Neutrino oscillations and neutrinoless double beta decay

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The relation between neutrino oscillation parameters and neutrinoless double beta decay is studied, assuming normal and inverse hierarchies for Majorana neutrino masses. For a normal hierarchy the crucial dependence on U_{e3} is explored. The link with tritium beta decay is also briefly discussed.

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There is now convincing evidence for neutrino masses and lepton mixings from oscillation experiments. Neutrino masses can be either of the Dirac type or of the Majorana type. In the case of Majorana masses the neutrinoless double beta decay ($0\nu\beta\beta$ decay) is allowed [1]. Such a decay has not yet been observed and only an upper limit on the related mass parameter M_{ee} (to be defined below) is available, $M_{ee}<0.2$ eV [2] (in this paper M_{ee} is always expressed in eV). Several experiments have been proposed to lower this limit by one or even two orders and eventually discover the $0\nu\beta\beta$ decay, thus revealing the Majorana nature of neutrinos. In particular, GENIUS I (1 t) will test M_{ee} down to 2×10^{-2} and GENIUS II (10 t) down to 2×10^{-3}. Therefore, the subject of the relation between oscillation parameters and M_{ee} has been studied by many authors [3–5]. Here we turn to the question in order to clarify the link between the lepton mixings and the predictions for M_{ee}.

In fact, the random extraction of the relevant neutrino parameters is very useful in this case. In particular, we will see that for $U_{e3}\lesssim0.05$ the four solutions to the solar neutrino problem may give quite different predictions for the mass parameter M_{ee}. Since also the bound $U_{e3}<0.2$ [6] is expected to be lowered in the future, phenomenological relations between M_{ee} and U_{e3} are welcome. We consider normal and inverse hierarchies for the Majorana masses of three active neutrinos. Recent evidence for cosmological dark energy eliminates most of the motivations for considering the degenerate spectrum, which was relevant before for hot dark matter (see for example [7]).

Let us now define the mass parameter M_{ee} as

$$M_{ee}=|U_{e1}^2 e^{2i\alpha}m_1+U_{e2}^2 e^{2i\beta}m_2+U_{e3}^2 m_3|, \tag{1}$$

where U_{ei} $(i=1,2,3)$ are the moduli of the elements in the first row of the lepton mixing matrix U. This matrix can be parametrized as the standard form of the Cabibbo-Kobayashi-Maskawa (CKM) matrix (with one phase $-\delta$ in entry 1-3) times diag$(e^{i\varphi_1},e^{i\varphi_2},1)$. Two relative phases $\alpha=\varphi_1+\delta$ and $\beta=\varphi_2+\delta$ appear in M_{ee}. Moreover, m_i are positive Majorana masses. From Ref. [8] we get

$$U_{e2}^2=(\sin^2\theta_s)(1-U_{e3}^2), \tag{2}$$

where θ_s is the solar neutrino mixing angle, and due to the unitarity of U we have

$$U_{e1}^2=1-U_{e2}^2-U_{e3}^2=(\cos^2\theta_s)(1-U_{e3}^2). \tag{3}$$

Therefore, neglecting U_{e3}^2 with respect to 1, Eq. (1) can be written as

$$M_{ee}=|(\cos^2\theta_s)e^{2i\alpha}m_1+(\sin^2\theta_s)e^{2i\beta}m_2+U_{e3}^2 m_3|. \tag{4}$$

In this way M_{ee} depends on seven neutrino parameters. As said above the mixing U_{e3} is bounded (by the CHOOZ experiment),

$$U_{e3}<0.2. \tag{5}$$

In order to determine the masses m_i we have to distinguish between the normal mass hierarchy, $m_1\ll m_2\ll m_3$, and the inverse mass hierarchy, $m_1\simeq m_2\gg m_3$. In the normal hierarchy case

$$m_3=\sqrt{\Delta m_a^2+m_1^2}, \tag{6}$$

$$m_2=\sqrt{\Delta m_s^2+m_1^2}, \tag{7}$$

and for m_1 we take $10^{-5}\sqrt{\Delta m_s^2}<m_1<10^{-1}\sqrt{\Delta m_s^2}$, with $\Delta m_a^2=(1-6)\times10^{-3}$ eV^2 for atmospheric neutrinos, and Δm_s^2 reported in Table I (in eV^2) together with $\sin\theta_s$ for solar neutrinos. These values come from Ref. [9]. LMA, SMA, LOW are the large mixing angle, small mixing angle, low mass matter Mikheyev-Smirnov-Wolfenstein (MSW) solutions, and VO is the vacuum solution. The best global fit of solar neutrino data is given by the LMA solution, although the other solutions are not ruled out [7]. The value $\sin\theta_s=0.71$ $(\theta_s=\pi/4)$ means maximal mixing. For inverse hierarchy one has $m_1\simeq m_2\simeq\sqrt{\Delta m_a^2}$.

Let us consider first the normal hierarchy. We extract random values for the parameters $\sin\theta_s$, Δm_s^2, Δm_a^2, and m_1, within the ranges reported in Table I and above. Then also m_3 and m_2 are obtained through Eqs. (6),(7). The mixing U_{e3} is extracted according to Eq. (5) and phases cover the full range $[0,\pi]$. In total we take random values for seven parameters, so that the determination of M_{ee} from Eq. (4) is achieved. The results of the calculation (2500 points extracted) are in Figs. 1 and 2, where we plot $\log_{10}M_{ee}$ versus U_{e3}. For $U_{e3}>0.1$ the SMA, LOW and VO solutions give similar values for M_{ee}, while the LMA solution provides

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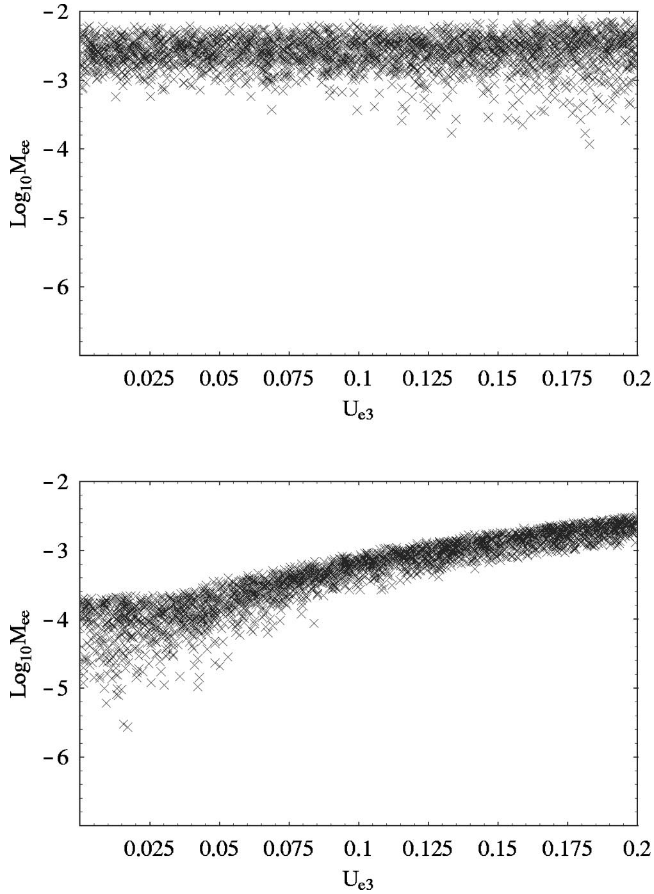

FIG. 1. $\mathrm{Log}_{10}M_{ee}$ vs U_{e3} for the LMA and SMA solutions with the normal hierarchy.

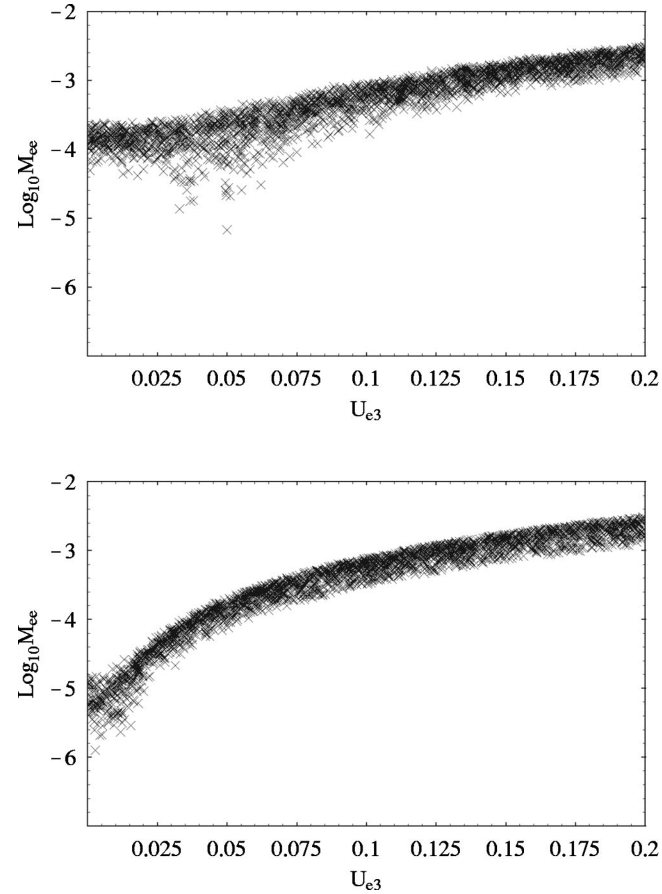

FIG. 2. $\mathrm{Log}_{10}M_{ee}$ vs U_{e3} for the LOW and VO solutions with the normal hierarchy.

also higher values. However, the LMA solution gives M_{ee} almost constant, because the m_3-term is negligible even for $U_{e3}\simeq 0.2$, while for the other solutions M_{ee} decreases for smaller U_{e3}, till the m_3-term becomes negligible. In particular, for $U_{e3}\lesssim 0.05$ the LMA solution is clearly distinguished from the SMA solution (and also from the others). A similar behavior happens for the LOW solution with respect to the VO solution, for $U_{e3}\lesssim 0.02$. In order to clarify this aspect we have checked the results on a linear plot. In Fig. 3 we report the lower LMA bound and the upper SMA bound as well as the lower LOW bound and the upper VO bound for M_{ee}. The lower LMA bound can be obtained from the expression $M_{ee}\simeq m_2 \sin^2\theta_s - m_3 U_{e3}^2$ and the upper SMA bound from $M_{ee}\simeq m_1 + m_3 U_{e3}^2$. In a similar way, the lower LOW bound can be obtained from $M_{ee}\simeq m_2 \sin^2\theta_s - m_3 U_{e3}^2$ and the upper VO bound from $M_{ee}\simeq m_2 \sin^2\theta_s + m_3 U_{e3}^2$. We can thus predict, for $U_{e3}\lesssim 0.05$, that $4\times 10^{-4} < M_{ee} < 8\times 10^{-3}$ for the LMA solution, while $M_{ee} < 4\times 10^{-4}$ for the SMA solution (and also the LOW solution). For $U_{e3}\lesssim 0.01$ we get $3\times 10^{-5} < M_{ee} < 4\times 10^{-4}$ for the LOW solution, while $M_{ee} < 3\times 10^{-5}$ for the VO solution. We now comment about the cancellations appearing in Figs. 1 and 2. They are obtained when the m_2-term and/or the m_1-term are comparable with the m_3-term. Of course, this happens for different U_{e3} values (and the relevant phase tuned around $\pi/2$), according to the different solar solutions. Note also that in the SMA case the m_1-term can easily exceed the m_2-term. In the other cases, for $U_{e3}\simeq 0$, we have $M_{ee}\simeq(\sin^2\theta_s)m_2 \simeq(\sin^2\theta_s)\sqrt{\Delta m_s^2}$.

For the inverse hierarchy two main results can be drawn out. One is with respect to the normal hierarchy, namely in the region $10^{-2} < M_{ee} < 10^{-1}$ only the inverse hierarchy is possible, while $M_{ee} < 10^{-2}$ for the normal hierarchy. The other result is that the SMA solution gives clean bounds, $3\times 10^{-2} < M_{ee} < 8\times 10^{-2}$. All solutions have the same upper bound $M_{ee} < 8\times 10^{-2}$, not depending on U_{e3}, which is

TABLE I. Neutrino oscillation parameters.

	LMA	SMA	LOW	VO
Δm_s^2	$(0.15-1.5)\times 10^{-4}$	$(0.4-1)\times 10^{-5}$	$(0.3-2.5)\times 10^{-7}$	$(0.3-10)\times 10^{-10}$
$\sin\theta_s$	$0.40-0.71$	$0.02-0.05$	$0.53-0.71$	$0.43-0.71$

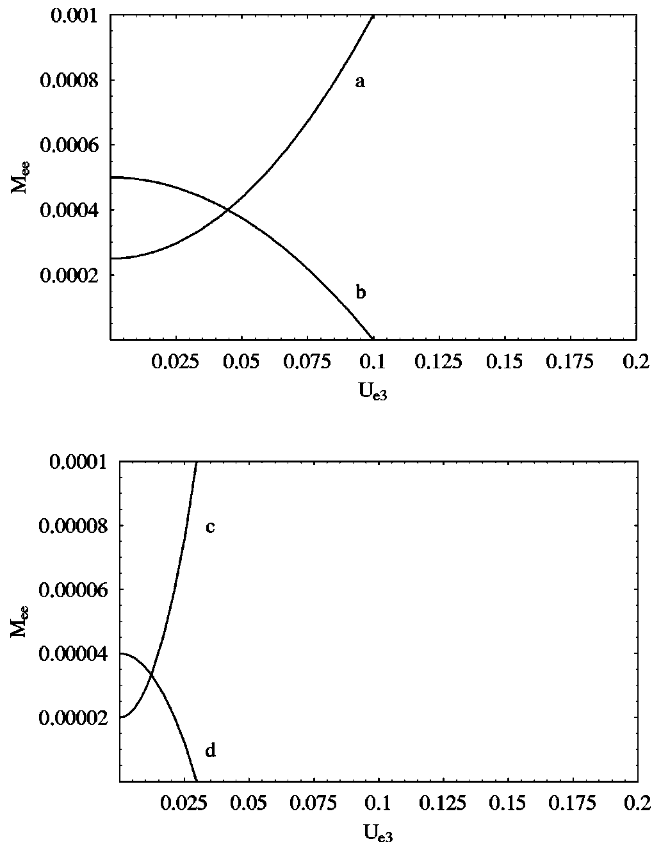

FIG. 3. Upper SMA bound (a) and lower LMA bound (b) as well as upper VO bound (c) and lower LOW bound (d) for M_{ee} with the normal hierarchy.

easily understood since the m_3-term is negligible for inverse hierarchy. The basic features of the inverse hierarchy case can be obtained by using the approximation

$$M_{ee} \simeq (\cos^2\theta_s \pm \sin^2\theta_s) m_{1,2} \tag{8}$$

in the CP-conserving case ($\alpha=0,\ \beta=0,\pi/2$). In fact, the plus sign ($\beta=0$) gives $M_{ee}\simeq m_{1,2}\simeq\sqrt{\Delta m_a^2}$, while the minus sign ($\beta=\pi/2$) gives $M_{ee}\simeq\sqrt{\Delta m_a^2}\cos 2\theta_s$. For small mixing $\theta_s\simeq 0$ one has $M_{ee}\simeq\sqrt{\Delta m_a^2}$, while for large mixing $\theta_s \simeq \pi/4$ the value $M_{ee}\simeq 0$ is allowed by cancellations, so that the full range $0\leqslant M_{ee}\leqslant\sqrt{\Delta m_a^2}$ is covered. Of course, if maximal mixing is excluded, a lower bound appears also for the LMA, LOW and VO solutions.

Now we discuss the mass parameter m_β, related to tritium beta decay, which is defined as

$$m_\beta^2 = U_{e1}^2 m_1^2 + U_{e2}^2 m_2^2 + U_{e3}^2 m_3^2. \tag{9}$$

Using Eqs. (2),(3) and neglecting again U_{e3}^2 with respect to 1, we obtain

$$m_\beta^2 = (\cos^2\theta_s) m_1^2 + (\sin^2\theta_s) m_2^2 + U_{e3}^2 m_3^2. \tag{10}$$

There are no cancellations for m_β, so that it is sufficient to evaluate M_{ee} in $U_{e3}\simeq 0$ and $U_{e3}\simeq 0.2$. In the normal hierarchy case, for $U_{e3}\simeq 0.2$ we get $m_\beta\simeq U_{e3}m_3\simeq U_{e3}\sqrt{\Delta m_a^2}$. For $U_{e3}\simeq 0$ the SMA solution gives $m_\beta\simeq m_1\ll\sqrt{\Delta m_s^2}$, while the other solutions give $m_\beta\simeq(\sin\theta_s)m_2\simeq(\sin\theta_s)\sqrt{\Delta m_s^2}$. For inverse hierarchy all solutions give $m_\beta\simeq m_{1,2}\simeq\sqrt{\Delta m_a^2}$, not depending on U_{e3}. The experimental limit on m_β is now $m_\beta < 2.2$ eV (see [10]) and it is hard to lower this limit by one order. However, the maximum value allowed by the previous discussion is $m_\beta\simeq 8\times 10^{-2}$ eV, so that the impact of neutrino oscillations on the prediction for m_β cannot be checked.

In conclusion, we have studied the prediction for M_{ee} obtained by varying neutrino parameters, within the experimental ranges, for the normal and inverse mass hierarchy cases. For normal hierarchy the main result is that for $U_{e3} \lesssim 0.05$ the LMA solution is clearly distinguished from the other solutions. Moreover, for $U_{e3}\lesssim 0.01$ the LOW solution is distinguished from the VO solution. This means that if the LMA solution is confirmed, and even if U_{e3} is very small (similar to V_{cb} or V_{ub}), the GENIUS II (10 t) experiment should find the $0\nu\beta\beta$ decay unless neutrinos are Dirac particles. Instead, if another solution is confirmed and $U_{e3} \lesssim 0.1$, then the GENIUS project will not be able to decide about the neutrino nature. For inverse hierarchy M_{ee} could be higher by one order, with respect to normal hierarchy, and the $0\nu\beta\beta$ decay be possibly found also by the GENIUS I (1 t) and MOON experiments.

In this paper we have taken $0<\theta_s\leqslant\pi/4$. However, for LOW and VO solutions, part of the range $\pi/4<\theta_s<\pi/2$ (the so-called dark side of neutrino parameter space [11]) is allowed (see for example [12]), so that for normal hierarchy the related regions in M_{ee} can overlap also for $U_{e3}\simeq 0$. Of course, progress in the determination of neutrino oscillation parameters will sharpen the predictions on M_{ee} for both hierarchies.

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