New approach to the classification and solving of Einstein-Maxwell-dilaton gravity and its application for a particular set of exactly solvable models

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(Received 23 May 2001; revised manuscript received 18 June 2001; published 26 September 2001)

We prove the full separability of the static dyonic Einstein-Maxwell-dilaton system for three basic geometries that in turn yields the simple procedure of getting what we call the classes of integrability. It reveals the sector structure of EMD theory — in particular, it demonstrates that each graviton-dilaton scale relation determines a unique coupling-potential pair. Illustrating these concepts, we study the so-called linear class, which has a number of remarkable features: it comprises numerous EMD models including string-inspired, Liouville, trigonometric, polynomial, etc., and the majority of them remain nontrivial even if both charges are zeros; in addition to the usual electric-magnetic duality it obeys a certain duality between Maxwell-dilaton coupling and the dilaton potential. We single out some models inside this class and obtain the families of exact dyonic solutions. In a certain limit they can be interpreted as the Reissner–Nordström–de Sitter (with "renormalized" dyonic charge) plus small logarithmic corrections. The latter change the global structure of the nonperturbed solution by shifting and splitting of horizons, breaking down extremality and "dressing" the naked singularity. Finally, a certain cosmological-type model brings some insight concerning the appearance of a cosmological electrostatic field in the low-energy limit of string theory.

DOI: 10.1103/PhysRevD.64.084026 PACS number(s): 04.20.Jb, 04.40.Nr, 04.65.+e

I. INTRODUCTION

The Einstein-Maxwell-dilaton (EMD) system described by the action

$$2k_D^2 S=\int d^D x\sqrt{-g}\,Z[R+B(\partial\phi)^2+\Xi F^2+\Lambda], \qquad (1)$$

with Z, B, Ξ, and Λ being functions of ϕ, $F_{\mu\nu}=\nabla_\mu\mathcal{A}_\nu-\nabla_\nu\mathcal{A}_\mu$, is nowadays the most important field-theoretical model; e.g., it appears in the low-energy limit of string theory. In general, the arena of such systems is the microworld where averaged charges cannot be made negligible. In the neutral case $\Xi\equiv 0$ this system is primarily used in cosmology [1] beginning from the (Jordan-Thirry-)Brans-Dicke models or when studying the fundamental aspects of black hole physics. Theorists are highly interested in its exact solutions because no satisfactory perturbation theory has been constructed and numerical solutions can be regarded only as additional arguments, whereas if one knows an exact solution then further studies are straightforward.

To settle all the conventions let us first write the equations of motions following from the action above:

$$G_{\mu\nu}-g_{\mu\nu}\left[\left(\frac{B}{2}-\frac{Z_{,\phi\phi}}{Z}\right)(\partial\phi)^2+\frac{\Xi F^2+\Lambda}{2}-\frac{Z_{,\phi}}{Z}\Box\phi\right]+\left(B-\frac{Z_{,\phi\phi}}{Z}\right)\partial_\mu\phi\,\partial_\nu\phi+2\Xi F_{\mu\alpha}F_\nu^{\ \alpha}=\frac{Z_{,\phi}}{Z}\nabla_\mu\nabla_\nu\phi,$$

$$2ZB\Box\phi+(ZB)_{,\phi}(\partial\phi)^2-Z_{,\phi}R-(Z\Xi)_{,\phi}F^2=(Z\Lambda)_{,\phi},$$

$$Z\Xi\,\nabla^\mu F_{\mu\nu}+(Z\Xi)_{,\phi}\partial^\mu\phi F_{\mu\nu}=0, \qquad (2)$$

where G is the Einstein tensor and the subscript "ϕ" stands for the derivative with respect to dilaton field. Then, to simplify further considerations, without loss of generality we can assume $Z(\phi)=1$ (that can be always achieved by virtue of the metric conformal rescaling if $D>2$) and $B(\phi)=-\beta/2$, β is a constant — we wish to keep it unfixed as a regulator of the dilaton's rescaling which may involve the imaginary unit. Further, we are interested in four-dimensional (4D) static solutions hence we will work with the metric ansatz

$$ds^2=-e^{U(r)}dt^2+e^{-U(r)}dr^2+e^{A(r)}d\Omega^2_{(k)}, \qquad (3)$$

$$d\Omega^2_{(k)}\equiv\begin{cases} d\theta^2+\sin^2\theta d\varphi^2, & k=1,\\ d\theta^2+\theta^2 d\varphi^2, & k=0,\\ d\theta^2+\sinh^2\theta d\varphi^2, & k=-1,\end{cases} \qquad (4)$$

thus k enumerates these three geometries — spherical, flat, and hyperbolic — we will work with all of them simultaneously and uniformly. Similarly, the electromagnetic potential 1-form is assumed as

$$\mathcal{A}=\begin{cases} \omega(r)dt-P\cos\theta d\varphi, & k=1\ ,\\ \omega(r)dt-\frac{1}{2}P\theta^2 d\varphi, & k=0\ ,\\ \omega(r)dt-P\cosh\theta d\varphi, & k=-1\ ,\end{cases} \qquad (5)$$

with constant P being the magnetic charge. With all this in hand, the field equations take the form (we adopt curvature conventions from Ref. [2])

$$A''+A'(A'+U')-\hat{\Xi}\,e^{-2A-U}=\Lambda e^{-U}+2ke^{-A-U}, \qquad (6)$$

$$\beta\phi''+\beta\phi'(A'+U')+\hat{\Xi}_{,\phi}e^{-2A-U}+\Lambda_{,\phi}e^{-U}=0, \qquad (7)$$

$$2A''+A'^2+\beta\phi'^2=0, \tag{8}$$

$$\Xi\,\omega'=Qe^{-A}, \tag{9}$$

where $\hat{\Xi}\equiv 2(Q^2\,\Xi^{-1}+P^2\Xi)$, $'\equiv\partial_r$, integration constant Q stands for electric charge up to a coefficient.

II. SEPARABILITY AND SECTOR STRUCTURE OF EMD THEORY

In the theory of systems of differential equations the full separability is a maximal achievement because then one has the system of mutually noninvolved equations that is convenient for further studies toward the full integrability, if possible, and is crucial for physical understanding. To demonstrate the full separability of the static EMD system, first one should switch the independent variable from r to ϕ (due to the dilaton being an invertible function of r). Then by straightforward linear rearrangement of Eqs. (6) and (7), with the use of rest ones, it can be shown that the essential system (6)–(8) is equivalent to the following one:

$$2k(p-1)+\frac{2e^{\tilde{A}}}{p}(\Lambda+e^{-p\tilde{A}}\hat{\Xi})+e^{\tilde{U}+2Y}\left(\frac{\beta}{p\tilde{A}^2_{,\phi}}-\frac{\tilde{U}_{,\phi}}{\tilde{A}_{,\phi}}-\frac{p-1}{2}\right)=0, \tag{10}$$

$$2k(p-1)+\frac{2e^{\tilde{A}}}{p}\left(\Lambda+\frac{p}{2\beta}\Lambda_{,\phi}\tilde{A}_{,\phi}\right)+e^{\tilde{U}+2Y}\left(\frac{1}{\tilde{A}_{,\phi}}\right)_{,\phi}+\frac{2e^{-(p-1)\tilde{A}}}{p}\left(\hat{\Xi}+\frac{p}{2\beta}\hat{\Xi}_{,\phi}\tilde{A}_{,\phi}\right)=0, \tag{11}$$

$$\phi'=\pm\frac{e^{Y-\tilde{A}/2}}{\tilde{A}_{,\phi}},\quad Y(\phi)\equiv-\frac{\beta}{p}\int\frac{d\phi}{\tilde{A}_{,\phi}}+Y_0, \tag{12}$$

where $U(r)\equiv\tilde{U}(\phi(r))$, $A(r)\equiv\tilde{A}(\phi(r))$, and $p\equiv D-2=2$ (though, this system is valid for EMD in arbitrary $D>2$ provided $P\equiv 0$ at $D\neq 4$). If $\tilde{A}_{,\phi}\neq$const then $\tilde{U}$ is algebraically given by Eq. (11) so one can easily exclude it from Eq. (10) to receive the core equation of the EMD theory — the class equation

$$\frac{H_{,\phi}}{\tilde{A}_{,\phi}}+\left(\frac{\beta}{p\tilde{A}^2_{,\phi}}+\frac{p-1}{2}\right)H+k(p-1)+\frac{e^{\tilde{A}}}{p}(\Lambda+e^{-p\tilde{A}}\hat{\Xi})=0, \tag{13}$$

where

$$H\equiv\frac{1}{p(1/\tilde{A}_{,\phi})_{,\phi}}\left[kp(p-1)+e^{\tilde{A}}\left(\Lambda+\frac{p}{2\beta}\Lambda_{,\phi}\tilde{A}_{,\phi}\right)+e^{-(p-1)\tilde{A}}\left(\hat{\Xi}+\frac{p}{2\beta}\hat{\Xi}_{,\phi}\tilde{A}_{,\phi}\right)\right].$$

This equation is a third-order with respect to $\tilde{A}$ and a *linear* second-order ordinary differential equation (ODE) with respect to Λ and $\hat{\Xi}$. Therefore, with each $\tilde{A}$ it is associated the appropriate class of integrability determined by the equation above, which determines a self-consistent Ξ-Λ pair. Thus, we came to the system of autonomous equations consecutively yielding A,ϕ,U. Regrettably, the class equation is a nonlinear ODE so the direct task is still hard to accomplish without supplementary symmetries or assumptions. From the physical viewpoint, the class equation is highly important by itself. To see that, first note that the dependence $A(\phi)$ is occasionally more universal than, e.g., $U(\phi)$ or $A(U)$: being related to the radius of a (compact) product space, A determines, in fact, the geometrical scale of gravity. Therefore, the function $\tilde{A}$ symbolizes the relation between the gravitational and dilatonic scales, and the class equation claims that with each such relation is associated a unique Ξ-Λ pair. Thus, classes of integrability can shed light upon the fundamental nature of the latter which is precisely known neither in string theory nor in cosmology. Moreover, dealing with low-energy string theory, one can use the formalism to accomplish the inverse task: deduce the form of Ξ (which is so far known only perturbatively) by implying physical assumptions for the Λ term. However, all this is possible only if the dependence $A(\phi)$ is known explicitly. Therefore, the primary aim now is to study the physically relevant $\tilde{A}$'s and properties of the models they yield.

III. AN EXAMPLE: LINEAR CLASS

By virtue of Eq. (13), each function $A(\phi)$ uniquely determines one or another class (sector) of EMD gravity. In turn, each class comprises of the plethora of models with specific Ξ and Λ. For instance, the class that predominates in supergravity and superstring theories [2–4] is given by

$$\tilde{A}=d_1\phi-\ln d_2+d_3\ln(e^{d_4\phi}-1), \tag{14}$$

where the constants d_i are all fixed except d_2 which is related to the mass-charge parameter [5]. Further, in this hierarchy there exists a one exceptional class: if $\tilde{A}\sim\phi$ then $\tilde{U}$ disappears in Eq. (11), so the latter turns to the linear first-order ODE with respect to Λ and Ξ, whereas Eq. (13) becomes meaningless. This class approximates supergravity classes (when $e^{d_4\phi}$ is much larger or much smaller than one), in addition it is of interest by itself, so worthy of study in detail.

A. Overview of models and solutions

We begin with

$$\tilde{A}=d_1\phi-\ln d_2,\quad \phi=\begin{cases}\dfrac{2d_1}{\beta+d_1^2}\ln r, & d_1^2+\beta\neq 0,\\ \chi r/\sqrt{\beta}, & d_1=i\sqrt{\beta},\end{cases} \tag{15}$$

with d_i and χ being arbitrary constants, $d_2 \equiv e^{d_1 \phi_0}$. Here the first equation is imposed whereas the expression for $\phi(r)$ comes after integration of Eq. (12), assuming the "+" branch for definiteness. As for U and ω then their problem is trivial [6]. Equation (11) becomes the integrability class equation that determines the admissible Ξ and Λ:

$$\frac{e^{d_1\phi}}{d_2}\left(\Lambda+\frac{d_1}{\beta}\Lambda_{,\phi}\right)+\frac{d_2}{e^{d_1\phi}}\left(\hat{\Xi}+\frac{d_1}{\beta}\hat{\Xi}_{,\phi}\right)=-2k. \quad (16)$$

In addition to the usual electric-magnetic $\{\Xi \leftrightarrow 1/\Xi,\, Q \leftrightarrow P\}$ duality this linear ODE is invariant under the duality transformations between Maxwell-dilaton coupling and dilaton potential, and between physical and tachyonic sectors of the theory $\{\Lambda \leftrightarrow \hat{\Xi},\, d_2 \leftrightarrow 1/d_2,\, d_1 \leftrightarrow -d_1,\, \beta \leftrightarrow -\beta\}$. Here we will not address the separate good issue — does this duality play any special physical role. In addition, analyzing the D-dimensional analogue of Eq. (16) one should emphasize that despite it does not undergo sufficient changes, the abovementioned duality appears to be broken at $D \neq 4$; it is curious that the electric-magnetic duality is also broken if $D \neq 4$.

Thus, the expressions (15) and (16) (with Ref. [6] kept in mind) yield a complete general-in-class solution. Now, to demonstrate how large this sector is, let us consider its most key or important specimens.

(*a*) *Exponential (string-inspired) coupling*: $\Xi = a_1 e^{b_1 \phi}$. The physically interesting cases are as follows (but not limited to). If one assumes in action (1) that ($D=4$): $\beta = 16/(D-2)$, $a_1 = -k_D^2/2$, $b_1 = -4g_2/(D-2)$ then $g_2=1$ corresponds to field theory limit of superstring model (more precisely, compactified effective theory if $D=4$), $g_2 = \sqrt{1+(D-2)/n}$ corresponds to the toroidal T^n reduction of $(D+n)$-spacetime to D-spacetime, $g_2=0$ is a usual Einstein-Maxwell system. The previously done work is: Gibbons and Maeda [8] received solutions for $\Lambda=0$ and arbitrary D and g_2, see also Refs. [4,9], several solutions for potentials of special type were obtained in Refs. [2,10,11]. Also a lot of qualitative and numerical work has been done [12–14]. To summarize the present knowledge about exact static spherically symmetric solutions, we first note that most of these have been obtained in the case of a vanishing potential, and, second, with the exception of Refs. [2,11] the cases of dyonic solutions and of solutions with nontrivial dilaton potentials have not been treated. Integrating Eq. (16) one reveals the following cases.

(1) $\beta - d_1^2 \neq 0$, $\beta \pm b_1 d_1 - 2d_1^2 \neq 0$. Then the dilaton potential is given by

$$\Lambda = a_2 e^{-(\beta/d_1)\phi} - \frac{2k\beta d_2}{\beta - d_1^2} e^{-d_1\phi} - \frac{2a_1 d_2^2 P^2(\beta + b_1 d_1)}{\beta + b_1 d_1 - 2d_1^2} e^{(b_1 - 2d_1)\phi} - \frac{2d_2^2 Q^2(\beta - b_1 d_1)}{a_1(\beta - b_1 d_1 - 2d_1^2)} e^{-(b_1+2d_1)\phi}, \quad (17)$$

for the sake of uniformity we will not make redefinitions of newborn arbitrary constants a_2, d_1, d_2, etc. Incidentally, note that Λ is essentially exponential hence it can be either large or incredibly small value — the former takes place in microworld whereas the latter does in cosmology. Further, when obtaining U we reveal a number of additional subcases generalizing Ref. [11].

(i) $\beta - 3d_1^2 \neq 0$, $\beta + d_1^2 \neq 0$, $\beta \pm 2b_1 d_1 - d_1^2 \neq 0$. Then Λ is given by Eq. (17) whereas the complete solution is

$$e^U = \frac{(\beta + d_1^2)^2}{2d_1^2}\left[\frac{c r^{1-2d_1^2/(\beta+d_1^2)}}{\beta + d_1^2} - \frac{2k d_1^2 d_2 r^{2\beta/(\beta+d_1^2)}}{\beta^2 - d_1^4} - \frac{a_2 r^{2d_1^2/(\beta+d_1^2)}}{\beta - 3d_1^2} - \frac{4P^2 a_1 d_1^2 d_2^2 r^{2(\beta + b_1 d_1 - d_1^2)/(\beta+d_1^2)}}{(\beta + b_1 d_1 - 2d_1^2)(\beta + 2b_1 d_1 - d_1^2)} - \frac{4Q^2 d_1^2 d_2^2 r^{2(\beta - b_1 d_1 - d_1^2)/(\beta+d_1^2)}}{a_1(\beta - b_1 d_1 - 2d_1^2)(\beta - 2b_1 d_1 - d_1^2)}\right],$$

$$\omega - \omega_0 = \frac{Q d_2(\beta + d_1^2)}{a_1(\beta - 2b_1 d_1 - d_1^2)} r^{1-2d_1(b_1+d_1)/(\beta+d_1^2)}, \quad (18)$$

and A and ϕ are given precisely by Eq. (15).

(ii) $\beta - 3d_1^2 = 0$, $\beta + d_1^2 \neq 0$, $\beta \pm 2b_1 d_1 - d_1^2 \neq 0$. Choose the positive root then Λ is given by Eq. (17) at $d_1 = \sqrt{\beta/3}$ and the solution is

$$e^U = 2\sqrt{r}\left[c + a_2 \ln r - k d_2 r - \frac{8\beta d_2^2 Q^2 r^{1/2(1-\sqrt{3/\beta}b_1)}}{a_1(\sqrt{\beta} - \sqrt{3}b_1)^2} - \frac{8\beta a_1 d_2^2 P^2 r^{1/2(1+\sqrt{3/\beta}b_1)}}{(\sqrt{\beta} + \sqrt{3}b_1)^2}\right], \quad e^A = \frac{\sqrt{r}}{d_2},$$

$$e^{2\phi} = r^{\sqrt{3/\beta}}, \quad \omega - \omega_0 = \frac{2\sqrt{\beta}\, d_2\, Q/a_1}{\sqrt{\beta} - \sqrt{3}\, b_1} r^{1/2(1-\sqrt{3/\beta}b_1)}. \quad (19)$$

Other subcases when expressions (18) but not (17) become singular can be treated similarly using Ref. [6] and Eq. (15). Their common feature is the appearance of logarithm in e^U.

(2) $\beta - d_1^2 = 0$, $\beta \pm b_1 d_1 - 2d_1^2 \neq 0$. We choose the root $d_1 = \sqrt{\beta}$ then following Eq. (16) the dilaton potential is given by

$$\Lambda = (a_2 - 2k\sqrt{\beta} d_2 \phi) e^{-\sqrt{\beta}\phi} + 2d_2^2 e^{-2\sqrt{\beta}\phi}\left[\frac{a_1 P^2(\sqrt{\beta} + b_1)}{\sqrt{\beta} - b_1} e^{b_1\phi} + \frac{Q^2(\sqrt{\beta} - b_1)}{a_1(\sqrt{\beta} + b_1)} e^{-b_1\phi}\right], \quad (20)$$

and the only solution (no subcases) is

$$e^U=c-2kd_2r(\ln r-2)+\frac{4\beta d_2^2}{b_1}\left[\frac{a_1P^2}{\sqrt{\beta}-b_1}r^{b_1/\sqrt{\beta}}-\frac{Q^2/a_1}{\sqrt{\beta}+b_1}r^{-b_1/\sqrt{\beta}}\right]+a_2r,\quad e^A=r/d_2,$$

$$e^{\sqrt{\beta}\phi}=r,\quad \omega-\omega_0=\frac{\sqrt{\beta}d_2Q}{a_1b_1}r^{-b_1/\sqrt{\beta}}. \tag{21}$$

(3) $\beta-d_1^2\neq 0$, $\beta+b_1d_1-2d_1^2=0$, $\beta-b_1d_1-2d_1^2\neq 0$. To avoid wearisome square-root branches let us impose $b_1=2d_1-\beta/d_1$, and work with d_1 bearing in mind its relation to the given parameter b_1. We have

$$\Lambda=(a_2-4a_1d_1d_2^2P^2\phi)e^{-(\beta/d_1)\phi}-\frac{2k\beta d_2}{\beta-d_1^2}e^{-d_1\phi}-\frac{2d_2^2Q^2(\beta-d_1^2)}{a_1(\beta-2d_1^2)}e^{[(\beta-4d_1^2)/d_1]\phi}, \tag{22}$$

whereas the solution (at $3\beta-5d_1^2\neq 0$) is expressed as

$$e^U=(\beta+d_1^2)\left[fr^{2d_1^2/(\beta+d_1^2)}+cr^{1-2d_1^2/(\beta+d_1^2)}-\frac{kd_2r^{2\beta/(\beta+d_1^2)}}{\beta-d_1^2}-\frac{d_2^2(\beta+d_1^2)Q^2r^{(4\beta-6d_1^2)/(\beta+d_1^2)}}{a_1(3\beta-5d_1^2)(\beta-2d_1^2)}\right],$$

$$\omega=\frac{d_2Q(\beta+d_1^2)}{a_1(3\beta-5d_1^2)}r^{(3\beta-5d_1^2)/(\beta+d_1^2)},$$

with A and ϕ given by Eq. (15), where we have denoted

$$f\equiv-\frac{\beta+d_1^2}{d_1^2(\beta-3d_1^2)}\left[\frac{a_2}{2}+\frac{a_1d_2^2P^2}{(\beta+d_1^2)(\beta-3d_1^2)}\times[\beta^2-2\beta d_1^2(2\ln r+3)+d_1^4(12\ln r-7)]\right].$$

The subcase $3\beta-5d_1^2=0$, i.e., $\{b_1,d_1\}=\{\pm\sqrt{\beta/15},\pm\sqrt{3\beta/5}\}$, can be treated similarly (nothing special with it except that ω turns out to be a linear function of $\ln r$).

(4) $\beta-d_1^2\neq 0$, $\beta+b_1d_1-2d_1^2\neq 0$, $\beta-b_1d_1-2d_1^2=0$. This case is in some sense a counterpart of the previous one (a3) but the roles of Q and P are interchanged. It can be treated absolutely similarly as well as other cases when expressions $\beta-d_1^2$ and $\beta\pm b_1d_1-2d_1^2$ are equal zero pairwise (the case when they are zeros all together is inconsistent). In all these cases Λ resemble the potential from (a3), i.e., happen to be the combinations of exponents of dilaton coupled to linear functions of dilaton. Instead of handling them it is better to outline some other interesting Ξ-Λ pairs that belong to the class.

(b) *Gravity coupled to neutral scalar field*: $\Xi\equiv 0$. Historically the neutral scalar field is the oldest system (massless neutral scalar field was considered by Fisher in 1948 [15], see also Ref. [16]). Integration of Eq. (16) at $Q=P=0$ reveals the following three cases:

(1) $\beta-d_1^2\neq 0$. It is the (generalized) Liouville model

$$\Lambda=a_2e^{-\beta\phi/d_1}-\frac{2k\beta d_2}{\beta-d_1^2}e^{-d_1\phi}, \tag{23}$$

qualitatively studied in Ref. [7], and from Eq. (15) and Ref. [6] we have the two following subcases.

(i) $\beta-3d_1^2\neq 0$. Here Λ is as above whereas the solution is given precisely by Eq. (18) at $Q=P=0$.

(ii) $\beta-3d_1^2=0$. Choosing for definiteness the root $d_1=\sqrt{\beta/3}$ we have

$$\Lambda=e^{-\sqrt{3\beta}\phi}(a_2-3kd_2\,e^{2\sqrt{\beta/3}\phi}), \tag{24}$$

so this is special case $Q=P=0$ of the solution (19).

(2) $\beta+d_1^2=0$. The Λ in this case is a special case of Eq. (23) but we want to treat it separately because this is the only way to combine exponents into trigonometric functions. Using Eq. (15) and [6] we obtain

$$\Lambda=a_2\,e^{i\sqrt{\beta}\phi}-\frac{kd_2}{e^{i\sqrt{\beta}\phi}},\quad e^U=\frac{kd_2r+c}{i\chi e^{i\chi r}}-\frac{a_2e^{i\chi r}}{2\chi^2}, \tag{25}$$

where c is another arbitrary constant and A and ϕ are given by Eq. (15). The complete story is that at $k\neq 0$ this solution contains the simplest trigonometric potentials. If we assume $a_2=-kd_2$, then we come to the cosine-Einstein model and similarly for sine-Einstein ($a_2=kd_2$, $d_2=i\overline{d}_2$):

$$\Lambda=-2kd_2\cos(\sqrt{\beta}\phi),\quad \Lambda=-2k\overline{d}_2\sin(\sqrt{\beta}\phi), \tag{26}$$

and for sinh and cosh just by Wick rotation. The solutions of such models on flat or fixed background geometry (sine-Gordon, etc.) have been got long ago but to our knowledge so far nobody has managed to obtain any self-gravitating solution despite tremendous efforts were made in view of evident importance of the subject.

(3) $\beta-d_1^2=0$. Choosing for definiteness a positive root we obtain the Liouville model coupled to a linear term

$$\Lambda=e^{-\sqrt{\beta}\phi}(a_2-2k\sqrt{\beta}\,d_2\,\phi), \tag{27}$$

its solution is given by Eq. (21) at zero charges, and with this model the neutral case is exhausted.

(c) *Cosmological constant potential*: $\Lambda=\Lambda_0$. The importance of EMD gravity with cosmological constant Λ_0 is obvious — for instance, it is related to the well-known "cosmological constant problem" and in use in string theory and supergravity related models. In addition, as a special case it comprises the massless dilaton, which comes from the tree level approximation of string theory without central charges, and recently is the most explored case if Ξ is a single exponent (minimal string-induced coupling), as mentioned above (see more about the massless dilaton below). Some nonminimal dilaton couplings (that can appear from, e.g., string threshold corrections) were approximately studied in Ref. [17]. What about Ξ's for our class? We have from Eq. (16)

$$\hat{\Xi} = a_1 e^{-(\beta/d_1)\phi} - \frac{2k\beta}{d_2(\beta+d_1^2)} e^{d_1\phi} - \frac{\beta\Lambda_0}{d_2^2(\beta+2d_1^2)} e^{2d_1\phi}, \tag{28}$$

where $\beta+d_1^2\neq 0$. We mentioned above that the linear class approximates the supergravity one, therefore, by last equation we have outlined the exact form of the Maxwell-dilaton coupling (up to integration constants) without the use of direct methods (e.g., string perturbation theory) but rather by requiring the self-consistency of supergravity low-energy limit (recall the proven uniqueness-inside class of Ξ when Λ is imposed). Of course, thereby we have imposed the form of dilaton potential but this is just a physical approximation. The metric (3) at $\beta+2d_1^2\neq 0$ and $\beta+3d_1^2\neq 0$ is

$$e^U = kd_2 r^{2\beta/(\beta+d_1^2)} + (\beta+d_1^2)\left[\frac{\Lambda_0(\beta+d_1^2)r^2}{(\beta+2d_1^2)(\beta+3d_1^2)} - \frac{2\mu d_2^{3/2}}{d_1^2} r^{(\beta-d_1^2)/(\beta+d_1^2)} - \frac{a_1 d_2^2}{2d_1^2} r^{-2d_1^2/(\beta+d_1^2)}\right], \tag{29}$$

and Eq. (15) holds, as usual. Further, if we consider purely electric case then the coupling is simply $\Xi = 2Q^2/\hat{\Xi}$, $a_1 = -2Q^2$, and

$$\omega = \frac{Qd_2}{r} - \frac{k\beta r}{Q(\beta+d_1^2)} - \frac{\beta\Lambda_0(\beta+d_1^2) r^{(\beta+3d_1^2)/(\beta+d_1^2)}}{2d_2 Q(\beta+2d_1^2)(\beta+3d_1^2)}. \tag{30}$$

The cases when only magnetic or both charges, as well as the complex cases when $\beta+2d_1^2$ and/or $\beta+3d_1^2$ are zeros, can be done by analogy.

(d) *Massless dilaton*: $\Lambda=0$. As was mentioned above recently this is the most explored case if Ξ is a single exponent (nonstandard dilaton couplings were approximately studied in Ref. [17]). We have from Eq. (16) the two cases

$$\hat{\Xi} = \begin{cases} a_1 e^{-(\beta/d_1)\phi} - \dfrac{2k\beta/d_2}{\beta+d_1^2} e^{d_1\phi}, & d_1^2+\beta\neq 0, \\ 2\, e^{i\sqrt{\beta}\phi}\left(a_1 + \dfrac{ik\sqrt{\beta}}{d_2}\phi\right), & d_1 = i\sqrt{\beta}. \end{cases} \tag{31}$$

We will consider the following subcases.

(1) $\beta\pm d_1^2\neq 0$. Then $\hat{\Xi}$ is given by the first from Eqs. (31), A and ϕ are given precisely by Eq. (15), and

$$e^U = r^{2\beta/(\beta+d_1^2)} d_2\left[k + \frac{\beta+d_1^2}{2d_1^2}\left(\frac{c}{r} - \frac{a_1 d_2}{r^2}\right)\right],$$

$$\omega = -\int \frac{4d_2 Q P^2 r^{2(d_1^2-\beta)/(\beta+d_1^2)} f^{-1} dr}{1+\sqrt{1-(4PQr^{2\beta/(\beta+d_1^2)} f^{-1})^2}}, \tag{32}$$

where $f\equiv a_1 - 2k\beta r^2/d_2(\beta+d_1^2)$. The purely electric solution is given by the appropriate expressions at $\Lambda_0=0$ from the paragraph (c) above.

(2) $\beta-d_1^2=0$. If one assumes $d_1=\pm\sqrt{\beta}$ then the first from Eq. (31) generates several trigonometric Maxwell-dilaton couplings if one adjusts a_1 (d_2), Q and P. Its solution for $d_1=\sqrt{\beta}$ is ($f\equiv a_1/r - kr/d_2$):

$$e^U = c - d_2^2 f, \quad \omega = \int \frac{4d_2 Q P^2 dr/r}{f\pm\sqrt{f^2-(4PQ)^2}}, \tag{33}$$

and A,ϕ are exactly as in Eq. (21). Note that in single-charged cases the square root in ω disappears so the latter can be resolved in ordinary functions.

(e) $\Lambda = a_2\sinh^2\phi$. This potential is also of interest for string theory though as a trial one. Its qualitative study was done in Ref. [14]. First consider the branch $d_1=\pm i\sqrt{\beta}$. Choosing the plus root we have two subcases (here $f\equiv 2i\cosh\phi + \sqrt{\beta}\sinh\phi$):

$$\hat{\Xi} = \frac{a_2 f^2 e^{2i\sqrt{\beta}\phi}}{d_2^2(\beta+4)} + e^{i\sqrt{\beta}\phi}\left(a_1 + \frac{2ik\sqrt{\beta}}{d_2}\phi\right), \tag{34}$$

$$\hat{\Xi} = e^{-2\phi}\left[a_1 - \frac{a_2+4kd_2}{d_2^2}\phi - \frac{a_2}{2d_2^2}e^{-2\phi}\right], \quad \beta=-4. \tag{35}$$

Further, at arbitrary $d_1\neq\pm i\sqrt{\beta}$, Eq. (16) produces so ugly $\hat{\Xi}$ that there is no sense to present it here. Instead we tried to find some simple case more or less resembling the string model of Ref. [14] $\Xi\sim e^{-2\phi}$, $\beta=4$. For instance, if we assume the purely magnetic case and impose $d_1=\sqrt{\beta}$, $\beta=4$ we obtain the model which approximates the string one at large P or d_2:

$$\Xi = a_1 e^{-2\phi} + \frac{a_2 e^{4\phi}(4-3e^{2\phi})}{48 d_2^2 P^2} - \frac{ke^{2\phi}}{2d_2 P^2}, \quad e^{2\phi}=r,$$

$$e^U = c + kd_2 r + \frac{a_2 r}{24}(r^2-4r+6) - \frac{2d_2^2 P^2 a_1}{r}, \tag{36}$$

and A is exactly as in Eq. (21).

(f) *Quadratic potential*: $\Lambda = a_2\phi^2$. This classical potential (whose study in gravity can be traced back as far as Ref. [15]) nowadays has been revived as a test one in string theory [13,14], but all the studies so far were conducted at numerical or qualitative level only. For the sake of brevity we consider only the case $\beta+d_1^2\neq 0$ then Eq. (16) yields

$$\hat{\Xi} = \frac{a_1}{e^{(\beta/d_1)\phi}} + \frac{(\phi-\phi_+)(\phi-\phi_-)}{\alpha^3 d_2^2/a_2} e^{2d_1\phi} - \frac{2k\beta/d_2}{\beta+d_1^2} e^{d_1\phi}, \tag{37}$$

where $\alpha \equiv \beta + 2d_1^2 \neq 0$, $\phi_\pm = (2d_1^2/\alpha\beta)(\pm\sqrt{\beta + d_1^2} - d_1)$. Again, the solution is unnecessarily bulky so it is better to assume $d_1 = \sqrt{\beta}$, $\beta = 1$ (we still have rescaling freedom due to a_2), then we obtain

$$e^U = c + kd_2 r + \frac{a_2 r^2}{54}[19 + 6\ln r\,(3\ln r - 5)] - \frac{a_1 d_2^2}{r},$$

with A and ϕ being given by Eq. (21), and ω is as in Eq. (33) if $f \equiv a_1/r - kr/d_2 - (a_2/27d_2^2)r^2(9\ln^2 r + 12\ln r - 4)$.

B. Physical properties of solutions

After we have enumerated the exact solutions for most interesting cases it is time to proceed to studies of the physical relevance of some above-mentioned families of solutions. However, the methods used below can be extrapolated on all the models that belong to the linear-class sector of EMD gravity. It is important to note that the common feature of the linear-class solutions is that they always have at least one physically interpretable limit — all the solutions converge to the Reissner-Nordström (–de Sitter) when the parameter d_1 approaches infinity. Therefore, their physical interpretation can be easily deduced from the series expansions in the d_1-parametric space. The examples of such a procedure will be given below.

(*a*) *Exponential (string-inspired) coupling*: $\Xi = a_1 e^{b_1\phi}$. We will study the case (a 1 i) because this is the largest family of solutions: d_1 is not fixed. Moreover, we will be concentrated on the properties of this solution at large values of $|d_1|$. In view of future considerations, we redefine the constants

$$c = -4\mu d_2^{3/2}, \quad a_1 = -1, \quad a_2 = \Lambda_0, \tag{38}$$

and switch to the distant observer's frame of reference

$$e^{A(r)} = d_2^{-1} r^{2d_1^2/(\beta + d_1^2)} \to r^2, \qquad d_2^{(\beta + d_1^2)/2d_1^2} t \to t. \tag{39}$$

In these new coordinates, assuming that $|d_1| \gg \max\{1, |\beta|, |b_1|\}$, we obtain that to the order $O[1/d_1]$ the metric (18) takes the habitual form

$$ds^2 = -e^{U_+}dt^2 + e^{-U_-}dr^2 + r^2 d\Omega_{(k)}^2, \tag{40}$$

where

$$e^{U_+} = e^{U_-} = k - \frac{2\mu}{r} + \frac{\Lambda_0 r^2}{6} + \frac{\Delta - \Theta\ln(r/\eta)}{r^2}, \tag{41}$$

and the following notations:

$$\Delta = Z^2 - \frac{5}{2}b_1 d_1^{-1}W, \quad \Theta = 2b_1 d_1^{-1}W, \quad \eta = d_2^{-1/2},$$

$$W = Q^2 - P^2, \quad Z^2 = Q^2 + P^2,$$

are used. Also, the $O[d_1^{-1}]$-asymptotical dilaton potential (17) and ω (18) are

$$\Lambda = \Lambda_0 + \frac{b_1 d_2^2 W}{d_1 e^{2d_1\phi}}, \qquad \omega = \frac{Q\{1 - 2b_1[1 + \ln(\sqrt{d_2}r)]/d_1\}}{r}. \tag{42}$$

The first four terms in Eq. (41) is the Reissner-Nordström–de Sitter with the only difference that the effective dyonic charge is the standard one Z plus a small correction. The last term, proportional to Θ, is something new — its influence will be studied below. From now we will work with the spherical case $k=1$, in addition we will neglect Λ_0 for simplicity. Then the information about the global structure can be read off from the intersection of two curves described by the algebraic equation

$$r^2 - 2\mu r + \Delta \equiv (r - \delta_+)(r - \delta_-) = \Theta\ln(r/\eta), \tag{43}$$

where $\delta_\pm = \mu(1 \pm \sqrt{1 - \Delta/\mu^2})$. It is useful to keep in mind that Θ is small ($O[d_1^{-1}]$) that simplifies subject matter.

Case $\mu^2 > \Delta$. If $\Theta = 0$ (i.e., $d_1 = \infty$) this case corresponds to the Reissner-Nordström black hole. Otherwise, to determine horizons we have to solve the transcendental Eq. (43) with real δ's. Fortunately, it can be done analytically with the use of Θ's smallness. Solving it, we obtain that we still have two horizons but their radii acquire a correction:

$$r_{H^\pm} = \delta_\pm + \frac{\Theta\ln(\delta_\pm/\eta)}{2(\delta_\pm - \mu)},$$

and the corresponding Hawking temperatures are

$$T_{H^\pm} = \frac{\delta_\pm - \mu}{2\pi\delta_\pm^2} - \frac{\Theta}{4\pi\delta_\pm^3}\left(1 + \frac{(\delta_\pm - 2\mu)\ln(\delta_\pm/\eta)}{\delta_\pm - \mu}\right), \tag{44}$$

an absolute value is implied.

Case $\mu^2 = \Delta$. Without the Θ perturbation we have extreme Reissner-Nordström black hole. It turns out that the series expansion used in the previous case fails so we have to invent another one. The nonperturbed horizon appears at $x = \mu$. We are interested in small deviations from the nonperturbed case so it is natural to expand Eq. (43) with respect to r up to the second order near this point. We obtain that Eq. (43) becomes the quadratic equation $(1 + \Theta/2\mu^2)r^2 - 2\mu(1 + \Theta/\mu^2)r + \mu^2 = \Theta[\ln(\mu/\eta) - \frac{3}{2}]$, from which one concludes that extremality is broken: the extreme horizon is shifted and split into two ones, with the radii

$$r_{H^\pm} = \mu + \frac{\Theta}{2\mu} \pm \sqrt{\Theta\ln(\mu/\eta)}.$$

Here, the term proportional to Θ shifts the horizon outward or inward (depending on the sign of $b_1 W/d_1$) whereas the term proportional to $\sqrt{\Theta}$ describes the split. It is curious that in the particular case $\eta = \mu$ the extremality is again restored up to $O[d_1^{-2}]$. The corresponding Hawking temperatures are

$$T_{H^\pm} = \frac{\sqrt{\Theta\ln(\mu/\eta)}}{2\pi\mu^2}\left(1 \mp \frac{2}{\mu}\sqrt{\Theta\ln(\mu/\eta)}\right). \tag{45}$$

Case $\mu^2<\Delta$. If $\Theta=0$ then the solution describes the naked Reissner-Nordström singularity. There is a strong hope that the Θ-perturbation "dresses" the singularity, i.e., creates a horizon around it. To prove it, one has to reveal the conditions at which the parabola and logarithmic curve (43) have the intersection point(s) even if the former does not cross an x axis. The intuitive solution for this is to require the minimum point of the parabola to be as close as possible to the x axis, hence, to the logarithmic curve, because the latter is small. The distance from the minimum point of the parabola to the x axis equals to $\Delta-\mu^2$, so Δ must be equal to μ^2 plus a small positive correction, say

$$\Delta=\mu^2+|\mathrm{const}\, d_1^{-1}|. \tag{46}$$

Again, we expand Eq. (43) near the minimum point of parabola and obtain the quadratic equation, $(1+\Theta/2\mu^2)x^2-2\mu(1+\Theta/\mu^2)x+\Delta=\Theta[\ln(\mu/\eta)-\frac{3}{2}]$. If it has complex roots then the singularity is naked otherwise it is hidden under at least one horizon. One can check that this equation in general case does not have real roots. However, if

$$\Delta=\mu^2-\frac{2\Theta\mu^2\ln(\mu/\eta)}{\Theta-4\mu^2}=\mu^2+\frac{\Theta}{2}\ln(\mu/\eta),$$

i.e., of the form (46), provided $b_1 d_1^{-1} W\ln(\mu/\eta)$ is nonpositive, then the imaginary part vanishes, so one does have the purely real double root. It means that we have found an example when a singularity is dressed by the single horizon. Its radius and Hawking temperature are

$$r_H=\mu+\frac{\Theta}{2\mu},\quad T_H=\frac{\Theta\ln(\mu/\eta)}{4\pi\mu^3},$$

the latter being of order $O[d_1^{-1}]$, rather than $O[d_1^{-1/2}]$ as in previous case.

(*b*) *Gravity coupled to neutral scalar field*: $\Xi\equiv 0$. We will study the physical interpretation of the neutral case with nonfixed d_1, i.e., (b 1 i). Actually, the latter is nothing but the case (a 1 i) at $Q=P=0$ but the asymptotic analysis of the neutral solutions is slightly different. The reason is that the neutral solutions at large $|d_1|\gg\max\{1,|\beta|\}$ converge to Schwarzschild–de Sitter even faster, so that the first nonzero correction is of order $O[d_1^{-2}]$ instead of previous $O[d_1^{-1}]$. Keeping in mind the redefinitions (38), to this order the dilaton potential becomes

$$\Lambda=\Lambda_0\left(1-\frac{\beta}{d_1}\phi\right)+\frac{2k\beta d_2}{d_1}e^{-d_1\phi}, \tag{47}$$

the metric is again of the form (40) but now $e^{U_+}\neq e^{U_-}$:

$$e^{U_\pm}=k-\frac{2\mu\left[1\pm\dfrac{\beta\ln(r/\eta)}{d_1^2}\right]}{r}+\frac{\Lambda_0 r^2}{6}\pm\frac{\beta f_\pm\ln(r/\eta)}{d_1^2}, \tag{48}$$

where $f_+\equiv 2k$, $f_-\equiv\Lambda_0 r^2/3$. The asymptotic form of this metric at large r differs from de Sitter, however, this deviation is physically negligible due to infinitesimality of d_1^{-2}, so the initial (exact) solution is still of physical interest. The other neutral scalar field cases do not seem to have any physically interpretable asymptotics.

(*c*) *Cosmological constant potential*: $\Lambda=\Lambda_0$. We will study the electrical case, i.e., the model with Maxwell-dilaton coupling $\Xi=2Q^2/\hat{\Xi}$ where $\hat{\Xi}$ is as in Eq. (28). The family of solutions is given by Eqs. (29), (30), and (15). Again, we are interested in analysis of the solutions at large nonfixed values of d_1, i.e., when $|d_1|\gg\max\{1,|\beta|\}$. Switching to the infinite-observer frame of reference (40), one can deduce that the metric converge to Reissner-Nordström–de Sitter so fast that the first nonzero correction is of order $O[d_1^{-2}]$ not $O[d_1^{-1}]$. To this order, the metric, field strength and Maxwell-dilaton coupling become, respectively,

$$e^{U_\pm}=k-\frac{2\mu}{r}+\frac{Q^2}{r^2}+\frac{\Lambda_0}{6}r^2+\frac{2\beta f_\pm\ln(r/\eta)}{d_1^2}, \tag{49}$$

$$\omega'=-\frac{Q}{r^2}\left[1-\frac{\beta}{d_1}\ln(r/\eta)\right]-\frac{k\beta}{d_1^2 Q}, \tag{50}$$

$$\Xi=-1-\frac{\beta}{d_1}\phi+\frac{k\beta}{d_1^2 d_2 Q^2}e^{d_1\phi}, \tag{51}$$

where $f_+=k-\mu/r+(\Lambda_0/6)r^2$, $f_-=\mu/r-Q^2/r^2$, η is as above. The new feature is that the electrostatic field strength (50) gets the additional constant term that describes the cosmological electrostatic background field. The magnitude of this field turns to be rather small, $\sim d_1^{-2}$, that makes it nonobservable on local scales. The direction of the field depends on geometry via k. Thus, recalling remarks after Eq. (28), this brings some evidence that the theories with the Maxwell-dilaton coupling of kind "exponent + something" [for instance, the low-energy string theory with the nonminimal coupling caused by threshold corrections that resemble Eq. (51) [17]] and with nonvanishing dilaton potential (for instance, the certain supergravities and the low-energy string theory with loop corrections) might generate the cosmological electrostatic field. Of course, this field is static only in static observer's frame of reference.

IV. CONCLUSION

Let us summarize the goals achieved in the paper. First, we have proven the full separability of the static EMD gravity of general type for three basic geometries. In turn, it revealed the hidden sector structure of the theory. It appeared that the theory has infinite number of sectors, and solution from one sector cannot be linked with that from another sector by means of perturbative series in parametric space.

Then we concentrated on the concrete class with simplest dilaton-graviton scale relation $\tilde{A}\sim\phi$ (15), which has a number of remarkable features: it always has at least one physical limit (Reissner-Nordström–de Sitter); in addition to the usual electric-magnetic duality it obeys a certain duality between Maxwell-dilaton coupling and dilaton potential; it comprises numerous EMD models including string-inspired, Liouville, trigonometric, polynomial, etc., and major of them remain non-trivial even if both charges are zeros. We singled out some models inside this class and obtained the families of exact dyonic solutions. Within certain range of values of the parameter d_1 some of them can be interpreted as the Reissner-Nordström–de Sitter (with ‘‘renormalized’’ dyonic charge) plus small logarithmic corrections. The latter change the global structure of the nonperturbed solution by shifting and splitting of horizons, breaking down extremality, and ‘‘dressing’’ the naked singularity. Also, the model (c) with cosmological constant and its solutions bring some evidence concerning the appearance of cosmological electrostatic field from the low-energy limit of string theory.

In addition, it is worthwhile to mention that the presented string-induced models and solutions (a) can be regarded as some kind of the indirect counterexample to the conjecture made by Garfinkle, Horowitz, and Strominger (GHS) [4] that the Reissner-Nordström is not even an approximate solution of string theory. That conjecture was based on the model with the minimal coupling and vanishing dilaton potential. However, the RN limit does appear if one considers certain nontrivial dilaton potentials, similar to those above. Therefore, it seems that situations crucially depend on concrete forms of the coupling and potential. In fact, in low-energy string theory (even without central charges) the loop corrections do induce the nontrivial dilaton potential. In addition, one should not forget that the threshold corrections in low-energy string theory affect a form of Maxwell-dilaton coupling. In turn, the nonminimal terms in the coupling also may drastically change properties of solutions, as can be seen from the case (c). Unfortunately, the forms of both Maxwell-dilaton coupling and dilaton potential are not reliably known at this time.

Of course, we have studied just a few models belonging to only one particular class. Other Ξ or Λ that might appear from a concrete problem can be paired up within this (or any another) class in a similar manner. Despite this pairing is somewhere artificial procedure the generated exact solutions are better than numerical studies from scratch (especially if one recalls the abovementioned existence of nonperturbative sectors), besides exact solutions can verify or falsify qualitative approaches and results. If one wishes to go beyond the linear class then one should start with the general class equation (13).

ACKNOWLEDGMENTS

I am grateful to Edward Teo (National University of Singapore) for suggesting the theme and fruitful remarks. Also, I acknowledge the correspondence by David Wiltshire (University of Adelaide) that helped to improve the paper.

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