

Strong coupling limit of open string: Born-Infeld analysis

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We consider a large coupling limit of a Born-Infeld action in a curved background of an arbitrary metric and a two form field. Following Gibbons, Hori, and Yi, we go to the Hamiltonian description. The Hamiltonian can be dualized and the dual action admits a stringlike configuration as its solution. We interpret it as a *closed* string configuration. The procedure can be viewed as a novel way of bringing out the appropriate degrees of freedom, a closed string, for an open string under the strong coupling limit. We argue that this interpretation implies a large number of dual pairs of gauge and gravity theories whose particular examples are AdS/CFT and matrix theory conjectures.

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The fact that open strings have end points has given rise to many interesting results in the recent progress of string theory. Most notably it led to the discovery of D-branes [1]. It is plausible that there is yet to be discovered physics associated with the end points, some of which could be important. In the studies of D-branes it has been fruitful to consider extreme values, such as zero or infinity, of various parameters of the system under consideration. This is, for example, what one does in AdS/CFT correspondence [2–4], a large- N duality between gauge theory and gravity theory.

In this Rapid Communication we consider the dynamics of the end points of an open string by associating a “quark” pair with them. In particular we consider a strong coupling limit where the string coupling constant approaches *infinity*. Then it seems natural, at least at a naive level, to expect that the two end points get “stuck together,” which will in turn suggest a novel mechanism in which the original open string may be converted into a closed string. It is the aim of this paper to investigate this possibility and its implications.

Part of the motivation for this work came from [5] (see [6–8] for related discussions) where it was observed that the gauge-supergravity duality may in fact be deduced as a low energy limit of a “duality” between two different *stringy* descriptions of D-branes, open and closed. There, the “duality” was taken to be a starting point as an axiomatic assumption. Given that we have an open string description on one side and a closed string description on the other side, a conversion mechanism is likely to be relevant for the understanding of the relation between the two descriptions. Here we propose that the axiomatic duality should be associated with a strong-weak duality of an open string in the sense we will discuss in the latter part of this Rapid Communication.

In the first part, we are concerned with how one might see the conversion of open strings into closed strings. It would be ideal if this picture could be realized quantitatively at the level of full string theory. However, given the limitations of the full string theory techniques for a strongly coupled system, it will be easier to turn to the low energy effective action of open strings, a Born-Infeld (BI) action.

The simplicity gained by resorting to the effective action does not come without a price: one has to face the issue of justifying the results because, as is well known, a Born-

Infeld type action has its limitations (for reviews of a BI action, see [9,10]). We discuss some of these issues in due course.

If the physics of the strong coupling limits indeed converts open strings into closed strings, it should be a general phenomenon that could and should happen to a generic open string system. Long ago the authors of [11] considered a bosonic Born-Infeld action. (See [12,13] for more recent discussions of a strong coupling limit.) They argued that the Lagrangian in the strongly coupled limit admits a solution that provides a simple description for *closed* strings. The dynamical generation of closed strings has also appeared in [14–16] in the tachyonic context. (See also [17,18] and [19,20].) In particular it was the $U(1)$ confinement mechanism [21,22] that is responsible for the appearance of closed strings in the work of [14]. For our purpose it is intriguing to note that all the mathematical manipulations of [14] carry over when we replace the tachyon potential $V(T)$ by the usual tension τ . The limit $V \rightarrow 0$ can be viewed as to correspond to the strong coupling limit where the tension vanishes, $\tau \rightarrow 0$.

The fact that the manipulations remain the same (other than $V \rightarrow \tau$) strengthens our belief that a confinement mechanism should, in fact, be a general feature of open string systems, not restricted to a tachyonic system. Therefore one should be able to see the same feature for a BI action in a more general background such as a curved one or a background with a B field.¹ We will see that this is true. The necessary manipulation is a generalization of the steps presented in [14].

Although our calculations are limited to a low energy limit, we view the results as evidence for the “open-closed string duality” and study its implications. In particular we note that AdS/CFT may be the low-energy realization of the duality. We also argue that the duality may explain the matrix theory conjectures.

To generalize the calculations in Sec. 4 of [14], consider a BI Lagrangian in the presence of a metric $g_{\mu\nu}$ and a

¹Various BI actions in a curved background with a (non-)constant B field were previously considered in [23].

constant² two form field $B_{\mu\nu}$,

$$\mathcal{L} = -\tau\sqrt{-\text{Det}(g_{\mu\nu} + M_{\mu\nu})}, \quad (1)$$

where we have introduced a shorthand notation, $M_{\mu\nu} \equiv F_{\mu\nu} + B_{\mu\nu}$. For simplicity we impose the following conditions on the metric: $g_{0i} = 0 = g_{i0}$. The full discussion is presented in the Appendix. The determinant can be rewritten as

$$-\text{Det}(g_{\mu\nu} + M_{\mu\nu}) = -\text{Det}(h_{ik})g_{00} - M_i D^{ik} M_k \quad (2)$$

where $M_i \equiv M_{0i}$. Above we have introduced similar notation as that in [14], $h_{ij} = g_{ij} + M_{ij}$, $D^{ij} = (-1)^{i+j} \Delta^{ji}(h)$, $D = \text{Det}(h)h^{-1}$, where Δ represents the minor matrix of h . Since the B field is a background we consider the canonical momenta of E_i but not of M_{0i} . The Hamiltonian is then $\mathcal{H}_B = \pi^i E_i - \mathcal{L} = \pi^i M_i - \pi^i B_i - \mathcal{L}$, where $B_i \equiv B_{0i}$. The canonical momenta are

$$\pi^i = \frac{\delta \mathcal{L}}{\delta \dot{A}_i} = \frac{\tau}{\sqrt{-\text{Det}(g_{\mu\nu} + M_{\mu\nu})}} \frac{1}{2} (M_k D^{ki} + D^{ik} M_k). \quad (3)$$

After some algebra one can show

$$\begin{aligned} \mathcal{H}_B &= -\frac{\tau g_{00} \text{Det}(h)}{\sqrt{-\text{Det}(g_{\mu\nu} + M_{\mu\nu})}} - \pi^i B_i \\ &= \sqrt{-g_{00} \{ \pi^i g_{ij} \pi^j + [(F_{ij} + B_{ij}) \pi^j]^2 + \tau^2 \text{Det}(h) \}} - \pi^i B_i. \end{aligned}$$

In the strong coupling limit, one can drop the last term inside the square root. Performing a Legendre transformation [24] $\mathcal{L}'_B = \mathcal{H}_B - \frac{1}{2} F_{ij} K^{ij}$, with

$$K^{ij} = 2 \frac{\delta \mathcal{H}}{\delta F_{ij}} = -\frac{g_{00}}{\mathcal{H}} (M^i_k \pi^k \pi^i - M^j_k \pi^k \pi^j) \quad (4)$$

leads to

$$\begin{aligned} \mathcal{L}'_B &= \mathcal{H}_B - \frac{1}{2} M_{ij} K^{ij} + \frac{1}{2} B_{ij} K^{ij} \\ &= \sqrt{-g_{00} \pi^2 - K^{ij} K_{ij} / 2} - \pi^i B_i + \frac{1}{2} B_{ij} K^{ij}. \end{aligned} \quad (5)$$

Therefore Eq. (1) admits a compact dual description,

$$\mathcal{L}' = \sqrt{-\frac{1}{2} \mathcal{K}^{\mu\nu} \mathcal{K}_{\mu\nu} + \frac{1}{2} \mathcal{B}_{\mu\nu} \mathcal{K}^{\mu\nu}}, \quad (6)$$

where

$$\mathcal{K} = -g_{00} \pi_i dt \wedge dx^i + \frac{1}{2} K_{ij} dx^i dx^j. \quad (7)$$

The same equations that were satisfied by \mathcal{K} in the flat case [14] are also satisfied here: from the definition of \mathcal{K} it is easy to show that it satisfies a constraint $\mathcal{K} \wedge \mathcal{K} = 0$. The Bianchi

²Or one could consider a nonconstant, *closed* two form field as in [23].

identity, $dF = 0$, is now translated into the equation of motion of \mathcal{K} ,

$$0 = d(\mathcal{K} / \sqrt{-\mathcal{K}^2/2}). \quad (8)$$

There is another constraint equation that \mathcal{K} must satisfy: $\partial_\mu \mathcal{K}^{\mu\nu} = 0$. This corresponds to the equation of motion and the Gauss constraint of the original description.³ With these conditions one can write down the following solution:

$$\mathcal{K}^{\mu\nu} = \int \delta(X - Z(\sigma)) dZ^\mu \wedge dZ^\nu. \quad (9)$$

Substitution of Eq. (9) into Eq. (6) gives a Nambu-Goto type action in the same background,

$$\begin{aligned} S &= \int d^2 \sigma \sqrt{\text{Det}(\partial_a X^\mu \partial_b X^\nu g_{\mu\nu})} \\ &+ \frac{1}{2} \int d^2 \sigma \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu B_{\mu\nu}. \end{aligned} \quad (10)$$

We view Eq. (10) as a *closed* string action.

We have considered a Born-Infeld action and its strong coupling limit. Following the literature, we went to the Hamiltonian formulation to study the physics of the strong coupling limit. The Hamiltonian can be dualized to yield an action that allows a connection to a Nambu-Goto type string action, Eq. (10). We interpret this as a low-energy realization of the conversion of an open string into a closed string.

As we discussed in the Introduction, the transition of an open string into a closed string should be a general phenomenon. Therefore it should be possible to extend the results to supersymmetric cases.⁴ We will not pursue this issue here but will simply assume that such an extension exists. For this reason and others that will follow we mostly concentrate on supersymmetric cases below.

Since the discussions have been kept to the level of a low-energy effective action, there are various limitations to the claims one can make based on the results. For example, the solution (9) should not be viewed as to represent all of the stringy configurations of closed strings including the massive modes. To be able to make such a statement (or a similar one), one would probably have to consider a BI type action in a background that contains all the massive closed string fields, whose proper discussion would require a full string theory or string field theory. Rather one should view the procedure as a novel and effective way of bringing out a closed string as appropriate degrees of freedom for a *massless* open string in the extreme coupling.

However, this interpretation still faces criticism that the action of our starting point, Eq. (1), is incomplete because it

³The equation of motion (8) admits a scaling symmetry $\mathcal{K} \rightarrow f\mathcal{K}$, $\mu \rightarrow \mu/f$ where f is an arbitrary function. f should be a constant for the similar reason discussed in [11].

⁴For that, it will be of great use to employ superfield machinery such as the techniques of a nonlinear realization of supersymmetry [25–28] or the superembedding formulation [29,30].

does not contain certain higher derivative terms and therefore it could be used only for slowly varying configurations. Although more complete resolution of this problem would have to wait until the arrival of a superspace formulation of a D-brane action through a partial breaking of supersymmetry, it might be useful to recall that there could be a field redefinition that removes some (but not all) of the higher derivative terms. An example of a field redefinition that removes certain higher derivative terms appeared in the discussion of a 3-brane action in [27]. Another example is [31] where a field redefinition is introduced for a comparison of a four-dimensional super Yang-Mills action and a BI action. After such a field redefinition, if necessary, the resulting action might still allow a connection to a string configuration. It is also relevant to note that the equation $\partial_\mu \mathcal{K}^{\mu\nu} = 0$ does not depend on the detailed form of the action. It is the Bianchi identity of the dual field,⁵ $G = *\mathcal{K}$, and the dual field will satisfy the Bianchi identity irrespective of the detailed form of the original action.

There is another issue worth discussing. The picture seems to be contradictory to the fact that we have open string *boundary conditions* to start with. In other words, the open strings might remain as open strings even under the strong coupling limit since their attachments to the branes are realized as the boundary conditions. The resolution of this puzzle might come from the fact that in general, the boundary conditions must be consistent with the given background. This apparently innocuous statement has not been much appreciated, partially because we are more used to a flat background where no moduli parameters take extreme values such as infinity. In such backgrounds, one can impose Neumann or periodic boundary conditions without any obstacle. However, a more general background with some of its moduli parameters taking extreme values may restrict the choice of boundary conditions. After all, boundary conditions themselves should be considered as a part of the background and as such they should not contradict the rest of the data of the background. There is a familiar background that can provide a concrete example: consider open strings in a flat background with a constant B field. The boundary condition is $g_{\mu\nu} \partial_n X^\nu + 2\pi\alpha' B_{\mu\nu} \partial_t X^\nu|_{\text{bd}} = 0$. One *cannot* take $B \rightarrow \infty$ imposing the Neumann boundary condition at the same time. With these discussions we will assume that the conversion is true at the level of the low-energy effective action. Furthermore we will assume that it will remain true at the level of the full string discussion. We now turn to its implications.

Recently it has been shown [32,33] that closed strings can be decoupled from open strings in a background where the background electric field approaches its critical value. One of the lessons of these works is that the conventional lore that open strings need closed strings is in fact a background-dependent statement. One may take one step further and consider a general construction of open-closed string theory with open string fields only. (Closed strings *without* explicit closed string fields have been discussed in the past [34–37].)

⁵By the *dual* field we mean \mathcal{K} or G (which of the two should be clear from the context), while the original field refers to F .

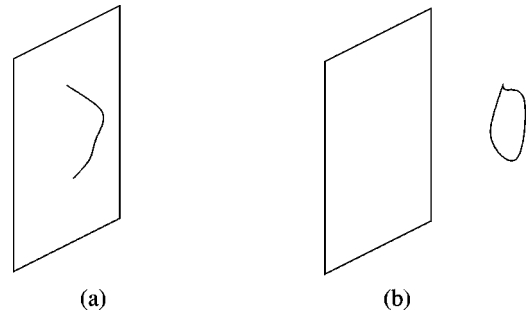


FIG. 1. (a) open string description; (b) closed string description.

In other words, instead of putting closed strings explicitly in the kinematic setup one may start only with open string world sheet Lagrangian.

The reason for considering such a kinematic setup is that it seems better suited for the possible proof of the duality between an open string description and a closed string description. Let us start with an open string theory with a very *small* coupling constant. The next step is to apply S duality. At the level of a low-energy effective action it is a well known operation, as we discussed in the previous section. After S -dualizing the system one can employ the argument that the appropriate degrees of freedom are now those of a closed string. Therefore, if true, the conversion will lead to a very general concept of duality between the two descriptions (which in turn reduces to low-energy duality between field theories and gravity theories). The dual closed string description will be *strongly* coupled: the duality under consideration is different from the familiar world sheet open-closed duality because the latter is considered in the usual kinematic setup and furthermore in the dual channel the description is still *weakly* coupled. It is amusing to note that, as proposed in [5], this picture implies that the geometry in which the dual closed string propagates is the same⁶ (but with strong coupling) as the one for the original open string. (See Fig. 1.)

We are ready to discuss how the conversion “derives” the AdS/CFT duality. AdS/CFT can be motivated by taking a viewpoint that there are two different but dual stringy descriptions of the same objects, D-branes. One description is via open strings with mixed Dirichlet and Neumann boundary conditions. In the other description one considers type-IIB closed string theory expanded around the D-brane soliton solutions.⁷ Upon taking a low-energy near horizon limit, the viewpoint leads, e.g., in the case of D3 branes, to the duality between $\mathcal{N}=4$, $D=4$ Super Yang-Mills (SYM) theory and $\mathcal{N}=8$, $D=5$ gauged supergravity. To make the discussion slightly more general, consider an open string attached to an arbitrary odd-dimensional brane. The open string propagates in the curved background produced by the presence of the

⁶The relevance of curved backgrounds for a BI action was discussed in [38,39,5,40,41] in connection with AdS/CFT.

⁷As discussed above, it should not be confused with the familiar channel duality. In [5] it was called fundamental-solitonic duality to avoid confusion.

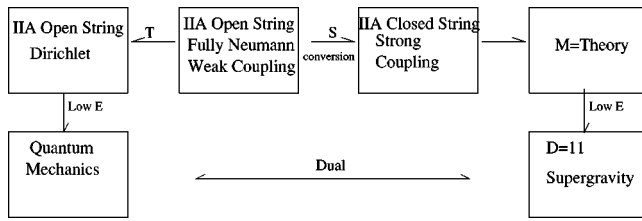


FIG. 2. Duality chain for BFSS matrix theory.

brane. We start with a weakly coupled open string theory in a pure open string formulation⁸ and go to a strong coupling limit by S duality. After the duality the appropriate degrees of freedom are a closed string in the same background. The closed string should be of type IIB and strongly coupled. In the case of a D3 brane, the resulting closed string is in the D3 brane background, but can be considered as *weakly* coupled due to the $SL(2, Z)$ self-duality [42]: the weakly coupled IIB closed string is connected by a chain of dualities to the starting point, a weakly coupled open string. In the low energy this leads to the duality between $\mathcal{N}=4$ SYM theory and IIB supergravity on $AdS_5 \times S^5$. Similarly, starting with an open string in the background of D2/D4⁹ branes we will get AdS/CFT conjecture concerning $AdS_{4,7} \times S^{7/4}$. (See Fig. 2.)

One can give similar arguments for matrix theories. Consider an open superstring with the fermionic coordinates of opposite chiralities. Take fully Neumann boundary conditions. Apply the conversion procedure, i.e., start with a very weak coupling and consider S duality to go to a strong coupling limit. The closed string that appears will be of type IIA. In particular it will be strongly coupled. The strong coupling limit of IIA is M theory [44]. On the other hand we can T -dualize the original open string to an open string with Dirichlet boundary conditions for the nine space dimensions. Therefore, one end of this operation is an open string with Dirichlet boundary conditions and the other end is M theory. Once we go to a low-energy limit, the two theories respectively reduce to quantum mechanics and eleven-dimensional supergravity: we have the matrix theory conjecture of M theory [45]. On the other hand, if we start with an open string whose fermionic coordinates have the *same* chiralities, we will get the Ishibashi-Kawai-Kitazawa-Tsuchiya matrix theory conjecture [46].

⁸Or one may consider a decoupling limit where the asymptotic closed strings decouple. The limit results from a slight modification of the familiar scaling limits of the moduli parameters. Consider open strings with an extremely weak coupling. With a vanishing coupling constant the asymptotic closed strings will be decoupled. One then takes a large- N limit such that $g_s N$ becomes fixed. However, $g_s N$ is taken to be *small* to avoid suppressing massive stringy excitations. The condition $g_s \rightarrow 0$ suppresses the dynamical generation of closed strings, which otherwise would propagate off the branes. However, the open string theory is still an interacting theory on the world volume because of the nonvanishing value of $g_s N$, which is the effective coupling of the open strings on the world volume.

⁹The relevance of the world volume theories of D2/D4 branes was discussed in [43].

AdS/CFT conjecture was motivated in [5] by starting from an axiomatic assumption that there are two dual descriptions of D-branes, open and closed. We have argued here that the duality may originate from the conversion of an open string into a closed string under a large coupling limit where the coupling constant approaches infinity. For that, we considered a (*nontachyonic*) generalization of the analysis of [14] to a curved background with a constant B field. Although we studied only a bosonic Born-Infeld action we believe that a similar analysis should be possible for a supersymmetric case, which is an interesting open problem. We noted that the duality under consideration implies, in low-energy limits, very general dual relations between gauge theories and gravity theories. In particular, it seems that AdS/CFT and matrix theory conjectures come from the same root, the conversion of an open string into a closed string under a strong coupling limit. It will be interesting to promote the discussions to a full string theory analysis.

Note added. After this work was published, a paper [47] appeared which has partially related discussions.

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APPENDIX

Earlier we imposed $g_{0i} = 0$ for simplicity. Here we relax the condition. Since the calculations for the B part remain the same we will not consider them. With $g_{0i} \neq 0$ the determinant can be rewritten as

$$-\text{Det}(g_{\mu\nu} + F_{\mu\nu}) = -g_{00} \text{Det}(h_{ik}) - E_i^{(+)} D^{ik} E_k^{(-)}, \quad (\text{A1})$$

where

$$E_i^{\pm} = E_i \pm g_{0i}, \quad h_{ij} = g_{ij} + F_{ij}, \\ D^{ij} = (-1)^{i+j} \Delta^{ji}(h), \quad D = \text{Det}(h) h^{-1}, \quad (\text{A2})$$

where Δ represents the minor matrix of h . The canonical momenta are

$$\pi^i = \frac{\tau}{\sqrt{-\text{Det}(g_{\mu\nu} + F_{\mu\nu})}} \frac{1}{2} (E_k^{(+)} D^{ki} + D^{ki} E_k^{(-)}). \quad (\text{A3})$$

After some algebra one can show that

$$\mathcal{H} = \frac{\tau}{\sqrt{-\text{Det}(g_{\mu\nu} + F_{\mu\nu})}} \left[-\text{Det}(h) g_{00} + \frac{1}{2} g_{0i} (E_k^{(+)} D^{ki} - D^{ik} E_k^{(-)}) \right]. \quad (\text{A4})$$

The Hamiltonian in terms of the canonical variables can be obtained by solving the following equation that it satisfies:

$$\mathcal{H}^2 - 2g_{0i} u^i F_{ij} \pi^j \mathcal{H} + \frac{1}{G_{00}} [\pi^i g_{ij} \pi^j + (F_{ij} \pi^j)^2] = 0, \quad (\text{A5})$$

where $(F_{ij}\pi^j)^2 \equiv g^{ik}F_{ij}\pi^j F_{km}\pi^m$ and τ has been set to zero. We have also introduced u^{ij} , the inverse matrix of g_{ij} . In general, u^{ij} is not the same as g^{ij} the (ij) th component of $g^{\mu\nu}$ although that was true in the case we considered earlier, i.e., in the case of $g_{i0}=0$. As before we perform a Legendre transformation with

$$K^{ij} = - \frac{g_{0l}u^{li}\pi^j - g_{0l}u^{lj}\pi^i + \frac{2}{G^{00}}(F_l^i\pi^l\pi^j - F_l^j\pi^l\pi^i)}{2\mathcal{H} - 2g_{0l}u^{li}F_{ij}\pi^j} \quad (\text{A6})$$

where $F_l^i \equiv G^{ik}F_{kl}$. It is straightforward to show

$$\mathcal{L}' = - \frac{2}{G^{00}} \frac{\pi^i g_{ij} \pi^j}{2\mathcal{H} - 2g_{0l}u^{li}F_{ij}\pi^j}. \quad (\text{A7})$$

Using the same definitions for the components of $\mathcal{K}^{\mu\nu}$, i.e.,

$$\mathcal{K}^{0i} = -\pi^i, \quad \mathcal{K}^{ij} = K^{ij}, \quad (\text{A8})$$

one can show, after rather lengthy algebra, that

$$\frac{1}{2}\mathcal{K}^{\mu\nu}\mathcal{K}_{\mu\nu} = - \left(\frac{2}{G^{00}} \right)^2 \frac{(\pi^i g_{ij} \pi^j)^2}{(2\mathcal{H} - 2g_{0l}u^{li}F_{ij}\pi^j)^2}. \quad (\text{A9})$$

Therefore the dual Lagrangian has the same form as before,

$$\mathcal{L}' = \sqrt{-\frac{1}{2}\mathcal{K}^{\mu\nu}\mathcal{K}_{\mu\nu}} \quad (\text{A10})$$

and the discussions below Eq. (7) remain the same.

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