Bulk viscosity of neutron-star matter

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(Received 14 March 2001; published 4 September 2001)

Hyperon bulk viscosity is the most significant transport coefficient affecting the growth or decay of any neutron-star mode of oscillation which has, even to second order in its amplitude, a periodic density fluctuation. This paper evaluates both the direct Urca and strangeness-changing four-baryon weak-interaction terms in the coefficient.

DOI: 10.1103/PhysRevD.64.084003 PACS number(s): 04.40.Dg, 04.30.Db, 97.60.Jd

I. INTRODUCTION

The coefficient of bulk viscosity may be anomalously large, in a fluid undergoing periodic density fluctuations with angular frequency ω, if the approach to local equilibrium between physically distinct degrees of freedom has a relaxation time τ such that $\omega\tau$ is within a few orders of magnitude of unity [1]. An example of this general phenomenon occurs in the core of a neutron star, where the bulk modulus derived from the translational degrees of freedom of the various Fermi-liquid components depends on the degree of local equilibrium between them. The physically important relaxation times are those associated with weak-interaction equilibrium. Strong-interaction equilibrium occurs so rapidly that its relaxation times are negligibly small. Approximate calculations of the bulk viscosity produced by the nonleptonic weak process $\Sigma^- + p \rightleftharpoons n + n$, assuming a bare-particle current-current interaction, were published many years ago [2,3]. The present paper, giving further results for matter above the Σ^- and Λ-hyperon threshold densities, has been prompted by more recent developments, and may be relevant to the unstable growth of *r*-modes in neutron stars through gravitational radiation reaction [4–6].

Modern equations of state, with interacting fermion components, usually have hyperon formation threshold densities of about 5×10^{14} g cm^{-3} [7–10]. The Σ^- and Λ thresholds are quite closely spaced and their order is not completely certain because a repulsive, isoscalar-exchange, Σ^--nucleon interaction [8] may be strong enough to move the Σ^- threshold above that for the Λ. The Λ component is important because its relaxation time, for $\Lambda + n \rightleftharpoons n + n$, can be obtained from observational data [11–15] and so is much less uncertain than the bare-particle current-current interaction relaxation time assumed for $\Sigma^- + p \rightleftharpoons n + n$. Its existence almost certainly allows direct Urca processes [16,17] with associated relaxation times, at a reference temperature $T_o = 10^{10}$ K, about five orders of magnitude longer than for the nonleptonic processes, but still within the interval of physical interest.

Calculation of the bulk viscosity in terms of relaxation times is a classical problem. For the degenerate fermion systems concerned, the temperature can be neglected as an independent thermodynamic variable and is significant only through its effect on the relaxation times. It is assumed to be such that $T \geqslant T_{ci}(\rho)$ in at least a moderate fraction of the stellar volume, where T_{ci} is the *i*th Fermi superfluid transition temperature at matter density ρ. (The index i represents both baryons and electrons.) There are equilibrium values of the number densities N_i and of the pressure P, denoted by N_i^o and P^o. The existence of density fluctuations $N_i = N_i^o + \delta N_i$ implies a non-equilibrium state. The Fourier components $\delta N_i(\boldsymbol{k},\omega)$ of small number density fluctuations and the fluid velocity $\boldsymbol{v}$ are related with the various net transition rates per unit volume by the following linear equations:

$$-i\omega\delta N + iN^o\boldsymbol{k}\cdot\boldsymbol{v} = 0 \tag{1}$$

$$-i\omega\delta N_e + iN_e^o\boldsymbol{k}\cdot\boldsymbol{v} - I_{np} - I_{\Lambda p} - I_{\Sigma n} - I_{\Sigma\Lambda} = 0 \tag{2}$$

$$-i\omega\delta N_p + iN_p^o\boldsymbol{k}\cdot\boldsymbol{v} - I_{np} - I_{\Lambda p} + I_{\Sigma p} + I_s = 0 \tag{3}$$

$$-i\omega\delta N_n + iN_n^o\boldsymbol{k}\cdot\boldsymbol{v} + I_{np} - I_{\Sigma n} - I_{\Lambda n} - 2I_{\Sigma p} - I_s = 0 \tag{4}$$

$$-i\omega\delta N_\Lambda + iN_\Lambda^o\boldsymbol{k}\cdot\boldsymbol{v} + I_{\Lambda p} - I_{\Sigma\Lambda} + I_{\Lambda n} - I_s = 0 \tag{5}$$

$$-i\omega\delta N_\Sigma + iN_\Sigma^o\boldsymbol{k}\cdot\boldsymbol{v} + I_{\Sigma n} + I_{\Sigma\Lambda} + I_{\Sigma p} + I_s = 0 \tag{6}$$

and by the conditions of charge neutrality and baryon conservation,

$$\delta N_p = \delta N_\Sigma + \delta N_e \tag{7}$$

$$\delta N = \delta N_p + \delta N_n + \delta N_\Lambda + \delta N_\Sigma\,. \tag{8}$$

The μ-meson number density immediately above the hyperon formation thresholds can be approximately half the magnitude of N_e^o, but the associated net transition rates have been neglected in Eqs. (1)–(8). Their inclusion would require an additional equation, completely analogous with Eq. (2), and an additional term in Eq. (7). (Leptonic direct Urca $\mu - e$ transitions are not allowed.) The presence of a further number density variable would greatly complicate, but not change significantly, Eqs. (41)–(43) which give a simple algebraic expression for the bulk viscosity induced by the nonleptonic weak interaction. Strong-interaction equilibrium imposes the constraint

$$\delta\mu_\Sigma + \delta\mu_p = \delta\mu_\Lambda + \delta\mu_n \tag{9}$$

on chemical potential fluctuations $\delta\mu_i$ away from their equilibrium values μ_i^o. The net transition rates per unit volume are for the following processes and each expression defines its associated relaxation time:

$$\Sigma^- p \to \Lambda n \quad I_s \neq 0 \tag{10}$$

$$n \to p e^- \bar{\nu} \quad I_{np} = \frac{\delta\mu_n - \delta\mu_p - \delta\mu_e}{\tau_{np}\alpha_n} \tag{11}$$

$$\Sigma^- \to \Lambda e^- \bar{\nu} \quad I_{\Sigma\Lambda} = \frac{\delta\mu_\Sigma - \delta\mu_\Lambda - \delta\mu_e}{\tau_{\Sigma\Lambda}\alpha_\Sigma} \tag{12}$$

$$\Lambda \to p e^- \bar{\nu} \quad I_{\Lambda p} = \frac{\delta\mu_\Lambda - \delta\mu_p - \delta\mu_e}{\tau_{\Lambda p}\alpha_\Lambda} \tag{13}$$

$$\Sigma^- \to n e^- \bar{\nu} \quad I_{\Sigma n} = \frac{\delta\mu_\Sigma - \delta\mu_n - \delta\mu_e}{\tau_{\Sigma n}\alpha_\Sigma} \tag{14}$$

$$\Lambda n \to n n \quad I_{\Lambda n} = \frac{\delta\mu_\Lambda - \delta\mu_n}{\tau_{\Lambda n}\alpha_\Lambda} \tag{15}$$

$$\Sigma^- p \to n n \quad I_{\Sigma p} = \frac{\delta\mu_\Lambda - \delta\mu_n}{\tau_{\Sigma p}\alpha_\Sigma}. \tag{16}$$

The baryon parameters $\alpha_i = \partial\mu_i / \partial N_i = \pi^2/(p_i^o m_i^\star)$, where p_i^o is the Fermi momentum and $m_i^\star$ the effective mass, are the Fermi-surface inverse densities of states. [The electron inverse density of states is $\alpha_e = \pi^2/(p_e^o \mu_e^o)$.] The strong-interaction rate I_s is dependent on almost completely infinitesimal deviations from the constraint given by Eq. (9), but cannot be neglected. It will be convenient to assume that Eq. (9) is exact, although with the consequence that I_s is indeterminate. Equations (11)–(14) include all possible direct Urca processes. If one or more of these processes are allowed, the ubiquitous modified Urca process transition rates [17] can be treated as negligibly small.

The bulk viscosity is the real part of ζ obtained from a calculation of the pressure fluctuation as a function of ω:

$$\delta P(\omega) = \sum_i \gamma_i \delta N_i = \sum_{i,j} N_j^o \frac{\partial\mu_j}{\partial N_i} \delta N_i, \tag{17}$$

which, with elimination of the δN_i, can be expressed in the form

$$\delta P(\omega) = \delta P(0) - i\zeta \boldsymbol{k}\cdot\boldsymbol{v}, \tag{18}$$

so as to define ζ [1]. Obviously, Eqs. (1)–(9) are not independent: constraints (7) and (8) are implicit in Eqs. (2)–(6) because all the processes (10)–(16) are charge and baryon-number conserving.

Calculation of ζ from Eqs. (17) and (18) requires, in principle, a very complete specification of the equation of state for the various interacting Fermi systems. Hyperon formation threshold chemical potentials are not needed in the evaluation of relaxation times, but both Eq. (17) and the relations between chemical potential and number density fluctuations

$$\delta\mu_i = \sum_j \frac{\partial\mu_i}{\partial N_j} \delta N_j \tag{19}$$

contain a set of partial derivatives which are not usually specified with published equations of state. In general, the off-diagonal partial derivatives should not be neglected. Their calculation, as an example, for a specific model of interacting neutrons and protons, the Skyrme pseudopotential density adopted by Lattimer *et al.* [18], has demonstrated that diagonal and off-diagonal partial derivatives can be of the same order of magnitude. Precise evaluation of ζ is therefore difficult, quite apart from uncertainties in the nonleptonic relaxation times, but the effect of bulk viscosity can be so large that accurate values of the coefficient are not always essential.

In order to remove as much uncertainty as possible, an expression for the relaxation time $\tau_{\Lambda n}$ is obtained, in Sec. II, from an observationally based effective coupling constant. Expressions for the direct Urca relaxation times are also given, but based on well-established weak currents [19,20]. A minor extension of the calculations gives the neutrino luminosities for these processes. A complete algebraic solution for ζ, with six relaxation times, would be possible by numerical techniques if the chemical potential partial derivatives were known. The linear approximation assumed in Eqs. (1)–(6) will always be satisfactory provided the fluctuations are such that the chemical potential differences in Eqs. (11)–(16) are small compared with $k_B T$, a condition which should be satisfied at temperatures within an order of magnitude of T_o. However, the direct Urca relaxation times at T_o are about six orders of magnitude longer than either $\tau_{\Lambda n}$ or the probable value of $\tau_{\Sigma p}$. This difference makes possible compact approximate algebraic solutions for ζ which are satisfactory for deciding whether or not bulk viscosity produces significant damping in a given circumstance for particular regions of ρ and T. Unless otherwise stated, the system of units is such that $\hbar = c = 1$.

II. RELAXATION TIMES

The Λ mean life in large-A hypernuclei is determined principally by the processes $\Lambda + n, p \to n + n, p$. There may be a contribution from more complex processes in which two particle-hole pairs are created [21], but the channel $\Lambda \to p\pi^-$ is very strongly suppressed by the Pauli exclusion principle. The sparse measurements that have been made for large-A hypernuclei [11,12] give a transition rate for $\Lambda + n \to n + n$ nearly an order of magnitude greater than for $\Lambda + p \to n + p$. Measurements of the Λ mean life in nuclear matter (^{209}Bi, ^{238}U) are more recent and numerous, and use two different techniques [13–15]. Averaging these results gives a mean life of $1.6 \pm 0.2 \times 10^{-10}$ s in the large-A limit. It can be assumed, with little error apart from the neglect of multiple particle-hole processes [21], that the transition rate for $\Lambda + n \to n + n$ is the sole contributor to this lifetime. A description in terms of a phenomenological interaction expressed as the most simple possible scalar coupling of the fermion fields, $G_\Lambda \overline{\psi_\Lambda}\psi_n \overline{\psi_n}\psi_n + \text{H.c.}$, gives a constant $G_\Lambda = 1.29 G_F$, where G_F is the Fermi weak constant [19]. (The Pauli principle correction has been made by neglecting the difference between Λ-hyperon and neutron single-particle potentials and by adopting a neutron Fermi momentum of

260 MeV/c that corresponds with the known neutron density in large-A nuclei.) This procedure enables the derivation of the relaxation time $\tau_{\Lambda n}$ in neutron-star matter, with details of the phase space calculation following those of Ref. [3], by scaling from large-A hypernuclei. There are, of course, some uncertainties. At higher matter densities, it is quite possible that multiple particle-hole pair creation [21] may be relatively more important. However, proton and hyperon fractions in neutron star matter are small so that errors caused by our neglect of processes such as $\Lambda+p\rightarrow n+p$ and $\Lambda+\Lambda\rightarrow n+\Lambda$ are unlikely to be significant.

Analogously with Eq. (11) of Ref. [3], the transition rate per unit volume is

$$I_{\Lambda n}=\frac{G_\Lambda^2}{6\pi^3\beta^2}p_\Lambda^o m_\Lambda^\star m_n^{\star 3}\delta\mu \tag{20}$$

where $\beta^{-1}=k_BT$, and the chemical potential imbalance $\delta\mu=\delta\mu_\Lambda-\delta\mu_n$ is assumed to be such that $\beta\delta\mu\ll 1$. The relaxation time is defined by Eq. (15),

$$\frac{1}{\tau_{\Lambda n}}=\frac{G_\Lambda^2}{6\pi\beta^2}m_n^{\star 3}. \tag{21}$$

It is several orders of magnitude shorter than $\tau_{\Sigma p}$,

$$\frac{1}{\tau_{\Sigma p}}=\frac{G_F^2}{12\pi\beta^2}m_n^{\star 2}m_p^\star \sin^2\theta_c\cos^2\theta_c(C_VC_V'+3C_AC_A')^2 \tag{22}$$

where $\sin\theta_c=0.223$, $C_V=1$, $C_V'=-1$, $C_A=-1.267$, $C_A'=-0.340$. [See the baryon summary table in Ref. [19] for the source of these parameters. This definition is not quite identical with that of Ref. [3] because Eq. (16) which defines $\tau_{\Sigma p}$ contains the Σ-hyperon density of states in place of that for the neutron. It is also unfortunately the case that Eq. (7) of Ref. [3] contains an error equivalent to the presence of a minus sign within the bracketed term in Eq. (22).] An almost complete but fortuitous cancellation of the bracketed terms in Eq. (22) accounts for most of the several orders of magnitude difference found on evaluation of $\tau_{\Sigma p}$ and $\tau_{\Lambda n}$. This difference in order of magnitude is also interesting because the transition rate for $\Lambda+n\rightarrow n+n$, unlike that for $\Sigma^-+p\rightarrow n+n$, has no contribution from an elementary bare-particle current-current interaction based on W-boson exchange, but must depend on a very wide class of possible weak-hadronic exchange processes. Because most of these processes also contribute to $\Sigma^-+p\rightarrow n+n$, there must be serious doubts about their neglect both here and in Ref. [3]. For these reasons, the view expressed in this paper is that the relaxation time given by Eq. (22) is essentially meaningless and that the true $\tau_{\Sigma p}$ is unknown but is probably of the same order of magnitude as $\tau_{\Lambda n}$. However, the value given by Eq. (22) is used later in this paper, but merely for the sake of definiteness.

The weak currents from which the direct Urca relaxation times have been obtained are well-known [19,20], apart from the induced pseudoscalar and weak magnetism terms which are neglected here. Straightforward calculation, with the assumption of empty neutrino phase spaces, gives the net transition rate

$$I_{ij}=\beta(\delta\mu)_{ij}(I_{\rightarrow})_{ij} \tag{23}$$

corresponding with any of Eqs. (11)–(14), in which $\delta\mu$ is the chemical potential imbalance for the process concerned and $I_{\rightarrow}$ is its forward transition rate per unit volume (not the net rate). The relaxation time is

$$\frac{1}{\tau_{ij}}=\frac{G_F^2}{(2\pi)^3\beta^4}\frac{m_j^\star\mu_e^o}{p_i^o}U_2|M_{ij}|^2 \tag{24}$$

and assumes that the direct Urca condition $p_i^o<p_j^o+p_e^o$ can be satisfied. [The relaxation time for the corresponding μ-meson process would be identical with Eq. (24) because the μ-meson chemical potential is equal to μ_e^o. However, a direct Urca condition, being dependent on momentum, may be satisfied, at a given chemical potential, by electrons but not necessarily μ-mesons.] The various factors are

$$|M_{np}|^2=2\cos^2\theta_c(1+3C_A^2), \tag{25}$$

$$|M_{\Sigma\Lambda}|^2=\frac{4}{3}\cos^2\theta_c(3C_A''^2), \tag{26}$$

$$|M_{\Lambda p}|^2=3\sin^2\theta_c(1+3C_A'''^2), \tag{27}$$

$$|M_{\Sigma n}|^2=2\sin^2\theta_c(1+3C_A'^2). \tag{28}$$

The factor $C_A'''=0.718$, and the result $C_A''=0.788$ has been derived from the semileptonic Σ^--decay branching ratios given in Ref. [19]. The dimensionless factor $U_2=20.6$ in Eq. (24) is the $s=2$ case of the integral

$$U_s=\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\int_0^{\infty}g^s\,dg\,\frac{1}{\exp(x)+1}\times\frac{1}{\exp(y)+1}\,\frac{1}{\exp(g-x-y)+1}. \tag{29}$$

These relaxation times are longer, by a factor of the order of $(\beta m_n^\star)^2$, than that given by Eq. (21). This is about six orders of magnitude at the reference temperature $T_o=10^{10}$ K and is a consequence of the small neutrino phase space in any semileptonic process.

The direct Urca neutrino luminosity can be obtained by a minor extension of these calculations. For any one of the processes (11)–(14), the luminosity is

$$L_{ij}=\frac{8G_F^2}{(2\pi)^5\beta^6}m_i^\star m_j^\star\mu_e^o U_3|M_{ij}|^2 \tag{30}$$

which includes both neutrino and antineutrino emission, for example, $e^-p\rightarrow n\nu$ and $n\rightarrow pe^-\overline{\nu}$. [The luminosities for μ-meson direct Urca processes are identical with those given by Eq. (30).] The integral in Eq. (30) has the numerical value

$U_3=87.2$. These expressions give luminosities identical with those published earlier by Prakash *et al.* [16].

For most published equations of state, any hyperon fractional number density N_i^o/N^o increases rapidly, as a function of matter density, from threshold to an approximately constant value and then declines slowly as other hyperon thresholds are passed. The number densities selected as typical for approximate evaluation of relaxation times have been obtained from the upper left-hand diagram in Fig. 7 of the paper by Knorren *et al.* [7]. At a total baryon density $N^o=3.9\times10^{38}$ cm^{-3}, individual number densities are $N_n^o=2.34\times10^{38}, N_p^o=7.8\times10^{37}, N_\Lambda^o=4.7\times10^{37}, N_\Sigma^o=3.1\times10^{37}$, $N_e^o+N_\mu^o=4.7\times10^{37}$ cm^{-3}. The corresponding Fermi momenta are $p_n^o=375, p_p^o=260, p_\Lambda^o=219, p_\Sigma^o=191, p_e^o=192$ MeV/c. With the assumption that the effective masses are identical with free-particle masses, the relaxation times at the reference temperature T_o are $\tau_{\Lambda n}=8.9\times10^{-8}$, $\tau_{\Sigma p}=7.4\times10^{-5}, \tau_{np}=2.0\times10^{-2}, \tau_{\Sigma\Lambda}=4.0\times10^{-2}, \tau_{\Lambda p}=3.4\times10^{-1}, \tau_{\Sigma n}=8.3\times10^{-1}$ s. The direct Urca luminosities given by Eq. (30) are enormous at T_o, for example, $L_{np}=2.4\times10^{33}$ erg cm^{-3} s^{-1}. The neutrino or antineutrino mean free path λ determined by the processes (11)–(14) is simply related to the relaxation times τ_{ij}. It is given by

$$\lambda^{-1}=\sum \frac{2\beta^2 p_i^o m_i^\star I_\lambda}{c\tau_{ij}U_2} \tag{31}$$

where the summation is over those of processes (11)–(14) that satisfy a direct Urca condition. The integral is

$$I_\lambda=\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\ \frac{1}{\exp(x)+1}\times\frac{1}{\exp(y)+1}\ \frac{1}{\exp(-x-y-x_\nu)+1} \tag{32}$$

($I_\lambda\simeq0.5x_\nu^2$, for $x_\nu\gg1$) where x_ν is the neutrino (or antineutrino) energy in units of k_BT. Again, μ-neutrino and e-neutrino mean free paths are identical provided the same set of direct Urca conditions are satisfied. Neutral weak-current processes are always kinematically allowed but have little effect on λ owing to the smallness of the scattered neutrino phase space relative to that of the charged lepton. Their contribution has been neglected here. The reference temperature mean free path is obviously small, of the order of 2×10^3 cm at a typical neutrino energy of $3k_BT$. Thus the empty neutrino phase space assumption is not valid at T_o: Eqs. (24) and (30) become valid only at temperatures $T\lesssim10^9$ K. The computation of cooling rates at higher temperatures is an involved problem, but one whose solution is not usually necessary. In most cases, it is sufficient to know that cooling is extremely rapid in comparison with other regions which do not support direct Urca processes. The influence of the relative positions of the Λ and Σ^--hyperon thresholds on temperature is considered further in Sec. IV.

III. BULK VISCOSITY

Owing to our serious lack of understanding of the nonleptonic weak-interaction matrix elements, an accurate and complete evaluation of their contribution to the bulk viscosity coefficient is not possible quite apart from its dependence on details of the equation of state above the hyperon thresholds. This contrasts with the $npe\mu$ direct Urca process for which a more accurate bulk viscosity has been obtained [22,23] for a specific equation of state. However, such a solution is not always necessary. The coefficient ζ can vary over so many orders of magnitude that an approximate evaluation may be adequate for many problems. It should be possible to show, in many cases, either that the damping is too weak to be significant or that it is so strong that its accurate evaluation is not necessary.

At, or below, the reference temperature $T_o=10^{10}$ K, the nonleptonic and direct Urca relaxation times form two distinct groups, differing by five or more orders of magnitude. Therefore, there exists a wide interval of ω such that $\omega\tau_{\Lambda n}$ and $\omega\tau_{\Sigma p}$ are several orders of magnitude smaller than unity. The equality

$$\delta\mu_\Lambda=\delta\mu_n \tag{33}$$

can be assumed, though $I_{\Lambda n}$ and $I_{\Sigma p}$ then become indeterminate. Asymptotic regions $\omega_<$ and $\omega_>$ can be defined within this interval such that $\omega_<\tau_U\ll1$ and $\omega_>\tau_U\gg1$, where

$$\frac{1}{\tau_U}=\alpha_\Lambda\sum\frac{1}{\tau_{ij}\alpha_i}, \tag{34}$$

the summation here being over all the processes (11)–(14) that satisfy a direct Urca condition. [Given Eq. (33), Eqs. (11)–(14) have a common numerator.] An algebraic solution for ζ from Eqs. (1), (17) and (18) requires five independent equations chosen to avoid the indeterminate net transition rates. Equations (2), (7)–(9) and (33) have been selected. All solutions given here neglect the off-diagonal chemical potential derivatives present in Eqs. (17) and (19) and it should be noted that the expressions obtained [Eqs. (36)–(39), (42) and (43)], although of complex form, are therefore simply approximations. The bulk viscosity coefficient is the real part of ζ, where

$$\zeta=N^o\mu_n^o\left(\frac{u_>^2}{c^2}-\frac{u_<^2}{c^2}\right)\frac{z}{\omega}\frac{1}{1-iz}. \tag{35}$$

This expression is of general validity, giving the bulk viscosity associated with any relaxation time. It has been obtained from Eqs. (1) and (18) by recourse to the hydrodynamic equation relating the time derivative of the enthalpy flux with the pressure gradient,

$$-i\omega N^o\mu_n^o\boldsymbol{v}+ic^2\boldsymbol{k}\delta P=0,$$

in the linear approximation adopted here. The parameters $u_<$ and $u_>$ are the phase velocities of sound in the asymptotic regions defined here. Their evaluation for Urca processes, with $z=z_U$, gives the following forms:

$$\mu_n^o\left(\frac{u_>^2}{c^2}\right)_U=(\gamma_\Sigma+A\alpha_\Sigma)\left(\frac{2N^o\alpha_\Lambda\alpha_n-N_e^oB_p}{N^o(B_p+B_\Sigma)}\right)+(\gamma_p+A\alpha_p)\left(\frac{2N^o\alpha_\Lambda\alpha_n+N_e^oB_\Sigma}{N^o(B_p+B_\Sigma)}\right)+\gamma_e\frac{N_e^o}{N^o}, \tag{36}$$

$$\mu_n^o\left(\frac{u_<^2}{c^2}\right)_U=(\gamma_\Sigma+A\alpha_\Sigma)\left(\frac{\alpha_\Lambda\alpha_n(2\alpha_e+\alpha_p)}{C+\alpha_e(B_p+B_\Sigma)}\right)+(\gamma_p+A\alpha_p)\times\left(\frac{\alpha_\Lambda\alpha_n(2\alpha_e+\alpha_\Sigma)}{C+\alpha_e(B_p+B_\Sigma)}\right)+\gamma_e\left(\frac{\alpha_\Lambda\alpha_n(\alpha_\Sigma-\alpha_p)}{C+\alpha_e(B_p+B_\Sigma)}\right), \tag{37}$$

$$z_U=\frac{\omega\tau_U\alpha_\Lambda(B_p+B_\Sigma)}{C+\alpha_e(B_p+B_\Sigma)}. \tag{38}$$

The constants are:

$$A=\frac{\gamma_\Lambda\alpha_n+\gamma_n\alpha_\Lambda}{2\alpha_\Lambda\alpha_n},$$
$$B_p=2\alpha_\Lambda\alpha_n+\alpha_p(\alpha_\Lambda+\alpha_n),$$
$$B_\Sigma=2\alpha_\Lambda\alpha_n+\alpha_\Sigma(\alpha_\Lambda+\alpha_n),$$
$$C=\alpha_\Sigma\alpha_\Lambda\alpha_p+\alpha_\Sigma\alpha_\Lambda\alpha_n+\alpha_\Sigma\alpha_p\alpha_n+\alpha_\Lambda\alpha_p\alpha_n. \tag{39}$$

The viscosity given by this untidy expression reflects the coupling of electrons with the baryon component which itself, at these values of ω, is in almost exact nonleptonic weak-interaction equilibrium. There is a high degree of cancellation within the expression such that it can be well approximated by making the replacement

$$\mu_n^o\left(\frac{u_>^2}{c^2}-\frac{u_<^2}{c^2}\right)_U\approx\gamma_e\frac{N_e^o}{N^o}, \tag{40}$$

though with z_U still defined by Eq. (38).

A similar procedure is possible at higher values of ω, where $\omega\tau_U$ for the Urca processes given by Eq. (34) is at least several orders of magnitude greater than unity. Here, the Urca net transition rates are negligibly small in Eqs. (2)–(6). The electrons are weak-interaction uncoupled from the baryons. In this region, asymptotic $\omega_<$ and $\omega_>$ can be defined such that $\omega_<\tau_H\ll1$ and $\omega_>\tau_H\gg1$, where τ_H is defined by

$$\frac{1}{\tau_H}=\frac{1}{\tau_{\Lambda n}}+\frac{\alpha_\Lambda}{\tau_{\Sigma p}\alpha_\Sigma}. \tag{41}$$

[The same chemical potential imbalance produces $I_{\Lambda n}$ and $I_{\Sigma p}$; see Eq. (9).] The five equations selected, in this case, for algebraic solution are Eqs. (2), (7)–(9), and the sum of Eqs. (3) and (4). The bulk viscosity coefficient is the real part of ζ, again given by Eq. (35) but with a relaxation time defined by Eq. (41) and with different forms for the parameters $u_>$ and z. The velocity $u_<$ in this case is, as expected, identical with the previous $u_>$, given by Eq. (36). The parameters $u_>$ and z are

$$\mu_n^o\left(\frac{u_>^2}{c^2}\right)_H=\gamma_\Lambda+(\gamma_\Lambda-\gamma_\Sigma+\gamma_e)\frac{N_e^o}{N^o}+(\gamma_n-\gamma_\Lambda)\times\left(\frac{(N_p^o+N_n^o)(\alpha_\Sigma+2\alpha_\Lambda+\alpha_p)-N^o\alpha_\Lambda-N_e^o(\alpha_\Lambda+\alpha_\Sigma)}{N^o(\alpha_\Sigma+\alpha_\Lambda+\alpha_p+\alpha_n)}\right)+(\gamma_\Sigma+\gamma_p-2\gamma_\Lambda)\times\left(\frac{N^o\alpha_\Lambda+N_e^o(\alpha_\Lambda+\alpha_\Sigma)-(\alpha_\Lambda-\alpha_n)(N_p^o+N_n^o)}{N^o(\alpha_\Sigma+\alpha_\Lambda+\alpha_p+\alpha_n)}\right), \tag{42}$$

$$z_H=\frac{\omega\tau_H\alpha_\Lambda(\alpha_\Sigma+\alpha_\Lambda+\alpha_p+\alpha_n)}{B_p+B_\Sigma}. \tag{43}$$

The evaluation of Eqs. (36)–(39), (42) and (43) assumes exactly those conditions adopted in Sec. II for the relaxation time calculations. It is not intended that these conditions should apply over a wide range of density: they represent the restricted interval within which only Λ and Σ^- hyperons are present.

The coefficient of bulk viscosity at any temperature T, for which the relaxation time calculations of Sec. II are valid, is the sum

$$Re(\zeta)=N^o\mu_n^o\left(\frac{u_>^2}{c^2}-\frac{u_<^2}{c^2}\right)_U\frac{z_U}{\omega}\frac{1}{1+z_U^2}+N^o\mu_n^o\left(\frac{u_>^2}{c^2}-\frac{u_<^2}{c^2}\right)_H\frac{z_H}{\omega}\frac{1}{1+z_H^2}$$
$$\approx\frac{4.9\times10^{30}(T_o/T)^4}{1+2.0\times10^{-6}\omega^2(T_o/T)^8} \tag{44}$$
$$+\frac{2.7\times10^{26}(T_o/T)^2}{1+2.9\times10^{-15}\omega^2(T_o/T)^4}, \tag{45}$$

in units of $\mathrm{g\,cm^{-1}\,s^{-1}}$. The evaluation represented by Eq. (45) assumes that the Fermi liquids are normal. For lower temperatures, $T<T_{ci}$, it would be necessary, in proceeding to evaluate Eq. (44), to make valid relaxation time calculations for superfluids. The temperature dependence of the bulk viscosity would then, of course, be completely different. The npe^- direct Urca bulk viscosity for normal Fermi liquids, with empty neutrino phase spaces, has been calculated previously by Haensel and Schaeffer [22]. Their result,

which is based on an equation state different from that assumed here, has the same temperature dependence and is no more than a factor of 2 larger than the first term in Eq. (45), considered in the $\omega\tau_U \gg 1$ limit. The calculations have recently been extended to include the effects of neutron and proton superfluidity [23].

At $T<T_{ci}$, relaxation times are increased by an energy-gap dependent exponential factor. The Urca term is valid, as observed at the end of Sec. II, only for $T \lesssim 10^9$ K where the empty neutrino phase space assumption can be made. Its magnitude of $2.4\times10^{32}\omega^{-2}$ g cm^{-1} s^{-1} at 10^9 K indicates that the true Urca bulk viscosity at higher temperatures must be extremely large. On the other hand, the nonleptonic weak term is valid for all $T>T_{ci}$. At $T>10^{-1}T_o$, and physically important values of ω, it is almost completely ω-independent.

IV. CONCLUSIONS

Equations (44) and (45) give the bulk viscosity coefficient for a normal Fermi liquid containing both Λ and Σ^- hyperons. The numerical values are based on a particular equation of state but nevertheless should be of the correct order of magnitude. These hyperon thresholds are expected [7–10] to lie below those possible thresholds for quark deconfinement or a K^--condensate. The contribution of such phases to bulk viscosity is a separate problem not considered here [24]. Also not considered are the thresholds for Σ^o, Σ^+ and Ξ^- hyperons which almost certainly lie above those for Λ and Σ^-. There is no reason to suppose that the bulk viscosity in such a region should differ qualitatively from Eqs. (44) and (45).

Modified Urca processes have been neglected here, although the modified Urca npe^- process has been the basis for some earlier calculations of neutron-star matter bulk viscosity [25]. The transition rates concerned are some orders of magnitude lower than for the corresponding direct Urca process and give, for ω of physical importance, proportionally smaller bulk viscosities. Also, the existence of a Λ-hyperon component ensures, except for very small fractional concentrations, that at least two Urca conditions of the form $p_i^o < p_j^o + p_e^o$, those for processes (12) and (13), will be satisfied [16]. The conditions for Eqs. (11) and (14) are also satisfied by the equation of state adopted in Sec. II and might be satisfied in a further region below the Λ-threshold, containing Σ^--hyperons, because the negatively charged baryon component necessarily increases the proton fraction. The existence of a direct Urca process in the lower-density npe^- region of the core is much less certain because a relatively large proton fraction, $N_p^o/N^o = 0.11-0.15$, is required [26,27]. But at even lower densities, in the inner crust of the star, a form of direct Urca process is allowed (see Appendix A of Ref. [28]). Finally, there is no reason to doubt that direct Urca conditions are satisfied at very high densities, where many different hyperons are present. These considerations lead to a particularly simple picture of neutron-star internal cooling. This should be extremely fast except, possibly, in any limited region of the core not containing hyperons (with the possible exception of Σ^-) in which the npe^- Urca condition is not satisfied.

At $T<T_{ci}$, relaxation times are lengthened by an energy-gap dependence. For an Urca process, this is approximately equal to the greater of the exponential factors $\exp(\beta\Delta_i)$,$\exp(\beta\Delta_j)$ for the two baryons concerned. In the case of a nonleptonic process, the lengthening is approximately equal to the product of the two largest exponential factors $\exp(\beta\Delta_i)$ for the baryons involved. The scale of the problem represented by our lack of knowledge of the gaps $\Delta_i(\rho)$ can be appreciated by reference to existing work on the calculation of $\Delta_{n,p}$ at much lower densities [29].

It is unfortunate that the significance of the strangeness-changing four-baryon weak interaction as a generator of bulk viscosity has not been widely appreciated, possibly because it is otherwise observed only in the decay of hypernuclei. The relevance of this bulk viscosity for the unstable growth of *r*-modes in neutron stars has been pointed out only recently [30], using a coefficient derived from much earlier work [3]. The very rapid cooling associated with direct Urca neutrino luminosities indicates that the bulk viscosity given by Eqs. (44) and (45) needs to be considered primarily as a function of T at constant ω. At low temperatures, neutron shear viscosity is a separate and important factor [31] but with respect to bulk viscosity; the nonleptonic term is the more important owing to its shorter relaxation time. It has a maximum at a temperature such that $z=1$ (equivalent to $\omega\tau_H \approx 1$) and a low-temperature limit, expressed directly in terms of τ_H, given by

$$\mathrm{Re}(\zeta) = \frac{8.3\times10^{33}}{\omega^2\tau_H} \tag{46}$$

in units of g cm^{-1} s^{-1}, which is valid for any τ_H such that $\omega\tau_H \gg 1$, including temperatures $T<T_{ci}$ below superfluid transitions.

Given the existence of any oscillation with finite periodic density fluctuation, the energy dissipated per unit volume and time is the time-average of

$$\frac{1}{N^o}\int P\,\delta N = \mathrm{Re}(\zeta)\int \left|\frac{1}{N^o}\frac{\partial\delta N}{\partial t}\right|^2 dt = \frac{\omega^2}{2}\left|\frac{\delta N}{N^o}\right|^2 \mathrm{Re}(\zeta), \tag{47}$$

by reference to Eqs. (1) and (18). The rate of change of temperature in any small volume is determined almost exactly by Eq. (47) and by the direct Urca neutrino luminosity. Thermal conduction from regions not supporting direct Urca processes is negligible by comparison. Urca process dissipation here is obviously negligible because the energy dissipated per transition is small compared with the typical neutrino energy of $3k_BT$ which accounts for the luminosity. The luminosities given by Eq. (30) can be re-expressed, individually, as

$$L_{ij} = \frac{1}{\alpha_i\tau_{ij}}\frac{2U_3}{\beta^2 U_2},$$

exactly for normal systems, and approximately in the case of superfluids. The total neutrino and antineutrino luminosity is

$$L_\nu = \sum L_{ij} = \frac{1}{\alpha_\Lambda \tau_U} \frac{2U_3}{\beta^2 U_2},$$

excluding μ-meson processes. The steady temperature maintained by a time-independent density fluctuation amplitude, found by equating L_ν with the energy dissipation given by Eq. (47), satisfies

$$\left| \frac{\delta N}{N^o} \right|^2 = 7.8 \times 10^{-3} \left(\frac{\tau_H}{\tau_U} \right) \left(\frac{T}{T_o} \right)^2, \tag{48}$$

from Eqs. (34) and (46), given that $\alpha_\Lambda = 4.9 \times 10^{-43}$ erg cm^3. The relaxation times are related by $\tau_H / \tau_U = 8.9 \times 10^{-6} (T/T_o)^2$ at $T > T_{ci}$, obtained from the individual relaxation times of Sec. II, but are modified, at $T < T_{ci}$, by the exponential factors referred to previously. However, Eq. (48) shows that no more than a very small density fluctuation amplitude is necessary to maintain a temperature of the order of $10^{-1} T_o$.

Our assumption of a constant density fluctuation amplitude here is not inappropriate because it is determined by the mode of oscillation of the whole star and is independent of the conditions existing in any specific small volume. Temperature evolution for the case of *r*-mode growth in neutron stars [4–6] is complex. First of all, any consideration of *r*-modes has to assume an equation of state with known hyperon formation thresholds. The instability producing growth is the result of gravitational radiation reaction but its energy dissipation has to be considered as a function of matter density using Eq. (45). It would also be necessary to know, or make specific assumptions about, individual superfluid transition temperatures. Superfluidity itself produces further dissipation mechanisms, for example, the mutual friction derived from the scattering of electrons by the magnetic flux entrained in neutron vortex cores [32]. Such a program is beyond the scope of this paper which merely attempts to describe those properties of hyperon bulk viscosity which are relevant.

ACKNOWLEDGMENTS

I thank Dr. L. Lindblom and Dr. B. J. Owen for a most useful comment on the role of the direct Urca processes.

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