

Cold big-bang cosmology as a counterexample to several anthropic arguments

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A general Friedmann big-bang cosmology can be specified by fixing a half-dozen cosmological parameters such as the photon-to-baryon ratio η_γ , the cosmological constant Λ , the curvature scale R , and the amplitude Q of (assumed scale-invariant) primordial density fluctuations. There is currently no established theory as to why these parameters take the particular values we deduce from observations. This has led to proposed “anthropic” explanations for the observed value of each parameter as the *only* value capable of generating a universe that can host intelligent life. In this paper, I explicitly show that the requirement that the universe generates Sun-like stars with planets does *not* fix these parameters, by developing a class of cosmologies (based on the classical “cold big-bang” model) in which some or all of the cosmological parameters differ by orders of magnitude from the values they assume in the standard hot big-bang cosmology, without precluding in any obvious way the existence of intelligent life. I also give a careful discussion of the structure and context of anthropic arguments in cosmology, and point out some implications of the cold big-bang model’s existence for anthropic arguments concerning specific parameters.

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I. INTRODUCTION

Current fundamental physical theories and cosmological models incorporate a number of “parameters” or “constants” which could theoretically assume different values while leaving the mathematical structure of those theories unchanged. Responses to the question of why any given parameter or constant assumes the particular value it does fall into three rough categories:

(1) Like the laws of physics themselves, the value of the parameter or constant in question is fundamental, and simply a part of the nature of the universe itself. Whether or not it “could have” been different is unclear but irrelevant, since it assumes a unique value that is fixed in space and time.

(2) The parameter or constant can ultimately be derived from a fundamental physical theory with no free parameters. Thus the observed “parameter” could not have been different, as it is a purely mathematical object.

(3) The parameter varies between members of a spatial-temporal ensemble, of which the region of the universe we observe is part of one member. The parameter not only could have been different but *is* different in other ensemble members, and the value we observe depends upon which ensemble member we happen to inhabit.

Explanations of the third type have raised considerable interest in cosmology for two related reasons. First, a number of inflationary cosmology and quantum cosmology theories explicitly fail to predict unique values for cosmological parameters such as the photon-to-baryon ratio η_γ , the cosmological constant Λ , or the amplitude of density perturbations Q ; rather, these theories yield only a probability distribution for the parameters, which take different values in causally disconnected “subuniverses” [1,2]. Second, the observed values of some parameters or constants (such as the cosmological constant [3]) seem to require an incredible degree of fine-tuning if they are to admit explanations of the first or second type.

This fine-tuning might be avoided by explanations of the

third type, which allow for a decoupling between the “expected” or “natural” value of a parameter and the value we observe, by incorporating the fact that some parameter values may preclude the existence of observers that could measure those values. These “anthropic” considerations imply that the probability of measuring a given value for some parameter is not simply given by the probability distribution of values assumed by that parameter among members of the ensemble, but is modified by the probability of observers arising in each member. Values of parameters that are very different from the expected values might then be explained as being typical values among the set of values that allow the formation of observers.

Arguments of this sort have been used to explain the observed value of Λ [3–9], η_γ [10], Q [11], the curvature scale [12–14], and the density ratio of non-baryonic dark matter to baryons η_{dm} [15]. Generally, these arguments consider the viability of life in a universe in which one of these parameters assumes a value ten times smaller or larger than the observed values *with all other parameters fixed*. If the formation of observers is strongly suppressed in each alternative universe, and if the *a priori* distribution of parameter values is fairly flat, it is claimed that the observed value has been explained. For example, Tegmark and Rees [11] (TR) explain the observed value of $Q \sim 10^{-5}$ by showing that galaxies could not cool sufficiently if $Q \leq 10^{-6}$, and would be so dense as to disrupt most planetary orbits if $Q \geq 10^{-4}$.

Two substantial worries arise with respect to such arguments. First, a parameter value differing by many orders of magnitude (rather than just one) may correspond to qualitatively different physical processes that allow a rather different universe in which life could still arise. Second, if more than one parameter is to be explained anthropically, then several parameters must be varied at once, and there is a risk that degeneracies will occur in which changing one constant counteracts the adverse effect of changing another. This paper argues that these two worries are substantial and serious, by developing a specific set of cosmological models in which

one or more of the basic cosmological parameters can be altered by many orders of magnitude, without preventing the formation of observers (conservatively assumed to be similar to us) in any obvious way.

To develop this argument, I first discuss in some detail the logical structure of the “weak” anthropic arguments in question. I then discuss specific contexts (inflation, quantum cosmology, etc.) in which they may arise. Next I develop a general class of cosmologies based on the classical “cold big-bang” (CBB) cosmology and argue that observers like us could plausibly arise in these cosmologies. I then discuss several specific anthropic arguments in light of the cold big-bang cosmology.

II. THE STRUCTURE OF ANTHROPIC ARGUMENTS IN COSMOLOGY

Anthropic arguments concerning fundamental parameters in a theory generally are invoked to explain the particular measured value of a given parameter in relation to the large range of other possible values that it seems reasonable to expect that the parameter *could* have assumed. The weak form of the anthropic principle (the only form this paper discusses) explains particular values of some set of parameters using a two-part argument. First, the argument requires that the parameters in question not only *could have* been different, but in fact *do* assume a range of values in a physically realized ensemble of systems, some of which contain observers capable of measuring the parameter, and all of which are similar except for variations in the values of the parameters in question. The second part of the argument is the self-evident statement that only systems capable of producing observers can have their parameters measured. Thus observers will never measure a parameter to have a value which would preclude the existence of observers. (For example, assuming that atoms are required for life to exist, no living observer will ever measure a value of the electron charge incompatible with the existence of atoms.)

The argument is simple, but immediately raises the issue of to *which* parameters describing *which* physical systems the argument might be applied. For the purposes of this paper, let us categorize parameters into three groups. First are parameters that are known to vary in space or time and may be derivable from more fundamental parameters; for example, the “solar constant” or the “Hubble constant.” Second, the constants¹ used in the currently accepted fundamental physical theories (the standard model of particle physics and general relativity) [16]. Third, the key parameters describing the current “standard model” of cosmology (defined presently). This study concerns parameters in the third category, and addresses anthropic arguments in cosmology which attempt to explain their values.

A. The general argument applied to cosmology

I take the cosmological “standard model” to be a Friedmann-Robertson-Walker (FRW) big-bang cosmology

¹Excepting Λ (which is assumed to vary), I will hereafter reserve the term “constant” for parameters with fixed values.

(i.e., the cosmology generated by solving Einstein’s equations assuming large-scale homogeneity and isotropy), characterized by a spatial curvature scale \mathcal{R} (evaluated at the Planck time, in units of the Planck length), a photon-to-baryon ratio η_γ , an (electronic²) lepton-to-baryon ratio η_L , a ratio η_{dm} of non-baryonic dark matter to baryonic matter (evaluated when both are nonrelativistic), a cosmological constant Λ (in Planck units), and a scale-invariant power spectrum of Gaussian primordial density perturbations with an amplitude Q on the horizon scale. This definition restricts the types of cosmologies that can be considered, but encompasses those for which anthropic arguments have been made in the literature.

Anthropic arguments in cosmology have raised new interest due to the possibility that the cosmological parameters are *not* uniquely derivable from more fundamental considerations, nor “fixed” as part of the initial conditions of the universe, but vary among subuniverses with probability distribution $P(\bar{\alpha}_1, \dots, \bar{\alpha}_N)$. Here, the set $\{\alpha_i\}$ stands for some subset of $\{\mathcal{R}, \eta_\gamma, \eta_L, \eta_{\text{dm}}, \Lambda, Q\}$, and $d^N P(\bar{\alpha}_1, \dots, \bar{\alpha}_N)/d\bar{\alpha}_1 \dots d\bar{\alpha}_N$ is the differential probability that a randomly chosen baryon³ resides in a subuniverse in which the parameters α_i ($i=1, \dots, N$) take a given set of values in the range $[\bar{\alpha}_i, \bar{\alpha}_i + d\alpha_i]$. (The sorts of systems that constitute subuniverses are discussed below in Sec. II B.) But P is *not* the probability distribution of observed values of α_i , which is instead (by anthropic reasoning) proportional to $\mathcal{P}(\bar{\alpha}_1, \dots, \bar{\alpha}_N) \equiv P(\bar{\alpha}_1, \dots, \bar{\alpha}_N) \xi(\bar{\alpha}_1, \dots, \bar{\alpha}_N)$, where $\xi(\bar{\alpha}_1, \dots, \bar{\alpha}_N)$ is the total number, integrated over time, of observers per baryon that are capable of making independent measurements of α_i in a region of space in which the cosmological parameters are given by $\bar{\alpha}_i$. One can straightforwardly normalize \mathcal{P} to yield a true probability distribution, provided that $P\xi$ is integrable over the space of values attained by the α_i in the ensemble. (The method of defining probabilities I have chosen is probably most similar to that of Vilenkin [17]; other authors have formulated anthropic arguments in ways that are similar in spirit but different in detail.)

The function P will presumably follow from the (currently unknown) fundamental physics describing the universe as a whole. In this paper, I concentrate on ξ (though I draw some conclusions about P); determining ξ requires a criterion for the existence of an observer and a method by which to calculate the density of such observers for a given set of α_i and their values $\bar{\alpha}_i$. Determining what sort of physical configurations could give rise to a being capable of measuring cosmological parameters is a rather difficult task that I will sidestep by confining my criteria to those which are obviously essential for the existence of life similar to humans. This assumption is conservative in the present context in that it grants the anthropic argument maximal predic-

²I assume for simplicity that the other species have lepton numbers small compared to the electronic leptons.

³Any number which is conserved during the cosmological expansion could be used here.

time power, and is tantamount to assuming that the frequency of independent human-type observers greatly exceeds that of all other types of observers the universe may produce. The specific criterion adopted here is to require the formation of a main-sequence star with a moderate fraction of heavy elements such as C, N, O, etc. The star must burn steadily and without significant disturbance (e.g., which would disrupt planetary orbits) for more than an “evolutionary time scale” τ_{ev} ; I take $\tau_{\text{ev}} = 5$ Gyr (the single available observation for the time scale on which observers arise after the formation of a star). Adopting this criterion, I set

$$\xi(\bar{\alpha}_1, \dots, \bar{\alpha}_i) = \int_{Z_{\min}}^{Z_{\max}} dZ \int_{\tau_{\text{ev}}}^{\infty} d\tau \int_0^{t_{\max} - \tau_{\text{ev}}} dt \frac{dn(t, \tau, Z; \bar{\alpha}_i)}{dt d\tau dZ}. \quad (1)$$

Here, $dn(t, \tau, z; \bar{\alpha}_i)/dt d\tau dZ$ is the differential formation rate (per baryon) at time t of stars with metallicity Z which will live undisturbed for time τ ; t_{\max} is the lifetime⁴ of the sub-universe in question, and Z_{\min} and Z_{\max} define the range of allowed metallicities. Thus $\xi(\bar{\alpha}_1, \dots, \bar{\alpha}_i)$ is the total number of stars per baryon with $Z_{\min} \leq Z \leq Z_{\max}$ that live ≥ 5 Gyr in relative isolation, in a subuniverse with cosmological parameters $\bar{\alpha}_i$ [18]. For example, in the observable universe $\xi \sim 0.01 m_p / M_{\odot}$ since about 1% of baryons form single stars a solar mass or less.

Having defined the ingredients, I now discuss the hope of what I will term the “anthropic program.” The hope is that given *a priori* calculations of $d^N \mathcal{P}(\bar{\alpha}_1, \dots, \bar{\alpha}_N) / d\bar{\alpha}_1 \dots d\bar{\alpha}_N$ and $\xi(\bar{\alpha}_1, \dots, \bar{\alpha}_N)$, their product $d^N \mathcal{P}(\bar{\alpha}_1, \dots, \bar{\alpha}_N) / d\bar{\alpha}_1 \dots d\bar{\alpha}_N$ will have a well-defined global maximum at some set of parameters $\bar{\alpha}_i^{\max}$. For each parameter α_k , one might then integrate \mathcal{P} over the other α_i s ($k \neq i$) to obtain a one-dimensional probability $d\mathcal{P}(\bar{\alpha}_k) / d\bar{\alpha}_k$. If the peak surrounding the global maximum is very sharp, $\mathcal{P}(\bar{\alpha}_k)$ can be used to define a range of $\bar{\alpha}_k$ containing (say) 99% of the probability. If (and only if) each *observed* value of $\bar{\alpha}_k^{\text{obs}}$ falls inside its “highly probable” region, and if we *assume* that we observe the value that a typical observer does, then anthropic argument has explained their values. In this case, for example, the “natural” value of Λ would be reconciled with its observed value (which if nonzero is many orders of magnitudes smaller). Note that this procedure is very different from the calculation of the *conditional* probability $\mathcal{P}(\alpha_k | \bar{\alpha}_1^{\text{obs}}, \dots, \bar{\alpha}_{k-1}^{\text{obs}}, \dots, \bar{\alpha}_{k+1}^{\text{obs}}, \dots, \bar{\alpha}_N^{\text{obs}})$ of measuring a single $\alpha_k = \bar{\alpha}_k$ with the other parameters fixed (i.e., by their observed values). Making an anthropic argument using such a conditional probability is only logically consistent under the *assumption* that the anthropic program will be successful, and that one can look at variations in a single parameter

while keeping others fixed at their maximally probable values (presumed to be close to the observed values).

The anthropic program, however, can fail in three clear ways. First, there may *not* be any well-defined maximum: there may be degeneracies among two or more parameters such that there are multi-dimensional surfaces of constant \mathcal{P} spanning a relatively large region of parameter space. In this case, anthropic arguments still do not provide a satisfactory explanation of the observed value of those parameters in light of the large range allowed by fundamental physics and anthropic constraints. Second, the well-defined global maximum of $d^N \mathcal{P} / d\bar{\alpha}_1 \dots d\bar{\alpha}_N$ may *not* lie near the observed values $\bar{\alpha}_i^{\text{obs}}$. This would imply either a flaw in some element of the computation of \mathcal{P} , or that we live in a region of parameter space with low probability, or that there is something fundamentally wrong with the whole anthropic approach (e.g., that there are *not* other subuniverses with different values of the parameters in question). Third, \mathcal{P} may have two or more well-defined local maxima. This would not be a problem in *principle* as long as one of the peaks was much higher than the rest. But a similar and important practical difficulty can arise if ξ has multiple local maxima. This is because anthropic arguments in the literature typically make simplifying assumptions regarding \mathcal{P} , on the grounds that ξ must have a peak with a width which is very narrow compared to the scale over which \mathcal{P} varies (e.g., [6]). But if multiple peaks (even narrow ones) occur in widely separated regions of parameter space, \mathcal{P} becomes crucial and the implications of computations based on anthropic reasoning become more ambiguous.

This paper argues that multiple regions of large ξ do, in fact, exist in the set of parameters $\alpha_i = \{\eta_\gamma, \mathcal{Q}, \eta_{\text{dm}}, \eta_L, \mathcal{R}, \Lambda\}$, by providing a cosmological model in which many of these parameters can take quite different values than those we observe, without preventing (according to the criteria define above) the existence of observers. Before developing this cosmology and its implications for the anthropic arguments, I will first outline the cosmological contexts in which anthropic arguments are typically made.

B. Contexts for cosmological anthropic arguments

Any attempt to implement the anthropic program described above requires that the “universe” (i.e., everything that exists throughout all time) contains an ensemble⁵ of regions that may be treated as individual Friedmann-Robertson-Walker (FRW) cosmologies. A number of such “meta-cosmologies” have been proposed. For example, the “oscillating universe” model consists of a series of finite-volume, finite-age cosmologies (e.g., [19]). Here, each “big crunch” is followed by a new “big bang” in which the parameters α_i might be newly drawn from the proba-

⁴ $t_{\max} \rightarrow \infty$ is allowed for subuniverses that do not recollapse, but I assume that each baryon is incorporated into a finite number of stars.

⁵This is *not* an ensemble of imagined identical systems in the Gibbs-Einstein sense, but rather a canonical or micro-canonical ensemble of weakly or non-interacting subsystems of a large or infinite physical system.

bility distribution $P(\bar{\alpha}_1, \dots, \bar{\alpha}_N)/N_b(\bar{\alpha}_1, \dots, \bar{\alpha}_N)$, where $N_b(\bar{\alpha}_1, \dots, \bar{\alpha}_N)$ is the total number of baryons in a cosmology with parameters $\bar{\alpha}_i$. Then $\xi(\bar{\alpha}_1, \dots, \bar{\alpha}_N)$ can be straightforwardly taken to be the number of stars that form (with $Z_{\min} \lesssim Z \lesssim Z_{\max}$ and a lifetime greater than τ_{ev}) in the cosmology with parameters $\bar{\alpha}_i$, divided by N_b . Current astronomical data weighs strongly against a closed, recollapsing cosmology, so this context is of value primarily because it is fairly unambiguous.

The regions constituting members of the ensemble could be separated in space rather than time. For example, an infinite (or extremely large) universe in which the parameters α_i vary spatially could be partitioned into finite regions⁶ of differing $\bar{\alpha}_i$ which are uniform enough to be treated as individual FRW cosmologies with the same initial time, each with a fixed number of baryons. The relative numbers of regions described by FRW cosmologies with parameters $\bar{\alpha}_i^1$ and $\bar{\alpha}_i^2$ would then give $P(\bar{\alpha}_i^1)/P(\bar{\alpha}_i^2)$, and probabilities as described in Sec. II A can be defined unambiguously as long as the universe can be coordinatized in such a way that there is a time after which no stars form. Open or critical globally FRW universes with small density inhomogeneities would (for example) satisfy these criteria.

Anthropic arguments can also be made in quantum cosmology (e.g., [7,8]). Here the universe begins in a superposition of states which “decoheres” into an ensemble of classical cosmologies with different properties. The hope is that for a compelling initial condition, the wave function can be represented as a superposition of (or at least dominated by) FRW-type cosmologies with different values of α_i . If these cosmologies were closed, then the situation would closely resemble the first example of the oscillating universe, and the probability $P(\bar{\alpha}_1, \dots, \bar{\alpha}_N)/N_b(\bar{\alpha}_1, \dots, \bar{\alpha}_N)$ of a component of the ensemble being described by parameters $\bar{\alpha}_i$ would be proportional to the square of the amplitude of the term corresponding to that cosmology in the initial superposition. Note, however, that it is not entirely clear how to extract probabilities if the superposition contains both open and closed cosmologies, nor is it clear that the squared amplitudes can be straightforwardly interpreted as relative frequencies in an ensemble of classical cosmologies. But assuming that these problems are not fatal, quantum cosmology does provide a possible framework for anthropic arguments.

Applying anthropic arguments to FRW cosmologies embedded in an arbitrary global geometry is much more difficult because there may not be a unique globally defined initial time at which to begin the integration of Eq. (1). This is the case in models of “eternal inflation” in which inflation does not end globally (which would provide an initial time for the subsequent FRW cosmology), but always continues in

some regions. The global structure of the universe approaches an ensemble of thermalized regions separated by inflating regions [20], and the values of cosmological parameters describing each region can vary throughout the ensemble. This is a natural context for anthropic arguments, but as discussed at length in Ref. [21] (see also [22]), it is a subtle matter to unambiguously define probabilities in such cosmologies because the probabilities for many proposed schemes depend strongly upon the coordinate choice: depending on this choice, a $t=0$ hypersurface can intersect one, many, or no thermalized regions having different parameter values. Vanchurin *et al.* [21] propose a scheme that apparently circumvents this problem and gives unambiguous probabilities by calculating probabilities within any one thermalized region. For this to work the parameters must vary continuously in such a way that there is a finite range of values over which the probability has nonzero measure, so that the (arbitrarily) chosen thermalized region will contain many sub-regions with different parameters spanning that range. Reference [23] extends this scheme to compute probabilities for situations in which parameters take on different discrete values in different thermalized regions.

III. THE COLD BIG-BANG COSMOLOGY

As described above, modern cosmology provides several plausible (if speculative) contexts in which the universe could consist of an ensemble of regions, each describable as an FRW cosmology with different initial conditions. The thesis of the present study is that regions with quite different parameters may support life, thereby greatly complicating or invalidating several anthropic arguments. I support this thesis by developing a big-bang type cosmology in which some or all of the parameters η_γ , Q , η_{dm} , η_L , \mathcal{R} , and Λ may differ by at least several orders of magnitude from the current “standard model” of cosmology in which they take approximate values of $|\mathcal{R}| \approx 2 \times 10^{29} (1 - \Omega - \Omega_\Lambda)^{1/2}$, $\eta_\gamma \approx 2 \times 10^9$, $\eta_L \sim 1$ [24], $\eta_{\text{dm}} \approx 5 - 10$, $\Lambda \approx 3 \times 10^{-122}$ (i.e., $\Omega_\Lambda \approx 0.7$), and $Q \sim 10^{-5}$.

A. Initial conditions

Consider an FRW cosmology with physical baryon number density $N(t_i)$ at some initial time t_i after which the comoving baryon number is conserved, and at which the photon-to-baryon ratio is small ($\eta_\gamma \lesssim 10$) relative to that we currently observe. Choosing $N(t_i) = 10^{35} \text{ cm}^{-3}$ ensures also that nucleons are nonrelativistic, that known baryonic species other than nucleons have decayed, and that for $\eta_\gamma \gtrsim 0.01$ a state of nuclear statistical equilibrium holds [25,26]. The expansion is dominated by relativistic matter for

$$N_i \gtrsim 10^{39} (\eta_L^{4/3} + \eta_\gamma^{4/3})^{-3} \text{ cm}^{-3} \quad (2)$$

or cosmic time

$$t \lesssim 10^{-4} (\eta_L^{4/3} + \eta_\gamma^{4/3})^2 \text{ s.} \quad (3)$$

A cosmology with $\eta_L \sim \eta_\gamma \sim 1$ could result from efficient baryogenesis after an inflationary epoch, or might simply be

⁶The volume of space with a given set of α_i need not be finite, because the ensemble may contain an infinite number of finite regions with the same α_i . I assume that “edge effects” associated with the boundaries separating these regions are not important.

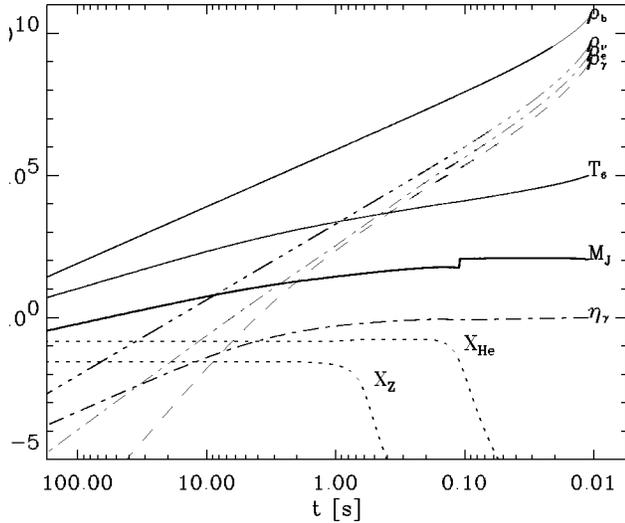


FIG. 1. Early evolution of a sample FRW cosmology with (initial) $\eta_\gamma=1.0$, $\eta_L=2.5$, and $\eta_{\text{dm}}\ll 1$. Baryon, photon, electron, and neutrino densities are given in g cm^{-3} . The Jeans mass is in solar units, the temperature is in units of 10^6 K, and X_{He} and X_Z indicate mass fractions of baryons bound into helium and into elements heavier than lithium.

“assumed” as the initial state for an FRW cosmology (see Sec. IV). Cosmic expansion dominated by nonrelativistic matter will steadily decrease η_γ (though it is constant for $\eta_\gamma\gg 1$), while non-equilibrium processes may increase it slightly [25–27].

I leave the rest of the cosmological parameters relatively unconstrained, except for generally assuming $\eta_L\sim\eta_\gamma$ and $\eta_{\text{dm}}\leq 100$ (both for convenience) and $Q\ll 1$ (to avoid complications involved with significant primordial black hole formation). I also assume that \mathcal{R} and Λ^{-1} are large enough for the expansion to be radiation or matter dominated, until explicitly stated otherwise (see Secs. VII and VIII).

B. Nucleogenesis

The evolution of a big-bang cosmology with these initial conditions through the epoch of nucleogenesis is described in detail in Ref. [26]. If $\eta_\gamma\geq 0.01$ at $N\sim 10^{35}$ cm^{-3} , the medium will be hot enough to reach nuclear statistical equilibrium, and will be dominated by free neutrons and protons (their ratio depending upon η_L). In this case a standard nucleogenesis calculation, generalized to treat degenerate leptons, yields the products of primordial nucleogenesis at late times [26]. For certain combinations of η_γ and η_L , nucleogenesis yields a helium fraction of 25%, just as in the standard hot big-bang (HBB) model. However, for $\eta_\gamma\leq 10$, nucleogenesis also produces heavier elements (metals), yielding metallicity of $Z\geq 0.1 Z_\odot$ for a 25% helium yield. By varying η_γ and η_L , almost any desired yield of primordial helium and metals can be obtained. As a particular example, for $\eta_\gamma\approx 1$ and $\eta_L\approx 2.5$, the cosmic medium would emerge with $\sim 15\%$ helium by mass, and \sim solar metallicity (see Fig. 1). Thus the cosmic medium in a CBB cosmology can start out with the same level of enrichment as gas in the

HBB model which has been processed by stars. (Of course the ratios of different heavy elements will be different in the CBB model, but C, N, and O would be produced in abundance). Primordial metal synthesis is suppressed by either high η_L or high η_γ .

C. Initial perturbations

Whereas in the standard HBB model structure formation cannot begin until the time of matter-radiation equality ($\sim 10^{12}$ s), significantly lower η_γ allows much earlier and more efficient structure formation. In making the present argument I have assumed that there are scale-invariant primordial density perturbations of amplitude Q on the horizon scale. It is interesting to note that structure may form in CBB models even *without* primordial perturbations (i.e., $Q=0$), because of phase transitions either in the QCD era [27] or (if $\eta_\gamma\ll 1$) later as the cosmic medium approaches the density of solid hydrogen [28,29]; in either case the cosmic medium shatters into “chunks” with random velocities that induce density perturbations of the form $\delta M/M=(M/m)^{-7/6}$ on a mass scale M , where m is the chunk mass [30,31]. If these chunks survive they can directly coagulate into the first structures, which hierarchically generate larger ones [29]; if the chunks dissipate they leave behind the density fluctuations that can see later structure formation [27]. The former case would lead to qualitatively different formation of the first structures; in the latter case the general picture of early structure formation would be affected quantitatively but not qualitatively. For simplicity and continuity with the HBB case, and to investigate general values of η_γ I will focus on the case of “primordial” scale-invariant perturbations with $\delta M/M\propto M^{-2/3}$.

D. Structure formation

The early history of star formation in a cold FRW cosmology of the described type has been worked out in some detail by Carr [33]. The key point is that the Jeans mass when it first falls below the mass enclosed by the horizon, M_J^i , is $\sim (2-10)(\eta_L+\eta_\gamma)^2 M_\odot$ (Hogan 1982) at $t\sim 10^4$ s, rather than $\sim 10^3 M_\odot$ at $t\sim 10^{13}$ s in the HBB. Stars in a CBB cosmology can therefore begin to form soon after regions of stellar mass enter the horizon.

Density perturbations on scales smaller than the initial Jeans length are converted into acoustic waves they enter the horizon and do not grow subsequently (hence are suppressed relative to larger modes), therefore the first collapsed regions have mass of order M_J^i . Subsequently, larger and larger regions collapse hierarchically. Carr argues that for objects below a critical mass $M_c(Q)$, the cooling time exceeds the free-fall time, and a single object tends to form; for $M>M_c$ fragmentation is expected and the collapsing object can form a cluster. For $M<M_c$, Carr further argues that the successive hierarchical collapse of regions will lead to a mass function of protostellar clumps peaked near M_c . (Carr’s analysis gives $dN/dM\propto M^{-1}$ where N is the number density of collapsing clumps of mass M ; a Press-Schechter analysis would give the qualitatively similar dN/dM

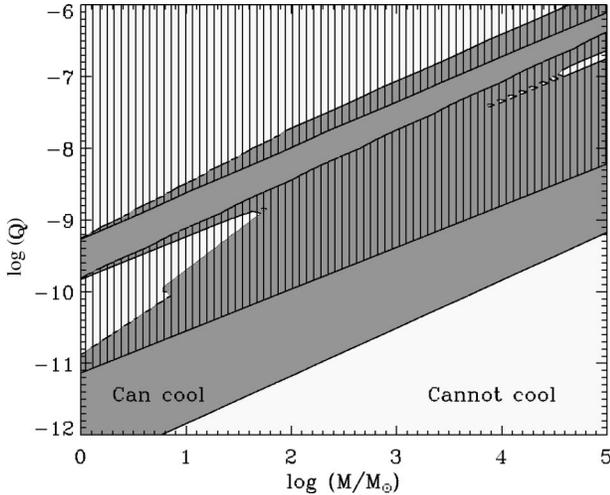


FIG. 2. Cooling of primordial clouds in a CBB for $\eta_\gamma = 1, \eta_{\text{dm}} \ll 1$. The characteristic temperature T and density ρ as masses of $100M_{100} M_\odot$ begin collapse yield an absorption $\kappa(T, \rho)$ using the fits of Bell and Lin [32]. Condensations in the region indicated by vertical lines are optically thick; others are optically thin. The shaded regions indicate condensations for which the dynamical time exceeds the cooling time.

$\propto M^{-4/3}$.) For each clump, M provides an upper limit to the mass of one or more stars which form from its collapse; for clumps with $M > M_c$, the stellar mass function depends on the details of the (presumed⁷) fragmentation but objects of $M \ll M_c$ seem likely (since that is what appears to have occurred in observed globular clusters and galaxies).

If we accept this basic picture, we may consider in slightly more detail the early history of the example cosmology plotted in Fig. 1, with $\eta_\gamma = 1.0$, $\eta_L = 2.5$, and $\eta_{\text{dm}} \ll 1$. At $t < 0.1$ s, the Jeans mass is constant at $M_J^i \sim 100 M_\odot$ and fluctuations below this mass are strongly suppressed. Depending upon $Q_8 \equiv Q/10^{-8}$, two scenarios then suggest themselves.

First, if $10^{-3} \leq Q_8 \leq 1$, the first collapsed regions of $\sim M_J^i \sim 10^2 M_\odot$ cool faster than they collapse (see Fig. 2), presumably forming stars of much smaller mass. These primordial groups of stars collapse beginning at time $\sim 10^9 Q_8^{-1.5}$ s, and hierarchical clustering continues, with masses $\sim 10^{10} Q_8^{1.5} M_\odot$ collapsing around $t \sim 5$ Gyr. In this scenario, depending on the details of fragmentation, a substantial fraction of the cosmic baryon mass can form $\sim 1 M_\odot$ stars at very early times.

A second, qualitatively different scenario would result from $10 \leq Q_8 \leq 10^4$, in which case masses well above $M_J^i \sim 100 M_\odot$ can form single systems (see Fig. 2). The mass function of the first objects not expected to fragment (by

⁷It is not entirely clear to the author whether or not fragmentation should, in fact, occur in such systems (see, e.g., the arguments of Layzer [34] and the recent simulations by Abel, Bryan, and Norman [35]); but as discussed below solar mass primordial objects can be obtained without invoking fragmentation if dark matter of sufficient mass and density exists.

Carr's criterion) will be dominated by objects of $\approx 2000 - 5 \times 10^6 M_\odot$ which begin to collapse at cosmic time 10^9 s. In the absence of significant rotation, those of $\geq 10^5 M_\odot$ should collapse directly to black holes; the rest should form supermassive stars. Either type of object will emit large amounts of radiation at (probably) Eddington luminosity, converting a fraction ϵ of its rest mass (or of a mass of accreted matter similar to its own mass) into energy over a time $t_e \sim 4\epsilon \times 10^8$ yr (Rees 1978). Assuming t_e greatly exceeds the formation time for the objects, this leads to a photon/baryon ratio of

$$\eta_\gamma \approx 5\epsilon^{5/4} \times 10^{10}. \quad (4)$$

Such a large energy release would probably evaporate small structures and suppress further structure formation until later, when larger masses go nonlinear and structure can form much as in the HBB but with the remnants of the initial supermassive objects as dark matter. The same basic scenario should occur for any $\eta_\gamma \leq 50$ and $10 \leq Q_8 \leq 10^4$, though later structure formation will depend on Q_8 .

The presence of primordial dark matter can modify this picture substantially, but the effect depends crucially on the mass m_{dm} of the dark matter particles, and qualitatively on η_{dm} . Dark matter in a region of a given size can only collapse if the dark particles free-stream less than that distance during a dynamical time of the system. This gives a dark matter “Jeans mass” in terms of the temperature T_{dm} of the dark matter particles, of $M_J^{\text{dm}} \sim (kT_{\text{dm}}/Gm_{\text{dm}})^{3/2} \rho^{-1/2}$. Considering only the dark matter contribution to the density and assuming that the expansion is roughly adiabatic since the formation of the dark matter, I find $M_J^{\text{dm}} \sim 4(m_{\text{dm}}/1 \text{ GeV})^{-2} M_\odot$. Consider the case $\eta_{\text{dm}} \gg 1$. If $M_J^{\text{dm}} \ll M_J^b$ (where the latter is computed neglecting dark matter), then the dark matter will just decrease the initial (baryonic+dark) Jean mass, M_J^i , by a factor $(1 + \eta_{\text{dm}})^{1/2}$ so that smaller primordial objects can form. If, however, $M_J^{\text{dm}} \gg M_J^b$, then only primordial objects of mass M_J^{dm} or larger can form, since the Hubble drag due to the dominant dark matter component prevents the growth of smaller baryonic perturbations. Now consider $\eta_{\text{dm}} \ll 1$. Dark matter perturbations on a mass scale M with $M_J^{\text{dm}} \ll M \ll M_J^b$ cannot grow because of Hubble drag due to baryons, but will not free-stream away. This is important because the baryonic jeans mass decreases with time, reaching $\sim 1 M_\odot$ at $t \sim 30$ s in our example cosmology. Without dark matter fluctuations on this scale have been suppressed early on; but with dark matter, the small-scale fluctuations are preserved, and have amplitude $\sim Q \eta_{\text{dm}} / (1 + \eta_{\text{dm}})$. Thus solar mass objects (or smaller) can still collapse fairly early, as long as η_{dm} is non-negligible. Since the collapse time scales with M but M_J falls slightly more slowly than $1/t$, these objects will collapse *after* the larger objects of M_J^i , but can still survive if they form in regions that are underdense on a scale M_J^i but overdense on the solar mass scale.

Whether the first stars form as $\sim 100 M_\odot$ groups or as individual Jeans mass objects, they will soon find themselves

in a growing hierarchy of stellar systems, and we must check that any nascent planetary systems or proto-planetary disks are not disrupted through stellar encounters. Using standard linear theory and spherical collapse, a mass perturbation of mass $100M_{100} M_{\odot}$ turns around at roughly $t \approx 10^9 Q_8^{-1.5} M_{100}$ s, and virializes at radius $r_V \approx 10^{15} Q_8^{-1} M_{100}$ cm at density $\rho_V \approx 5 \times 10^{-11} Q_8^3 M_{100}^{-2} \text{ g cm}^{-3}$. If such a clump fragments into $100M_{100}/(1 + \eta_{\text{dm}})$ protostars of solar mass, each protostar forms from a volume with characteristic radius $\sim 2 \times 10^{14} Q_8^{-1} M_{100}^{2/3} (1 + \eta_{\text{dm}})^{1/3}$ cm, and can therefore contract by a factor of ten or more to produce a protoplanetary disk of ≥ 1 AU for $Q_8 \leq 1$. The protostars will have velocity dispersion $\approx Q^{1/2} c$, and the system will quickly relax after a time of

$$\tau_{\text{rlx}} \approx 10^9 Q_8^{-1.5} M_{100}^2 (1 + \eta_{\text{dm}})^{-1} \chi \text{ s}, \quad (5)$$

where $\chi \equiv (\log 40)/[\log 40M_{100}/(1 + \eta_{\text{dm}})]$ [36]. The mean time between (proto)stellar encounters with impact parameter $b \approx 1$ AU (which would disrupt the formation or orbits of Earth-like planets) is (see [36], p. 541):

$$\tau_{\text{coll}} \approx 3.2 \log \left(\frac{40M_{100}}{1 + \eta_{\text{dm}}} \right) \left(\frac{\Theta^2}{1 + \Theta} \right) \tau_{\text{rlx}}, \quad (6)$$

where

$$\Theta \equiv \frac{G M_{\odot}}{2v^2 b} \approx 0.7 Q_8^{-1} (\text{AU}/b). \quad (7)$$

Thus $\tau_{\text{coll}} \leq 5$ Gyr, which would bode ill for any forming planets. However, approximately 1/100th of the cluster's stars would evaporate each relaxation time [36], so at least some stars will avoid collisions (by being evaporated) for $Q_8 = 1$, and the entire cluster will evaporate before significant collisions occur if $Q_8 \leq 1$ (or $\eta_{\text{dm}} \geq 1$). This evaporation occurs as the relaxation of the cluster moves stars into the high-energy tail of the Maxwell distribution, and does not require close stellar encounters to proceed. After the evaporation (or if formed alone in its halo), a given (proto)star will likely find itself in a larger mass condensation; but since $\tau_{\text{coll}}/\tau_{\text{evap}}$ increases (logarithmically) with M , it cannot experience a planet-disrupting encounter before this new mass condensation evaporates, and so on.⁸

E. Summary of CBB models

In summary, I have argued that for values of η_L and η_{γ} of order unity, an FRW cosmology can begin with solar or greater metallicity and with a Jeans mass $M_J \leq 100 M_{\odot}$ at

⁸Nearby supernovae might also disrupt protoplanetary disks. When the first supernovae explode at $\sim 10^7$ yr, most stars would be in clusters of $\sim 10^7 Q_8^{3/2} (1 + \eta_{\text{dm}})^{-1}$ stars with radii $\sim 100 Q_8^{1/2}$ pc, i.e. large globular-cluster type objects. It is not presently clear whether or not this might be fatal to forming planets. Note also that supernovae may also help unbind clusters by removing gas.

very early times. Adding dark matter with mass ≥ 1 GeV yields objects with baryonic mass $M \leq 100/[\eta_{\text{dm}}(1 + \eta_{\text{dm}})^2] M_{\odot}$ and preserves density fluctuations in $\leq 1 M_{\odot}$ regions. Structure formation depends crucially on the primordial perturbation amplitude Q . For $10^{-11} \leq Q \leq 10^{-8}$ and $\eta_{\text{dm}} \leq 1$ the first collapsed objects are formally unstable to fragmentation. In this picture there are three distinct ways solar mass stars can form: first, by the fragmentation of the first $\sim 100 M_{\odot}$ objects; second, as solitary objects in large dark matter halos (if $\eta_{\text{dm}} \geq 1$); third, as solar-mass overdensities embedded in “void” regions, when the Jeans mass drops to a solar mass. These stars generally form beginning at time ~ 100 yr, and should be able to survive without experiencing encounters which disrupt their planetary systems, as long as $Q \leq 10^{-8}$ and/or $\eta_{\text{dm}} \geq 1$. For $Q \geq 10^{-7}$ very massive primordial stars and quasars could form, inhibiting structure formation until much later, when it would form as in the HBB (but with Population III remnants as dark matter and with arbitrary primordial metallicity).

In the notation introduced in Sec. II A, these arguments suggest

$$\xi(\eta_{\gamma} \sim 1, 10^{-11} \leq Q \leq 10^{-5}) \sim \xi(\eta_{\gamma} \sim 10^9, Q \sim 10^{-5}), \quad (8)$$

where ξ is the number of solar mass stars per baryon, and where the parameters not listed can be (but are not necessarily) the same in both cases. While the argument for M_{\odot} stars in a CBB is not incontrovertible, it seems doubtful that a much stronger argument for M_{\odot} star formation could be made in the HBB model without the benefit of *observations* of solar mass stars and the assumption that the HBB model describes the observable universe.

IV. ARGUMENTS CONCERNING THE ENTROPY PER BARYON

I have outlined a cosmological model with $\eta_{\gamma} \sim 1 \ll \eta_{\gamma}^{\text{obs}} \sim 10^9$, which appears to allow the formation of life-supporting stars. This serves as a counterexample to any argument that attempts to rule out cosmologies with $\eta_{\gamma} \ll \eta_{\gamma}^{\text{obs}}$ using anthropic arguments. It is therefore interesting to discuss how cosmologies with $\eta_{\gamma} \sim 1$ might come about, and what arguments have been forwarded against them.

A major challenge in cosmology is to understand the origin of the observed nonzero baryon number, given that almost all models incorporate an early baryon-nonconserving GUT phase and/or an inflationary phase, both of which erase baryon number. One of the more attractive scenarios for generating the baryon number is the Affleck-Dine (AD) mechanism [37,38], which emerges somewhat naturally from supersymmetric models and is compatible with the rather low reheating temperatures that may be required by some inflationary models [10]. The simplest versions of this mechanism, however, tend to generate $\eta_{\gamma} \sim 1$ rather than the much greater observed value. This finding led to a number of explanations involving either a modification of the theory which suppresses its efficiency [38], or entropy generation after baryogenesis [39], or an anthropic argument such as

that by Linde [10]. I will discuss Linde’s argument first, then make a few comments on the general possibility of low- η_γ cosmologies since they are a crucial ingredient of the remainder of the paper.

In Linde’s scenario, the universe is comprised of a vast number of exponentially large and causally disconnected regions carrying random values of the field $\tilde{\phi}$, which determines the photon-to-baryon ratio after AD baryogenesis. “Typical” domains with $\tilde{\phi} \sim \pm m_{\text{plank}}$ generate $\eta_\gamma \sim \pm 1$ but much more rare subuniverses could carry $\eta_\gamma \sim \pm 10^9$. Fixing Q , Linde argues that $\eta_\gamma \ll 10^9$ would lead to extremely dense galaxies, with (density) $\propto \eta_\gamma^{-3}$. This could prevent the survival of planetary systems (see also TR and Sec. V below) and thus anthropically limit η_γ to large values.

This argument is subverted in two ways by the (theoretical) existence of the CBB cosmologies outlined in Sec. III. First, even if $Q \sim 10^{-5}$, low η_γ can lead to most of the early cosmic medium collapsing into Population III objects which generate entropy and can increase η_γ to $\sim 10^9$, thus restoring a semblance of the “standard” picture of galaxy formation at much later times (see Sec. III and [40]. For a discussion of the possibility that *our* Universe is of this type, see [26–29,33,41,42]). Second, if Q varies in addition to η_γ and $Q \ll 10^{-5}$ is allowed, solar mass stars can (plausibly) form primordially in the CBB. While the stars may exist in clusters that are extremely dense, I have argued that these clusters would evaporate into larger, less dense structures before stellar collisions destroy planetary systems around their component stars. Thus there is no clear obstacle to the generation of life-supporting stars in cosmologies with $\eta_\gamma \sim 1$, and the anthropic argument cannot explain the observation of $\eta_\gamma \sim 10^9$.

A possible objection to the assumptions of this paper is that it might be difficult to produce cosmologies with $\eta_\gamma \sim 1$ and $\eta_L \sim 1$ in the ensemble. This is because electroweak “sphaeleron” interactions which violate $B+L$ but preserve $B-L$ (where $B \equiv \eta_\gamma^{-1}$ and $L \equiv \eta_L B$) are in equilibrium at temperatures ≥ 100 GeV and tend to wash out $B+L$, giving $B/L < 0$ and requiring $B-L \neq 0$ for there to be any baryons left [43,44]. Generation of large $B-L$ is quite possible in Affleck-Dine baryogenesis, but a negative lepton number can cause problems in the CBB model because abundant antineutrinos lead to neutron domination during nucleogenesis and hence to a metal-dominated medium [27,33]. It is unclear whether or not such a cosmology could support life like our own. There are, however, at least three possible ways to avoid this difficulty. First, Davidson *et al.* [45] have argued that the AD condensate can survive long enough (before decaying into baryons) to suppress sphaeleron interactions below their critical temperature. Second, so-called “ B -balls” can form from the condensate and protect the baryon number from erasure [46], then later decay into baryons. Third, since $B-L$ is preserved family-by-family, it is possible to obtain $B/L_e > 0$ while $B/L < 0$, by compensating for the positive electron lepton number (desired for neutron suppression) using large negative τ and/or μ lepton numbers. For example, setting $L_\mu = B/3$, $L_e = B/3 + 1$, and $L_\tau = B/3 - 25/13$ yields, after $B+L$ erasure, $B = 6/13$, $L = 15/13$, and hence L/B

$= 2.5$ (see Ref. [43], p. 189 for details). Thus the desired B/L can be obtained by tuning $B/3 - L_i$ for each family i . This is possible as long as weak interactions are not fast enough to equalize L_e , L_τ , and L_μ .

Thus we see that given an ensemble of subuniverses with different cosmological parameters, members with $\eta_\gamma \sim 1$ are quite possible, and anthropic arguments do not rule out their observation even if all other parameters are fixed. And as discussed below, if other parameters vary, values of η_γ a few orders of magnitude different are also allowed.

V. ARGUMENTS CONCERNING THE AMPLITUDE OF PRIMORDIAL FLUCTUATIONS

A second anthropic argument which is directly contradicted by the claim that a cosmology with $\eta_\gamma \sim 1$ and $Q \lesssim 10^{-8}$ can support life is the argument that only $Q \sim 10^{-5}$ is anthropically allowed (TR). In essence, TR argue that for $Q \ll 10^{-5}$ structures are too cold and diffuse to cool efficiently into galaxies, whereas for $Q \gg 10^{-5}$ galaxies are too dense for planetary systems to survive for 5 Gyr. However (and as noted by TR), limits on Q depend on the other cosmological parameters; for example, while lower Q impedes the formation of structures that can cool, lower η_γ enhances it. Indeed, the lower limit on Q of TR depends on $\eta_\gamma^{4/3}$ if all other parameters are fixed [their Eq. (11)], so a very low value of Q can be accommodated if η_γ is lowered in tandem.

Against this possibility, TR offer two comments. First, that Q must be large enough that the characteristic energy of virialized structures exceeds the atomic energy scale of ~ 1 Ryd, lest cooling be very inefficient. This gives $Q \geq 10^{-8}$ [their Eq. (6)]. But this assumes that only atomic transitions can cool the gas; for the high densities and low temperatures of the first objects in the example cosmology of Sec. III, molecular vibrational and rotational cooling, and dust cooling, all with characteristic energy scales orders of magnitude below atomic energy scales, would be very efficient, particularly since the objects can have arbitrarily high metallicity. TR’s second objection is that lowering η_γ increases the likelihood of planetary disruptions, the frequency of which increases roughly as $\eta_\gamma^{-4} Q^{7/2}$ [see TR’s Eq. (18) or Eq. (6) of Sec. III given $M_{100} \propto \eta_\gamma^2$]. Like the cooling constraint, this allows lower η_γ in combination with lower Q , but with a different scaling, and the region of η_γ satisfying both cooling and disruption constraints vanishes if Q is too low. But as argued in Sec. III, for sufficiently low Q or high η_{dm} stellar clusters should always evaporate before planet-disrupting collisions can occur, removing the disruption constraint and allowing very low η_γ and Q .

Thus it seems that while TR have provided plausible arguments why Q could not vary by one or two orders of magnitude without suppressing the formation of Sun-like stars *if* the other cosmological parameters are fixed at their observed values, their calculations do allow somewhat different (by 1–2 orders of magnitude) values of Q if η_γ is also somewhat different (this is another argument against a strict anthropic constraint on η_γ). Furthermore, the argu-

ments of this paper indicate that variations in Q (and η_γ) of many orders of magnitude are allowed because qualitatively new physics becomes important.

VI. ARGUMENTS CONCERNING THE RATIO OF BARYONIC TO DARK MATTER

While little is known about the nature of the (probably) cold, dark, (probably) noninteracting dark matter that is postulated as part of the standard cosmological model, it is known that it seems to have $\sim 4-10$ times the mass density of baryons inferred from primordial nucleogenesis constraints. For most dark matter candidates there is no obvious reason why the dark and baryonic matter densities should be similar, so one might appeal to anthropic arguments for an explanation.

Linde [15] constructs such an argument to explain why the axion-to-baryon density ratio could take a value of $\eta_{\text{dm}} \sim 10$ rather than the $\eta_{\text{dm}} \sim 10^8$ expected if the axion field ϕ is in the natural range of $\sim 10^{16}-10^{17}$ GeV. Assuming the other cosmological parameters—and in particular η_γ and Q —to be fixed, Linde considers a hypothetical subuniverse with ϕ ten times its “observed” value, which leads to $\eta_{\text{dm}} \sim 400-1000$. By his reasoning, structures should then be $\sim 10^8$ times more dense than observed galaxies and thus presumably incapable of supporting intelligent life.

As discussed in Sec. V, however, changes in the ratio $(1 + \eta_{\text{dm}})/\eta_\gamma$ of matter to radiation can be largely compensated for by changes in Q , since the virial density is $\propto Q^3(1 + \eta_{\text{dm}})^4/\eta_\gamma^4$ [TR, Eq. (5)] and hence the number density of stars is $\propto Q^3 \eta_{\text{dm}}^{-3} \eta_\gamma^{-4}$ (for $\eta_{\text{dm}} \gg 1$). So in Linde’s example, the increase in density due to the increase in η_{dm} could be offset entirely by a decrease in Q of a factor 10^{-2} . According to TR’s analysis the structures would still be able to cool for this combination of Q and η_{dm} (though for much higher η_{dm} —and hence much lower Q —atomic cooling would fail). This weakens Linde’s anthropic argument for low η_{dm} . The degeneracy seems also to allow for values of $\eta_{\text{dm}} \leq 1$, since the matter-to-photon ratio would only change by a factor of ≤ 10 . Note, however, that $\eta_{\text{dm}} \ll 1$ would qualitatively change the HBB because structure formation may begin to become “top down” since fluctuations in the baryons below the Jeans mass at matter domination ($\sim 10^{15} M_\odot$) would be suppressed.

The CBB model can also form stars when $\eta_{\text{dm}} \ll 1$ if fragmentation of primordial objects is effective, and it is possible to construct a CBB model with $\eta_{\text{dm}} \gg 100$ by properly tuning other parameters. But because variations about the standard HBB model can already give cosmologies with $\eta_{\text{dm}} \sim 1000$ or $\eta_{\text{dm}} \ll 1$ that can form stars efficiently (as long as Q can also vary), the CBB scenario does not add anything particularly useful.

Because of the degeneracy between η_{dm} and Q , it seems that the observed value of η_{dm} cannot be explained by anthropic means unless the probability distribution P is peaked more strongly toward higher values of Q than toward higher values of η_{dm} , i.e., unless $P(\eta_{\text{dm}} \sim 1000, Q \sim 10^{-7}) \ll P(\eta_{\text{dm}} \sim 10, Q \sim 10^{-5})$ [where here and henceforth $P(\alpha_k$

$\sim \bar{\alpha}_k$) should be interpreted as $dP(\bar{\alpha}_k)/d\bar{\alpha}_k$, integrated over α_k within an order of magnitude of $\bar{\alpha}_k$.] This can be seen either as evidence against an anthropic argument for η_{dm} or, if preferred, as a constraint on $P(\eta_{\text{dm}}, Q)$.

VII. ARGUMENTS CONCERNING THE COSMOLOGICAL CONSTANT

Anthropic arguments have been forwarded a number of times [3–9] to explain the vast difference between the “natural” value of the cosmological constant (roughly the inverse Planck length squared, l_{pl}^{-2}), and its small or vanishing observed value of $\lesssim 3 \times 10^{-122} l_{\text{pl}}^{-2}$. Because the energy density of clustering matter decays more quickly than vacuum energy as the medium expands, in any FRW cosmology with $\Lambda > 0$ structure formation ceases after some time $t(\Lambda; \bar{\alpha}_i)$ at which vacuum energy dominates the cosmic energy density (α_i are the cosmological parameters other than Λ). It is argued that this gives an anthropic “upper bound” on Λ : if this time occurs before the collapse of structures capable of forming solar mass stars and planets, no observer like us can measure the corresponding value of Λ . In the notation of this paper, if the first structures form at cosmic density $\rho_{\text{fs}}(\bar{\alpha}_i)$ in a cosmology with parameters $\bar{\alpha}_i$, then $\xi(\Lambda; \bar{\alpha}_i) = 0$ for $\Lambda \geq 8\pi G \rho_{\text{fs}}/c^2$ (see Ref. [4] for a more precise criterion). More sophisticated versions of this argument attempt to calculate $\mathcal{P}(\Lambda; \bar{\alpha}_i^{\text{obs}})$, i.e., the conditional probability of an observer measuring a value Λ , with the other α_i fixed at their “observed” values. In these papers [5,6,9], \mathcal{P} is computed by calculating $\xi(\Lambda; \bar{\alpha}_i^{\text{obs}})$ and multiplying by an assumed $P(\Lambda)$; they find a probability function which peaks at Λ comparable to—but somewhat larger than—the value indicated by observations.

It is simple to see how the anthropic upper bound to Λ can change if cosmological parameters other than Λ vary between members of the ensemble of subuniverses postulated by the anthropic program. As noted, for example, by TR, since ρ_{fs} varies with the α_i (excluding Λ), so does the anthropic upper bound to Λ . To make this ambiguity concrete, consider the hypothetical cosmology with $\eta_\gamma = 1.0$, $\eta_L = 2.5$, $Q \leq 10^{-8}$, and $\eta_{\text{dm}} \leq 10$ developed in Sec. III. In this cosmology, the first (solar mass) stars may form at time $t_{\text{fs}} \sim 10^9 Q_8^{-1.5}$ s when the cosmic density is $\rho_{\text{fs}} \sim 10^{-12} Q_8^3 \text{ g cm}^{-3}$. Once this first generation of stars (or star clusters) forms, subsequent domination of the cosmic expansion by vacuum energy should not affect the development of life (as also argued by Weinberg [4]).⁹ Allowing vacuum domination soon after the turnaround of these first structures yields an anthropic upper bound of $\Lambda \lesssim 4 Q_8^3 \times 10^{-105}$ (in Planck units), about 17 orders of magnitude larger than the upper bound on the observed value of Λ .

⁹Weinberg also noted the requirement that the stellar clusters be massive enough to retain metals generated in supernovas. This requirement is unnecessary in the present case because ample metals can be generated primordially.

In Sec. III I argued that $\xi(\eta_\gamma \sim 10^9, Q \sim 10^{-5}) \sim \xi(\eta_\gamma \sim 1, Q \leq 10^{-9})$, i.e., similar numbers of life-nurturing stars per baryon might plausibly arise in the example CBB model and in the HBB. Weinberg [4] has conjectured that $P(\Lambda) = \text{const.}$, therefore *if* we assume that independent of Λ the two cosmologies have similar *a priori* probability, i.e.,

$$P(\eta_\gamma \sim 10^9, Q \sim 10^{-5}) \sim P(\eta_\gamma \sim 1, Q \leq 10^{-9}), \quad (9)$$

Then it would be about 10^{17} times more likely for an observer to find themselves in a cold cosmology with an enormous Λ than in a cosmology like the one we observe. Thus the anthropic explanation for a small value of Λ fails *if* the ensemble of cosmologies comprising the universe contains cosmologies with values of Q and η_γ much smaller than those we observe, unless the Λ -independent probability of forming those cosmologies is many orders of magnitude smaller than that of forming standard HBB cosmologies.

VIII. ARGUMENTS CONCERNING THE CURVATURE SCALE

As for Λ , anthropic arguments have been invoked to explain the difference between the “natural” value for the curvature scale of a Planck length ($\mathcal{R} \sim 1$), and its observed value of $\mathcal{R} \geq 10^{29}$. This large difference has been termed the “flatness problem,” and is one of the prime motivations for considering inflationary models. Anthropic arguments can, however, still be made either in the absence of inflation, or within open inflation models.

If an arbitrary FRW cosmology is closed, a straightforward anthropic constraint on \mathcal{R} arises from the requirement that the time before the big crunch must exceed the time scale for the development of intelligent life $\tau_{\text{ev}} \sim 5$ Gyr.

For open cosmologies, anthropic arguments quite similar to those concerning Λ have been formulated [12–14]. In simplest form, these arguments require that structures capable of forming stars and planets form before the cosmic medium becomes curvature dominated; this gives an anthropic lower limit on \mathcal{R} . Curvature domination occurs roughly when

$$\mathcal{R}[a(t)/a(t_{\text{pl}})]l_{\text{pl}} \sim ct, \quad (10)$$

where t is cosmic time, $a(t)$ is the scale factor and l_{pl} and t_{pl} are the Planck length and time. This yields the anthropic constraint

$$\mathcal{R} \geq \left(\frac{t_{\text{eq}}}{t_{\text{fs}}}\right)^{2/3} \left(\frac{t_{\text{pl}}}{t_{\text{eq}}}\right)^{1/2} \left(\frac{ct_{\text{fs}}}{l_{\text{pl}}}\right), \quad (11)$$

where t_{eq} is the time when relativistic domination ends [see Eq. (3)], and t_{fs} is the time when the first star-forming bound structures form. For the standard cosmological model with $\eta_\gamma \sim 10^9$ and $Q \sim 10^5$, the first structures form at $t_{\text{fs}} \sim 10^8 - 10^9$ yr and $t_{\text{eq}} \sim 10^4$ yr, giving the constraint $\mathcal{R} \geq 10^{29}$.

If we allow very different values of Q and η_γ , then the constraint weakens considerably. Considering the example

cosmology of Sec. III in which stars form beginning at $t \sim 10^9 Q_8^{-3/2}$ s and using Eq. (3) for t_{eq} , the constraint becomes

$$\mathcal{R} \geq 10^{24} Q_8^{-1/2}. \quad (12)$$

Thus the curvature scale can be $\sim 10^5$ times lower in a cold cosmology, and measurement of a small value of \mathcal{R} and a cold cosmology will be much more probable than measurement of larger \mathcal{R} and a hot cosmology, in any universe in which the *a priori* probability of hot and cold cosmologies is similar and in which the *a priori* probability distribution of \mathcal{R} is peaked at small values (the latter being the only situation in which anthropic arguments about \mathcal{R} in open cosmologies are useful).

IX. WHAT IF η_γ DOES NOT VARY?

The difficulties for the anthropic program pointed out in this paper all depend on the assumption that low- η_γ cosmologies exist in the ensemble comprising the universe which is posited in any cosmological anthropic argument. Can all of the difficulties be avoided if it is assumed that η_γ is the same in all ensemble members? Perhaps, but this is by no means clear. For example (and to note yet another worry regarding the anthropic program), consider cosmologies with $Q \sim 0.01$ and $\Lambda \sim 10^{-114}$. The cosmology would (by TR’s arguments) be dominated by black holes, with (at best) extremely dense $10^{16} M_\odot$ galaxies forming just after matter domination and just as Λ -domination begins. This cosmology would be rather inhospitable to life, i.e., $\xi(Q \sim 0.01, \Lambda \sim 10^{-114}) \ll \xi(Q \sim 10^{-5}, \Lambda \sim 10^{-124})$. However, differences in P might compensate: consider a universe in which $d^2 P(\Lambda, Q^2)/d\Lambda dQ^2$ is flat [as per Weinberg’s conjecture applied to Linde’s [1] fiducial chaotic inflation model with $V(\Lambda) = \lambda \phi^4/4$]. Then $P(Q \sim 0.01, \Lambda \sim 10^{-114}) \sim 10^{16} \xi(Q \sim 10^{-5}, \Lambda \sim 10^{-124})$, and such cosmologies must produce only one life-supporting star (perhaps by some extremely baroque and unlikely process) per $10^{18} M_\odot$ of baryons to compete with our cosmology. It seems difficult to rule out such a possibility.

X. SUMMARY AND DISCUSSION

In Sec. II A I denoted by $\xi(\bar{\alpha}_1, \dots, \bar{\alpha}_N)$ the number of solar-mass, metal-rich stars per baryon that would form over the lifetime of a universe described by a FRW model specified by N parameters α_i with values $\bar{\alpha}_i$. If there is a probability $P(\bar{\alpha}_1, \dots, \bar{\alpha}_N)$ that a given baryon finds itself in a cosmology described by $\bar{\alpha}_i$, the probability $\mathcal{P} \equiv P(\bar{\alpha}_1, \dots, \bar{\alpha}_N) \xi(\bar{\alpha}_1, \dots, \bar{\alpha}_N)$ should describe the probability that a randomly chosen observer measures the values $\bar{\alpha}_i$. I then defined the “anthropic program” as the computation of \mathcal{P} ; if this probability distribution has a single peak at a set $\bar{\alpha}_i^{\text{pk}}$ and if these are near the measured values $\bar{\alpha}_i^{\text{obs}}$, then it could be claimed that the anthropic program has “explained” the values $\bar{\alpha}_i^{\text{obs}}$ of the parameters of our cosmology by showing that it is extremely likely for typical observers to

measure values very near $\bar{\alpha}_i^{\text{pk}} \approx \bar{\alpha}_i^{\text{obs}}$, so that (assuming we are typical observers) it is not surprising that we do.

In Sec. III I developed a class of cosmologies in which many of the α_i take values quite different from those deduced from observations. Setting $\alpha_i = \{\eta_\gamma, Q, \eta_{\text{dm}}, \eta_L, \mathcal{R}, \Lambda\}$, the observable universe can apparently be described by a FRW model with $\bar{\alpha}_i^{\text{obs}} = \{\sim 10^9, \sim 10^{-5}, \sim 10, \ll 10^9, \gg 10^{30}, \leq 10^{-56} \text{ cm}^{-2}\}$. Observationally, we see that $\sim 1\%$ of baryons form stars, i.e., $\xi(\bar{\alpha}_1^{\text{obs}}, \dots, \bar{\alpha}_N^{\text{obs}}) \sim 0.01 m_p / M_\odot$. Let $\tilde{\alpha}_i \equiv \bar{\alpha}_i / \bar{\alpha}_i^{\text{obs}}$ (i.e., the α_i in units of their observed values). Then in Sec. III I argued that similar values for ξ can arise if $\tilde{\alpha}_i \sim \{10^{-9}, 1, 0, 1, 1, 1\}$ if supermassive population III objects form primordially and heat the cosmic medium up to $\eta_\gamma \sim 10^9$. Alternatively, solar-mass stars might form primordially (with primordially formed metals), plausibly giving similar values of ξ for $\tilde{\alpha}_i \sim \{10^{-9}, 10^{-4}, 0, 1, 1, 1\}$ (if primordial clouds fragment into M_\odot stars) or $\tilde{\alpha}_i \sim \{10^{-9}, 10^{-4}, 1, 1, 1, 1\}$ (if not). Furthermore, structure forms very early in such cosmologies, implying that ξ can be high even in drastically different cosmologies such as (for example) that described by $\tilde{\alpha}_i \sim \{10^{-9}, 10^{-4}, 0, 1, 10^5, 10^{17}\}$. Less dramatically, but quite illustratively, the calculations of TR show that one can construct a whole four-dimensional region of parameter space in which ξ is within a few orders of magnitude, defined by $\xi(y, x, 1, 1, w, z) \sim (\text{const})$ and parametrized by x , where $10^{-3} \lesssim x \lesssim 100$, $0.1x^{7/8} \lesssim y \lesssim 10x^{3/4}$, $w \gtrsim 0.1yx^{-1/2}$, and $z \lesssim y^{-4}x^3$.

At minimum, the existence of many independent maxima and planes of degeneracy in ξ —widely separated in parameter space—should be discouraging for proponents of the anthropic program: it implies that it is quite important to know the probabilities P , which generally depend on poorly constrained models of the early universe. The hope that anthropic considerations would lead to only a small allowed region of parameter space—i.e., a small, sharp peak in $\xi(\alpha)$ —is not realized, and it seems that anthropic arguments alone cannot simultaneously constrain multiple cosmological parameters, even when many assumptions that are quite sympathetic to the anthropic program are made.

Drawing further conclusions from the arguments I have presented requires assumptions about P . Given strong *a pri-*

ori confidence in a particular form of P , one could rule out the anthropic program itself if that form of P favors any of the cosmologies I have developed here which are unlike our own; this might imply, for example, that we simply happen to be “improbable” observers and/or that there are *not* other universes in which the parameters vary as assumed, or that it is misguided to consider as possible only “observers” quite like ourselves. For those committed to the anthropic program, the CBB cosmologies could be used constructively to constrain P : any model in which P is very strongly weighted toward one of the “alternative” sets of $\bar{\alpha}_i$ is ruled out.

This paper has largely addressed anthropic arguments concerning cosmological parameters, but many of the issues raised here clearly apply to anthropic arguments concerning more fundamental constants. The construction of specific counterexamples to these arguments (i.e., cosmologies with very different fundamental constants yet with observers), analogous to the CBB cosmologies developed here, would seem to be an enormously more difficult technical task and would require a much more careful assessment of what could reasonably constitute an observer [47]. This difficulty would, however, be greatly exceeded by the difficulty of rigorously arguing that *no* such alternative cosmology exists [48].

In conclusion, I have noted that one cannot simultaneously anthropically explain the values of several parameters if the argument for each parameter requires all others to be fixed. It is possible, then, to explain at most one parameter using such an argument. To explain more than one parameter, more than one parameter must be varied among the ensemble members, and degeneracies can arise in the probability distributions. In the case that all six of the parameters specifying the standard FRW cosmology are allowed to vary, I find that it is possible to construct a cosmology in which all of the parameters vary by (at least) several orders of magnitude from their “observed” values, yet in which stars, planets, and intelligent life can plausibly arise. This greatly complicates, and reduces the explanatory power of, anthropic arguments in cosmology.

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