CPT, *T*, and Lorentz violation in neutral-meson oscillations

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Tests of CPT and Lorentz symmetry using neutral-meson oscillations are studied within a formalism that allows for indirect CPT and T violation of arbitrary size and is independent of phase conventions. The analysis is particularly appropriate for studies of CPT and T violation in oscillations of the heavy neutral mesons D, B_d, and B_s. The general Lorentz- and CPT-breaking standard-model extension is used to derive an expression for the parameter for CPT violation. It varies in a prescribed way with the magnitude and orientation of the meson momentum and consequently also with sidereal time. Decay probabilities are presented for both uncorrelated and correlated mesons, and some implications for experiments are discussed.

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I. INTRODUCTION

The original discovery of CP violation in the neutral-kaon system [1] has led to numerous theoretical and experimental studies of discrete symmetries in neutral-meson oscillations [2]. Much of the effort has been focused on the K system, but the advent of high-statistics experiments involving the heavy neutral mesons, in particular the B_d mesons [3], has opened the door for a broader class of investigations.

In a neutral-meson system, the violation of CP symmetry includes the possibility of CPT violation [4,5]. For the K system, CPT violation in oscillations can be parametrized by a complex quantity δ_K that is known to be small or zero [6]. Under the *ad hoc* assumption that δ_K is a constant complex number, experiments have established that its real and imaginary parts are no greater than about 10^{-4} [7,8].

The assumption of constant nonzero δ_K is known to fail in conventional quantum field theory. The nature of δ_K is determined by the properties of the theory under Lorentz transformations. For any realistic Lorentz-invariant quantum field theory such as the standard model, the CPT theorem shows that δ_K must be zero [4]. If instead Lorentz violation is allowed, then using an explicit and general standard-model extension [9] to calculate δ_K reveals that it varies with the meson 4-momentum [10,11]. This variation has recently been exploited by the KTeV Collaboration in placing a qualitatively new bound on CPT violation in the neutral-K system [12].

For systems involving the heavy mesons D, B_d, B_s several CPT tests have been proposed [13–15], and bounds have been obtained in some recent experiments with the B_d system [16]. All these results rely on the assumption of a nonzero constant complex parameter for CPT violation. However, as in the K system, this assumption fails in realistic quantum field theories: either the parameter vanishes by the CPT theorem, or it depends on the 4-momentum of the meson.

The present work provides a general treatment of CPT violation in neutral-meson oscillations in the context of quantum field theory allowing for Lorentz violation. A convenient formalism is adopted that is independent of phase conventions and allows for CPT and T violation of arbitrary size in any neutral-meson system. The complex parameter for CPT violation is calculated in the general Lorentz-violating standard-model extension, revealing a well-defined variation with the magnitude and orientation of the meson momentum and a corresponding variation with sidereal time. Some experimentally relevant decay probabilities and asymmetries are derived for both uncorrelated and correlated mesons. The results obtained here complement the analyses in earlier works, which described some essential physics [10] and obtained expressions valid for small CPT violation in the K, D, B_d, and B_s systems [11].

Section II provides background information and fixes some notational conventions. A suitable parametrization of the effective Hamiltonian for the time evolution of a neutral-meson state with CPT and T violation of arbitrary size is presented in Sec. III. The calculation of the complex parameter for CPT violation is given in Sec. IV. Implications for experiment are considered in Sec. V. The Appendix contains a brief description of other formalisms adopted in the literature. Throughout this work, a strong-interaction eigenstate is denoted generically by P^0, where P^0 is one of K^0, D^0, B_d^0, B_s^0, and the corresponding opposite-flavor antiparticle is denoted $\overline{P^0}$.

II. BASICS

An arbitrary neutral-meson state is a linear combination of the Schrödinger wave functions for the meson P^0 and its antimeson $\overline{P^0}$. This combination can be represented as a two-component object $\Psi(t)$, with time evolution governed by a 2×2 effective Hamiltonian Λ according to the Schrödinger-type equation [6]

$$i\partial_t\Psi = \Lambda\Psi. \tag{1}$$

Throughout this paper, subscripts P are understood on Ψ, on the components of the effective Hamiltonian Λ, and on related quantities such as meson masses and lifetimes.

The physical propagating states are the eigenstates of Λ, analogous to the normal modes of a classical two-dimensional oscillator [17]. In this work, these states are generically denoted as $|P_a\rangle$ and $|P_b\rangle$. They evolve in time as

$$|P_a(t)\rangle = \exp(-i\lambda_a t)|P_a\rangle,$$

$$|P_b(t)\rangle = \exp(-i\lambda_b t)|P_b\rangle. \quad (2)$$

The complex parameters λ_a, λ_b are the eigenvalues of Λ. They can be decomposed as

$$\lambda_a \equiv m_a - \tfrac{1}{2} i\gamma_a, \quad \lambda_b \equiv m_b - \tfrac{1}{2} i\gamma_b, \quad (3)$$

where m_a, m_b are the propagating masses and γ_a, γ_b are the associated decay rates. For the K system, contact with the standard notation can be made via the identification $m_a = m_S$, $m_b = m_L$, $\gamma_a = \gamma_S$, $\gamma_b = \gamma_L$. For the D system, there is no well established convention and I use the notation in Eq. (3). For the B_d and B_s systems, the relation to the standard notation can be taken as $m_a = m_L$, $m_b = m_H$, $\gamma_a = \Gamma_L$, $\gamma_b = \Gamma_H$.

For calculational purposes, it is useful to introduce a separate notation for the sums and differences of these parameters:

$$\lambda \equiv \lambda_a + \lambda_b = m - \tfrac{1}{2} i\gamma,$$

$$\Delta\lambda \equiv \lambda_a - \lambda_b = -\Delta m - \tfrac{1}{2} i\Delta\gamma, \quad (4)$$

where $m = m_a + m_b$, $\Delta m = m_b - m_a$, $\gamma = \gamma_a + \gamma_b$, $\Delta\gamma = \gamma_a - \gamma_b$. Note in particular the choice of sign in the definition of $\Delta\gamma$, which coincides with that in the K system but is the negative of the quantity $\Delta\Gamma$ often adopted in the B_d system. The reader can therefore make direct contact with results in the latter convention by identifying $\Delta\gamma \equiv -\Delta\Gamma$ in any equation in this work.

The off-diagonal components of Λ control the flavor oscillations between P^0 and $\overline{P^0}$. Indirect CPT violation occurs if and only if the difference of diagonal elements of Λ is nonzero, $\Lambda_{11} - \Lambda_{22} \neq 0$. Indirect T violation occurs if and only if the magnitude of the ratio of off-diagonal components of Λ differs from 1, $|\Lambda_{21}/\Lambda_{12}| \neq 1$.

A priori, the effective Hamiltonian Λ can be parametrized by eight independent real quantities. Four of these can be specified in terms of the masses and decay rates, two describe CPT violation, and one describes T violation. The remaining parameter, determined by the relative phase between the off-diagonal components of Λ, is physically irrelevant. It can be dialed at will by rotating the phases of the P^0 and $\overline{P^0}$ wave functions by equal and opposite amounts. The freedom to perform such rotations exists because the wave functions are eigenstates of the strong interactions, which preserve strangeness, charm, and beauty. Under a rotation of this type involving a phase factor of $\exp(i\chi)$ for the P^0 wave function, the off-diagonal elements of Λ are multiplied by equal and opposite phases, becoming $\exp(2i\chi)\Lambda_{12}$ and $\exp(-2i\chi)\Lambda_{21}$.

III. FORMALISM

Since relatively little experimental information is available about CPT and T violation in the heavy neutral-meson systems, a general parametrization of Λ is appropriate. It is desirable to have a parametrization that is model independent, valid for arbitrary size CPT and T violation, independent of phase conventions, and expressed in terms of mass and decay rates insofar as possible. A parametrization of this type was originally introduced by Lavoura in the context of the kaon system [18,19]. For simplicity, it is also attractive to arrange matters so that the quantities controlling T and CPT violation are denoted by single symbols that are distinct from other frequently used notation. In this section, a parametrization convenient to the four meson systems and satisfying all the above criteria is presented and related to formalisms often used in the literature.

For a complex 2×2 matrix, it is possible to write the two diagonal elements as the sum and difference of two complex numbers. It is also possible to write the off-diagonal elements as the product and ratio of two complex numbers. Using these two facts, which ultimately permit the clean representation of T- and CPT-violating quantities, a general expression for Λ can be taken as

$$\Lambda = \tfrac{1}{2}\Delta\lambda \begin{pmatrix} U+\xi & VW^{-1} \\ VW & U-\xi \end{pmatrix}, \quad (5)$$

where the parameters $UVW\xi$ are complex. The factor $\Delta\lambda/2$ has been extracted from Λ to make these parameters dimensionless and to avoid factors of 2 in the expressions below.

The requirements that the trace of the matrix is $\operatorname{tr}\Lambda = \lambda$ and that the determinant is $\det\Lambda = \lambda_a\lambda_b$ impose the identifications

$$U \equiv \lambda/\Delta\lambda, \quad V \equiv \sqrt{1-\xi^2} \quad (6)$$

on the complex parameters U and V. The free parameters in Eq. (5) are therefore W and ξ. These can be regarded as four independent real quantities: $W = w\exp(i\omega)$, $\xi = \operatorname{Re}\xi + i\operatorname{Im}\xi$. One of these four real numbers, the argument ω of W, is arbitrary and physically irrelevant. It changes under the phase redefinitions discussed at the end of the previous section. The other three are physical. The modulus w of W controls T violation, with $w=1$ if and only if T is preserved [20]. The two remaining real numbers, $\operatorname{Re}\xi$ and $\operatorname{Im}\xi$, control CPT violation and both are zero if and only if CPT is preserved. The quantities w and ξ can be expressed in terms of the components of Λ as [21]

$$w = \sqrt{|\Lambda_{21}/\Lambda_{12}|}, \quad \xi = \Delta\Lambda/\Delta\lambda, \quad (7)$$

where $\Delta\Lambda = \Lambda_{11} - \Lambda_{22}$.

In this $w\xi$ formalism, the three parameters for CP violation w, $\operatorname{Re}\xi$, $\operatorname{Im}\xi$ are dimensionless and independent of phase conventions. They are phenomenologically introduced and therefore are independent of specific models. However, this does *not* imply that they are necessarily constant num-

TABLE I. Comparison of formalisms for neutral-meson mixing.

Formalism	Parameters depend on phase convention?	λ, $\Delta\lambda$ given as	CPT parameter (complex)	T parameter (real)
$w\xi$	No	λ, $\Delta\lambda$	ξ	w
$M\Gamma$	Yes (M_{12},Γ_{12})	See Eq. (A2)	$(M_{11}-M_{22})-\frac{1}{2}i(\Gamma_{11}-\Gamma_{22})$	$\dfrac{\lvert M_{12}^*-i\Gamma_{12}^*/2\rvert}{\lvert M_{12}-i\Gamma_{12}/2\rvert}$
$DE_1E_2E_3$	Yes (E_1,E_2)	$-2iD$, $2\sqrt{E_1^2+E_2^2+E_3^2}$	E_3	$i(E_1E_2^*-E_1^*E_2)$
$DE\theta\phi$	Yes (ϕ)	$-2iD$, $2E$	$\cos\theta$	$\lvert\exp(i\phi)\rvert$
$pqrs$	Yes (p,q,r,s)	λ, $\Delta\lambda$	$(ps-qr)$	$\lvert pr/qs\rvert$
$\epsilon\delta$	Yes (ϵ,δ)	λ, $\Delta\lambda$	δ	Re ϵ, if $C\!\!\!/P\!\!\!/$ small

bers. Indeed, the assumption of constancy for ξ frequently made in the literature is a special choice that strongly restricts the generality of the parametrization and which according to the CPT theorem is inconsistent with the fundamental structure of Lorentz-invariant quantum field theory. In fact, if the requirement of exact Lorentz symmetry is relaxed, then ξ cannot be a constant quantity within the framework of quantum field theory but instead must vary with the momentum of the meson. Since CPT violation is a profound effect, it is unsurprising that the parameter ξ has features different from w. The choice of the notation ξ (rather than, say, X) in Eq. (5) has been made to emphasize this crucial fact.

The physical states with definite mass and lifetimes are the eigenstates of Λ. In the $w\xi$ formalism, they take the form

$$|P_a\rangle=\mathcal{N}_a(|P^0\rangle+A|\overline{P^0}\rangle),$$

$$|P_b\rangle=\mathcal{N}_b(|P^0\rangle+B|\overline{P^0}\rangle), \tag{8}$$

where

$$A=(1-\xi)W/V, \quad B=-(1+\xi)W/V. \tag{9}$$

The normalizations $\mathcal{N}_a$, $\mathcal{N}_b$ in Eq. (8) can be chosen as desired. For unit-normalized states, the normalizations are

$$\mathcal{N}_a=\exp(i\eta_a)/\sqrt{1+|A|^2},$$

$$\mathcal{N}_b=\exp(i\eta_b)/\sqrt{1+|B|^2}, \tag{10}$$

where η_a and η_b are phases that can be chosen freely. For the analysis of physical observables in the following sections, the values of these phases are irrelevant [22].

Some insight into the advantages of the $w\xi$ formalism can be obtained by comparing it to alternative formalisms available in the literature. The Appendix summarizes some of the more popular ones, and Table I provides a comparative synopsis of their features. The first column identifies the formalism through the standard notation for its parameters. The second column indicates the phase-convention dependence of its parameters. The third column lists the connection between the physical quantities λ, $\Delta\lambda$ and their expression in the given formalism. The fourth column specifies the complex combination of parameters that governs CPT violation in the specified formalism, while the last column gives the real number controlling T violation. Note that the final entry on the last line holds only for small CPT and T violation and assumes a phase convention with Im $\epsilon=0$.

Exact relationships exist between the $w\xi$ formalism and the other formalisms listed in Table I, but they can be involved and may change with the choice of phase conventions. Expressing the complex parameter ξ for CPT violation in the other parametrizations gives

$$\begin{aligned}\xi&=\tfrac{1}{2}[(M_{11}-M_{22})-\tfrac{1}{2}i(\Gamma_{11}-\Gamma_{22})]\\&\quad\times\{(M_{12}-\tfrac{1}{2}i\Gamma_{12})(M_{12}^*-\tfrac{1}{2}i\Gamma_{12}^*)\\&\quad+\tfrac{1}{4}[(M_{11}-M_{22})-\tfrac{1}{2}i(\Gamma_{11}-\Gamma_{22})]^2\}^{-1/2}\\&=\frac{E_3}{\sqrt{E_1^2+E_2^2+E_3^2}}\\&=\cos\theta\\&=\frac{(ps-qr)}{(ps+qr)}\\&\approx 2\delta.\end{aligned} \tag{11}$$

The last line is valid only for small δ and ϵ and only in a special phase convention, but shows that ξ can be identified with 2δ for an appropriate choice of phase convention in the K system. In any case, for the D, B_d, and B_s systems, ξ appears simpler to use than δ or any of the other parametrizations.

A similar exercise for the real parameter w for T violation yields

$$\begin{aligned} w &= |(M_{12}^* - \tfrac{1}{2} i\Gamma_{12}^*)/(M_{12} - \tfrac{1}{2} i\Gamma_{12})|^{1/2} \\ &= |(E_1 + iE_2)/(E_1 - iE_2)|^{1/2} \\ &= |\exp(i\phi)| \\ &= \sqrt{|qs/pr|} \\ &\approx 1 - 2\,\mathrm{Re}\,\epsilon. \end{aligned} \tag{12}$$

The last line is again valid only for small δ and ϵ and only in a special phase convention.

The above equations reveal that the $w\xi$ formalism is most closely related to the $DE\theta\phi$ formalism, but offers a more direct link to λ, $\Delta\lambda$, an abbreviated notation for CPT violation, and a single symbol for the phase-independent physical parameter for T violation. On the more practical side, the use of ξ also avoids confusion with the standard use of the track orientation angles θ, ϕ for the meson in the detector, which is a useful asset in the presence of orientation-dependent CPT-violating effects. Overall, advantages of the $w\xi$ formalism include its model independence, its use of mass and decay rates as physical parameters, its validity for arbitrary-size CPT and T violation, and its independence of phase conventions. In the present work, use of the $w\xi$ formalism simplifies the results of the study of CPT violation.

IV. THEORY FOR *CPT* VIOLATION

The CPT theorem guarantees CPT invariance of Lorentz-symmetric quantum field theories, including the usual standard model of particle physics. To construct a description of CPT violation viable at the level of quantum field theory, it is therefore of interest to consider the possibility of small violations of Lorentz invariance. A general standard-model extension allowing for Lorentz and CPT violation is known [9]. It could emerge, for example, as the low-energy limit of a fundamental theory at the Planck scale [23]. This standard-model extension provides a quantitative microscopic theory for Lorentz and CPT violation that is applicable to a wide class of experiments in addition to the studies of neutral-meson oscillations considered in the present work. Among these are, for example, comparative tests of QED in Penning traps [24–27], spectroscopy of hydrogen and antihydrogen [28,29], measurements of muon properties [30,31], clock-comparison experiments [32–35], observations of the behavior of a spin-polarized torsion pendulum [36,37], measurements of cosmological birefringence [38,9,39,40], and observations of the baryon asymmetry [41]. However, none of these tests are sensitive to the sector of the standard-model extension involved in the experiments with neutral-meson oscillations, essentially because the latter are flavor changing [10].

Using the general standard-model extension, a perturbative calculation can be performed to obtain the leading-order CPT-violating contributions to Λ. These emerge as the expectation values of interaction terms in the standard-model Hamiltonian [13]. The CPT-unperturbed wave functions $|P^0\rangle$ and $|\overline{P^0}\rangle$ are the appropriate states for constructing the expectation values. The hermiticity of the perturbation Hamiltonian ensures reality of the dominant contributions to the difference $\Delta\Lambda = \Lambda_{11} - \Lambda_{22}$ of the diagonal terms of Λ and therefore constrains the form of Λ. It can be shown that [10]

$$\Delta\Lambda \approx \beta^\mu \Delta a_\mu, \tag{13}$$

where $\beta^\mu = \gamma(1,\vec{\beta})$ is the 4-velocity of the meson state in the observer frame. The effect of Lorentz and CPT violation in the standard-model extension appears in Eq. (13) via the factor $\Delta a_\mu = r_{q_1} a_\mu^{q_1} - r_{q_2} a_\mu^{q_2}$, where $a_\mu^{q_1}$, $a_\mu^{q_2}$ are CPT- and Lorentz-violating coupling coefficients for the two valence quarks in the P^0 meson, and where r_{q_1} and r_{q_2} are quantities resulting from quark-binding and normalization effects [13]. The coefficients $a_\mu^{q_1}$, $a_\mu^{q_2}$ for Lorentz and CPT violation have mass dimension one and emerge from terms in the Lagrangian for the standard-model extension of the form $-a_\mu^q \overline{q} \gamma^\mu q$, where q specifies the quark flavor.

The 4-velocity and hence 4-momentum dependence in Eq. (13) confirms the failure of the usual assumption of a constant parameter for CPT violation. This dependence has substantial implications for experiments, since CPT observables will typically vary with the momentum magnitude and orientation of the mesons. As a result, the CPT reach of an experiment is affected by the meson momentum spectrum and angular distribution [10,11].

A significant consequence of the 4-momentum dependence arises from the rotation of the Earth relative to the constant vector $\Delta\vec{a}$. This leads to sidereal variations in some observables [10,11]. The point is that the analysis leading to Eq. (13) is performed in the laboratory frame, which rotates with the Earth. The resulting sidereal time dependence can be exhibited explicitly by converting the expression for $\Delta\Lambda$ to a nonrotating frame.

Denote the spatial basis in the nonrotating frame by $(\hat{X},\hat{Y},\hat{Z})$ and that in the laboratory frame by $(\hat{x},\hat{y},\hat{z})$. Following Ref. [33], define the nonrotating-frame basis $(\hat{X},\hat{Y},\hat{Z})$ to be compatible with celestial equatorial coordinates [42] with $\hat{Z}$ aligned along the Earth's rotation axis. The $\hat{z}$ axis in the laboratory frame can be chosen for maximal convenience. For collimated mesons, it may be useful to take it as the beam direction. In a collider, the direction of the colliding beams could be adopted. For a nonzero signal involving sidereal variations, $\cos\chi = \hat{z}\cdot\hat{Z}$ is nonzero, and $\hat{z}$ precesses about $\hat{Z}$ with the Earth's sidereal frequency Ω. A complete map between the two bases is given by Eq. (16) of Ref. [33]. For convenience in what follows, take θ and ϕ to be conventional polar coordinates defined about the $\hat{z}$ axis in the laboratory frame. If the $\hat{z}$ axis is chosen along the axis of a detector, then θ, ϕ are the usual detector polar coordinates.

Any coefficient $\vec{a}$ for Lorentz violation with laboratory-frame components (a^1,a^2,a^3) has nonrotating-frame components (a^X,a^Y,a^Z) given by Eq. (12) of Ref. [11]. This relation determines the sidereal variation of $\Delta\vec{a}$ and, using Eq. (13), of $\Delta\Lambda$. The complete momentum and sidereal-time dependence of the parameter ξ for CPT violation in any of the

P systems can then be obtained. Noting that the laboratory-frame 3-velocity of a P meson has the form $\vec{\beta} = \beta(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ and the momentum magnitude is $p \equiv |\vec{p}| = \beta m_P \gamma(p)$, where $\gamma(p) = \sqrt{1+p^2/m_P^2}$ as usual, the expression for ξ is found to be

$$\xi \equiv \xi(\hat{t},\vec{p}) \equiv \xi(\hat{t},p,\theta,\phi)$$
$$= \frac{\gamma(p)}{\Delta\lambda}\{\Delta a_0 + \beta\Delta a_Z(\cos\theta\cos\chi - \sin\theta\cos\phi\sin\chi)$$
$$+\beta[\Delta a_Y(\cos\theta\sin\chi + \sin\theta\cos\phi\cos\chi)$$
$$-\Delta a_X \sin\theta\sin\phi]\sin\Omega\hat{t} + \beta[\Delta a_X(\cos\theta\sin\chi$$
$$+\sin\theta\cos\phi\cos\chi) + \Delta a_Y\sin\theta\sin\phi]\cos\Omega\hat{t}\}, \qquad (14)$$

where $\hat{t}$ denotes the sidereal time.

In deriving Eq. (14), only leading-order terms in a_μ have been kept but no other assumption about the size of ξ has been made. The result (14) is therefore a generalization of Eq. (13) in Ref. [11], which was obtained for the K system under the assumption of small δ_K. In particular, Eq. (14) holds for the heavy-meson systems where the possibility of large $|\xi| \gtrsim 1$ remains experimentally admissible at present.

Note that the expressions (13) and (14) explicitly show that the real and imaginary parts of ξ are connected through the mass and lifetime differences of the two physical eigenstates P_a, P_b [13]. The relationship is

$$\mathrm{Re}\,\xi = -2\Delta m\,\mathrm{Im}\,\xi/\Delta\gamma. \qquad (15)$$

However, in the interest of generality this result is used only sparingly in this work.

V. EXPERIMENT

To illustrate some implications of the result (14), this section derives some experimentally relevant decay amplitudes, probabilities, and asymmetries. For simplicity, attention is restricted to the case of semileptonic decays into a final state f or its conjugate state $\bar{f}$. Although studying these decays suffices for present purposes, other decays are also likely to be relevant in practice, and it would be of interest to perform a more complete study. Another simplification adopted here is the neglect of any violations of the $\Delta Q = \Delta S$, $\Delta Q = \Delta C$, or $\Delta Q = \Delta B$ rules. A careful consideration of these and other more mundane complications would certainly be important in a definitive experimental analysis [19]. However, since there is no reason to expect such complications to exhibit observable momentum or sidereal-time dependences, the extraction of a compelling positive signal for CPT violation should be feasible.

Under these assumptions, the basic transition amplitudes for semileptonic decays can be taken as

$$\langle f|T|P^0\rangle = F, \quad \langle f|T|\overline{P^0}\rangle = 0,$$
$$\langle\bar{f}|T|\overline{P^0}\rangle = \bar{F}, \quad \langle\bar{f}|T|P^0\rangle = 0. \qquad (16)$$

Note that this parametrization allows for direct CPT violation, which is proportional to the difference $F^* - \bar{F}$, as well as direct T violation.

To determine the time-dependent decay amplitudes and probabilities, it is useful to obtain an explicit expression for the time evolution of the neutral-P states. The wave functions $|P^0\rangle$ and $|\overline{P^0}\rangle$ can be constructed in terms of $|P_a\rangle$ and $|P_b\rangle$, and their evolution with the meson proper time t can then be incorporated via Eq. (2). This gives

$$\begin{pmatrix} P^0(t,\hat{t},\vec{p}) \\ \overline{P^0}(t,\hat{t},\vec{p}) \end{pmatrix} = \begin{pmatrix} C+S\xi & SVW \\ SVW^{-1} & C-S\xi \end{pmatrix}\begin{pmatrix} P^0 \\ \overline{P^0} \end{pmatrix}. \qquad (17)$$

The functions C and S depend on the meson proper time t and are given by

$$C = \cos(\tfrac{1}{2}\Delta\lambda t)\exp(-\tfrac{1}{2}i\lambda t)$$
$$= \tfrac{1}{2}(e^{-i\lambda_a t} + e^{-i\lambda_b t}),$$
$$S = -i\sin(\tfrac{1}{2}\Delta\lambda t)\exp(-\tfrac{1}{2}i\lambda t)$$
$$= \tfrac{1}{2}(e^{-i\lambda_a t} - e^{-i\lambda_b t}). \qquad (18)$$

In addition to the proper-time dependence in S and C, Eq. (17) also contains sidereal-time and momentum dependence from $\xi(\hat{t},\vec{p})$. Since the meson decays occur quickly on the scale of sidereal time, it is an excellent approximation to treat sidereal time $\hat{t}$ as a parameter independent of the meson proper time t. It is therefore appropriate to take ξ as independent of t but varying with $\hat{t}$. This approximation is implemented in what follows.

A. Uncorrelated mesons

For the case of uncorrelated meson decays, the time-dependent decay probabilities can be obtained by combining Eqs. (17) and (16). This gives

$$P_f(t,\hat{t},\vec{p}) \equiv |\langle f|T|P(t),\hat{t},\vec{p}\rangle|^2$$
$$= \tfrac{1}{2}|F|^2 e^{-\gamma t/2}[(1+|\xi|^2)\cosh\Delta\gamma t/2$$
$$+(1-|\xi|^2)\cos\Delta m t - 2\,\mathrm{Re}\,\xi\sinh\Delta\gamma t/2$$
$$-2\,\mathrm{Im}\,\xi\sin\Delta m t],$$
$$\overline{P}_{\bar{f}}(t,\hat{t},\vec{p}) \equiv |\langle\bar{f}|T|\overline{P}(t,\hat{t},\vec{p})\rangle|^2 = P_f(\xi\to-\xi, F\to\bar{F}),$$
$$P_{\bar{f}}(t,\hat{t},\vec{p}) \equiv |\langle\bar{f}|T|P(t,\hat{t},\vec{p})\rangle|^2$$
$$= \tfrac{1}{2}|\bar{F}|^2 w^2|1-\xi^2|$$
$$\times e^{-\gamma t/2}(\cosh\Delta\gamma t/2 - \cos\Delta m t),$$
$$\overline{P}_f(t,\hat{t},\vec{p}) \equiv |\langle f|T|\overline{P}(t,\hat{t},\vec{p})\rangle|^2 = P_{\bar{f}}(w\to 1/w, \bar{F}\to F), \qquad (19)$$

where the dependence on sidereal time $\hat{t}$ and momentum $\vec{p}$ is inherited from that of ξ in Eq. (14). Inspection of these equations reveals that nonzero indirect CPT violation changes the shape of the first two probabilities, while both CPT and T violation merely scale the latter two. I emphasize that these expressions are valid for CPT and T violation of arbitrary size. They are also manifestly independent of the choice of phase convention [43].

To extract the CPT and T violation from the decay probabilities (19), it is useful to construct appropriate asymmetries. For the case of T violation, the dependence on sidereal time and meson momentum has relatively little effect. The last two probabilities in Eq. (19) have the same CPT but different T dependences, and their difference divided by their sum is sensitive to the parameter w for T violation but independent of the parameter ξ for CPT violation and hence independent of sidereal time and meson momentum. In contrast, for the case of CPT violation the situation is more involved and several new features appear.

As a simple example illustrating some of the effects, consider the case where $F^* = \overline{F}$, i.e., neglible direct CPT violation. The usual procedure is to assume constant nonzero ξ (which is inconsistent with quantum field theory, as discussed above) and define an asymmetry $\mathcal{A}_{CPT}(t)$ for CPT violation as

$$\mathcal{A}_{CPT}(t) = \frac{\overline{P}_{\overline{f}}(t) - P_f(t)}{\overline{P}_{\overline{f}}(t) + P_f(t)}. \tag{20}$$

The comparable definition in the present context is still useful but results in an asymmetry depending also on sidereal time and meson momentum:

$$\mathcal{A}_{CPT}(t,\hat{t},\vec{p}) \equiv \frac{\overline{P}_{\overline{f}}(t,\hat{t},\vec{p}) - P_f(t,\hat{t},\vec{p})}{\overline{P}_{\overline{f}}(t,\hat{t},\vec{p}) + P_f(t,\hat{t},\vec{p})} = \frac{2\,\mathrm{Re}\,\xi \sinh \Delta\gamma t/2 + 2\,\mathrm{Im}\,\xi \sin \Delta m t}{(1+|\xi|^2)\cosh \Delta\gamma t/2 + (1-|\xi|^2)\cos \Delta m t}, \tag{21}$$

where the $\hat{t}$, $\vec{p}$ dependence of ξ is understood.

In practice, the efficient practical application of this and related asymmetries depends on the nature of the experiment. Appropriate averaging over one of more of the variables t, $\hat{t}$, p, θ, ϕ either before or after constructing the asymmetry (21) can aid the clean extraction of bounds on Δa_μ. For instance, under certain circumstances it may be useful to sum the data over ϕ and use an asymmetry like Eq. (21) but defined with the ϕ average of Eq. (19). The form of Eq. (14) shows that binning the data in $\hat{t}$ typically provides information on Δa_X and Δa_Y, while binning in θ permits the separation of the spatial and timelike components of Δa_μ. The p dependence can also be useful [10,11].

As a specific example, already used in the K system [11,12], suppose the mesons involved are highly collimated in the laboratory frame. Then, the 3-velocity can be written $\vec{\beta} = (0,0,\beta)$ and the expression (14) for ξ simplifies to

$$\xi(\hat{t},\vec{p}) = \frac{\gamma}{\Delta\lambda}[\Delta a_0 + \beta \Delta a_Z \cos\chi + \beta \sin\chi(\Delta a_Y \sin \Omega\hat{t} + \Delta a_X \cos \Omega\hat{t})]. \tag{22}$$

Binning in $\hat{t}$ therefore provides sensitivity to the equatorial components Δa_X, Δa_Y, while averaging over $\hat{t}$ eliminates them altogether. Indeed, a conventional measurement that ignores the dependence on sidereal time and meson momentum is typically sensitive only to the average magnitude

$$|\overline{\xi}| = \overline{\gamma}|\Delta a_0 + \overline{\beta}\Delta a_Z \cos\chi|/|\Delta\lambda|, \tag{23}$$

where $\overline{\beta}$ and $\overline{\gamma}$ are averages weighted over the meson-momentum spectrum. This shows explicitly that previous analyses performed under the assumption of constant CPT parameter produce results dependent on the type of experiment.

If CPT violation is small so $\xi < 1$, the asymmetry (21) takes the form

$$\mathcal{A}_{CPT}(t,\hat{t},\vec{p}) \approx \frac{2\,\mathrm{Re}\,\xi \sinh \Delta\gamma t/2 + 2\,\mathrm{Im}\,\xi \sin \Delta m t}{\cosh \Delta\gamma t/2 + \cos \Delta m t}. \tag{24}$$

A further assumption that could be countenanced involves the approximation of small $\Delta\gamma t/2$, i.e., $t < 2/\Delta\gamma$. This gives

$$\mathcal{A}_{CPT}(t,\hat{t},\vec{p}) \approx \frac{\mathrm{Re}\,\xi \Delta\gamma t + 2\,\mathrm{Im}\,\xi \sin \Delta m t}{1 + \cos \Delta m t}. \tag{25}$$

It is tempting also to neglect as small the term involving $\mathrm{Re}\,\xi$, but this is potentially invalid because $\mathrm{Re}\,\xi \propto \mathrm{Im}\,\xi/\Delta\gamma$ according to Eq. (15). Imposing the prediction (15) instead gives

$$\mathcal{A}_{CPT}(t,\hat{t},\vec{p}) \approx \frac{2\,\mathrm{Im}\,\xi(\sin \Delta m t - \Delta m t)}{1 + \cos \Delta m t}. \tag{26}$$

The extraction of complete information about Δa_μ requires clean CPT tests involving asymmetries such as Eq. (21) that are independent of the parameter w for T violation. However, the dependence on sidereal time of certain CPT-violating effects offers the possibility of extracting clean CPT bounds on spatial components of Δa_μ even using observables that mix T and CPT effects [11]. An example is provided by the standard rate asymmetry δ_l for K_L semileptonic decays [44]:

$$\delta_l \equiv \frac{\Gamma(K_L \to l^+ \pi^- \nu) - \Gamma(K_L \to l^- \pi^+ \overline{\nu})}{\Gamma(K_L \to l^+ \pi^- \nu) + \Gamma(K_L \to l^- \pi^+ \overline{\nu})}, \tag{27}$$

which under the assumption of constant nonzero parameter for CPT violation (inconsistent with quantum field theory, as noted above) is determined by a combination of T and CPT effects that cannot be disentangled without further information. In the $w\xi$ formalism, the asymmetry (27) and its generalization to arbitrary P_b is found to be

$$\delta_l(\hat{t},\vec{p}) \equiv \frac{\Gamma(P_b \to f) - \Gamma(P_b \to \bar{f})}{\Gamma(P_b \to f) + \Gamma(P_b \to \bar{f})} = \frac{|1-\xi^2| - |1+\xi|^2 w^2}{|1-\xi^2| + |1+\xi|^2 w^2} \approx (1-w) - \mathrm{Re}\,\xi(\hat{t},\vec{p}), \tag{28}$$

where the last line assumes $w \approx 1$, $\xi \ll 1$, i.e., small T and CPT violation. Binning in sidereal time or momentum can therefore under suitable circumstances bound the spatial components of Δa_μ independently of T violation, even for observables involving both T and CPT violation.

B. Correlated mesons

Another situation of experimental importance is the case of correlated meson pairs, resulting from quarkonium production and decay. The normalized initial quantum state ensuing immediately after the strong decay of the quarkonium can be written as

$$|i\rangle = \frac{1}{\sqrt{2}}(|P^0(+)\rangle|\overline{P^0}(-)\rangle - |P^0(-)\rangle|\overline{P^0}(+)\rangle), \tag{29}$$

where $(+)$ indicates the meson travels in a specified direction in the quarkonium rest frame while $(-)$ indicates it travels in the opposite direction. Note that this initial state is independent of the choice of phase convention.

Let the meson moving in the $(+)$ direction have 3-momentum $\vec{p}_1$ in the laboratory frame and decay into a final state f_1 at proper time t_1. Similarly, let the other meson have 3-momentum $\vec{p}_2$ and decay into a final state f_2 at proper time t_2. As before, in tracking the sidereal-time dependence, it is an excellent approximation to regard the time interval between quarkonium production and detection of the decay products as negligible on the scale of the Earth's rotation period, so in what follows the creation of the state $|i\rangle$ and its evolution through the double decay process are taken to occur at fixed sidereal time $\hat{t}$.

The probability amplitude $A_{f_1 f_2}$ for the double decay can be regarded as a function of the decay times t_1, t_2, of the sidereal time $\hat{t}$, and of the two meson momenta $\vec{p}_1$, $\vec{p}_2$. It is given by

$$\begin{aligned} A_{f_1 f_2} &\equiv A_{f_1 f_2}(t_1,t_2,\hat{t},\vec{p}_1,\vec{p}_2) \\ &= \langle f_1 f_2|T|i\rangle \\ &= \frac{1}{\sqrt{2}}[\langle f_1|T|P^0(t_1,\hat{t},\vec{p}_1)\rangle\langle f_2|T|\overline{P^0}(t_2,\hat{t},\vec{p}_2)\rangle \\ &\quad - \langle f_1|T|\overline{P^0}(t_1,\hat{t},\vec{p}_1)\rangle\langle f_2|T|P^0(t_2,\hat{t},\vec{p}_2)\rangle]. \end{aligned} \tag{30}$$

The time evolutions of $|P^0(t,\hat{t},\vec{p})\rangle$ and $|\overline{P^0}(t,\hat{t},\vec{p})\rangle$ are determined by Eq. (17). In substituting these expressions into the decay amplitude (30), care is required to keep separate track of the CPT-violating parameters ξ_1 and ξ_2 for each meson, since they depend on the meson 3-momenta and therefore typically differ in accordance with Eq. (14).

It is convenient and feasible to write a single expression that holds for all double decay modes, including the various double-semileptonic combinations. For $a=1, 2$, define

$$\langle f_a|T|P^0\rangle = F_a, \quad \langle f_a|T|\overline{P^0}\rangle = \overline{F}_a, \tag{31}$$

and let $C_a = C(t_a)$, $S_a = S(t_a)$. Then, the probability amplitude is found to be

$$\begin{aligned} A_{f_1 f_2} = \frac{1}{\sqrt{2}}[&(F_1\overline{F}_2 + F_2\overline{F}_1)(\xi_1 S_1 C_2 - \xi_2 S_2 C_1) \\ &+ (F_1\overline{F}_2 - F_2\overline{F}_1)(C_1C_2 - (\xi_1\xi_2 + V_1V_2)S_1S_2) \\ &+ (F_1F_2W^{-1} - \overline{F}_1\overline{F}_2 W)(V_2C_1S_2 - V_1S_1C_2) \\ &+ (F_1F_2W^{-1} + \overline{F}_1\overline{F}_2W)(\xi_1V_2 - \xi_2V_1)S_1S_2], \end{aligned} \tag{32}$$

where the dependence on $\hat{t}$ and $\vec{p}_1$, $\vec{p}_2$ is understood. The quantities V_1, V_2 are defined in terms of ξ_1, ξ_2 by Eq. (6), while $W = w\exp(i\omega)$ as before.

Next, consider the special case of double-semileptonic decays and adopt the notation of Eq. (16). It is useful to introduce the definitions

$$t = t_1 + t_2, \quad \Delta t = t_1 - t_2. \tag{33}$$

In terms of these variables, some algebra yields the four possible decay amplitudes as

$$\begin{aligned} A_{f\bar{f}} &= \frac{F\overline{F}}{2\sqrt{2}}[(1-\xi_1\xi_2 - V_1V_2)\cos\tfrac{1}{2}\Delta\lambda t \\ &\quad + (1+\xi_1\xi_2+V_1V_2)\cos\tfrac{1}{2}\Delta\lambda\Delta t - i(\xi_1-\xi_2)\sin\tfrac{1}{2}\Delta\lambda t \\ &\quad - i(\xi_1+\xi_2)\sin\tfrac{1}{2}\Delta\lambda\Delta t]e^{-i\lambda t/2}, \\ A_{\bar{f}f} &= -A_{f\bar{f}}(\xi_1 \to -\xi_1, \xi_2 \to -\xi_2), \\ A_{ff} &= \frac{F^2}{2\sqrt{2}}W^{-1}[(\xi_1V_2 - \xi_2V_1)(\cos\tfrac{1}{2}\Delta\lambda t - \cos\tfrac{1}{2}\Delta\lambda\Delta t) \\ &\quad + i(V_1-V_2)\sin\tfrac{1}{2}\Delta\lambda t + i(V_1 \\ &\quad + V_2)\sin\tfrac{1}{2}\Delta\lambda\Delta t]e^{-i\lambda t/2}, \\ A_{\bar{f}\bar{f}} &= -A_{ff}(F \to \overline{F}, W \to W^{-1}, \xi_1 \to -\xi_1, \xi_2 \to -\xi_2), \end{aligned} \tag{34}$$

where the dependence on $\hat{t}$ and $\vec{p}_1$, $\vec{p}_2$ is again understood.

The expressions (34) are valid for CPT and T violation of arbitrary size and are independent of phase conventions. Nontrivial sensitivity to the sum and difference of ξ_1 and ξ_2 is manifest. The corresponding decay probabilities are straightforward to obtain but are somewhat cumbersome. They inherit the independence of phase conventions and the nontrivial sensitivity to $\xi_1 \pm \xi_2$. Since these factors depend on all four parameters Δa_μ for CPT violation, appropriate analysis of experimental data for correlated decays can provide four independent CPT tests.

The type of analysis needed depends on the experimental situation. The remarks following Eq. (21) about averaging and binning apply here, and there are also considerations specific to the case of correlated mesons. For example, if the quarkonium is produced at rest in the laboratory, perhaps by a symmetric collider, then the 3-momenta of the correlated mesons are equal in magnitude and opposite in direction. The sum

$$\xi_1 + \xi_2 = 2\gamma(p)\Delta a_0/\Delta\lambda \qquad (35)$$

is then independent of $\Delta\vec{a}$, so extracting an asymmetry sensitive to $\xi_1+\xi_2$ yields a clean bound on Δa_0. Similarly, the difference $\xi_1-\xi_2$ is independent of Δa_0, and binning in sidereal time permits bounds on the three components $\Delta\vec{a}$. If instead the quarkonium is produced in an asymmetric collider, the two 3-momenta of the correlated mesons are *not* back-to-back in the laboratory frame, so $\xi_1 \pm \xi_2$ are both sensitive to all components of Δa_μ. Four independent measurements of CPT violation can again be extracted.

Many of the interesting features can be illustrated in the approximation of small ξ_1, ξ_2, for which the expressions simplify to some extent. This approximation is certainly valid for the K system, and the recent results from OPAL, DELPHI, and BELLE [16] imply it is also valid for the B_d system. The situation for the D and the B_s systems is less clear, with large CPT violation remaining experimentally admissible, but many of the following considerations still apply.

Consider for definiteness the double decay into $f\bar{f}$. To leading order in ξ_1 and ξ_2, the decay probability $P_{f\bar{f}}$ is

$$\begin{aligned} P_{f\bar{f}} &= P_{f\bar{f}}(t,\Delta t,\hat{t},\vec{p}_1,\vec{p}_2) \\ &= \tfrac{1}{4}|F\bar{F}|^2 e^{-\gamma t/2}\{\cosh\tfrac{1}{2}\Delta\gamma\Delta t + \cos\Delta m\Delta t \\ &\quad - \mathrm{Re}(\xi_1+\xi_2)\sinh\tfrac{1}{2}\Delta\gamma\Delta t - \mathrm{Im}(\xi_1+\xi_2)\sin\Delta m\Delta t \\ &\quad + 2\,\mathrm{Im}[(\xi_1-\xi_2)\cos(\tfrac{1}{2}\Delta\lambda^*\Delta t)\sin(\tfrac{1}{2}\Delta\lambda t)]\}. \end{aligned} \qquad (36)$$

This expression shows the combination $\xi_1+\xi_2$ is associated with an odd function in Δt, while $\xi_1-\xi_2$ is associated with an even function in Δt. This distinction allows the separate extraction of $\xi_1 \pm \xi_2$. As an explicit example, the case of the sum $\xi_1+\xi_2$ is treated here.

In typical experimental situations for the correlated double-meson decay, the time sum t is unobservable but the difference Δt can be used as a fitting parameter. It is therefore appropriate to work with an integrated probability $\Gamma_{f\bar{f}}(\Delta t,\hat{t},\vec{p}_1,\vec{p}_2)$ obtained by integrating the probability (19) over t:

$$\Gamma_{f\bar{f}}(\Delta t,\hat{t},\vec{p}_1,\vec{p}_2) = \int_{|\Delta t|}^{\infty} dt P_{f\bar{f}}(t,\Delta t,\hat{t},\vec{p}_1,\vec{p}_2). \qquad (37)$$

An asymmetry $\mathcal{A}_{CPT,f\bar{f}}$ sensitive to the sum $\xi_1+\xi_2$ of parameters for CPT violation can then be defined as

$$\begin{aligned} \mathcal{A}_{CPT,f\bar{f}} &= \mathcal{A}_{CPT,f\bar{f}}(\Delta t,\hat{t},\vec{p}_1,\vec{p}_2) \\ &= \frac{\Gamma_{f\bar{f}}(\Delta t,\hat{t},\vec{p}_1,\vec{p}_2) - \Gamma_{f\bar{f}}(-\Delta t,\hat{t},\vec{p}_1,\vec{p}_2)}{\Gamma_{f\bar{f}}(\Delta t,\hat{t},\vec{p}_1,\vec{p}_2) + \Gamma_{f\bar{f}}(-\Delta t,\hat{t},\vec{p}_1,\vec{p}_2)}. \end{aligned} \qquad (38)$$

Calculation gives

$$\mathcal{A}_{CPT,f\bar{f}} = \frac{-\mathrm{Re}(\xi_1+\xi_2)\sinh\frac{1}{2}\Delta\gamma\Delta t - \mathrm{Im}(\xi_1+\xi_2)\sin\Delta m\Delta t}{\cosh\frac{1}{2}\Delta\gamma\Delta t + \cos\Delta m\Delta t}, \qquad (39)$$

which is valid to lowest order in CPT-violating quantities. For the B_d system, this expression generalizes the asymmetry obtained [15] under the assumption of constant parameter for CPT violation and used to place the recent experimental limits on CPT violation at BELLE [16].

For quarkonia produced in a symmetric collider the asymmetry (39) depends only on Δa_0 because the sum $\xi_1+\xi_2$ is given by Eq. (35). There is therefore no variation with $\hat{t}$, and the line spectrum of the mesons implies there is also no variation with $\vec{p}_1 = -\vec{p}_2$. In this case, a direct fit to the variation with Δt provides a bound on Δa_0.

In contrast, for quarkonia produced in an asymmetric collider the asymmetry (39) depends on all four parameters Δa_μ and also varies with $\hat{t}$ and $\vec{p}_1$, $\vec{p}_2$. For any given situation, forming an asymmetry of the type (38) after averaging Eq. (36) over suitable combinations of the variables $\hat{t}$, $\vec{p}_1$, $\vec{p}_2$ permits the extraction of four independent CPT bounds, one for each parameter Δa_μ. Independent tests of this kind for the B_d system should be feasible at both BaBar and BELLE, where the quarkonia are produced in asymmetric collisions and the meson pairs are boosted in the laboratory frame.

VI. SUMMARY

This work has studied some aspects of tests of CPT and Lorentz symmetry using neutral-meson oscillations. A formalism has been adopted for the treatment of arbitrarily large indirect CPT and T violation in the K, D, B_d, and B_s systems that is phase-convention independent. It involves a real parameter w for T violation and a complex parameter ξ for CPT violation. An expression for the latter, given as Eq. (14), is derived from the general Lorentz- and CPT-breaking standard-model extension. This equation reveals that CPT observables can vary with the magnitude and orientation of the meson momentum and hence also with sidereal time. To illustrate some of the implications for experiment, transition amplitudes, decay probabilities, and sample CPT-sensitive asymmetries for semileptonic decays are derived. Both uncorrelated and correlated mesons are considered, and some consequences for experiments are described.

The analysis shows that four independent experimental bounds are required to bound CPT violation completely in any single neutral-meson system. Since these parameters may differ between systems, separate experimental analyses are required in each case. No bounds are available in the D or B_s systems as yet. Certain combinations of the four key parameters Δa_μ have been constrained in the K and B_d systems by recent experiments [12,16], but no definitive analysis has yet been performed. Obtaining a complete set of linearly independent measurements in any of the meson systems has the potential to offer our first glimpse of physics at the Planck scale and would in any case provide crucial experimental information on the existence of CPT and Lorentz violation in nature.

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APPENDIX: STANDARD FORMALISMS

This appendix lists a few key properties of five standard formalisms for indirect T and CPT violation. All these can be traced to early work several decades ago in the context of the K system [6]. For most of these standard formalisms, several closely related variants exist in the literature, but for definiteness only one of each type is presented here.

The $M\Gamma$ formalism sets

$$\Lambda = M - \tfrac{1}{2} i\Gamma = \begin{pmatrix} M_{11} - \tfrac{1}{2} i\Gamma_{11} & M_{12} - \tfrac{1}{2} i\Gamma_{12} \\ M_{12}^* - \tfrac{1}{2} i\Gamma_{12}^* & M_{22} - \tfrac{1}{2} i\Gamma_{22} \end{pmatrix}. \tag{A1}$$

The off-diagonal quantities are all phase-convention dependent. The parameter for CPT violation is the combination $(M_{11}-M_{22})-i(\Gamma_{11}-\Gamma_{22})/2$. The parameter for T violation is $|(M_{12}^*-i\Gamma_{12}^*/2)/(M_{12}-i\Gamma_{12}/2)|$. The masses and decay rates are given by

$$\lambda = (M_{11}+M_{22}) - \tfrac{1}{2} i(\Gamma_{11}+\Gamma_{22}),$$

$$\Delta\lambda = 2\{(M_{12} - \tfrac{1}{2} i\Gamma_{12})(M_{12}^* - \tfrac{1}{2} i\Gamma_{12}^*) + \tfrac{1}{4}[(M_{11}-M_{22}) - \tfrac{1}{2} i(\Gamma_{11}-\Gamma_{22})]^2\}^{1/2}, \tag{A2}$$

where the definitions in Eq. (4) are understood to hold.

The $DE_1E_2E_3$ formalism sets

$$\Lambda = \begin{pmatrix} -iD+E_3 & E_1 - iE_2 \\ E_1 + iE_2 & -iD - E_3 \end{pmatrix}. \tag{A3}$$

All off-diagonal quantities are phase-convention dependent. The parameter for CPT violation is E_3. The parameter for T violation is $i(E_1E_2^* - E_1^*E_2)$. The masses and decay rates are given by $\lambda = -2iD$, $\Delta\lambda = 2\sqrt{E_1^2+E_2^2+E_3^2}$.

The $DE\theta\phi$ formalism sets

$$\Lambda = \begin{pmatrix} -iD + E\cos\theta & E\sin\theta e^{-i\phi} \\ E\sin\theta e^{i\phi} & -iD - E\cos\theta \end{pmatrix}. \tag{A4}$$

The parameter ϕ is phase-convention dependent. The parameter for CPT violation is $\cos\theta$. The parameter for T violation is $|\exp(i\phi)|$. The masses and decay rates are given by $\lambda = -2iD$, $\Delta\lambda = 2E$.

There are also formalisms that are introduced in terms of the relationship between the strong-interaction eigenstates P^0, $\overline{P^0}$ and the physical eigenstates P_a, P_b. A general one is the $pqrs$ formalism, which sets

$$|P_a\rangle = p|P^0\rangle + q|\overline{P^0}\rangle,$$

$$|P_b\rangle = r|P^0\rangle - s|\overline{P^0}\rangle, \tag{A5}$$

where p, q, r, s are complex parameters. In this formalism, one can show

$$\Lambda = \frac{1}{2(ps+qr)} \begin{pmatrix} \lambda(ps+qr) + \Delta\lambda(ps-qr) & 2\Delta\lambda pr \\ 2\Delta\lambda qs & \lambda(ps+qr) - \Delta\lambda(ps-qr) \end{pmatrix}. \tag{A6}$$

The complex parameters p, q, r, s are all phase-convention dependent. They are also substantially redundant, since only three of their eight real components have physical meaning. The normalization conventions for the wave functions represent two degrees of freedom, often fixed by the choice $|p|^2 + |q|^2 = |r|^2 + |s|^2 = 1$. The remaining three unobservable degrees of freedom are the absolute phases of $|P_a\rangle$ and $|P_b\rangle$ and the relative phase of $|P^0\rangle$ and $|\overline{P^0}\rangle$. The parameter for CPT violation is $(ps-qr)$. The parameter for T violation is $|pr/qs|$. The masses and decay rates are additional independent quantities, taken here as λ, $\Delta\lambda$.

The $\epsilon\delta$ formalism [45] is widely adopted for the K system. It can be regarded as a special case of the $pqrs$ formalism. For arbitrary-size T and CPT violation, the $\epsilon\delta$ formalism can be defined as

$$|P_a\rangle = \frac{(1+\epsilon+\delta)|P^0\rangle + (1-\epsilon-\delta)|\overline{P^0}\rangle}{\sqrt{2(1+|\epsilon+\delta|^2)}},$$

$$|P_b\rangle = \frac{(1+\epsilon-\delta)|P^0\rangle - (1-\epsilon+\delta)|\overline{P^0}\rangle}{\sqrt{2(1+|\epsilon-\delta|^2)}}. \qquad \text{(A7)}$$

In this formalism, Λ is given by Eq. (A6) with appropriate substitutions for the parameters p, q, r, s in terms of ϵ, δ, obtained from Eq. (A7). Both ϵ and δ depend on phase conventions. Nonzero values of ϵ and δ characterize T and CPT violation, respectively. For the special case of small ϵ and δ, which is a good approximation in the K system, one can show

$$\Lambda \approx \frac{1}{2}\begin{pmatrix} \lambda + 2\Delta\lambda\,\delta & \Delta\lambda(1+2\epsilon) \\ \Delta\lambda(1-2\epsilon) & \lambda - 2\Delta\lambda\,\delta \end{pmatrix}. \qquad \text{(A8)}$$

Even within this approximation ϵ is phase-convention dependent, although δ is not. The parameter for T violation can then be taken to be Re ϵ, for example. The masses and decay rates are independent quantities and here are specified by λ, $\Delta\lambda$.

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[21] Note that if $\xi = \pm 1$ then $V=0$, $\Delta\Lambda = \Delta\lambda$ and there are no oscillations, so in what follows $\xi \neq \pm 1$ and hence $V \neq 0$ is assumed.

[22] When CP is preserved, $\xi = 0$, $w=1$, and $|P_a\rangle$ and $|P_b\rangle$ reduce to CP eigenstates. In a phase convention with $\omega = \eta_a = \eta_b = 0$, the eigenstates then take the familiar form $|P_{a,b}\rangle = (|P^0\rangle \pm |\overline{P^0}\rangle)/\sqrt{2}$.

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