Lensing at cosmological scales: A test of higher dimensional gravity

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Recent developments in gravitational lensing astronomy have paved the way to genuine mappings of the gravitational potential at cosmological scales. We stress that comparing these data with traditional large-scale structure surveys will provide us with a test of gravity at such scales. These constraints could be of great importance in the framework of higher dimensional cosmological models.

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Recent phenomenological developments in cosmology have been inspired by the introduction of branes in the context of superstring theories [1,2]. It leads to concepts of higher dimensional spacetimes in which the interaction gauge fields are localized on a 3-brane [i.e., a (3+1)-dimension spacetime] whereas gravity propagates in all dimensions. In any of such string-inspired models, one expects the existence of Kaluza-Klein gravitons implying a nonstandard gravity on small scales and of light bosons, which can manifest themselves as a new fundamental small-scale force. Moreover, it seems quite generic that there also exist neighboring branes; the interbrane distance then appears as a new scale (exponentially large compared to the small distance scale) above which gravity is also nonstandard [3,4]. In this paper, we investigate how cosmological observations can test gravity at large distances, thus providing constraints on this new scale.

During the past 20 years, there has been a large amount of activity in the search for a deviation from Newtonian gravity [5,6] in the form of looking for a violation of the weak equivalence principle or of the inverse square law. It has been pointed out in particular that little was known about gravity on submillimeter scales [7]. On the other hand, in the weak-field limit, tests in the solar system (perihelion advance, bending and delay of electromagnetic waves, laser ranging of the Moon) and the bounds on the variation of the constants of nature have put severe constraints on the post-Newtonian parameters [6,8]. However, results of a confrontation between standard gravity and alternative theories at cosmological scales are sparse and no systematic studies have been performed (mainly because no general scheme, such as the parametrized post-Newtonian formalism, has been devised yet). Moreover, cosmological observations entangle gravity and many other astrophysical processes, which renders such cosmological tests *a priori* less robust than those in, e.g., the solar system. Nevertheless, comparisons between x-ray emissivity and gravitational lensing, which is an indirect test of the Newton law through the equation of hydrostatic equilibrium, show no dramatic discrepancy below 2 Mpc [9]. On larger scales, there is no other test on gravity than the mechanism of structure formation through gravitational instability, which is the subject of this paper.

In most high dimensional spacetime models, matter is confined to a 3-brane and gravity can propagate in all dimensions. The law of gravity takes its standard four-dimensional form for distances larger than a given length scale (of order of the compactification radius) [10], but at smaller distances, the effect of the extra dimensions starts to dominate, implying a deviation with respect to the Newtonian gravity. These models were extended to noncompact extra dimensions [1] where the bulk spacetime is described by an anti–de Sitter space. Testing gravity at small scales offers the possibility to investigate these structures (for a description of gravity at small distances in these models see, e.g., [11]). Recently, it was proposed in the framework of higher dimensional models that gravity can deviate from its Newton form also on large scales [3,4]. In the Gregory *et al.* model [3], a Randall-Sundrum (RS) -like solution is considered but with three branes in which space is anti–de Sitter in between the brane but not in the outer parts; this solution does not possess a normalizable zero mode. The graviton is shown to be unstable and its decay implies a modification of gravity on large scales. Kogan *et al.* [4] proposed a model where the extra dimensions are compact and large distance effects appear due to the existence of very light Kaluza–Klein states. And it was pointed out by Dubovsky *et al.* [12] that when one tries to give masses to a localized scalar, a potential with power-law behavior at large scales appears due to the existence of quasilocalized states.

Constraints on the size of large extra dimensions coming from astrophysical systems can be applied [13] but they do not test directly the gravity law. The goal of this paper is precisely to point out that some relevant cosmological observables potentially exist that enable us to test gravity on cosmological scales.

It has already been argued [14] that if the gravitational potential differs from its Newtonian form on large scales, it affects the evolution of cosmological density perturbations. The authors claim that it can be visible on the cosmic microwave background (CMB) anisotropy spectrum. It should be noted, however, that a more detailed implementation of these results may turn out not to be so easy to achieve mainly because the deviation from the Newton gravity has to be recast into a covariant cosmological form to treat the evolution of superhorizon modes.

In what follows, we assume that the background spacetime can be described by a Friedmann-Lemaître spacetime. As long as we are dealing with subhorizon scales, we can take the metric to be of the form

$$ds^2=-(1-2\Phi)dt^2+a^2(1+2\Phi)[d\chi^2+q^2(\chi)d\Omega^2], \tag{1}$$

where t is the cosmic time, $a(t)$ the scale factor, χ the comoving radial coordinate, $d\Omega^2$ the unit solid angle, and $q(\chi)=(\sin\chi,\chi,\sinh\chi)$ according to the curvature of the spatial sections. In a Newtonian theory of gravity, Φ is the Newtonian potential Φ_N determined by the Poisson equation

$$\Delta\Phi_N=4\pi G\rho a^2\delta, \tag{2}$$

where G is the Newton constant and Δ the three-dimensional Laplacian in comoving coordinates, ρ is the background energy density, and $\delta\equiv\delta\rho/\rho$ is the density contrast. If the Newton law is violated above a given scale r_s, then we have to change Eq. (2), and the force between two masses separated by a distance of r derives from $\Phi=\Phi_N f(r/r_s)$, where $f(x)\to 1$ when $x\ll 1$. This encompasses, for instance, the potential considered in [3,14] for which $f(x)=1/(1+x)$ (in that case $f\propto 1/x$ and 5D gravity is recovered at large distance). Using Eq. (2) it leads, with $\mathbf{r}=a\mathbf{x}$, to

$$\Phi(\mathbf{x})=-G\rho a^2\int d^3\mathbf{x}'\frac{\delta(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}f\left(\frac{|\mathbf{x}-\mathbf{x}'|}{x_s}\right), \tag{3}$$

which, making use of $\Delta[f(x)/x]=-4\pi\delta^{(3)}(\mathbf{x})+f_s(x/x_s)$ with $f_s(x/x_s)\equiv(\partial_x^2 f)/x$, gives

$$\Delta\Phi=\Delta\Phi_N-G\rho a^2\int d^3\mathbf{x}'\,\delta(\mathbf{x}'+\mathbf{x})f_s(x'/x_s). \tag{4}$$

For any stochastic field X, we define its power spectrum $\mathcal{P}_X$ by

$$\langle\hat{X}(\mathbf{k})\hat{X}^*(\mathbf{k}')\rangle\equiv(2\pi)^{-3/2}\mathcal{P}_X(k)\delta^{(3)}(\mathbf{k}-\mathbf{k}'), \tag{5}$$

where $\delta^{(3)}$ is the Dirac distribution, $\hat{X}$ the Fourier transform of X, and the brackets refer to an ensemble average [21]. If the Poisson equation is satisfied, then

$$\mathcal{P}_{\Delta\Phi_N}(k)=(4\pi G\rho a^2)^2\mathcal{P}_\delta(k). \tag{6}$$

In Fourier space, Eq. (4) reads

$$-k^2\hat{\Phi}(k)=4\pi G\rho a^2\hat{\delta}(k)f_c(kr_s) \tag{7}$$

from which we deduce that

$$\mathcal{P}_{\Delta\Phi}(k)=(4\pi G\rho a^2)^2\mathcal{P}_\delta(k)f_c^2(kr_s), \tag{8}$$

where $f_c(kr_s)\equiv 1-2\pi^2 f_s(kr_s)$, $f_s(kr_s)$ being the Fourier transform of $f_s(r/r_s)$ (see Fig. 1). A way to test the validity of the Newton law is thus to test the validity of Eq. (2), which is possible if one can measure δ and Φ independently.

From galaxy catalogs, one can extract a measure of the two-point correlation function of the cosmic density,

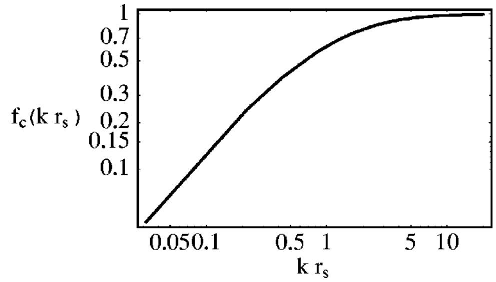

FIG. 1. Function $f_c(k\,r_s)$ as a function of $k\,r_s$ for $f(x)=1/(1+x)$.

$$\xi(r)\equiv\langle\delta(0)\delta(\mathbf{r})\rangle, \tag{9}$$

where the brackets refer here to a spatial average. It leads to a measure of

$$\mathcal{P}_\delta(k)=\frac{1}{2\pi^2}\int\xi(r)\frac{\sin kr}{kr}r^2dr. \tag{10}$$

On the other hand, weak lensing surveys offer a novel and independent window on the large-scale structures. The bending of light by a matter distribution is intrinsically a relativistic effect, which enables us to test gravity at extragalactic scales. Weak lensing measurements are based on the detection of coherent shape distortions of background galaxies due to the large-scale gravitational tidal forces. The apparent angular position $\vec{\theta}_{\rm I}$ of a lensed image can be related to the one, $\vec{\theta}_{\rm S}$, of the source (at radial distance $\chi_{\rm S}$) by [15,16]

$$\vec{\theta}_{\rm I}=\vec{\theta}_S+\frac{\mathcal{D}(\chi_S-\chi)}{\mathcal{D}(\chi_S)}\vec{\alpha}, \tag{11}$$

where $\mathcal{D}$ is the comoving angular diameter distance [16]. $\vec{\alpha}$, the deflection angle, depends on the gravitational potential integrated along the line of sight

$$\vec{\alpha}=\frac{2}{c^2}\int_0^{\chi_S}d\chi\nabla_x\Phi. \tag{12}$$

The deformation of a light bundle is obtained by differentiating Eq. (11),

$$A_a^b\equiv\begin{pmatrix}1-\kappa-\gamma_1 & \gamma_2\\ \gamma_2 & 1-\kappa+\gamma_1\end{pmatrix}=\frac{d\theta_a^S}{d\theta_b^I}. \tag{13}$$

κ and $\vec{\gamma}$ are, respectively, the convergence and the shear of the amplification matrix A_{ab}. The shear can be measured from galaxy ellipticities [17] from which one can reconstruct κ. The convergence is generated by the cumulative effect of large-scale structures along the line of sight [15,16]. In direction $\vec{\theta}$, it reads

$$\kappa(\vec{\theta})=\int_0^{\chi(z=\infty)}g(\chi)\Delta_2\Phi(\mathcal{D}(\chi)\vec{\theta},\chi)d\chi, \tag{14}$$

where Δ_2 is the two-dimensional Laplacian in the plane perpendicular to the line of sight; the function g depends on the distance distribution of the sources, $n(\chi_S)$, by

$$g(\chi)=\int_{\chi}^{\chi(z=\infty)} \mathrm{d}\chi_S n(\chi_S)\frac{\mathcal{D}(\chi_S-\chi)\mathcal{D}(\chi)}{\mathcal{D}(\chi_S)}. \tag{15}$$

$\kappa(\vec{\theta})$ is a function on the celestial sphere that can be decomposed, in the small-angle approximation, in Fourier modes,

$$\hat{\kappa}(\boldsymbol{l})=\int \frac{d^2\vec{\theta}}{2\pi}\kappa(\vec{\theta})e^{i\boldsymbol{l}\cdot\vec{\theta}} \tag{16}$$

so that, using the expression (14) and the definition of the angular power spectrum of κ as $\langle\hat{\kappa}(\boldsymbol{l})\hat{\kappa}^*(\boldsymbol{l}')\rangle=(2\pi)^{-1}\mathcal{P}_\kappa(l)\delta^{(2)}(\boldsymbol{l}-\boldsymbol{l}')$, we obtain

$$\mathcal{P}_\kappa(l)=\int d\chi\frac{g^2(\chi)}{\mathcal{D}^2(\chi)}\mathcal{P}_{\Delta\Phi}\left(\frac{l}{\mathcal{D}(\chi)}\right). \tag{17}$$

It clearly appears that cosmic shear measurements are a direct probe of the gravitational potential. So far cosmic shear signals have been detected up to a scale of about $2\,h^{-1}$ Mpc [17] (h being the Hubble constant in units of 100 km/s/Mpc). This method is in principle applicable to any scale up to $100\,h^{-1}$ Mpc. With galaxy surveys such as SDSS that will measure $\mathcal{P}_\delta$ up to $500\,h^{-1}$ Mpc [18], it enables comparisons of $\mathcal{P}_\delta$ and $\mathcal{P}_{\Delta\Phi}$ at cosmological scales, thus enabling direct tests of the gravity law.

To illustrate this discrepancy, we consider the growth of the perturbations on scales from ten to some hundreds of Mpc in a modified gravity scenario. For that purpose, we assume that the standard behavior of the scale factor is recovered (i.e., we have the standard Friedmann equations). Note that it has not been proven that in the RS scenarios the localization of gravity was compulsory to recover the standard Friedmann equation, but a heuristic argument can be given. In the RS models, one recovers a Minkowski space-time on the brane with Newtonian gravity at large scales only if a special condition between the brane and bulk cosmological constants holds [1]. It can be thought from the naive Newtonian derivation [19] that Friedmann equations should also hold (at least in a matter-dominated universe). At first glance, the Friedmann equations turn out to be nonstandard [2] and reduce to the standard ones only if a relation similar to the RS condition ensuring localization of gravity holds [20]. The effect of the existence of extra branes on the Friedmann equations has not been investigated yet.

In the weak-field limit, δ and the peculiar velocity $\mathbf{v}$ obey (for a pressureless fluid) the continuity and Euler equations [21]

$$\dot{\delta}+\frac{1}{a}\boldsymbol{\nabla}\cdot[(1+\delta)\mathbf{v}]=0, \tag{18}$$

$$\dot{\mathbf{v}}+\frac{1}{a}(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{v}+H\mathbf{v}=-\frac{1}{a}\boldsymbol{\nabla}\Phi, \tag{19}$$

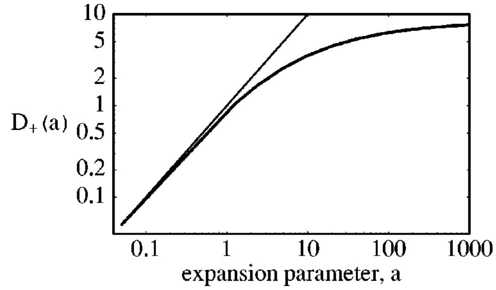

FIG. 2. Growth factor $D_+(a)$ as a function of a in an Einstein–de Sitter universe for $k\,r_s=1$ (thick line) compared with the standard growth rate, $D_+\propto a$ (thin line).

where a dot refers to a derivative with respect to t. $H\equiv\dot{a}/a$ is the Hubble parameter. The equation of evolution of the density contrast, δ_k, taking advantage of the fact that the relation between δ and Φ is local in Fourier space [see Eq. (7)], is then

$$\ddot{\delta}_k-2H\dot{\delta}_k-\frac{3}{2}H^2\Omega(t)f_c\left(k\frac{r_s}{a(t)}\right)\delta_k=0. \tag{20}$$

Looking for a growing mode as $\delta_k\propto t^{\nu_+(k)}$ in an Einstein–de Sitter matter-dominated universe ($\Omega=1$, $H=2/3t$) gives a growing solution such that $\nu_+(k)\to\frac{2}{3}$ for $kx_s\gg1$ and $\nu_+(k)\to0$ for $kx_s\ll1$. At large scales, the fluctuations stop growing mainly because gravity becomes weaker and weaker. In Fig. 2, we depict the numerical integration of Eq. (20) and the resulting power spectrum in Fig. 3 assuming that $f(x)=1/(1+x)$. Note that since x_s and the comoving horizon, respectively, scale as a^{-1} and $\sqrt{a}$ (in an Einstein–de Sitter universe), x_s enters the horizon at about $800\,h^{-1}$ Mpc if $r_s=50\,h^{-1}$ Mpc. Thus, all the modes with comoving wavelengths smaller than about 800 h^{-1} Mpc feel the modified law of gravity only when they are subhorizon. As a consequence, it is well justified for all the observable modes (i.e., up to $500\,h^{-1}$ Mpc) to consider the effect of the non-Newtonian gravity in the subhorizon regime only. For larger wavelengths (not relevant here but required for CMB calculation), a reformulation of the relativistic cosmo-

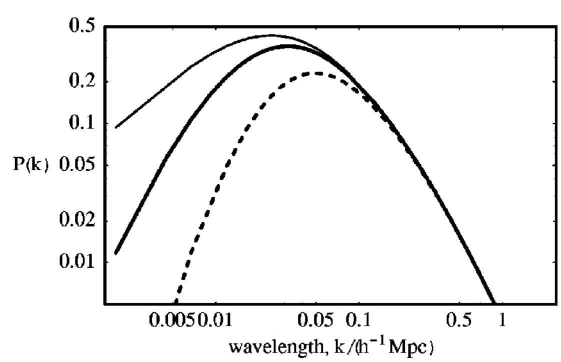

FIG. 3. Expected matter (thick line) power and gravitational potential Laplacian (dashed line) power spectra as functions of k compared to the standard cosmology case (thin line). We have assumed a CDM-like scenario (with $\Gamma=0.25$) and $r_s=50\,h^{-1}$ Mpc.

logical perturbation theory in the context of higher dimensional gravity would be needed.

Let us emphasize that, in Fig. 3, the deviation from the standard behavior of the matter power spectrum is model-dependent (it depends in particular on the cosmological parameters), but the discrepancy between the matter and gravitational potential Laplacian power spectra is a direct signature of a modified law of gravity. Note that the uncertainties in the source distribution $n(\chi_S)$ (that indeed could be quite large) would mainly affect the normalization of the measured power spectrum, not its shape. Biasing mechanisms (i.e., the fact that galaxies do not necessarily trace faithfully the matter field) cannot be a way to evade this test either since bias has been found to have no significant scale dependence at such scales [22].

Large-scale structure and gravitational lensing offer a new window for testing gravity, particularly the validity of the Poisson equation. Even if our method is more restricted in terms of tested length scales than a method based on CMB observation [14], it is worth stressing that comparison with CMB data involves many more parameters (cosmological parameters, initial power spectrum, etc.). Generically it is thus difficult to identify unambiguously the origin of a given feature in the CMB angular power spectrum (as an illustration, see the various propositions [23] to explain the ''low'' second acoustic peak of recent CMB data). The method proposed in this paper does not rely on a yet undetermined model of structure formation (and on an initial power spectrum) and obviously applies in a far more general context than the theoretical motivations from which models of higher dimensional gravity have emerged.

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