# $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ as an $E_6$ subgroup

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An extension of the standard model to the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ which is a subgroup of the electroweak-strong unification group $E_6$ is analyzed. The mass scales, the gauge boson masses, and the masses for the spin-1/2 particles in the model are calculated. The mass differences between the up and down quark sectors, between the quarks and leptons, and between the charged and neutral leptons in one family are explained as a consequence of mixing of the ordinary with exotic fermions implied by the model. By using experimental results we constrain the mixing angle between the two neutral currents and the mass of the additional neutral gauge boson to be $-0.0015 \leqslant \sin\theta \leqslant 0.0048$ and $M_{Z_2} \geqslant 600$ GeV at 95% C.L. The existence of a Dirac neutrino for each family with a mass of the order of the electroweak mass scale is predicted.

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## I. INTRODUCTION

The standard model (SM) local gauge group $G_{SM} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, with the $SU(2)_L \otimes U(1)_Y$ sector hidden [1] and $SU(3)_c$ confined [2], is in excellent agreement with all present experiments. However, the SM cannot explain some issues such as hierarchical masses and mixing angles, charge quantization, $CP$ violation, etc., and many physicists believe that it does not represent the final theory, but serves as an effective theory, originating from a more fundamental one. So, extensions of the SM are always worthy to be considered.

So far there are not yet any theoretical or experimental facts that point toward what lies beyond the SM, and the best approach may be to depart from it as little as possible. In this regard, $SU(3)_L \otimes U(1)_X$ as a flavor group has been introduced several times in the literature: first as a family independent theory as in the SM [3], and next as a family structure [4] that points to a natural explanation of the total number of families in nature. Several of the models studied in the literature are inconsistent in the sense that they are anomalous, vectorlike, or include more than three families with light neutral particles. Others include spin-1/2 particles with exotic electric charges [4], etc. In this paper we studied $SU(3)_L \otimes U(1)_X$ as an $E_6$ subgroup [6] and we do some phenomenological calculations in order to set the different mass scales in the model and calculate the masses for all the spin-1/2 particles in one family. In our analysis several see-saw mechanisms are implemented.

The paper is organized in the following way. In Sec. II we introduce the model as an anomaly-free theory, based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, and show that it is a subgroup of the electroweak-strong unification group $E_6$. In Sec. III we describe the scalar sector needed to break the symmetry and to produce masses to all the fermion fields in the model. In Sec. IV we analyze the gauge boson sector paying special attention to the two neutral currents present in the model and their mixing. In Sec. V we analyze the fermion mass spectrum. In Sec. VI we use experimental results in order to constrain the mixing angle between the two neutral currents and the mass scale of the new neutral gauge boson, and in Sec. VIII we present our conclusions. A technical Appendix with details of the diagonalization of the $5 \times 5$ mass matrix for the spin-1/2 neutral leptons in the model is presented at the end.

## II. THE FERMION CONTENT OF THE MODEL

First let us see what the fermion content of the model is.

### A. $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ as an anomaly free model

In what follows we assume that the electroweak gauge group is $SU(3)_L \otimes U(1)_X \supset SU(2)_L \otimes U(1)_Y$, that the left-handed quarks (color triplets) and left-handed leptons (color singlets) transform as the 3 and $\overline{3}$ representations of $SU(3)_L$, respectively, that $SU(3)_c$ is vectorlike as in the SM and, contrary to the models in Ref. [4], that the several anomalies are canceled individually in each family. So, we start with

$$Q_L = \begin{pmatrix} u \\ d \\ D \end{pmatrix}_L, \quad \psi_L = \begin{pmatrix} e^- \\ \nu_e \\ N_1^0 \end{pmatrix}_L,$$

where $D_L$ is an $SU(2)_L$ singlet exotic quark of electric charge $-1/3$ and $N_{1L}^0$ is also an $SU(2)_L$ singlet exotic lepton of zero electric charge.

If the $\{SU(3)_c,\ SU(3)_L,\ U(1)_X\}$ quantum numbers are $\{3,3,X_Q\}$ for $Q_L$, and $\{1,\overline{3},X_L\}$ for $\psi_L$, two more lepton multiplets with quantum numbers $\{1,\overline{3},X_i\}$, $i=1,2$ must be introduced in order to cancel the $[SU(3)_L]^3$ anomaly; each one of those multiplets must include one $SU(2)_L$ doublet and one singlet of exotic leptons. The quarks fields $u_L^c$, $d_L^c$ and $D_L^c$ color antitriplets and $SU(3)_L$ singlets must also be introduced in order to cancel the $[SU(3)_c]^3$ anomaly. Then the hypercharges ($X_Q$, $X_L$, $X_1$, $X_2$, etc.) must be chosen in order to cancel the anomalies $[SU(3)_c]^2U(1)_X$, $[SU(3)_L]^2U(1)_X$, $[\mathrm{grav}]^2U(1)_X$ and $[U(1)_X]^3$, where $[\mathrm{grav}]^2U(1)_X$ stands for the gravitationl anomaly [5].

We use for the symmetry-breaking chain $SU(3)_L \to SU(2)_L\otimes U(1)_Z$ the branching rule $3\to 2(1/6)+1(-1/3)$, where the numbers in parentheses are the $Z$ hypercharge value [which in turn implies $\overline{3}\to 2(-1/6)+1(1/3)$]. Then, by using for the electric charge generator $Q=T_{3L}+Z+X$, where $T_{3L}=\pm 1/2$ is the $SU(2)_L$ third isospin component, we have that $X_Q=0$, $X_u=-2/3$ and $X_D=X_d=1/3$. For those values the anomaly $[SU(3)_c]^2U(1)_X$ is automatically canceled.

By the same token $X_L=-1/3$. Then the $[SU(3)_L]^2U(1)_X$ anomaly cancellation condition reads $X_1+X_2=1/3$ and the $[U(1)_X]^3$ anomaly cancellation condition reads $X_1^3+X_2^3=7/27$. The solution to this pair of equations is $X_1=-1/3$ and $X_2=2/3$ (or vice versa because the two equations are 1,2 symmetric).

We end up with the following anomaly free multiplet structure for this model:

| $Q_L=\begin{pmatrix} u \\ d \\ D \end{pmatrix}_L$ | $u_L^c$ | $d_L^c$ | $D_L^c$ |
|---|---|---|---|
| $(3,3,0)$ | $(\bar{3},1,-\frac{2}{3})$ | $(\bar{3},1,\frac{1}{3})$ | $(\bar{3},1,\frac{1}{3})$ |

| $\psi_L=\begin{pmatrix} e^- \\ \nu_e \\ N_1^0 \end{pmatrix}_L$ | $\phi_L=\begin{pmatrix} E^- \\ N_2^0 \\ N_3^0 \end{pmatrix}_L$ | $\chi_L=\begin{pmatrix} N_4^0 \\ E^+ \\ e^+ \end{pmatrix}_L$ |
|---|---|---|
| $(1,\bar{3},-\frac{1}{3})$ | $(1,\bar{3},-\frac{1}{3})$ | $(1,\bar{3},\frac{2}{3})$ |

### B. $SU(3)_L\otimes SU(3)_c\otimes U(1)_X$ as an $E_6$ subgroup

Models constructed with $E_6$ as the grand unyield theory (GUT) group [6] utilize the complex 27-dimensional representation for the left-handed fermions in one family. The models can be divided into two classes, depending on whether they emphasize the $SO(10)$ or the $SU(3)_L\otimes SU(3)_c\otimes SU(3)_R$ subgroups. For models in the second category the fundamental representation of $E_6$ has the following branching rule under $[SU(3)]^3\equiv SU(3)_L\otimes SU(3)_c\otimes SU(3)_R$:

$$27\to(3,3,1)\oplus(\overline{3},1,3)\oplus(1,\overline{3},\overline{3})$$

where the particle content of each term is

$$(3,3,1)=(u,d,D)_L;\ (\overline{3},1,3)=\begin{pmatrix} N^0 & E^- & e^- \\ E^+ & N^{0c} & \nu_e \\ e^+ & \nu_e^c & M^0 \end{pmatrix}_L,\quad (1,\overline{3},\overline{3})=(u^c,d^c,D^c)_L.$$

When $SU(3)_R$ breaks into $U(1)_a\otimes U(1)_b$ we have the branching rule $3\to(a)(b)+(-a)(b)+(0)(-2b)$, which shows that by taking $b=1/3$ we can identify $U(1)_b=U(1)_X$ and it implies that $(\overline{3},1,3)=\psi_L\oplus\phi_L\oplus\chi_L$. This allows us to identify $N_{3L}^0=\nu_{eL}^c$ as the right-handed neutrino field (in the weak basis) and $N_{4L}^0=N_{2L}^{0c}$, as far as we stay in the breaking chain

$$[SU(3)]^3\to SU(3)_L\otimes SU(3)_c\otimes SU(2)_R\otimes U(1)' \to SU(3)_L\otimes SU(3)_c\otimes U(1)_X.$$

Let us mention that the lower-dimensional $E_6$ irreducible representations (irreps) are 1, 27, and 78, where irreps 1 and 78 are real representations and irrep 27 is complex and chiral [6]. In the electroweak-strong unification group based on $E_6$ the fermion fields are placed in irrep 27 and the gauge fields in irrep 78.

## III. THE SCALAR SECTOR

Our aim is to break the symmetry in the way,

$$SU(3)_c\otimes SU(3)_L\otimes U(1)_X\to SU(3)_c\otimes SU(2)_L\otimes U(1)_Y \to SU(3)_c\otimes U(1)_Q,$$

and at the same time give masses to the fermion fields in our model. With this in mind let us introduce the following set of Higgs scalars: $\phi_1=(1,\overline{3},-1/3)$ with a vacuum expectation value (VEV) $\langle\phi_1\rangle^T=(0,0,V)$; $\phi_2(1,\overline{3},-1/3)$ with a VEV $\langle\phi_2\rangle^T=(0,v/\sqrt{2},0)$, and $\phi_3(1,\overline{3},2/3)$ with a VEV $\langle\phi_3\rangle^T=(v'/\sqrt{2},0,0)$, with the hierarchy $V>v\sim v'\sim 250$ GeV, the electroweak breaking scale. The scale of $V$ could be fixed phenomenologically. At first glance it looks like only two Higgs triplets are necessary for the symmetry breaking, but as can be seen, they are not enough to reproduce a realistic fermion mass spectrum in the model.

## IV. THE GAUGE BOSON SECTOR

There are a total of 17 gauge bosons in our model. One gauge field $B^\mu$ associated with $U(1)_X$, the eight gluon fields associated with $SU(3)_c$ which remain massless, and another eight associated with $SU(3)_L$ and that we write for convenience in the following way:

$$\frac{1}{2}\lambda_\alpha A^\mu_\alpha = \frac{1}{\sqrt{2}}\begin{pmatrix} D^\mu_1 & W^{+\mu} & K^{+\mu} \\ W^{-\mu} & D^\mu_2 & K^{0\mu} \\ K^{-\mu} & \overline{K}^{0\mu} & D^\mu_3 \end{pmatrix},$$

where $D^\mu_1 = A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6}$, $D^\mu_2 = -A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6}$, and $D^\mu_3 = -2A^\mu_8/\sqrt{6}$. $\lambda_i$, $i=1,2,\ldots,8$ are the eight Gell-Mann matrices normalized as $\mathrm{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}$, which allows us to write the charge operator as

$$Q = \frac{\lambda_3}{2} + \frac{\lambda_8}{2\sqrt{3}} + XI_3,$$

where $I_3$ is the $3\times 3$ unit matrix.

After breaking the symmetry with $\langle \phi_i \rangle$, $i=1,2,3$, and using for the covariant derivative for triplets $D^\mu = \partial^\mu - i(g/2)\lambda_\alpha A^\mu_\alpha - ig'XB^\mu$, we get the following mass terms for the charged gauge bosons of the electroweak sector: $M^2_{W^\pm} = (g^2/4)(v^2 + v'^2)$, $M^2_{K^\pm} = (g^2/4)(2V^2 + v'^2)$, $M^2_{K^0(\overline{K}^0)} = (g^2/4)(2V^2 + v^2)$. For the three neutral gauge bosons we get a mass term of the form

$$M = V^2\left(\frac{g'B^\mu}{3} - \frac{gA^\mu_8}{\sqrt{3}}\right)^2 + \frac{v^2}{8}\left(\frac{2g'B^\mu}{3} - gA^\mu_3 + \frac{gA^\mu_8}{\sqrt{3}}\right)^2 + \frac{v'^2}{8}\left(gA^\mu_3 - \frac{4g'B^\mu}{3} + \frac{gA^\mu_8}{\sqrt{3}}\right)^2.$$

By diagonalizing $M$ we get the physical neutral gauge bosons that are defined through the mixing angle $\theta$ and $Z_\mu$, $Z'_\mu$ by

$$Z^\mu_1 = Z_\mu \cos\theta + Z'_\mu \sin\theta,$$

$$Z^\mu_2 = -Z_\mu \sin\theta + Z'_\mu \cos\theta,$$

$$\tan(2\theta) = \frac{-\sqrt{12}C_W(1-T^2_W/3)^{1/2}[v'^2(1+T^2_W) - v^2(1-T^2_W)]}{3(1-T^2_W/3)(v^2+v'^2) - C^2_W[8V^2 + v^2(1-T^2_W)^2 + v'^2(1+T^2_W)^2]},$$

where the photon fields $A^\mu$, $Z_\mu$, and $Z'_\mu$ are given by

$$A^\mu = S_W A^\mu_3 + C_W\left[\frac{T_W}{\sqrt{3}}A^\mu_8 + (1-T^2_W/3)^{1/2}B^\mu\right], \tag{1}$$

$$Z^\mu = C_W A^\mu_3 - S_W\left[\frac{T_W}{\sqrt{3}}A^\mu_8 + (1-T^2_W/3)^{1/2}B^\mu\right], \tag{2}$$

$$Z'^\mu = -(1-T^2_W/3)^{1/2}A^\mu_8 + \frac{T_W}{\sqrt{3}}B^\mu, \tag{3}$$

$S_W = \sqrt{3}g'/\sqrt{3g^2 + 4g'^2}$ and $C_W$ are the sine and cosine of the electroweak mixing angle, respectively, and $T_W = S_W/C_W$. Also we can identify the $Y$ hypercharge associated with the SM gauge boson as

$$Y^\mu = \left[\frac{T_W}{\sqrt{3}}A^\mu_8 + (1-T^2_W/3)^{1/2}B^\mu\right]. \tag{4}$$

In the limit $\sin\theta \to 0$, $M_Z = M_{W^\pm}/C_W$, this limit is obtained either by demanding $V \to \infty$ or $v'^2 = v^2(C^2_W - S^2_W) \equiv v^2 C_{2W}$.

### A. Charged currents

The interactions among the charged vector fields with leptons are

$$\begin{aligned} H^{CC} = \frac{g}{\sqrt{2}}[ & W^+_\mu(\overline{u_L}\gamma^\mu d_L - \overline{\nu_{eL}}\gamma^\mu e^-_L - \overline{N^0_{2L}}\gamma^\mu E^-_L \\ & - \overline{E^+_L}\gamma^\mu N^0_{4L}) + K^+_\mu(\overline{u_L}\gamma^\mu D_L - \overline{N^0_{1L}}\gamma^\mu e^-_L - \overline{N^0_{3L}}\gamma^\mu E^-_L \\ & - \overline{e^+_L}\gamma^\mu N^0_{4L}) + K^0_\mu(\overline{d_L}\gamma^\mu D_L - \overline{N^0_{1L}}\gamma^\mu \nu_{eL} - \overline{N^0_{3L}}\gamma^\mu N^0_{2L} \\ & - \overline{e^+_L}\gamma^\mu E^+_L)] + \mathrm{H.c.}, \end{aligned} \tag{5}$$

which implies that the interactions with the $K^\pm$ and $K^0(\overline{K^0})$ bosons violate the lepton number and the weak isospin. Notice also that the first two terms in the previous expression constitute the charged weak current of the SM as far as we identify $W^\pm$ as the $SU(2)_L$ charged left-handed weak bosons.

### B. Neutral currents

The neutral currents $J_\mu(EM)$, $J_\mu(Z)$, and $J_\mu(Z')$ associated with the Hamiltonian $H^0 = eA^\mu J_\mu(EM) + (g/C_W)Z^\mu J_\mu(Z) + (g'/\sqrt{3})Z'^\mu J_\mu(Z')$ are

$$\begin{aligned} J_\mu(EM) &= \frac{2}{3}\overline{u}\gamma_\mu u - \frac{1}{3}\overline{d}\gamma_\mu d - \frac{1}{3}\overline{D}\gamma_\mu D - \overline{e^-}\gamma_\mu e^- \\ &\quad - \overline{E^-}\gamma_\mu E^- \\ &= \sum_f q_f \overline{f}\gamma_\mu f, \end{aligned}$$

$$J_\mu(Z)=J_{\mu,L}(Z)-S_W^2 J_\mu(EM),$$
$$J_\mu(Z')=T_W J_\mu(EM)-J_{\mu,L}(Z'), \tag{6}$$

where $e=gS_W=g'C_W\sqrt{1-T_W^2/3}>0$ is the electric charge, $q_f$ is the electric charge of the fermion $f$ in units of $e$, $J_\mu(EM)$ is the electromagnetic current (vectorlike as it should be), and the left-handed currents are

$$\begin{aligned}J_{\mu,L}(Z)&=\frac{1}{2}(\overline{u_L}\gamma_\mu u_L-\overline{d_L}\gamma_\mu d_L+\overline{\nu_{eL}}\gamma_\mu \nu_{eL}-\overline{e_L^-}\gamma_\mu e_L^-\\&\quad+\overline{N_2^0}\gamma_\mu N_2^0-\overline{E^-}\gamma_\mu E^-)\\&=\sum_f T_{3f}\overline{f_L}\gamma_\mu f_L,\end{aligned}$$

$$\begin{aligned}J_{\mu,L}(Z')&=S_{2W}^{-1}(\overline{u_L}\gamma_\mu u-\overline{e_L^-}\gamma_\mu e_L^- -\overline{E_L^-}\gamma_\mu E_L^-\\&\quad-\overline{N_{4L}^0}\gamma_\mu N_{4L}^0)+T_{2W}^{-1}(\overline{d_L}\gamma_\mu d_L-\overline{E_L^+}\gamma_\mu E_L^+\\&\quad-\overline{\nu_{eL}}\gamma_\mu\nu_{eL}-\overline{N_{2L}^0}\gamma_\mu N_{2L}^0)-T_W^{-1}(\overline{D_L}\gamma_\mu D_L\\&\quad-\overline{e_L^+}\gamma_\mu e_L^+-\overline{N_{1L}^0}\gamma_\mu N_{1L}^0-\overline{N_{3L}^0}\gamma_\mu N_{3L}^0)\\&=\sum_f T_{9f}\overline{f_L}\gamma_\mu f_L,\end{aligned}\tag{7}$$

where $S_{2W}=2S_W C_W$, $T_{2W}=S_{2W}/C_{2W}$, $\overline{N_2^0}\gamma_\mu N_2^0=\overline{N_{2L}^0}\gamma_\mu N_{2L}^0+\overline{N_{2R}^0}\gamma_\mu N_{2R}^0=\overline{N_{2L}^0}\gamma_\mu N_{2L}^0-\overline{N_{2L}^{0c}}\gamma_\mu N_{2L}^{0c}=\overline{N_{2L}^0}\gamma_\mu N_{2L}^0-\overline{N_{4L}^0}\gamma_\mu N_{4L}^0$, and similarly $\overline{E}\gamma_\mu E=\overline{E_L^-}\gamma_\mu E_L^- -\overline{E_L^+}\gamma_\mu E_L^+$. In this way $T_{3f}=Dg(1/2,-1/2,0)$ is the third component of the weak isospin acting on the representation 3 of $SU(3)_L$ (the negative when acting on $\overline{3}$), and $T_{9f}=Dg(S_{2W}^{-1},T_{2W}^{-1},-T_W^{-1})$ is a convenient $3\times 3$ diagonal matrix acting on the representation 3 of $SU(3)_L$ (the negative when acting on $\overline{3}$). Notice that $J_\mu(Z)$ is just the generalization of the neutral current present in the SM, which allows us to identify $Z_\mu$ as the neutral gauge boson of the SM, which is consistent with Eqs. (1), (2), and (4).

The couplings of the physical states $Z_1^\mu$ and $Z_2^\mu$ are then given by

$$\begin{aligned}H^{NC}&=\frac{g}{2C_W}\sum_{i=1}^{2}Z_i^\mu\sum_f\{\overline{f}\gamma_\mu[a_{iL}(f)(1-\gamma_5)+a_{iR}(f)\\&\quad\times(1+\gamma_5)]f\}\\&=\frac{g}{2C_W}\sum_{i=1}^{2}Z_i^\mu\sum_f\{\overline{f}\gamma_\mu[g(f)_{iV}-g(f)_{iA}\gamma_5]f\},\end{aligned}\tag{8}$$

where

$$a_{1L}(f)=\cos\theta(T_{3f}-q_f S_W^2)-\frac{g'\sin\theta C_W}{g\sqrt{3}}(T_{9f}-q_f T_W),$$

$$a_{1R}(f)=-q_f S_W\left(\cos\theta S_W-\frac{g'\sin\theta}{g\sqrt{3}}\right),\tag{9}$$

$$a_{2L}(f)=-\sin\theta(T_{3f}-q_f S_W^2)-\frac{g'\cos\theta C_W}{g\sqrt{3}}(T_{9f}-q_f T_W),$$

$$a_{2R}(f)=+q_f S_W\left(\sin\theta S_W+\frac{g'\cos\theta}{g\sqrt{3}}\right)$$

and

$$g(f)_{iV}=a(f)_{iL}+a(f)_{iR},$$
$$g(f)_{iA}=a(f)_{iL}-a(f)_{iR},\tag{10}$$

so, after the algebra we get

$$g(f)_{1V}=\cos\theta(T_{3f}-2S_W^2 q_f)-\frac{g'\sin\theta}{g\sqrt{3}}(T_{9f}C_W-2q_f S_W),$$

$$g(f)_{2V}=-\sin\theta(T_{3f}-2S_W^2 q_f)-\frac{g'\cos\theta}{g\sqrt{3}}(T_{9f}C_W-2q_f S_W),$$

$$g(f)_{1A}=\cos\theta T_{3f}-\frac{g'\sin\theta}{g\sqrt{3}}T_{9f}C_W,$$

$$g(f)_{2A}=-\sin\theta T_{3f}-\frac{g'\cos\theta}{g\sqrt{3}}T_{9f}C_W,$$

to be compared with the SM values $g(f)_{1V}^{SM}=T_{3f}-2S_W q_f$ and $g(f)_{1A}^{SM}=T_{3f}$. The values of $g_{iV}$, $g_{iA}$ with $i=1,2$ are listed in Tables I and II. As we can see, in the limit $\theta=0$ the couplings of $Z_1^\mu$ to the ordinary leptons and quarks are the same as in the SM, and due to this we can test the new physics beyond the SM.

## V. FERMION MASSES

The Higgs scalars introduced in Sec. III not only break the symmetry in an appropriate way, but produce the following mass terms for the fermions of the model.

### A. Quark masses

For the quark sector we can write the following Yukawa terms:

$$\begin{aligned}\mathcal{L}_Y^Q&=Q_L^T C(h_u\phi_3 u_L^c+h_D\phi_1 D_L^c+h_d\phi_2 d_L^c+h_{dD}\phi_2 D_L^c\\&\quad+h_{Dd}\phi_1 d_L^c)+\text{H.c.},\end{aligned}\tag{11}$$

where $h_\eta$, $\eta=u,D,d,dD$, and $h_{Dd}$ are Yukawa couplings of order one and $C$ is the charge conjugate operator. From Eq.

TABLE I. The $Z_1^\mu \rightarrow \overline{f}f$ couplings.

| $f$ | $g(f)_{1V}$ | $g(f)_{1A}$ |
|---|---|---|
| $u$ | $\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)[\cos\theta - \sin\theta/(4C_W^2-1)^{1/2}]$ | $\frac{\cos\theta}{2} - \sin\theta/[2(4C_w^2-1)^{1/2}]$ |
| $d$ | $\cos\theta\left(-\frac{1}{2}+\frac{2S_W^2}{3}\right) - \frac{\sin\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}-\frac{S_W^2}{3}\right)$ | $-\frac{1}{2}\{\cos\theta + \sin\theta C_{2W}/[2(4C_W^2-1)^{1/2}]\}$ |
| $D$ | $\frac{2S_W^2\cos\theta}{3} + \sin\theta\left(1-\frac{5}{3}S_W^2\right) \Big/ (4C_W^2-1)^{1/2}$ | $C_W^2 \sin\theta/(4C_W^2-1)^{1/2}$ |
| $e^-$ | $\cos\theta\left(-\frac{1}{2}+2S_W^2\right) + \frac{3\sin\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}-S_W^2\right)$ | $-\frac{\cos\theta}{2} + \frac{\sin\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}-C_W^2\right)$ |
| $E^-$ | $\cos\theta(-1+2S_W^2) - \frac{S_W^2\sin\theta}{(4C_W^2-1)^{1/2}}$ | $C_W^2 \sin\theta/(4C_W^2-1)^{1/2}$ |
| $\nu_e$, $N_2^0$ | $\frac{1}{2}[\cos\theta + \sin\theta(1-2S_W^2)/(4C_W^2-1)^{1/2}]$ | $\frac{1}{2}[\cos\theta + \sin\theta(1-2S_W^2)/(4C_W^2-1)^{1/2}]$ |
| $N_1^0$, $N_3^0$ | $-C_W^2\sin\theta/(4C_W^2-1)^{1/2}$ | $-C_W^2\sin\theta/(4C_W^2-1)^{1/2}$ |
| $N_4^0$ | $-\frac{1}{2}[\cos\theta - \sin\theta/(4C_W^2-1)^{1/2}]$ | $-\frac{1}{2}[\cos\theta - \sin\theta/(4C_W^2-1)^{1/2}]$ |

(11) we get for the up-quark sector a mass term $m_u = h_u v'/\sqrt{2}$, and for the down-quark sector a mass matrix in the basis $(d,D)_L$ of the form

$$M_{dD} = \begin{pmatrix} h_d v/\sqrt{2} & h_{dD} v/\sqrt{2} \\ h_{Dd} V & h_D V \end{pmatrix}. \tag{12}$$

For the particular case $h_d = h_{dD} = h_{Dd} = h_D \equiv h$, the mass eigenvalues of the previous matrix are $m_d = 0$ and $m_D = h(V + v/\sqrt{2})$. Since there is not a physical reason for the Yukawas to be equal, let us calculate the mass eigenvalues as a function of $|M_{dD}| \equiv (h_d h_D - h_{dD} h_{Dd})$ the determinant of the Yukawas, and in the expansion $v/V$. Then the algebra shows that $m_D \simeq h_D V + h_d v/\sqrt{2} - |M_{dD}| v^2/\sqrt{2} V h_D + \cdots$ and $m_d \simeq v|M_{dD}|(1 + h_{dD} h_{Dd} v/\sqrt{2} V h_D^3 + \cdots)/\sqrt{2} h_D$. This expansion can be used to explain the experimental values for the quark masses in any one of the three families (for the third family, for example, we may choose the particular values $h_b = h_B = 0.8$, $h_{bB} = 0.9$, and $h_{Bb} = 0.7$ in order to get $m_b \sim 10^{-2} m_t$, etc.). Since the model does not allow us to calculate the Yukawa couplings, the fermion masses cannot be predicted in this model. But, contrary to what happens in the SM, in the context of this model we can implement, in a reasonable way, the weak isospin breaking in the quark sector for the three families (what we have here is a particular realization of the general analysis presented in Ref. [7]).

TABLE II. The $Z_2^\mu \rightarrow \overline{f}f$ couplings.

| $f$ | $g(f)_{2V}$ | $g(f)_{2A}$ |
|---|---|---|
| $u$ | $\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)[-\sin\theta - \cos\theta/(4C_W^2-1)^{1/2}]$ | $\frac{-\sin\theta}{2} - \cos\theta/2(4C_w^2-1)^{1/2}$ |
| $d$ | $-\sin\theta\left(-\frac{1}{2}+\frac{2S_W^2}{3}\right) - \frac{\cos\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}-\frac{S_W^2}{3}\right)$ | $-\frac{1}{2}\{-\sin\theta + \cos\theta C_{2W}/[2(4C_W^2-1)^{1/2}]\}$ |
| $D$ | $\frac{-2S_W^2\sin\theta}{3} + \cos\theta\left(1-\frac{5}{3}S_W^2\right)/(4C_W^2-1)^{1/2}$ | $C_W^2 \cos\theta/(4C_W^2-1)^{1/2}$ |
| $e^-$ | $-\sin\theta\left(-\frac{1}{2}+2S_W^2\right) + \frac{3\cos\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}-S_W^2\right)$ | $\frac{\sin\theta}{2} + \frac{\cos\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}-C_W^2\right)$ |
| $E^-$ | $-\sin\theta(-1+2S_W^2) - \frac{S_W^2\cos\theta}{(4C_W^2-1)^{1/2}}$ | $C_W^2 \cos\theta/(4C_W^2-1)^{1/2}$ |
| $\nu_e$, $N_2^0$ | $\frac{1}{2}[-\sin\theta + \cos\theta(1-2S_W^2)/(4C_W^2-1)^{1/2}]$ | $\frac{1}{2}[-\sin\theta + \cos\theta(1-2S_W^2)/(4C_W^2-1)^{1/2}]$ |
| $N_1^0$, $N_3^0$ | $-C_W^2\cos\theta/(4C_W^2-1)^{1/2}$ | $-C_W^2\cos\theta/(4C_W^2-1)^{1/2}$ |
| $N_4^0$ | $\frac{1}{2}[\sin\theta + \cos\theta/(4C_W^2-1)^{1/2}]$ | $\frac{1}{2}[\sin\theta + \cos\theta/(4C_W^2-1)^{1/2}]$ |

### B. Lepton masses

For the lepton sector we can write the following Yukawa terms:

$$\mathcal{L}_Y^l=\epsilon_{abc}[\psi_L^a C(h_1\phi_L^b\phi_3^c+h_2\chi_L^b\phi_1^c+h_3\chi_L^b\phi_2^c)+\phi_L^a C(h_4\chi_L^b\phi_1^c+h_5\chi_L^b\phi_2^c)]+\text{H.c.},\tag{13}$$

where $a,b,c$ are $SU(3)_L$ tensor indices and the Yukawas are again of order one. These new terms produce in the basis $(e,E)$ the mass matrix

$$M_{eE}=\begin{pmatrix}-h_3v/\sqrt{2} & -h_5v/\sqrt{2}\\ h_2V & h_4V\end{pmatrix}\tag{14}$$

with eigenvalues $m_E=h_4V-h_3v/\sqrt{2}-|M_{eE}|v^2/\sqrt{2}Vh_4+\cdots$ and $m_e\simeq v|M_{eE}|(1-h_2h_5v/\sqrt{2}Vh_4^3+\cdots)/\sqrt{2}h_4$, where $|M_{eE}|=h_2h_5-h_3h_4$, with similar consequences as in the down quark sector.

For the neutral leptons in the basis $(\nu_e,N_1,N_2.N_3,N_4)$ we get the mass matrix

$$M_N=\begin{pmatrix}0&0&0&h_1v'/\sqrt{2}&-h_2V\\0&0&-h_1v'/\sqrt{2}&0&h_3v/\sqrt{2}\\0&-h_1v'/\sqrt{2}&0&0&-h_4V\\h_1v'/\sqrt{2}&0&0&0&h_5v/\sqrt{2}\\-h_2V&h_3v/\sqrt{2}&-h_4V&h_5v/\sqrt{2}&0\end{pmatrix}.\tag{15}$$

For the particular case $h_2=h_4=h$ and $h_3=h_5=h'$ the mass eigenvalues are 0, $\pm h_1v'/\sqrt{2}$, $\pm\sqrt{h_1^2v'^2/2+h'^2v_2^2+2h^2V^2}$ which means that we have a Majorana neutrino of zero mass and two Dirac neutral particles with masses $h_1v'/\sqrt{2}$ and $(h_1^2v'^2/2+h'^2v_2^2+2h^2V^2)^{1/2}$. The eigenvector associated with the zero mass Majorana neutrino is $\alpha(-vh', -\sqrt{2}Vh,vh',\sqrt{2}Vh,v'h_1)$ where $\alpha=(2v^2/h'^2+4V^2h^2+v'^2h_1^2)^{-1/2}$ is a normalization factor. This amusing result implies, in the context of this model, that not only the known tiny mass neutral fermion must be a Majorana particle, but that it is not the electron neutrino but a mixture of five neutral particles with only one of them associated with the vertex $\overline{\nu_e}eW$.

But $h_2h_5\neq h_3h_4$, otherwise we will have a zero mass charged lepton. To diagonalize $M_N$ for the most general case ($h_i\simeq 1,i=1,2,\ldots,5$ but not equal) is not a simple task. A tedious algebra shows that, for the most general case, the five mass eigenvalues are $\pm h_1v'/\sqrt{2}$ (exact values), one small seesaw value of order $vv'/V$, and two very large values of order $(\pm V+\eta)$ where $\eta$ is a small seesaw quotient, which means that, in the context of this model, the five neutral leptons in each family split as a Dirac neutrino with a mass of the order of the electroweak breaking mass scale, a *pseudo Dirac* [8] neutrino with a very large mass, and a seesaw Majorana neutrino. In the Appendix we carry out the detailed calculation for the particular case $h_2=h_4\neq h_1\neq h_3\neq h_5$, in the expansion $vv'/V$. The five mass eigenvalues for that particular case are $\pm h_1v'/\sqrt{2}$, $\pm\sqrt{2}h_4V+\{(h_3+h_5)^2v^2+2[h_1v'\mp(h_3-h_5)v/\sqrt{2}]^2\}/(8\sqrt{2}h_4V)$ and $m_\nu=(h_5-h_3)vv'/2h_4V$. Notice that $m_\nu$ is suppressed not only for the seesaw quotient $vv'/V$ but also for the small difference of the Yukawas $(h_5-h_3)/2h_4$, which implies that $V$ can be lower than $10^{11}$ GeV, as it is for the simplest seesaw mechanism, the standard seesaw mechanism value.

## VI. CONSTRAINTS ON THE $(Z^\mu-Z'^\mu)$ MIXING ANGLE AND THE $Z_2^\mu$ MASS

To bound $\sin\theta$ and $M_{Z_2}$ we use parameters measured at the $Z$ pole from CERN $e^+e^-$ collider (LEP), SLAC Linear Collider (SLC), and atomic parity violation constraints which are given in Table III [9,10]. The expression for the partial width for $Z_1^\mu\to f\bar{f}$ is

$$\Gamma(Z_1\to f\bar{f})=\frac{N_CG_FM_{Z_1}^3}{6\pi\sqrt{2}}\rho\left[\frac{3\beta-\beta^3}{2}[g(f)_{1V}]^2+\beta^3[g(f)_{1A}]^2\right](1+\delta_f)R_{QCD+QED},\tag{16}$$

where $f$ is an ordinary SM fermion, $Z_1$ is the physical gauge boson observed at LEP, $N_C=3$ is the number of colors, $R_{QCD+QED}$ are the QCD and QED corrections, and $\beta=\sqrt{1-4m_b^2/M_{Z_1}^2}$ is a kinematic factor which is 1 for all the SM fermions except for the bottom quark for which $m_b=5.0$ GeV [11]. The factor $\delta_f$ is zero for all fermions except for the bottom quark for which the contribution coming from the top quark in the one-loop vertex radiative correction is parametrized as $\delta_b\approx 10^{-2}[-m_t^2/(2M_{Z_1}^2)+1/5]$ [12]. The $\rho$ parameter has two contributions, one is the oblique correction given by $\delta\rho\approx 3G_Fm_t^2/(8\pi^2\sqrt{2})$ and the other is the tree level contribution to the $(Z_\mu-Z'_\mu)$ mixing, which can be written as $\delta\rho_V\approx(M_{Z_2}^2/M_{Z_1}^2-1)\sin^2\theta$. Finally, $g(f)_{1V}$ and $g(f)_{1A}$ are listed in Table I. Using Eq. (16), $g(f)_{iV}$ and $g(f)_{iA}$ we can write expressions for CERN $e^+e^-$ collider LEP and SLAC Linear Collider (SLC) parameters, where we

TABLE III. Experimental data and SM values for the parameters.

| | Experimental results | SM |
|---|---|---|
| $\Gamma_Z$ (GeV) | $2.4952 \pm 0.0023$ | $2.4963 \pm 0.0016$ |
| $\Gamma(had)$ (GeV) | $1.7444 \pm 0.0020$ | $1.7427 \pm 0.0015$ |
| $\Gamma(l^+l^-)$ (MeV) | $83.984 \pm 0.086$ | $84.018 \pm 0.028$ |
| $R_e$ | $20.804 \pm 0.050$ | $20.743 \pm 0.018$ |
| $A_{FB}(e)$ | $0.0145 \pm 0.0025$ | $0.0165 \pm 0.0003$ |
| $R_b$ | $0.21653 \pm 0.00069$ | $0.21572 \pm 0.00015$ |
| $R_c$ | $0.1709 \pm 0.0034$ | $0.1723 \pm 0.0001$ |
| $A_{FB}(b)$ | $0.0990 \pm 0.0020$ | $0.1039 \pm 0.0009$ |
| $A_{FB}(c)$ | $0.0689 \pm 0.0035$ | $0.0743 \pm 0.0007$ |
| $A_{FB}(s)$ | $0.0976 \pm 0.0114$ | $0.1040 \pm 0.0009$ |
| $A_b$ | $0.922 \pm 0.023$ | $0.9348 \pm 0.0001$ |
| $A_c$ | $0.631 \pm 0.026$ | $0.6683 \pm 0.0005$ |
| $A_s$ | $0.82 \pm 0.13$ | $0.9357 \pm 0.0001$ |
| $A_e(\mathcal{P}_\tau)$ | $0.1498 \pm 0.0048$ | $0.1483 \pm 0.0012$ |
| $Q_W(Cs)$ | $-72.06 \pm 0.28 \pm 0.34$ | $-73.09 \pm 0.04$ |

are using the following values [10]: $m_t = 174.3$ GeV, $\alpha_s(m_Z) = 0.1192$, $\alpha(m_Z)^{-1} = 127.938$, and $S_W^2 = 0.2333$.

The effective weak charge $Q_W$ in atomic parity violation is given by

$$Q_W = -2[(2Z+N)c_{1u} + (Z+2N)c_{1d}], \tag{17}$$

where $c_{1q} = 2g(e)_{1A}g(q)_{1V}$, $Z$ is the number of protons, and $N$ is the number of neutrons in the atomic nucleus. The $Q_W$ determination for ${}^{133}_{55}C_s$ has been improved recently [13] to the value

$$Q_W({}^{133}_{55}C_s) = -72.06 \pm 0.28 \pm 0.34, \tag{18}$$

the result to be compared with the theoretical value for $Q_W$ which can be written as [14]

$$Q_W({}^{133}_{55}C_s) = -73.09 \pm 0.04 + \Delta Q_W, \tag{19}$$

where $\Delta Q_W$ includes the contributions of new physics. The discrepancy between the SM and the experimental data is given by [15]

$$\Delta Q_W = Q_W^{\exp} - Q_W^{SM} = 1.03 \pm 0.44. \tag{20}$$

Regarding the $(Z_\mu - Z'_\mu)$ mixing between two neutral currents, $\Delta Q_W$ can be written as [16]

$$\Delta Q_W = \left[\left(1 + 4\frac{S_W^2}{1-2S_W^2}\right) - Z\right]\delta\rho_V + \Delta Q'_W, \tag{21}$$

where $\Delta Q'_W$ is model dependent and it can be obtained for our model by using $g(e)_{1A}$ and $g(q)_{1V}$ from Table I. The analysis for our model gives

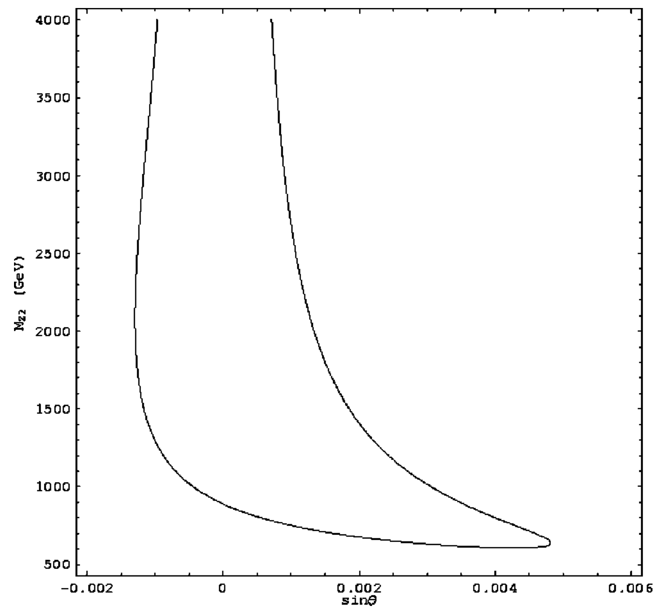


FIG. 1. Contour plot displaying the allowed region for $\sin\theta$ vs $M_{Z_2}$ at 95% C.L.

$$\Delta Q'_W = (-3.94Z - 6.40N)\sin\theta + (1.49Z + 1.81N)\frac{M_{Z_1}^2}{M_{Z_2}^2}. \tag{22}$$

With the expressions for $Z$ pole observables and $\Delta Q_W$ in terms of new physics and using experimental data from LEP and SLC [9] and atomic parity violation [13] as in Table III, we do a $\chi^2$ fit and we find the best allowed region for $\sin\theta$ vs $M_{Z_2}$ at 95% C.L.

In Fig. 1 we display the allowed region for $\sin\theta$ vs $M_{Z_2}$ at 95% confidence level. When $\sin\theta$ goes to zero, the contribution of $\delta\rho_V$ to $\Gamma(ff)$ decouples, which allows us to get a lower bound for $M_{Z_2}$; (the bounds come from $\Delta Q_W$). At 95% C.L. the allowed region gives

$$-0.0015 \leq \sin\theta \leq 0.0048,$$

$$600 \text{ GeV} \leq M_{Z_2}, \tag{23}$$

which implies that the mass of the new neutral gauge boson is compatible with the bound got in $p\bar{p}$ collisions at the Tevatron [17]. A model independent analysis, similar to the one presented here, has been reported by Erler and Langacker in Ref. [15].

## VII. CONCLUSIONS

We have presented an anomaly-free model based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ which is a subgroup of the electroweak-strong unification group $E_6$. We break the gauge symmetry down to $SU(3)_c \otimes U(1)_Q$ and at the same time give masses to all the fermion fields in the model in a consistent way by using three different Higgs scalars $\phi_i$, $i = 1,2,3$ which set two different mass scales; $v$

$\sim v' \sim 250$ GeV$\ll V$. By using experimental results from LEP, SLC, and atomic parity violation we bound the mixing angle and the mass of the additional neutral current to be $-0.0015 \leqslant \sin\theta < 0.0048$ and 600 GeV$\leqslant M_{Z_2}$ at 95% C.L.

The most outstanding result of our analysis is the mass spectrum of the fermion fields in the model which arises as a consequence of the mixing between ordinary fermions and their exotic counterparts without the need of different scale Yukawa couplings. Conspicuously, the five neutral leptons in the model split as a Dirac neutrino with a mass of the order of the electroweak breaking mass scale $v$, a *pseudo-Dirac* neutrino with a large mass of order $V$, and a tiny seesaw Majorana neutrino which is not the partner of the lightest charged lepton at the $W^{\pm}$ vertex, but a mixture of the five neutral states in the weak bases.

Notice that all the new states are vectorlike with respect to the SM quantum numbers. They consist of an isosinglet quark $D$ of electric charge $-1/3$, a lepton isodoublet $(E^-, N_2^0)_L^T$ with $N_{2L}^{0c} = N_{4L}^0$ in the weak basis, and two neutral lepton isosinglets $N_{1L}^0$ and $N_{3L}^0$. Since $V$ can be of the order of a few TeV, these exotic fermions could be accessible in forthcoming searches at the Fermilab Tevatron collider or at the Large Hadron Collider (LHC) under construction at CERN.

But two predictions of this model are as follows: first, the existence of a Dirac neutrino for each family with a mass of the order of the electroweak mass scale; second, the existence of a new neutral gauge boson with a mass just below the TeV scale. The experimental signature for these particles may be present already at the LEP II $e^+e^-$ collider data, or just around the corner of the Fermilab Tevatron output.

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## APPENDIX

In this appendix we diagonalize perturbatively the mass matrix for the neutral leptons which appears in Sec. V B. First let us simplify the notation with the following definitions: $a = h_1 v'/\sqrt{2}$, $B = -h_2 V$, $c = h_3 v/\sqrt{2}$, $b = -h_4 V$, and $C = h_5 v/\sqrt{2}$. Then the mass terms for the neutral fermions now reads: $M_N = B\nu_e N_4 + a\nu_e N_3 - aN_1N_2 + cN_1N_4 + bN_2N_4 + CN_3N_4 + \text{H.c.}$; which in the basis $(\nu_e, N_1, N_2, N_3, N_4)$ becomes

$$M_N = \begin{pmatrix} 0 & 0 & 0 & a & B \\ 0 & 0 & -a & 0 & c \\ 0 & -a & 0 & 0 & b \\ a & 0 & 0 & 0 & C \\ B & c & b & C & 0 \end{pmatrix}.$$

Making a unitary transformation of $M_N$ to the new basis,

$$\begin{aligned} E_1 &= \alpha[-(b+c),(B-C),(B-C),(b+c),0], \\ E_2 &= \beta[(b-c),(B+C),-(B+C),(b-c),0], \\ E_3 &= \beta[(B+C),-(b-c),(b-c),(B+C),0], \\ E_4 &= \alpha[(B+C),(b+c),(b+c),-(B-C),0], \\ E_5 &= e_5 = [0,0,0,0,1], \end{aligned} \tag{A1}$$

where $\alpha = 1/[2(b+c)^2 + 2(B-C)^2]^{1/2}$ and $\beta = 1/[2(b-c)^2 + 2(B+C)^2]^{1/2}$, we get the mass matrix in the following block diagonal form:

$$M_N = \begin{pmatrix} -a & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 1/2\beta \\ 0 & 0 & 0 & -a & 1/2\alpha \\ 0 & 0 & 1/2\beta & 1/2\alpha & 0 \end{pmatrix}.$$

For the nondiagonal $3\times 3$ block we use the approximation $B = b \gg c \sim C$ and the perturbative expansion $v^m v 4^{n-m}/V^{n-1}$ for $n = 1,2,3,\ldots$ and $m = 0,1,2,\ldots \leqslant n$. Then we get

$$M_{3N} = \begin{pmatrix} 0 & 0 & b \\ 0 & 0 & b \\ b & b & 0 \end{pmatrix} + \begin{pmatrix} a & 0 & \frac{(C-c)}{2} + \frac{(C+c)^2}{8b} \\ 0 & -a & \frac{-(C-c)}{2} + \frac{(C+c)^2}{8b} \\ \frac{(C-c)}{2} + \frac{(C+c)^2}{8b} & \frac{-(C-c)}{2} + \frac{(C+c)^2}{8b} & 0 \end{pmatrix}.$$

Rotating now $M4_{3N}$ with

$$R = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \end{pmatrix},$$

we finally get

$$\begin{pmatrix} 0 & \frac{a}{\sqrt{2}}+\frac{(C-c)}{2} & \frac{a}{\sqrt{2}}-\frac{(C-c)}{2} \\ \frac{a}{\sqrt{2}}+\frac{(C-c)}{2} & \sqrt{2}\left[b+\frac{(C+c)^2}{8b}\right] & 0 \\ \frac{a}{\sqrt{2}}-\frac{(C-c)}{2} & 0 & -\sqrt{2}\left[b+\frac{(C+c)^2}{8b}\right] \end{pmatrix},$$

which we diagonalize using matrix perturbation theory up to second order in the perturbation [18]. After the algebra is done we get the following three eigenvalues:

$$m_1 \simeq \frac{a(c-C)}{b},$$

$$m_2 \simeq \sqrt{2}b + \frac{1}{\sqrt{2}b}\left(\frac{a}{\sqrt{2}}+\frac{(C-c)^2}{2}\right)^2,$$

and

$$m_3 = \simeq -\sqrt{2}b + \frac{1}{\sqrt{2}b}\left(\frac{a}{\sqrt{2}}-\frac{(C-c)^2}{2}\right)^2.$$

Notice that for $h_3 = h_5$ $(c=C)$ we get a zero-mass Majorana neutrino.


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