New constraints on ultrashort-ranged Yukawa interactions from atomic force microscopy

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Many extensions to the standard model lead to the possibility of new forces which would produce Yukawa corrections to Newtonian gravity. Models in which the gravitational and gauge interactions are unified at ~ 1 TeV using large extra dimensions could produce Yukawa-type corrections to the Newtonian gravitational law at submillimeter distances. In some models with $n=3$ extra dimensions, deviations from Newtonian gravity would occur at separations ~ 5 nm, a distance scale accessible to an atomic force microscope (AFM). Here we present constraints on the Yukawa corrections derived from the latest AFM Casimir force measurement by Mohideen *et al.* which are up to 19 times stronger than those obtained from their previous experiment. We then discuss new designs for AFM experiments which have the potential to significantly improve upon these constraints.

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I. INTRODUCTION

Motivated in part by the hierarchy problem, a number of theoretical models have been developed recently which raise the possibility that the energy scale at which gravity is unified with the other fundamental forces could be as low as ~ 1 TeV [1–3]. In these models, large extra spatial dimensions (which for compact dimensions could be as large as ~ 1 mm) arise which might be accessible experimentally [4]. While the effects of these new dimensions are generally model dependent, constraints on extra dimensional physics which are relatively independent of theory can be obtained from experiments searching for deviations from Newton's law of gravity at submillimeter distances. At the same time these experiments can also constrain hypothetical long-range interactions which naturally arise in many extensions to the standard model which are unrelated to extra dimensional physics. Such experiments can be classified according to the dominant background force acting between the test bodies used. For separations $\gtrsim 10^{-4}$ m gravity produces the dominant background, and a number of groups are conducting experiments testing Newtonian gravity at ~ 0.1 mm scales. For smaller separations, Casimir forces provide the dominant background force, and the experiments setting the best limits in this regime are those testing the Casimir force law. In this paper, we will briefly review the theoretical motivation and phenomenology used in sub-millimeter force experiments. We then present new limits extracted from a recent experiment by the Riverside group [5], which uses an atomic force microscope (AFM) to test the Casimir force. After comparing these results with existing limits, we conclude by describing future possibilities for using AFM experiments to set limits on new forces and extra dimensional physics.

II. PHENOMENOLOGY

Since gravity is a theory of spacetime, it is inevitable that the effects of extra dimensions will modify Newton's law of gravity which is only valid in a 4-dimensional spacetime. Although Newtonian gravity has been well-tested over long distance scales, our understanding of gravity at submillimeter scales is quite limited [6]. Phenomenologically, the deviations from Newton's law arise naturally in two different ways [7]. In models with large compact dimensions such as originally proposed by Arkani-Hamed, Dimopoulos, and Dvali (ADD) [1], one finds that the potential energy between two point particles with masses m_1 and m_2 separated by a distance $r \gg R_*$ (the characteristic size of the compact dimensions) is given by the usual Newtonian gravitational potential energy with a Yukawa correction [4,8,9]:

$$V(r) = -\frac{Gm_1m_2}{r}(1+\alpha e^{-r/\lambda}) \quad (r \gg R_*), \tag{1}$$

where the dimensionless constant α depends on the nature of the extra dimensions, and where $\lambda \sim R_*$. For example, for a toroidal compactification with all n extra dimensions having equal size, $\alpha = 2n$ [4,8,9]. When the separation between the masses decreases to the point where $r \ll R_*$, the usual inverse square law of gravity changes to a new power-law:

$$V(r) = -\frac{G_{4+n}m_1m_2}{r^{n+1}} \quad (r \ll R_*), \tag{2}$$

where $G_{4+n} \sim GR_*^n$ is the fundamental gravitational constant in the full $4+n$ dimensional spacetime. The size R_* of the extra dimensions, according to the ADD models, is then related to the energy scale $M_* \sim (1/G_{4+n})^{1/(2+n)}$ at which the unification occurs by

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$$R_* \sim \frac{1}{M_*}\left(\frac{M_{Pl}}{M_*}\right)^{2/n} \sim 10^{32/n-17} \text{ cm}, \tag{3}$$

where the Planck mass is $M_{Pl}=1/G^{1/2}\sim 10^{19}$ GeV, and we have assumed $M_*\sim 1$ TeV [1]. For $n=1$, $R_*\sim 10^{15}$ cm is obviously excluded by solar system tests of Newtonian gravity [6]. If $n=2$, $R_*\sim 1$ mm, which is the scale currently being probed by a number of sub-millimeter gravity experiments [10–13]. For $n=3$, Eq. (3) gives $R_*\sim 5$ nm which is roughly the range accessible to experiments using atomic force microscopy like those discussed below. It is important to recognize, however, that while $M_*\sim 1$ TeV is a natural scale, it is little more than a reasoned guess. Therefore, the scales suggested by Eq. (3) should be taken simply as a heuristic guide for designing experiments. It should also be noted that the Yukawa correction given in Eq. (1) arises naturally in ways unrelated to extra dimensional physics. For example, Yukawa potentials arise from new forces generated by the exchange of bosons of mass $\mu=1/\lambda$ (e.g., scalar axion, graviphoton, dilaton) [6,14–16].

A second class of extra dimensional models, such as that originally proposed by Randall and Sundrum [2], allows the extra dimensions to be non-compact, but warped. In these models, the leading order correction to the Newtonian potential energy between two point masses takes on a power-law form [2,7,17]:

$$V(r) = -\frac{Gm_1m_2}{r}\left(1+\frac{2}{3k^2r^2}\right) \quad (r \gg 1/k), \tag{4}$$

where $1/k$ is the warping scale. The introduction of a new length scale r_0 such as $1/k$ or R_* into a theory allows one to generalize Eq. (4) to arbitrary powers N:

$$V(r) = -\frac{Gm_1m_2}{r}\left[1+\alpha_N\left(\frac{r_0}{r}\right)^{N-1}\right], \tag{5}$$

where α_N is a dimensionless constant. Power-law corrections to Newtonian gravity also arise from new interactions involving the exchange of 2 massless quanta. For example, $N=2$ may arise from the simultaneous exchange of two photons or two massless scalars [18], and $N=3$ characterizes the exchange of two massless pseudoscalars [19,20]. Potentials with $N=5$ arise from massless axion exchange [20], and also from the exchange of a (massless) neutrino-antineutrino $(\nu\bar{\nu})$ pair [21].

All of the extra dimensional models discussed above are speculative at this stage, and much work remains to be done to test their consistency and viability. However, they are important in demonstrating that if extra spatial dimensions exist, new fundamental energy scales must also exist, and that these scales will modify Newtonian gravity at some level. Furthermore, from the models that have been developed we see that these modifications generally take the form of Yukawa or power-law potentials. Therefore, while there remain many unanswered questions regarding models of extra-dimensional physics, the phenomenology supporting short distance gravity experiments rests upon a much firmer footing.

TABLE I. Current constraints on power-law potentials of the form given by Eq. (5).

N	Experiment	$\alpha_N r_0^{N-1}$	Limit reference
1	Gundlach *et al.* [23]	1×10^{-9}	[23]
2	Gundlach *et al.* [23]	7×10^{-7} m^1	[23]
3	Spero *et al.* [24]	1×10^{-8} m^2	[26]
4	Mitrofanov *et al.* [25]	1×10^{-10} m^3	[26]

As noted earlier, experiments testing Newtonian gravity over separations $\gtrsim 10^{-3}$ m have set significant limits on the Yukawa coupling α for ranges $\lambda \gtrsim 10^{-3}$ m [6,22]. In addition, precise gravity experiments have set the best limits on power-law interactions for $N<5$ (see Table I). Motivated in part by the new extra dimensional models discussed above, several experimental groups [10–13] are now probing Newtonian gravity at sub-millimeter scales and have begun to set significant constraints on new physics at ranges ~ 0.1 mm. It is expected that within the next few years such experiments will extend our understanding of gravity down to separations where Casimir forces form the significant background.

III. NEW FORCES AND TESTS OF THE CASIMIR FORCE

Coincidentally, the past few years have also witnessed the first precise tests of the Casimir force [27–29]. Lamoreaux [30] used a torsion balance to measure the Casimir force between a disk and spherical lens, while Ederth [31] measured the force between two crossed cylinders. The practical implications of the Casimir force have been considered recently with a MEMS (microelectromechanical) device by Chan *et al.* at Bell Labs [32]. Finally, a series of force measurements specifically devised to test various aspects of the Casimir force has been conducted by the Riverside group using an atomic force microscope [5,33–35]. This entire experimental effort has motivated theorists to more carefully examine the effects of surface roughness, temperature, and finite conductivity which produce significant deviations from the idealized case of the Casimir force between two perfectly conducting, smooth bodies [36–42]. The inclusion of these corrections has been shown to be crucial in obtaining agreement between theory and experiment [5,33–35,37,38,43,44].

While the goal of all these experiments is to improve our understanding of the origin and behavior of Casimir forces, they have also been used to set limits on new interactions [45–48]. These limits are in addition to those obtained in Refs. [49–51] from earlier, much less precise Casimir and van der Waals force measurements [52,53]. It can be shown that Casimir force measurements are relatively ineffective compared to longer ranged gravity experiments in setting limits on power-law interactions of the form given by Eq. (5). The reason is that for $N<5$, such power-law forces between macroscopic bodies are relatively insensitive to the separation of the bodies when these are placed in close proximity [46]. Thus, there is no advantage to performing very short distance experiments, which are in any case generally less precise than larger scale gravity experiments, to search for power-law corrections to Newtonian gravity.

On the other hand, experiments searching for Yukawa interactions of the form given by Eq. (1) are only sensitive when the separation r of the test bodies is of order λ [6]. Hence, to constrain the Yukawa coupling α for a given value of λ, one must devise experiments such that r is of order λ. Therefore, if one wishes to constrain extra dimensions of size $R_* \lesssim 10^{-5}$ m using force measurements, one must inevitably confront large Casimir forces which grow rapidly for $r \lesssim 10^{-5}$ m [54]. This is why Casimir force experiments yield the best constraints on α for these separations.

IV. LIMITS FROM RECENT MOHIDEEN AFM EXPERIMENT

We turn next to the focus of this paper, which is the recent measurement of the Casimir force performed by the Riverside group [5] who used an atomic force microscope. (We will henceforth refer to the work of the Riverside group as "Mohideen *et al.*") Like the previous AFM measurements carried out by this group [33–35], this experiment measured the force between a metallized polystyrene sphere mounted on an AFM tip and a substrate composed of a flat sapphire disk. However, this experiment differed from the previous efforts in several important respects: (i) The sphere and disk were each coated with a layer of gold of thickness $\Delta = 86.6$ nm. Previously, aluminum coatings had been used instead, but this also required an additional surface layer of Au/Pd on top of the Al to prevent oxidation. The use of only a single gold layer in the new experiment significantly simplifies the calculations of the forces involved, since one can treat the gold layer as infinitely thick as far as the Casimir force is concerned. Also, because Au is much more dense than the previously used layers, stronger limits on new forces can be obtained. (ii) The gold layers were significantly smoother than previous coatings. The root-mean-square amplitude of the gold surfaces was measured to be only 1.0 ± 0.1 nm which means that corrections arising from the surface roughness could be neglected in all force calculations. (iii) Electrostatic forces which plague nearly all short distance force measurements were reduced to $\ll 1\%$ of the Casimir force at the shortest separation. This meant that no subtractions were required to separate Casimir forces from spurious electrostatic background forces. The electrostatic force was used to arrive at an independent measurement of the surface separation including separation on contact of the two surfaces. (iv) The measurements were performed over smaller separations a, $62\,\text{nm} \leqslant a \leqslant 350\,\text{nm}$ which means that this experiment can search for Yukawa forces with smaller ranges λ. However, the absolute error of the force measurements was somewhat larger than in the previous experiments, $\Delta F = 3.5 \times 10^{-12}$ N. This is due to the thinner gold coating used in [5] which led to poor thermal conductivity of the cantilever. At the smallest measured separations, this error was still less than 1% of the measured Casimir force. Thus, this experiment can be used to significantly tighten constraints on new Yukawa interactions as we will now show.

We begin by calculating the force arising between the AFM tip and the substrate using Eq. (1). First, it is easy to show that the Newtonian contribution is negligible. Since the diameter of the sphere (including the gold layer) was $2R = 191.3\,\mu$m, while the disk diameter was $L = 1$ cm, it follows that $R \ll L$ which allows us to consider each atom of the sphere as if it were placed directly above the center of the disk. In this case, the vertical component of the Newtonian gravitational force acting between the disk and a sphere atom of mass m_1 located at a distance $l \ll L$ above the disk is

$$f_{N,z}(l) = \frac{\partial}{\partial l}\left[G m_1 \rho_{\text{disk}} 2\pi \int_0^L r\,dr \int_l^{l+D} \frac{dz}{\sqrt{r^2+z^2}} \right] \approx -2\pi G m_1 \rho_{\text{disk}} D \left[1 - \frac{D+2l}{2L} \right], \quad (6)$$

where ρ_{disk} is the disk density, $D = 1$ mm is the thickness of the disk, and only the first order terms in D/L and l/L have been retained. The Newtonian gravitational force acting between the disk and the sphere is obtained from Eq. (6) by integration over the sphere

$$F_{N,z} \approx -\frac{8}{3}\pi^2 G \rho_{\text{disk}} \rho_{\text{sphere}} D R^3 \left(1 - \frac{D}{2L} - \frac{R}{L} \right), \quad (7)$$

where ρ_{sphere} is the density of the sphere. Even if the sphere and disk were composed of solid vacuo-distilled gold with $\rho_{\text{disk}} = \rho_{\text{sphere}} = 18.88 \times 10^3$ kg/m^3, one finds from Eq. (7) that $F_{N,z} \approx 6 \times 10^{-16}\,\text{N} \ll \Delta F$, so the usual Newtonian gravitational force can be neglected.

The force arising from the Yukawa correction to Newtonian gravity given by the second term of Eq. (1) should be calculated taking into account the actual compositions of the test bodies. For a polystyrene sphere $\rho_{\text{sphere}} = 1.06 \times 10^3$ kg/m^3, for a sapphire disk $\rho_{\text{disk}} = 4.0 \times 10^3$ kg/m^3, and for the vacuo-distilled gold covering layers $\rho_{\text{Au}} = 18.88 \times 10^3$ kg/m^3. The Yukawa force F_Y arising from Eq. (1) can be easily obtained using the same procedure used to calculate the Newtonian gravitational force. The result is

$$F_Y(a) = -4\pi^2 G \alpha \lambda^3 e^{-a/\lambda} R \times [\rho_{\text{Au}} - (\rho_{\text{Au}} - \rho_{\text{disk}}) e^{-\Delta/\lambda}] \times [\rho_{\text{Au}} - (\rho_{\text{Au}} - \rho_{\text{sphere}}) e^{-\Delta/\lambda}], \quad (8)$$

where a is the sphere-disk separation distance, and $\Delta = 86.6$ nm is the thickness of the gold coatings.

The most stringent limits on new forces are obtained for the smallest possible separation. According to Ref. [5], the rms deviation of the measured force ($F_{\text{experiment}}$) from the theoretical value (F_{theory}) of the sum of all the known forces (Casimir and electrostatic) is given by

$$\sigma_F = \sqrt{\frac{\sum (F_{\text{experiment}} - F_{\text{theory}})^2}{N_{\text{trials}}}} = 3.8 \text{ pN}, \quad (9)$$

where $N_{\text{trials}} = 2583$ is the total number of force measurements. This is slightly larger than 3.5 pN, the experimental

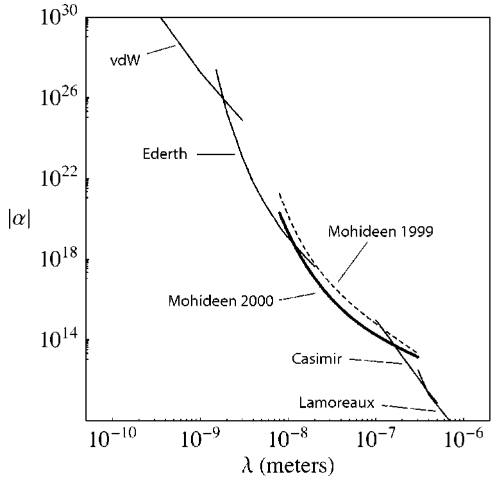

FIG. 1. Constraints on the Yukawa interactions of the form given by Eq. (1) from Casimir/van der Waals force measurements. ''Mohideen 2000'' refers to the limit obtained in this paper from the latest AFM experiment by the Riverside group [5], while ''Mohideen 1999'' refers to the limit [47] obtained from their previous AFM experiment which used aluminum surfaces [34]. ''vdW'' is the limit [51] obtained from older van der Waals force experiments [52], ''Ederth'' is the limit [48] from the recent experiment by Ederth [31] using two crossed cylinders, ''Casimir'' is the limit [49] from older Casimir force experiments [53], and ''Lamoreaux'' is the limit [45] from the Casimir force experiment by Lamoreaux [30].

uncertainty of the force measurement at $a=62$ nm. Therefore, to set constraints on the Yukawa coupling constant α, we will assume $|F_Y(a)|\leqslant\sigma_F=3.8$ pN. The resulting limit is represented by the bold curve labeled ''Mohideen 2000'' in Fig. 1. Note that for $\lambda\gg\Delta$ and $\lambda\gg a$, the exponential terms in Eq. (8) become unity, and as a result this curve is given by $\alpha=C/\lambda^3$, where $C=\sigma_F/(4\pi^2GR\rho_{disk}\rho_{sphere})$. However, the best constraints for a given experiment occur when $\lambda\sim a$, so the constraints for $\lambda\gg a$ are usually much weaker than those obtained by other experiments probing these separations.

Before comparing the constraint obtained here with limits obtained from other experiments, let us first discuss various sources of error which could affect our result. First, the uncertainty in the separation a arising from the surface roughness is approximately $\delta a\simeq 1$ nm which gives $\delta a/a\simeq 2\%$ for the closest separation. This will produce a significant effect when $\lambda\simeq\delta a\simeq 1$ nm, but as we will see below, other experiments produce better limits in this range. For $\lambda\simeq a$, where the most stringent limits are obtained, the resulting errors are only $\sim 2\%$ which can be neglected. Second, it is important that all known forces be included in F_{theory} in using Eq. (9). In short distance force experiments, electrostatic effects can be significant. However, in the Mohideen experiment, the measured residual potential difference of 3 mV between the gold sphere and disk leads to a force only 0.1% of the Casimir force at $a=62$ nm. Although this effect is small, it was included in F_{theory}.

The most significant difficulties arising in extracting limits from Casimir force experiments involve calculating the Casimir force between the interacting bodies. Because of practical difficulties, none of the most recent tests of the Casimir force use parallel plates for the interacting bodies, for which accurate calculations can be made. (The experiment by Carugno [13] uses parallel plates, but has not yet reached sufficient sensitivity to detect the Casimir force.) Rather, curved surfaces are used which requires the use of the proximity force theorem (PFT) to calculate the Casimir force [55]. Although the PFT produces reasonable results when applied to Casimir forces, it has not been rigorously proved in this context [42]. In spite of this, the errors which result when applying the PFT can be reliably estimated. For the configuration of a sphere above a disk used in [5], the error is of order a/R, i.e., much less than 1% (see Ref. [56] for the semi-classical proof of this result). Because of this we will assume that the use of the PFT in the analysis of the Mohideen experiment is valid.

Other difficulties in Casimir force calculations arise from the use of tabulated values of the complex dielectric constant as functions of frequency for the materials comprising the test bodies. It has been pointed out that if different interpolation schemes are applied to these discrete data, the resulting calculated Casimir force can differ by as much as 4% [42,57]. In addition, the tabulated dielectric properties were obtained using bulk samples and not the coatings which are actually used in the experiment. Even thick coatings (greater than 30 nm) can be sufficiently porous that their dielectric properties will deviate from the bulk values. It would then be best to directly measure the dielectric properties of the actual test bodies to accurately determine F_{Casimir} [58]. Since this cannot be done in the present context, and since there is no obvious discrepancy or inconsistency arising in the Mohideen experiment, we will not consider this problem here and leave it to future experiments to address this issue.

Finally, one must properly consider temperature corrections to the Casimir force. Recently this point was discussed by several authors with differing results [36,40,41,59,60]. According to the approach of Ref. [59], the dependence of the Casimir force on temperature is fundamentally different in the cases of ideal and real metals. For real metals the negative temperature corrections found in Ref. [59] are in contradiction with thermodynamical arguments, while the high temperature limit of the force is exactly half the value for an ideal metal. This difference does not disappear in the limit when the relaxation of a real metal goes to zero and its conductivity goes to infinity. As was noted in Ref. [61], the results of Ref. [59] are in contradiction with the experiment of Ref. [30]. In the approach of Ref. [60], all real metals, irrespective of their quality, are subjected to the same Casimir force as ideal metals for separations larger than several microns. A detailed analysis of the different approaches to calculating thermal corrections to the Casimir force is given in [36]. According to [36] the results of [59,60] are in error, while the correct results are found to be in agreement with Refs. [40,41] and with experiment. However, the separations used in the AFM experiments by Mohideen *et al.* are sufficiently small that temperature corrections to the Casimir force are negligible.

V. COMPARISON WITH OTHER LIMITS

As can be seen from Fig. 1, the Casimir force measurement between the gold surfaces carried out by Mohideen *et al.* strengthens the previously known constraints obtained from their earlier experiment [34] using aluminum surfaces (labeled by ''Mohideen 1999'') by as much as a factor of 19 within the range $4.3\times10^{-9}\,\mathrm{m}\leqslant\lambda\leqslant1.5\times10^{-7}\,\mathrm{m}$, with the most significant improvement occurring at $\lambda=(5-10)$ nm. These constraints are up to 4500 times more stringent than those obtained from older Casimir and van der Waals force measurements between dielectrics (curves labeled by ''Casimir'' and ''vdW'' respectively). In the same figure, the curve labeled by ''Lamoreaux'' exhibits the constraint obtained in Ref. [45] from the Casimir force measurement which used a torsion pendulum [30]. The curve denoted by ''Ederth'' gives the new constraints obtained in Ref. [48] from the recent experiment which measured the Casimir force between two crossed cylinders [31]. We see that curve ''Mohideen 2000,'' which exhibits the limits obtained in this paper, represents the most stringent bound in the interaction range $1.1\times10^{-8}\,\mathrm{m}\leqslant\lambda\leqslant1.5\times10^{-7}\,\mathrm{m}$.

Note that we have not included constraints from the recent experiment by Chan *et al.* [32]. The reason is that the amplitude of surface roughness for this experiment is so large (30 nm compared with 1 nm in [5]) that significant roughness corrections on both Casimir and hypothetical forces arise at separations $\lesssim100$ nm [37,46]. To calculate these corrections, one needs quantitative measurements of the roughness of the test bodies [37,46], but this information is not contained in Ref. [32]. However, the aim of this experiment was to demonstrate the first practical implication of the Casimir force rather than the precision comparison of theory and experiment.

It is evident from Fig. 1 that for separations $\lesssim10^{-5}$ m much work is needed to improve the sensitivity of force experiments to Yukawa interactions of the strength predicted by extra dimensional models ($\alpha\sim1-10$). As was shown in Ref. [45], the constraints following from the experiment in Ref. [30] can be improved by up to four orders of magnitude in the range $\lambda\sim10^{-4}$ m which is what is required to constrain physically interesting values of α. Still, experiments using atomic force microscopy [5,33–35] remain almost fifteen orders of magnitude below the sensitivity needed to reach the value $\alpha\sim1-10$ in the interaction range $\lambda\leqslant10^{-7}$ m. Furthermore, significant (and sometimes even more stringent) constraints on the extra dimensional mass scale M_* have been obtained from accelerator experiments [62–64], astrophysics [65–69], and cosmology [70–72]. However, one must remember that gravity and Casimir force experiments are also sensitive to other non-extra dimensional effects such as dilaton and moduli exchange which can lead to Yukawa forces with $\alpha\geqslant10$ [6,14–16]. Therefore, all experiments which can reduce the allowed region in the α-λ plane are meaningful.

VI. FUTURE AFM EXPERIMENTS

We believe that in the future the best ultrashort range limits will come from a new class of experiments designed specifically to search for new forces rather than to test the Casimir force. In such experiments the Casimir effect is an unwanted background that needs to be suppressed. One suppression technique proposed recently [54] relies on the observation that the Casimir force depends on the *electronic* properties of interacting samples, whereas any gravitational interactions (either conventional or new), and virtually all other proposed interactions, involve couplings to nuclei (as well as to electrons). Hence by comparing the interactions of two test masses which have very similar electronic properties, but different nuclei, with a common attracting source mass, we may be able to subtract out the common Casimir background. This would leave a residual nuclear-nuclear interaction between each of the samples and the source, and these should be different for the two samples. We refer to this technique as the *iso-electronic effect*, and it can be implemented in two ways. One is to choose as the test masses elements such as Cu and Au whose nuclei are quite different, but which are known to have very similar Casimir properties [42]. Alternatively, the test samples could be chosen to be isotopes of the same element whose nuclei would contain different numbers of neutrons, but whose electronic properties would be extremely similar. In either case, any residual differences in the electronic properties of the test masses would still have to be calculated, but this should in principle present no significant problems [73].

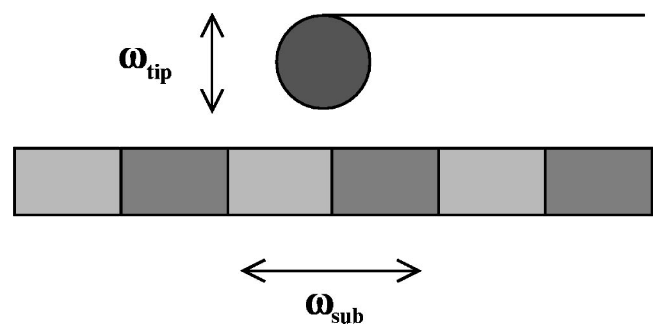

FIG. 2. Proposed experiment utilizing the iso-electronic effect [74,75]. A gold-coated sphere is attached to an oscillating AFM tip, while a substrate composed of alternating strips of different isotopes of the same element is oscillated horizontally beneath.

A possible design for an AFM experiment [74,75] which utilizes the iso-electronic effect, is shown schematically in Fig. 2. In this experiment, a gold-coated sphere would be attached to an AFM tip and oscillated with frequency ω_{tip}. The substrate would be composed of alternating strips of different isotopes of the same element, and would be oscillated horizontally with frequency ω_{sub}. The signal for a new force would then be a force on the tip that depends on ω_{sub} which should not be present if surface roughness and electrostatic effects are the same for the strips. Since the Yukawa force of the AFM tip on the substrate couples only to the mass within a range λ of the surfaces, the net sphere-substrate force will be proportional to the difference in the densities of the strips,

$$F_{\mathrm{Yukawa}}\propto(\rho_{\mathrm{sub}}-\rho'_{\mathrm{sub}}), \tag{10}$$

if the new force couples to mass as in the case of models with extra dimensions. For more general composition-dependent forces, the force difference will be even larger. Therefore, it is important to select elements having isotopes with the largest possible mass density differences. Possible candidates include ruthenium, osmium, nickel, and palladium, all of which can have isotope mass density differences exceeding $900\,\mathrm{kg/m^3}$. If such a design is to be successful, care must be taken to eliminate effects which could mimic the signal produced by a new force, such as insufficient vibration isolation, differences in surface roughness, and electrostatic effects arising from work function differences. Other possible designs for ultrashort distance experiments have been discussed in Refs. [12,54,74]. Our intent here is not so much to propose ultimate designs, but to suggest possibilities that would stimulate new experiments designed specifically to search for the effects of new forces and extra dimensions at ultrashort distances rather than to test the Casimir force law.

VII. CONCLUSIONS

It is clear that experiments which search for new forces in the Casimir regime will be faced with significant challenges. However, there is strong theoretical motivation to conduct such searches, and new techniques (including, possibly, those suggested here) will certainly be developed to overcome these obstacles. The AFM is a natural tool to be used in this effort as demonstrated by the experiments of Mohideen *et al.* [5,33–35]. Furthermore, this instrument can be easily adapted to experiments whose intent is to specifically search for new forces of both gravitational and nongravitational nature. Such laboratory force experiments have provided, and will continue to provide, important, and relatively model-independent constraints on new physics which complement those obtained from high-energy experiments and astrophysical observations.

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[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998).

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).

[3] A. Pérez-Lorenzana, hep-ph/0008333.

[4] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D **59**, 086004 (1999).

[5] B.W. Harris, F. Chen, and U. Mohideen, Phys. Rev. A **62**, 052109 (2000).

[6] E. Fischbach and C.L. Talmadge, *The Search for Non-Newtonian Gravity* (Springer-Verlag, New York, 1999).

[7] D.J.H. Chung, L. Everett, and H. Davoudiasl, Phys. Rev. D **64**, 065002 (2001).

[8] A. Kehagias and K. Sfetsos, Phys. Lett. B **472**, 39 (2000).

[9] E.G. Floratos and G.K. Leontaris, Phys. Lett. B **465**, 95 (1999).

[10] C.D. Hoyle, U. Schmidt, B.R. Heckel, E.G. Adelberger, J.H. Gundlach, D.J. Kapner, and H.E. Swanson, Phys. Rev. Lett. **86**, 1418 (2001).

[11] B. Schwarzschild, Phys. Today **53** (9), 22 (2000); R. Newman, Matters Grav. No. **15**, 18 (2000), gr-qc/0002027; No. **16**, 13 (2000), gr-qc/0009060.

[12] J.C. Long, A.B. Churnside, and J.C. Price, hep-ph/0009062; J.C. Long, H.W. Chan, and J.C. Price, Nucl. Phys. **B539**, 23 (1999).

[13] G. Carugno, Z. Fontana, R. Onofrio, and C. Rizzo, Phys. Rev. D **55**, 6591 (1997); G. Bressi, G. Carugno, A. Galvani, R. Onofrio, and G. Ruoso, Class. Quantum Grav. **17**, 2365 (2000).

[14] Y. Fujii, Int. J. Mod. Phys. A **6**, 3505 (1991).

[15] E.G. Adelberger, B.R. Heckel, C.W. Stubbs, and W.F. Rogers, Annu. Rev. Nucl. Part. Sci. **41**, 269 (1991).

[16] S. Dimopoulos and G.F. Giudice, Phys. Lett. B **379**, 105 (1996).

[17] J. Garriga and T. Tanaka, Phys. Rev. Lett. **84**, 2778 (2000).

[18] J. Sucher and G. Feinberg, in *Long-Range Casimir Forces*, edited by F.S. Levin and D.A. Micha (Plenum, New York, 1993).

[19] S.D. Drell and K. Huang, Phys. Rev. **91**, 1527 (1953); V.M. Mostepanenko and I.Yu. Sokolov, Sov. J. Nucl. Phys. **46**, 685 (1987).

[20] F. Ferrer and J.A. Grifols, Phys. Rev. D **58**, 096006 (1998).

[21] G. Feinberg and J. Sucher, Phys. Rev. **166**, 1638 (1968); S.D.H. Hsu and P. Sikivie, Phys. Rev. D **49**, 4951 (1994); E. Fischbach, Ann. Phys. (N.Y.) **247**, 213 (1996).

[22] G.L. Smith, C.D. Hoyle, J.H. Gundlach, E.G. Adelberger, B.R. Heckel, and H.E. Swanson, Phys. Rev. D **61**, 022001 (1999).

[23] J.H. Gundlach, G.L. Smith, E.G. Adelberger, B.R. Heckel, and H.E. Swanson, Phys. Rev. Lett. **78**, 2523 (1997).

[24] J.K. Hoskins, R.D. Newman, R. Spero, and J. Schultz, Phys. Rev. D **32**, 3084 (1985).

[25] V.P. Mitrofanov and O.I. Ponomareva, Sov. Phys. JETP **67**, 1963 (1988).

[26] E. Fischbach and D.E. Krause, Phys. Rev. Lett. **83**, 3593 (1999).

[27] V.M. Mostepanenko and N.N. Trunov, *The Casimir Effect and Its Applications* (Clarendon, Oxford, 1997).

[28] S.K. Lamoreaux, Am. J. Phys. **67**, 850 (1999).

[29] P.W. Milonni, *The Quantum Vacuum* (Academic, San Diego, 1994).

[30] S.K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997); **81**, 5475(E) (1998).
[31] T. Ederth, Phys. Rev. A **62**, 062104 (2000).
[32] H.B. Chan, V.A. Aksyuk, R.N. Kleiman, D.J. Bishop, and F. Capasso, Science **291**, 1941 (2001).
[33] U. Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998).
[34] A. Roy, C.-Y. Lin, and U. Mohideen, Phys. Rev. D **60**, 111101(R) (1999).
[35] A. Roy and U. Mohideen, Phys. Rev. Lett. **82**, 4380 (1999).
[36] G.L. Klimchitskaya and V.M. Mostepanenko, Phys. Rev. A **63**, 062108 (2001).
[37] G.L. Klimchitskaya, A. Roy, U. Mohideen, and V.M. Mostepanenko, Phys. Rev. A **60**, 3487 (1999).
[38] G.L. Klimchitskaya, U. Mohideen, and V.M. Mostepanenko, Phys. Rev. A **61**, 062107 (2000).
[39] V.B. Bezerra, G.L. Klimchitskaya, and V.M. Mostepanenko, Phys. Rev. A **62**, 014102 (2000).
[40] M. Bordag, B. Geyer, G.L. Klimchitskaya, and V.M. Mostepanenko, Phys. Rev. Lett. **85**, 503 (2000).
[41] C. Genet, A. Lambrecht, and S. Reynaud, Phys. Rev. A **62**, 012110 (2000).
[42] A. Lambrecht and S. Reynaud, Eur. Phys. J. D **8**, 309 (2000).
[43] S.K. Lamoreaux, Phys. Rev. Lett. **83**, 3340 (1999).
[44] U. Mohideen and A. Roy, Phys. Rev. Lett. **83**, 3341 (1999).
[45] M. Bordag, B. Geyer, G.L. Klimchitskaya, and V.M. Mostepanenko, Phys. Rev. D **58**, 075003 (1998).
[46] M. Bordag, B. Geyer, G.L. Klimchitskaya, and V.M. Mostepanenko, Phys. Rev. D **60**, 055004 (1999).
[47] M. Bordag, B. Geyer, G.L. Klimchitskaya, and V.M. Mostepanenko, Phys. Rev. D **62**, 011701(R) (2000).
[48] V.M. Mostepanenko and M. Novello, Phys. Rev. D **63**, 115003 (2001).
[49] V.M. Mostepanenko and I.Yu. Sokolov, Phys. Lett. A **132**, 313 (1988).
[50] V.M. Mostepanenko and I.Yu. Sokolov, Phys. Rev. D **47**, 2882 (1993).
[51] M. Bordag, V.M. Mostepanenko, and I.Yu. Sokolov, Phys. Lett. A **187**, 35 (1994).
[52] Y.N. Israelachvili and D. Tabor, Proc. R. Soc. London **A331**, 19 (1972).
[53] B.V. Derjaguin, I.I. Abrikosova, and E.M. Lifshitz, Q. Rev., Chem. Soc. **10**, 295 (1956).
[54] D.E. Krause and E. Fischbach, in *Gyros, Clocks, and Interferometers: Testing Relativistic Gravity in Space*, edited by C. Lämmerzahl, C.W.F. Everitt, and F.W. Hehl (Springer-Verlag, Berlin, 2001), pp. 292–309.
[55] J. Blocki, J. Randrup, W.J. Światecki, and C.F. Tsang, Ann. Phys. (N.Y.) **105**, 427 (1977).
[56] M. Schaden and L. Spruch, Phys. Rev. Lett. **84**, 459 (2000).
[57] M. Boström and Bo E. Sernelius, Phys. Rev. A **61**, 046101 (2000).
[58] S.K. Lamoreaux, Phys. Rev. A **59**, R3149 (1999).
[59] M. Boström and Bo E. Sernelius, Phys. Rev. Lett. **84**, 4757 (2000).
[60] V.B. Svetovoy and M.V. Lokhanin, Phys. Lett. A **280**, 177 (2001).
[61] S.K. Lamoreaux, quant-ph/0007029.
[62] B. Abbott *et al.*, Phys. Rev. Lett. **86**, 1156 (2001).
[63] ALEPH Collaboration, P. Abreu *et al.*, Phys. Lett. B **485**, 45 (2000); Eur. Phys. J. C **17**, 53 (2000); DELPHI Collaboration, M. Acciarri *et al.*, Phys. Lett. B **464**, 135 (1999); **470**, 268 (1999); **470**, 281 (1999); OPAL Collaboration, G. Abbiendi *et al.*, *ibid.* **465**, 303 (1999); Eur. Phys. J. C **13**, 553 (2000); **18**, 253 (2000).
[64] H1 Collaboration, C. Adloff *et al.*, Phys. Lett. B **479**, 358 (2000).
[65] C. Hanhart, D.R. Phillips, S. Reddy, and M.J. Savage, Nucl. Phys. **B595**, 335 (2001).
[66] V. Barger, T. Han, C. Kao, and R.-J. Zhang, Phys. Lett. B **461**, 34 (1999).
[67] S. Cullen and M. Perelstein, Phys. Rev. Lett. **83**, 268 (1999).
[68] S. Hannestad and G. G. Raffelt, Phys. Rev. Lett **87**, 051304 (2001).
[69] S. Cassisi, V. Castellani, S. Degl'Innocenti, G. Fiorentini, and B. Ricci, Phys. Lett. B **481**, 323 (2000).
[70] L.J. Hall and D. Smith, Phys. Rev. D **60**, 085008 (1999).
[71] M. Fairbairn, Phys. Lett. B **508**, 335 (2001).
[72] S. Hannestad, Phys. Rev. D **64**, 023515 (2001).
[73] A. Lambrecht (private communication).
[74] E. Fischbach, S.W. Howell, S. Karunatillake, D.E. Krause, R. Reifenberger, and M. West, Class. Quantum Grav. **18**, 2427 (2001).
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