Astrophysical limits on quantum gravity motivated birefringence

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We obtain observational upper bounds on a class of quantum gravity related birefringence effects, by analyzing the presence of linear polarization in the optical and ultraviolet spectrum of some distant sources. In the notation of Gambini and Pullin we find $\chi < 5 \times 10^{-5}$.

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I. INTRODUCTION

Recently two predictions from the leading approaches to quantum gravity have raised the expectations that the theories (or at least models based on these theories) could be tested in the near future [1,2]. The main prediction coming from superstrings is that the frequency dispersion relation is not linear and consequently photons at different energies travel at different speeds. In particular, Amelino-Camelia *et al.* have argued that energetic photons would travel slower than soft ones [2]. Thus, by comparing the time of arrival of rays at different energies emitted simultaneously from the same source, one could test the validity of this prediction. A first step in this direction is given in [3]. The second prediction comes from loop quantum gravity. It was observed by Gambini and Pullin [1] that if the weave states of quantum gravity have a definite parity, then light traveling in that medium will display birefringence, namely, photons with left and right circular polarizations will travel at different speeds. For a source located about 10^{10} ly away, and energies of the order of 200 keV, the predicted time of arrival difference for left and right photons, emitted simultaneously, is of the order of 10^{-5} s which, if correct, could be detected in the near future. In principle, as indicated in [2,1], a natural testing ground to test these predictions would be gamma ray bursts (GRB), since (a) they are at cosmological distances, (b) GRB emissions show fine-scale time structure down to the present instrumental resolution, and (c) they emit in a continuum spectrum (typically an E^{-2} law). However, partial analyses of GRB data taken from the BATSE catalog [4] (and other detectors) have not shown evidence for the presence of these effects [5,6]. Furthermore, there is evidence [6] of astrophysical processes in GRB's that affect their emission in a way that may completely swamp the signals expected from quantum gravity predictions. Thus, at present, GRB data do not appear to be useful to constrain these models, at least using time of arrival measurements.

We may, however, consider a different approach that, although restricted to the predictions of [1], does not require GRB data. This is based on the observation that a significant rotation of the plane of polarization of linearly polarized photons must occur long before any difference in time of arrival is even measurable. Of course, to measure a rotation, it is necessary to have a reference orientation. On the other hand, and this is the main idea of this paper, if we have evidence that the rotation, if present, is below a certain bound, we immediately obtain a bound on the model parameters characterizing the effect. In the next section we review the results of Gambini and Pullin, and indicate a way to relate the effect to astrophysical data pertaining to cosmological sources of polarized light. We close the paper with some further comments on our results.

II. POLARIZATION EFFECTS ON THE OBSERVED PHOTONS

In their model, based on loop quantum gravity, Gambini and Pullin [1] assume a nonparity invariant weave, with the result that the (vacuum) Maxwell fields satisfy equations of motion of the form

$$\begin{aligned}\partial_t \vec{E} &= \nabla \times \vec{B} + 2\chi l_P \Delta^2 \vec{B},\\ \partial_t \vec{B} &= -\nabla \times \vec{E} - 2\chi l_P \Delta^2 \vec{E},\end{aligned} \tag{1}$$

where l_P is the Planck length, and χ is a dimensionless constant, that characterizes both a parity nonconservation and a violation of Lorentz covariance. Combining these equations, together with $\nabla \cdot \vec{E} = \nabla \cdot \vec{B} = 0$, they obtain for $\vec{E}$ a wave propagation equation of the form

$$\partial_t^2 \vec{E} - \nabla^2 \vec{E} - 4\chi l_P \Delta^2 (\nabla \times \vec{E}) = 0 \tag{2}$$

and a similar equation for $\vec{B}$, where terms of higher order in χl_P have been dropped, on account of their assumed smallness. Plane wave solutions of (2), with wave vector $\vec{k}$, and given helicity, will be of the form,

$$\vec{E}_{\pm} = \mathrm{Re}((\hat{e}_1 \pm i\hat{e}_2) e^{i(\Omega_{\pm} t - \vec{k}\cdot\vec{x})}) \tag{3}$$

with $\hat{e}_1 \cdot \hat{e}_2 = 0$. Consistency with Eq. (2) implies

$$\Omega_{\pm} = \sqrt{k^2 \mp 4\chi l_P k^3} \simeq |k|(1 \mp 2\chi l_P |k|), \tag{4}$$

and, $\hat{e}_1 \cdot \vec{k} = \hat{e}_2 \cdot \vec{k} = 0$.

Thus, the model leads to the emergence of a birefringence effect, associated with quantum gravity corrections to the propagation of electromagnetic waves, because the group velocity associated with the dispersion relation (4) has two branches, one for each mode of circular polarization. In principle, this effect is very small, corresponding roughly to a shift of one Planck length per wavelength.

A possible way of detecting the effect, based on the difference in time of arrival associated with the difference in group velocities, was suggested in [1]. As indicated in the Introduction, in this paper we consider a different analysis, which is aimed at finding upper bounds on the magnitude of the effect. The main idea is more easily stated by restricting our attention to an astrophysical process where photons are emitted with linear polarization, and are detected after traveling a distance of cosmological relevance. We place the origin of coordinates at the emission point, and consider propagation along the z-direction. As a first approximation we disregard curvature effects. Real photons cannot be represented by plane waves, but rather by appropriate wave packets. If we assume that the photons are emitted at times t near $t=0$, with a central frequency Ω_0, and Gaussian frequency width $\delta\Omega$, their wave function may be represented by a Gaussian wave packet of the form,

$$\vec{E}=\mathrm{Re}\{\mathcal{A}e^{i\Omega_0(t-z)}[e^{-(z-v_+t)^2(\delta\Omega)^2}e^{-i\chi l_P z\Omega_0^2}\hat{e}_+ + e^{-(z-v_-t)^2(\delta\Omega)^2}e^{i\chi l_P z\Omega_0^2}\hat{e}_-]\} \tag{5}$$

where $\mathcal{A}$ is a constant, $\hat{e}_\pm=(\hat{e}_1\pm i\hat{e}_2)/\sqrt{2}$, and we have kept only the lowest, nontrivial orders in χ. The group velocities $v_\pm$, corresponding to the circular polarizations $\hat{e}_\pm$, are given by, $v_\pm=1\mp4\chi l_P|k|$.

Clearly, since the effect has not yet been observed, even if $v_+\neq v_-$, we must have $|v_+-v_-|\ll1$, and $v_+\simeq v_-\simeq1$. Then, the wave packets corresponding to both circular polarizations are centered near $z=t$, and at any distance z from the source, such that,

$$|v_+-v_-|t\,\delta\Omega\simeq 8\chi l_P z\Omega_0\delta\Omega\ll1 \tag{6}$$

we have

$$\vec{E}\simeq\mathrm{Re}\{\mathcal{B}e^{i\Omega_0(t-z)}e^{-(z-t)^2(\delta\Omega)^2}[\cos(\chi l_P z\Omega_0^2)\hat{e}_1 + \sin(\chi l_P z\Omega_0^2)\hat{e}_2]\}. \tag{7}$$

Therefore, for a sufficiently narrow (small $\delta\Omega$) packet, at distances satisfying (6), we recover the well known rotation of the polarization plane, proportional to the distance to the source, that characterizes optical birefringence. On the other hand, if we consider distances z such that,

$$8\chi l_P z\Omega_0\delta\Omega\gg1 \tag{8}$$

the wave function splits into two spatially separated pieces, corresponding to each one of the polarization modes. This is the situation envisaged in [1], where photons corresponding to each circular polarization would be detected with a time delay of the order of $\chi l_P z\Omega_0$. But, and this is one of the main points of our discussion, when (8) holds, the photons are no longer linearly polarized. In more precise terms, if we characterize a linear polarization detector by a (fixed) unit vector $\vec{n}$, such that $\vec{n}\cdot\vec{k}=0$, then we may obtain a measure of the amount of linear polarization in the direction of $\vec{n}$ by considering the quantity,

$$\mathcal{P}(\vec{n})=\langle|\vec{n}\cdot\vec{E}|^2\rangle/\langle|\vec{E}|^2\rangle \tag{9}$$

where $\langle\rangle$ indicates a suitable average, for instance, we may take $\langle X\rangle=\lim_{T\to\infty}(1/T)\int_0^T X\,dt$. When condition (6) is satisfied, we find,

$$\mathcal{P}(\vec{n})=\cos^2\phi=1/2[1+\cos(2\phi)] \tag{10}$$

where $\cos\phi=\vec{n}\cdot\hat{e}_1$. However, when z is large enough that (8) holds, we find $\mathcal{P}(\vec{n})=1/2$, *independent* of the direction of $\vec{n}$. Therefore, if one could be sure that the photons were emitted linearly polarized, a signal of the presence of the birefringency effect would be the absence of this polarization at the detector, even if no time delay measurement is possible, at the given level of detector discrimination. In fact, this reasoning may be further refined, because the linear polarization is "erased" before a long separation of the modes is achieved. To see this, consider a distance intermediate between (6) and (8). Suppose, for example, that the packets corresponding to right and left circular polarization essentially superpose each other only through half of their length. Then, at the detector, one first "sees" one polarization. As the packets begin their superposition, this turns to elliptic polarization, until a linear polarization is achieved, when the amplitudes of the packets are similar, and from that point on the situation is reversed, the polarization becomes again elliptic, and finally circular with the other mode. Clearly, the observed polarization will be such that, $\mathcal{P}(\vec{n})=1/2+\alpha\cos(2\phi)$, with $0\leq\alpha\leq1/2$, where $\alpha=1/2$ corresponds to full linear polarization and $\alpha=0$ to the absence of linear polarization, which requires only full separation of the polarization modes. Of course, all the previous argument relays in the knowledge of a mechanism that certainly produces linearly polarized photons. We may, however, turn the argument around, and ask ourselves under what conditions it would be possible to observe a linear polarization of photons, assuming *both* that they are emitted linearly polarized, *and* that a birefringency effect, such as the one proposed in [1], takes place. If we assume that linearly polarized photons are detected, and unambiguously identified with a source at cosmological distance z, without any significant interaction in between, we may be immediately sure that (6) is not strongly violated. Thus, if we can measure z and Ω_0, from (6) we find an upper bound on χ. But, at this point, if we review the previous derivations, we see that they refer only to essentially monochromatic waves. Actually, photons from a given cosmological source may be observed both in line and in continuous spectra. In the latter case, where the mechanism at the source responsible for the linear polarization gives rise to photons in a range of frequencies, but all polarized along the same direction, such as synchrotron emission, or polarization by reflection from an interstellar cloud, it is better to refer our result to the standard definition in terms of Stokes parameters. Introducing the quantities [7]

$$\mathcal{S}_0=\langle(E_x)^2\rangle+\langle(E_y)^2\rangle,\mathcal{S}_1=\langle(E_x)^2\rangle-\langle(E_y)^2\rangle,\mathcal{S}_2=\langle2E_xE_y\rangle \tag{11}$$

the polarization is given by

$$\mathcal{P}=\frac{[(\mathcal{S}_1)^2+(\mathcal{S}_2)^2]^{1/2}}{\mathcal{S}_0}. \tag{12}$$

In our case, even if we assume that (6) holds (otherwise no linear polarization would be observed), the averages must be taken over an ensemble of wave packets of the form (7). These are of the form, $\langle A\rangle=\int\mathcal{F}(\lambda)A(\lambda)d\lambda$, where $\lambda=2\pi/k$, and $\mathcal{F}(\lambda)$ is the distribution function for the ensemble that includes the characteristics of filters included in the detection device. We then find, up to an overall normalization factor,

$$\mathcal{S}_0=\int\mathcal{F}(\lambda)d\lambda$$

$$\mathcal{S}_1=\int\mathcal{F}(\lambda)\cos(8\pi^2\chi l_P z/\lambda^2)d\lambda \tag{13}$$

$$\mathcal{S}_2=\int\mathcal{F}(\lambda)\sin(8\pi^2\chi l_P z/\lambda^2)d\lambda.$$

This implies that no linear polarization may be observed if $\mathcal{F}$ does not change appreciably in a range of values of λ such that $8\pi\chi l_P z/\lambda^2$ changes in more than several times π. In other words, no net linear polarization will be observed if the plane of polarization of the different members of the ensemble (photon with different frequencies) is rotated in angles that cover a range larger than π. It is important to realize that what we have in mind is an experiment where polarized photons are actually observed, quite independently of the production mechanism. This observation puts a limit on the possible differential rotation of the polarization planes as a function of frequency, and, therefore, on the value of χ. Since the effect depends on λ^2, we obtain a very sensitive proof of the presence of a birefringence of the type proposed in [1]. In fact, there are many measurements available in the literature showing linear polarization in the light from quasars, or radio galaxies, with wavelengths in a continuous optical range.

We may take, for example, the results of Jannuzi *et al.* [8], that indicate a polarization larger than 10% in the ultraviolet for radio galaxy 3C 256, at a redshift of 1.82, and assume, for simplicity, no change in wavelength after emission, and a flat spectrum in the region of interest. For polarization measurements in the ultraviolet with a U filter we take, also for simplicity, again up to an overall normalization,

$$\mathcal{F}(\lambda)=\exp\bigl(-(\lambda-\lambda_0)^2/(\Delta\lambda)^2\bigr) \tag{14}$$

where $\lambda_0\simeq3500$ Å, with $\Delta\lambda\simeq500$ Å. Then, if, as a rough estimate, we take $z\simeq10^10$ light-years, we find that the observed polarization can be larger than about 10%, only if

$$\chi\leqslant5\times10^{-5}. \tag{15}$$

A different type of evidence may be obtained by noticing that there are objects at a cosmological distance that show a linearly polarized component throughout the visible spectrum (of the order of a few percent) with little dependence (less than 10°) of the polarization angle with wavelength [9]. In this case, directly from Eq. (7), since the rotation angle is, $\Delta\phi_{Pol}=4\pi^2\chi l_P z[(1/\lambda_1^2)-(1/\lambda_2^2)]$, if we take $\lambda_1=4000$ Å and $\lambda_2=8000$ Å, $z=10^10$ ly, and impose $\Delta\phi<10°$, we find,

$$\chi\leqslant10^{-4}. \tag{16}$$

III. FINAL COMMENTS

It has been suggested that the presence of a birefringence of space, of the type given by Eq. (4), might be put to test in the analysis of events where gamma rays are involved. This is because, assuming the characteristic parameter χ is of the order of 1, such high energies (or short wavelengths) are required in order to separate sufficiently in time the two polarization modes, so that their separate detection becomes technically feasible.

In the approach considered in this paper, we take Eq. (4) as an essentially phenomenological ansatz, and, instead of trying to measure χ, we analyze the possible consequences of the presence of such an effect as regards measurements of polarization already performed. Thus, although the lack of polarization, or the presence of a wavelength dependence on the polarization angle of cosmological sources, may be due to many effects, the detection of significant polarization, or lack of rotation, is possible only if the value of χ lies below a certain upper bound. Regarding the bounds indicated in (15) and (16), they should be considered as overly conservative. In fact, in the case of (15), just taking redshifts into account would make the effective wavelength shorter leading to a smaller upper bound on χ. Notice that by taking $\lambda_0=1500$ Å, the upper bound on χ is decreased by at least a factor of 10. Similarly, taking $\lambda_1=1500$ Å, the bound (16) is decreased by an order of magnitude. Further refinements on these bounds may be achieved by a more detailed analysis of $\Delta\phi$ in polarized sources. However, the results obtained in this paper already show that the presence of significant polarization in the light from cosmological sources provides important information on the possibility of a quantum gravity birefringence effect of the form (4).

We close this paper with the following remarks.

(i) Equations (1) give the simplest coupling between quantum gravity and Maxwell theory which includes a parity violation.

(ii) More important, although the results presented here are derived from Eqs. (1), dimensional analysis of a quantum gravity induced birefringence indicates that the change of phase of the linear polarization vector per unit length should be proportional to l_P/λ^2, since it should vanish in the classical limit $l_P\to0$. Thus, the result presented here should also apply to any model that gives rise to a quantum gravity induced birefringence.

(iii) At present there is no reliable way to estimate the value of χ. Our results put an upper bound for this value, which might be consistent with a more detailed calculation. Thus it would be very important to measure polarization effects for x and gamma rays from astrophysical sources of cosmological origin. The shorter wavelengths of these rays would either provide a much smaller upper bound for χ, or evidence of a quantum gravity effect.

(iv) Given the conservative upper bound on χ found in our analysis, the time delay in the situation envisaged in [1] would not be larger than 10^{-9} s, completely beyond present or near future technology.

(v) An extension of the polarization measurements to the x and γ ray part of the spectrum, even with current technology, could improve the estimate of the upper bound on χ by several orders of magnitude, or, perhaps give some indication of the existence of the effect.

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