1_7	1_8	1_5	1_2	1_3	1_4	1_1	$2'$	2
1_7	1_7	1_8	1_5	1_6	1_3	1_4	1_1	1_2	2	$2'$
1_8	1_8	1_5	1_6	1_7	1_4	1_1	1_2	1_3	$2'$	2
2	2	$2'$	2	$2'$	2	$2'$	2	$2'$	$1_1+1_3+1_5+1_7$	$1_2+1_4+1_6+1_8$
$2'$	$2'$	2	$2'$	2	$2'$	2	$2'$	2	$1_2+1_4+1_6+1_8$	$1_1+1_3+1_5+1_7$

TABLE LX. The group $Z_8 \tilde{\times} Z_2$, 16/11.

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_2	1_2	1_3	1_4	1_1	1_6	1_7	1_8	1_5	$2'$	2
1_3	1_3	1_4	1_1	1_2	1_7	1_8	1_5	1_6	2	$2'$
1_4	1_4	1_1	1_2	1_3	1_8	1_5	1_6	1_7	$2'$	2
1_5	1_5	1_6	1_7	1_8	1_1	1_2	1_3	1_4	2	$2'$
1_6	1_6	1_7	1_8	1_5	1_2	1_3	1_4	1_1	$2'$	2
1_7	1_7	1_8	1_5	1_6	1_3	1_4	1_1	1_2	2	$2'$
1_8	1_8	1_5	1_6	1_7	1_4	1_1	1_2	1_3	$2'$	2
2	2	$2'$	2	$2'$	2	$2'$	2	$2'$	$1_2+1_4+1_6+1_8$	$1_1+1_3+1_5+1_7$
$2'$	$2'$	2	$2'$	2	$2'$	2	$2'$	2	$1_1+1_3+1_5+1_7$	$1_2+1_4+1_6+1_8$

TABLE LXI. The group D_8, $(Z_8 \tilde{\times} Z_2)'$, 16/12 ($Q_8$, 16/14, has the same table).

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_2	1_2	1_1	1_4	1_3	2_3	2_2	2_1
1_3	1_3	1_4	1_1	1_2	2_1	2_2	2_3
1_4	1_4	1_3	1_2	1_1	2_3	2_2	2_1
2_1	2_1	2_3	2_1	2_3	$1_1+1_3+2_2$	2_1+2_3	$1_2+1_4+2_2$
2_2	2_2	2_2	2_2	2_2	2_1+2_3	$1_1+1_2+1_3+1_4$	2_1+2_3
2_3	2_3	2_1	2_3	2_1	$1_2+1_4+2_2$	2_1+2_3	$1_1+1_3+2_2$

TABLE LXII. The group $(Z_8 \tilde{\times} Z_2)''$, 16/13.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_2	1_2	1_1	1_4	1_3	2_3	2_2	2_1
1_3	1_3	1_4	1_1	1_2	2_3	2_2	2_1
1_4	1_4	1_3	1_2	1_1	2_1	2_2	2_3
2_1	2_1	2_3	2_3	2_1	$1_2+1_3+2_2$	2_1+2_3	$1_1+1_4+2_2$
2_2	2_2	2_2	2_2	2_2	2_1+2_3	$1_1+1_2+1_3+1_4$	2_1+2_3
2_3	2_3	2_1	2_1	2_3	$1_1+1_4+2_2$	2_1+2_3	$1_2+1_3+2_2$

TABLE LXIII. The group D_9, 18/4.

$\otimes$	1	$1'$	2_1	2_2	2_3	2_4
1	1	$1'$	2_1	2_2	2_3	2_4
$1'$	$1'$	1	2_1	2_2	2_3	2_4
2_1	2_1	2_1	$1+1'+2_2$	2_1+2_3	2_2+2_4	2_3+2_4
2_2	2_2	2_2	2_1+2_3	$1+1'+2_4$	2_1+2_4	2_2+2_3
2_3	2_3	2_3	2_2+2_4	2_1+2_4	$1+1'+2_3$	2_1+2_2
2_4	2_4	2_4	2_3+2_4	2_2+2_3	2_1+2_2	$1+1'+2_1$

TABLE LXIV. The group $(Z_3 \times Z_3)\tilde{\times} Z_2$, 18/5.

$\otimes$	1	$1'$	2_1	2_2	2_3	2_4
1	1	$1'$	2_1	2_2	2_3	2_4
$1'$	$1'$	1	2_1	2_2	2_3	2_4
2_1	2_1	2_1	$1+1'+2_1$	2_3+2_4	2_2+2_4	2_2+2_3
2_2	2_2	2_2	2_3+2_4	$1+1'+2_2$	2_1+2_4	2_1+2_3
2_3	2_3	2_3	2_2+2_4	2_1+2_4	$1+1'+2_3$	2_1+2_2
2_4	2_4	2_4	2_2+2_3	2_1+2_3	2_1+2_2	$1+1'+2_4$

TABLE LXV. The group D_{10}, 20/3.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4
1_2	1_2	1_1	1_4	1_3	2_4	2_3	2_1	2_1
1_3	1_3	1_4	1_1	1_2	2_1	2_2	2_3	2_4
1_4	1_4	1_3	1_2	1_1	2_4	2_3	2_1	2_1
2_1	2_1	2_4	2_1	2_4	$1_1+1_3+2_2$	2_1+2_3	2_2+2_4	$1_2+1_4+2_3$
2_2	2_2	2_3	2_2	2_3	2_1+2_3	$1_1+1_3+2_4$	2_1+2_3	2_2+2_4
2_3	2_3	2_2	2_3	2_2	2_2+2_4	2_1+2_3	$1_1+1_3+2_4$	2_1+2_3
2_4	2_4	2_1	2_4	2_1	$1_2+1_4+2_3$	2_2+2_4	2_1+2_3	$1_1+1_3+2_2$

TABLE LXVI. The group $Z_5 \tilde{\times} Z_4$, 20/5.

$\otimes$	1_1	1_2	1_3	1_4	4
1_1	1_1	1_2	1_3	1_4	4
1_2	1_2	1_3	1_4	1_1	4
1_3	1_3	1_4	1_1	1_2	4
1_4	1_4	1_1	1_2	1_3	4
4	4	4	4	4	$1_1+1_2+1_3+1_4+3\times 4$

TABLE LXVII. The group $Z_7 \tilde{\times} Z_3$, 21/2.

$\otimes$	1_1	1_2	1_3	3_1	3_2
1_1	1_1	1_2	1_3	3_1	3_2
1_2	1_2	1_3	1_1	3_1	3_2
1_3	1_3	1_1	1_2	3_1	3_2
3_1	3_1	3_1	3_1	$3_1+3_2+3_2$	$1_1+1_2+1_3+3_1+3_2$
3_2	3_2	3_2	3_2	$1_1+1_2+1_3+3_1+3_2$	$3_1+3_1+3_2$

TABLE LXVIII. The group D_{11}, 22/2.

$\otimes$	1	$1'$	2_1	2_2	2_3	2_4	2_5
1	1	$1'$	2_1	2_2	2_3	2_4	2_5
$1'$	$1'$	1	2_1	2_2	2_3	2_4	2_5
2_1	2_1	2_1	$1+1'+2_2$	2_1+2_3	2_1+2_4	2_3+2_5	2_4+2_5
2_2	2_2	2_2	2_1+2_3	$1+1'+2_4$	2_1+2_5	2_2+2_5	2_3+2_4
2_3	2_3	2_3	2_1+2_4	2_1+2_5	$1+1'+2_5$	2_1+2_4	2_2+2_3
2_4	2_4	2_4	2_3+2_5	2_2+2_5	2_1+2_4	$1+1'+2_3$	2_1+2_2
2_5	2_5	2_5	2_4+2_5	2_3+2_4	2_2+2_3	2_1+2_2	$1+1'+2_1$

TABLE LXIX. The group D_{12}, 24/10.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_2	1_2	1_1	1_4	1_3	2_1	2_2	2_3	2_4	2_5
1_3	1_3	1_4	1_1	1_2	2_5	2_4	2_3	2_2	2_1
1_4	1_4	1_3	1_2	1_1	2_5	2_4	2_3	2_2	2_1
2_1	2_1	2_1	2_5	2_5	$1_1+1_2+2_2$	2_1+2_3	2_2+2_4	2_3+2_5	2_3+2_5
2_2	2_2	2_2	2_4	2_4	2_1+2_3	$1_1+1_2+2_4$	2_1+2_5	$1_3+1_4+2_4$	2_3+2_5
2_3	2_3	2_3	2_3	2_3	2_2+2_4	2_1+2_5	$1_1+1_2+1_3+1_4$	2_1+2_5	2_2+2_4
2_4	2_4	2_4	2_2	2_2	2_3+2_5	$1_3+1_4+2_4$	2_1+2_5	$1_1+1_2+2_4$	2_1+2_3
2_5	2_5	2_5	2_1	2_1	2_3+2_5	2_3+2_5	2_2+2_4	2_1+2_3	$1_1+1_2+2_2$

TABLE LXX. The group S_4, 24/12.

$\otimes$	1	$1'$	2	3	$3'$
1	1	$1'$	2	3	$3'$
$1'$	$1'$	1	2	$3'$	3
2	2	2	$1+1'+2$	$3+3'$	$3+3'$
3	3	$3'$	$3+3'$	$1+2+3+3'$	$1'+2+3+3'$
$3'$	$3'$	3_2	$3+3'$	$1'+2+3+3'$	$1+2+3+3'$

TABLE LXXI. The group $SL_2(F_3)$, $Q\tilde{\times}Z_3$, 24/13.

$\otimes$	1_1	1_2	1_3	2_1	2_2	2_3	3
1_1	1_1	1_2	1_3	2_1	2_2	2_3	3
1_2	1_2	1_3	1_1	2_2	2_3	2_1	3
1_3	1_3	1_1	1_2	2_3	2_1	2_2	3
2_1	2_1	2_2	2_3	$1+3$	$1'+3$	$1''+3$	$2_1+2_2+2_3$
2_2	2_2	2_3	2_1	$1'+3$	$1''+3$	$1+3$	$2_1+2_2+2_3$
2_3	2_3	2_1	2_2	$1''+3$	$1+3$	$1'+3$	$2_1+2_2+2_3$
3	3	3	3	$2_1+2_2+2_3$	$2_1+2_2+2_3$	$2_1+2_2+2_3$	$1_1+1_2+1_3+3+3$

TABLE LXXII. The group $Z_8\tilde{\times}Z_3$, 24/14.

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2	2_3	2_4
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2	2_3	2_4
1_2	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_1	2_2	2_3	2_4	2_1
1_3	1_3	1_4	1_5	1_6	1_7	1_8	1_1	1_2	2_3	2_4	2_1	2_2
1_4	1_4	1_5	1_6	1_7	1_8	1_1	1_2	1_3	2_4	2_1	2_2	2_3
1_5	1_5	1_6	1_7	1_8	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4
1_6	1_6	1_7	1_8	1_1	1_2	1_3	1_4	1_5	2_2	2_3	2_4	2_1
1_7	1_7	1_8	1_1	1_2	1_3	1_4	1_5	1_6	2_3	2_4	2_1	2_2
1_8	1_8	1_1	1_2	1_3	1_4	1_5	1_6	1_7	2_4	2_1	2_2	2_3
2_1	2_1	2_2	2_3	2_4	2_1	2_2	2_3	2_4	$1_1+1_5+2_1$	$1_2+1_6+2_2$	$1_3+1_7+2_3$	$1_4+1_8+2_4$
2_2	2_2	2_3	2_4	2_1	2_2	2_3	2_4	2_1	$1_2+1_6+2_2$	$1_3+1_7+2_3$	$1_4+1_8+2_4$	$1_1+1_5+2_1$
2_3	2_3	2_4	2_1	2_2	2_3	2_4	2_1	2_2	$1_3+1_7+2_3$	$1_4+1_8+2_4$	$1_1+1_5+2_1$	$1_2+1_6+2_2$
2_4	2_4	2_1	2_2	2_3	2_4	2_1	2_2	2_3	$1_4+1_8+2_4$	$1_1+1_5+2_1$	$1_2+1_6+2_2$	$1_3+1_7+2_3$

TABLE LXXIII. The group $D_4\tilde{\times}Z_3$, 24/15.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_2	1_2	1_1	1_4	1_3	2_2	2_1	2_4	2_3	2_5
1_3	1_3	1_4	1_1	1_2	2_1	2_2	2_3	2_4	2_5
1_4	1_4	1_3	1_2	1_1	2_2	2_1	2_4	2_3	2_5
2_1	2_1	2_2	2_1	2_2	$1_1+1_3+2_1$	$1_2+1_4+2_2$	2_4+2_5	2_3+2_5	2_3+2_4
2_2	2_2	2_1	2_2	2_1	$1_2+1_4+2_2$	$1_1+1_3+2_1$	2_3+2_5	2_4+2_5	2_3+2_4
2_3	2_3	2_4	2_3	2_4	2_4+2_5	2_3+2_5	$1_2+1_4+2_1$	$1_1+1_3+2_2$	2_1+2_2
2_4	2_4	2_3	2_4	2_3	2_3+2_5	2_4+2_5	$1_1+1_3+2_2$	$1_2+1_4+2_1$	2_1+2_2
2_5	2_5	2_5	2_5	2_5	2_3+2_4	2_3+2_4	2_1+2_2	2_1+2_2	$1_1+1_2+1_3+1_4$

TABLE LXXIV. The group D_{13}, 26/2.

$\otimes$	1	$1'$	2_1	2_2	2_3	2_4	2_5	2_6
1	1	$1'$	2_1	2_2	2_3	2_4	2_5	2_6
$1'$	$1'$	1	2_1	2_2	2_3	2_4	2_5	2_6
2_1	2_1	2_1	$1+1'+2_2$	2_1+2_3	2_1+2_4	2_3+2_5	2_4+2_6	2_5+2_6
2_2	2_2	2_2	2_1+2_3	$1+1'+2_4$	2_1+2_5	2_2+2_6	2_3+2_6	2_4+2_5
2_3	2_3	2_3	2_1+2_4	2_1+2_5	$1+1'+2_6$	2_1+2_6	2_2+2_5	2_3+2_4
2_4	2_4	2_4	2_3+2_5	2_2+2_6	2_1+2_6	$1+1'+2_5$	2_1+2_4	2_2+2_3
2_5	2_5	2_5	2_4+2_6	2_3+2_6	2_2+2_5	2_1+2_4	$1+1'+2_3$	2_1+2_2
2_6	2_6	2_6	2_5+2_6	2_4+2_5	2_3+2_4	2_2+2_3	2_1+2_2	$1+1'+2_1$

TABLE LXXV. The group $(Z_3\times Z_3)\widetilde{\rtimes} Z_3$, 27/4.

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_2	1_2	1_3	1_1	1_5	1_6	1_4	1_8	1_9	1_7	3_1	3_2
1_3	1_3	1_1	1_2	1_6	1_4	1_5	1_9	1_7	1_8	3_1	3_2
1_4	1_4	1_5	1_6	1_7	1_8	1_9	1_1	1_2	1_3	3_1	3_2
1_5	1_5	1_6	1_4	1_8	1_9	1_7	1_2	1_3	1_1	3_1	3_2
1_6	1_6	1_4	1_5	1_9	1_7	1_8	1_3	1_1	1_2	3_1	3_2
1_7	1_7	1_8	1_9	1_1	1_2	1_3	1_4	1_5	1_6	3_1	3_2
1_8	1_8	1_9	1_7	1_2	1_3	1_1	1_5	1_6	1_4	3_1	3_2
1_9	1_9	1_7	1_8	1_3	1_1	1_2	1_6	1_4	1_5	3_1	3_2
3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	$3\times 3_2$	$\Sigma_{i=1}^{9}1_i$
3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	$\Sigma_{i=1}^{9}1_i$	$3\times 3_1$

TABLE LXXVI. The group $Z_9\widetilde{\rtimes} Z_3$,27/5 [note that this table is the same as for $(Z_3\times Z_3)\widetilde{\rtimes} Z_3$].

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_2	1_2	1_3	1_1	1_5	1_6	1_4	1_8	1_9	1_7	3_1	3_2
1_3	1_3	1_1	1_2	1_6	1_4	1_5	1_9	1_7	1_8	3_1	3_2
1_4	1_4	1_5	1_6	1_7	1_8	1_9	1_1	1_2	1_3	3_1	3_2
1_5	1_5	1_6	1_4	1_8	1_9	1_7	1_2	1_3	1_1	3_1	3_2
1_6	1_6	1_4	1_5	1_9	1_7	1_8	1_3	1_1	1_2	3_1	3_2
1_7	1_7	1_8	1_9	1_1	1_2	1_3	1_4	1_5	1_6	3_1	3_2
1_8	1_8	1_9	1_7	1_2	1_3	1_1	1_5	1_6	1_4	3_1	3_2
1_9	1_9	1_7	1_8	1_3	1_1	1_2	1_6	1_4	1_5	3_1	3_2
3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	$3\times 3_2$	$\Sigma_{i=1}^{9}1_i$
3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	$\Sigma_{i=1}^{9}1_i$	$3\times 3_1$

TABLE LXXVII. The group D_{14}, 28/3.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5	2_6
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5	2_6
1_2	1_2	1_1	1_4	1_3	2_1	2_2	2_3	2_4	2_5	2_6
1_3	1_3	1_4	1_1	1_2	2_6	2_5	2_4	2_3	2_2	2_1
1_4	1_4	1_3	1_2	1_1	2_6	2_5	2_4	2_3	2_2	2_1
2_1	2_1	2_1	2_6	2_6	$1_1+1_2+2_2$	2_1+2_3	2_2+2_4	2_3+2_5	2_4+2_6	$1_3+1_4+2_5$
2_2	2_2	2_2	2_5	2_5	2_1+2_3	$1_1+1_2+2_4$	2_1+2_5	2_2+2_6	$1_3+1_4+2_3$	2_4+2_6
2_3	2_3	2_3	2_4	2_4	2_2+2_4	2_1+2_5	$1_1+1_2+2_6$	$1_3+1_4+2_1$	2_2+2_6	2_3+2_5
2_4	2_4	2_4	2_3	2_3	2_3+2_5	2_2+2_6	$1_3+1_4+2_1$	$1_1+1_2+2_6$	2_1+2_5	2_2+2_4
2_5	2_5	2_5	2_2	2_2	2_4+2_6	$1_3+1_4+2_3$	2_2+2_6	2_1+2_5	$1_1+1_2+2_4$	2_1+2_3
2_6	2_6	2_6	2_1	2_1	$1_3+1_4+2_5$	2_4+2_6	2_3+2_5	2_2+2_4	2_1+2_3	$1_1+1_2+2_2$

TABLE LXXVIII. The group $D_5\times Z_3$, 30/2.

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	2_1	2_2	2_3	2_4	2_5	2_6
1_1	1_1	1_2	1_3	1_4	1_5	1_6	2_1	2_2	2_3	2_4	2_5	2_6
1_2	1_2	1_3	1_4	1_5	1_6	1_1	2_5	2_6	2_1	2_2	2_3	2_4
1_3	1_3	1_4	1_5	1_6	1_1	1_2	2_3	2_4	2_5	2_6	2_1	2_2
1_4	1_4	1_5	1_6	1_1	1_2	1_3	2_1	2_2	2_3	2_4	2_5	2_6
1_5	1_5	1_6	1_1	1_2	1_3	1_4	2_5	2_6	2_1	2_2	2_3	2_4
1_6	1_6	1_1	1_2	1_3	1_4	1_5	2_3	2_4	2_5	2_6	2_1	2_2
2_1	2_1	2_5	2_3	2_1	2_5	2_3	$1_1+1_4+2_1$	2_1+2_2	$1_3+1_6+2_4$	2_3+2_4	$1_2+1_5+2_6$	2_5+2_6
2_2	2_2	2_6	2_4	2_2	2_6	2_4	2_1+2_1	$1_1+1_4+2_2$	2_3+2_4	$1_3+1_6+2_3$	2_5+2_6	$1_2+1_5+2_5$
2_3	2_3	2_1	2_5	2_3	2_1	2_5	$1_3+1_6+2_4$	2_3+2_4	$1_2+1_5+2_6$	2_5+2_6	$1_1+1_4+2_2$	2_1+2_1
2_4	2_4	2_2	2_6	2_4	2_2	2_6	2_3+2_4	$1_3+1_6+2_3$	2_5+2_6	$1_2+1_5+2_5$	2_1+2_2	$1_1+1_4+2_1$
2_5	2_5	2_3	2_1	2_5	2_3	2_1	$1_2+1_5+2_6$	2_5+2_6	$1_1+1_4+2_2$	2_1+2_2	$1_3+1_6+2_4$	2_3+2_4
2_6	2_6	2_4	2_2	2_6	2_4	2_2	2_5+2_6	$1_2+1_5+2_5$	2_1+2_2	$1_1+1_5+2_1$	2_3+2_4	$1_3+1_6+2_3$

TABLE LXXIX. The group D_{15}, 30/4.

$\otimes$	1	$1'$	2_1	2_2	2_3	2_4	2_5	2_6	2_7
1	1	$1'$	2_1	2_2	2_3	2_4	2_5	2_6	2_7
$1'$	$1'$	1	2_1	2_2	2_3	2_4	2_5	2_6	2_7
2_1	2_1	2_1	$1+1'+2_2$	2_1+2_3	2_1+2_4	2_3+2_5	2_4+2_6	2_5+2_7	2_6+2_7
2_2	2_2	2_2	2_1+2_3	$1+1'+2_4$	2_1+2_5	2_2+2_6	2_3+2_7	2_4+2_7	2_5+2_6
2_3	2_3	2_3	2_1+2_4	2_1+2_5	$1+1'+2_6$	2_1+2_7	2_2+2_7	2_3+2_6	2_4+2_5
2_4	2_4	2_4	2_3+2_5	2_2+2_6	2_1+2_7	$1+1'+2_7$	2_1+2_6	2_2+2_5	2_3+2_4
2_5	2_5	2_5	2_4+2_6	2_3+2_7	2_2+2_7	2_1+2_6	$1+1'+2_5$	2_1+2_4	2_2+2_3
2_6	2_6	2_6	2_5+2_7	2_4+2_7	2_3+2_6	2_2+2_5	2_1+2_4	$1+1'+2_3$	2_1+2_2
2_7	2_7	2_7	2_6+2_7	2_5+2_6	2_4+2_5	2_3+2_4	2_2+2_3	2_1+2_2	$1+1'+2_1$

[1] S. Mandelstam, Nucl. Phys. **B213**, 149 (1983).
[2] S. Kachru and E. Silverstein, Phys. Rev. Lett. **80**, 4855 (1998).
[3] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
[4] P.H. Frampton, Phys. Rev. D **60**, 041901(R) (1999).
[5] P.H. Frampton and W.F. Shively, Phys. Lett. B **454**, 49 (1999).
[6] P.H. Frampton and C. Vafa, hep-th/9903226.
[7] P.H. Frampton, Phys. Rev. D **60**, 085004 (1999).
[8] P.H. Frampton, Phys. Rev. D **60**, 121901(R) (1999).
[9] P.H. Frampton and S.L. Glashow, Phys. Lett. B **190**, 157 (1987); Phys. Rev. Lett. **58**, 2168 (1987).
[10] P.H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).
[11] P.H. Frampton and T.W. Kephart, Phys. Rev. Lett. **50**, 1343 (1983); **50**, 1347 (1983); M.B. Green and J.H. Schwarz, Phys. Lett. **149B**, 117 (1984); L. Alvarez-Gaume and E. Witten, Nucl. Phys. **B234**, 269 (1984).
[12] Useful sources of information on the finite groups include D. E. Littlewood, *The Theory of Group Characters and Matrix Representations of Groups* (Oxford University Press, Oxford, 1940); M. Hamermesh, *Group Theory and Its Applications to Physical Problems* (Addison-Wesley, Reading, MA, 1962); J. S. Lomont, *Applications of Finite Groups* (Academic, New York, 1959) (reprinted by Dover, New York, 1993); A. D. Thomas and G.V. Wood, *Group Tables* (Shiva, Orpington, Kent, 1980).
[13] Another (nonminimal) approach to family symmetry appeared in the paper by P.H. Frampton, Phys. Lett. **88B**, 299 (1979).
[14] S. Pakvasa and H. Sugawara, Phys. Lett. **73B**, 61 (1978).
[15] P.H. Frampton and T.W. Kephart, Int. J. Mod. Phys. A **10**, 4689 (1995); Phys. Rev. D **51**, 1 (1995).
[16] J.C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R.N. Mohapatra and J.C. Pati, *ibid.* **11**, 566 (1975); R.N. Mohapatra and G. Senjanovic, *ibid.* **12**, 1502 (1975).
[17] H. Verlinde, Nucl. Phys. **B580**, 264 (2000).
[18] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999); J. Lykken and L. Randall, J. High Energy Phys. **06**, 014 (2000).
[19] W.D. Goldberger and M.B. Wise, Phys. Rev. D **60**, 107505 (1999); Phys. Rev. Lett. **83**, 4922 (1999); Phys. Lett. B **475**, 275 (2000).
[20] I. Antoniadis, Phys. Lett. B **246**, 377 (1990); I. Antoniadis and K. Benakli, *ibid.* **326**, 69 (1994); I. Antoniadis, K. Benakli, and M. Quiros, *ibid.* **331**, 313 (1994); J.D. Lykken, Phys. Rev. D **54**, 3693 (1996); I. Antoniadis *et al.*, Phys. Lett. B **436**, 257 (1998); I. Antoniadis, S. Dimopoulos, A. Pomarol, and M. Quiros, Nucl. Phys. **B544**, 503 (1999); K. Dienes, E. Dudas, and T. Gherghetta, Phys. Lett. B **436**, 55 (1998); Nucl. Phys. **B537**, 47 (1999); **B543**, 387 (1999); P.H. Frampton and A. Rasin, Phys. Lett. B **460**, 313 (1999); D. Ghilencea and G.G. Ross, *ibid.* **442**, 165 (1998); C.D. Carone, *ibid.* **454**, 70 (1999); C. Bachas, J. High Energy Phys. **11**, 023 (1998); G. Shiu and S.H.H. Tye, Phys. Rev. D **58**, 106007 (1998); Z. Kakushadze and S.H.H. Tye, Nucl. Phys. **B548**, 180 (1999); Z. Kakushadze, *ibid.* **B551**, 549 (1999); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 506 (1998).
[21] P.H. Frampton and T.W. Kephart, Phys. Lett. B **485**, 403 (2000).
[22] M. Douglas and G. Moore, hep-th/9603167.