Charge and color breaking conditions associated with the top quark Yukawa coupling

C. Le Mouël*
Department of Theoretical Physics, Aristotle University of Thessaloniki, 54006 Thessaloniki, Greece and Physique Mathématique et Théorique, UMR No. 5825–CNRS, Université Montpellier II, F–34095 Montpellier Cedex 5, France
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In the minimal supersymmetric extension of the standard model, the charge and color breaking (CCB) vacuum in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$ typically deviates largely from all D-flat directions, due to the large effects induced by the presence of the top quark Yukawa coupling. As a result, the critical CCB bound on the trilinear soft term A_t becomes more restrictive than the D-flat bound $A_t^2 \leqslant 3(m_{\tilde{t}_L}^2+m_{\tilde{t}_R}^2+m_2^2)$. For large $\tan\beta$, we consider the effect of a splitting between the soft squark masses $m_{\tilde{t}_L},m_{\tilde{t}_R}$ on this optimal CCB bound and give a useful approximation for it, accurate within 1% in all interesting phenomenological cases. The physical implications on the top squark mass spectrum and the one-loop upper bound on the lightest CP-even Higgs boson mass are also discussed in a model-independent way.

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I. INTRODUCTION

In the minimal supersymmetric standard model (MSSM), spontaneous symmetry breaking may occur into a dangerous charge and color breaking (CCB) vacuum, therefore, destabilizing the realistic physical electroweak (EW) vacuum [1,2]. To avoid this danger, CCB conditions must be imposed on the soft supersymmetry (SUSY) breaking terms which enter the scalar potential of the MSSM. Such conditions and their physical consequences have been extensively discussed in the literature in a large variety of directions in the MSSM scalar field space [see, e.g., [1–3] and references therein]. CCB vacua associated with the top quark Yukawa coupling represent a special class of dangerous vacua in the sense that they typically deviate largely from D-flat directions and can furthermore develop in the close vicinity of the EW vacuum. Such features were first investigated in Ref. [2] in the field direction $(H_1,H_2,\tilde{t}_L,\tilde{t}_R,\tilde{\nu}_L)$, neglecting possible deviations from the $SU(3)_c$ D-flat direction. In [3], we considered the restricted plane $(H_2,\tilde{t}_L,\tilde{t}_R)$ and proposed a new method to evaluate analytically, in a fully model-independent way, the vacuum expectation values (VEV's) of the CCB vacuum [3]. We showed that the effect of deviations from the $SU(3)_c$ D-flat direction, typically small in minimal supergravity (MSUGRA) models (as considered in Ref. [2]), cannot be neglected in a model-independent way and tend to make more restrictive the CCB condition on the trilinear soft term A_t. This new feature appears in models with a large splitting at the EW scale between the soft squark masses $m_{\tilde{t}_L},m_{\tilde{t}_R}$, as occurs for instance in some string effective field theories [4,5]. For large $\tan\beta$ and $m_{A^0} \gg m_{Z^0}$, it was also pointed out that the one-loop Higgs maximal mixing for the top squark masses is largely ruled out by CCB considerations [3], assuming a common soft squark mass $M_{\rm SUSY}=m_{\tilde{t}_L}=m_{\tilde{t}_R}$, and taking furthermore a simplified value for the Higgs maximal mixing $A_t=\sqrt{6}m_{\tilde{t}}$, which is accurate only for $m_{\tilde{t}} \equiv \sqrt{M_{\rm SUSY}^2+m_t^2} \gg m_t$ [6,7].

In the present paper, we first summarize our method to evaluate the optimal CCB condition on A_t in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$ and extend then the latter result by incorporating the possibility of a large mass splitting between the soft squark masses $m_{\tilde{t}_L},m_{\tilde{t}_R}$, taking also for comparison the exact one-loop Higgs maximal mixing. We still concentrate on the asymptotic regime $\tan\beta=+\infty$, which actually provides a benchmark CCB bound on A_t with properties useful for phenomenological applications. In particular, in the extended plane $(H_1,H_2,\tilde{t}_L,\tilde{t}_R)$, this benchmark value proves to put an upper bound on the CCB allowed values for the top squark mixing term $|\tilde{A}_t| \equiv |A_t+\mu/\tan\beta|$ [8]. We present an analytic approximation for it, accurate within 1% in all interesting phenomenological cases. Finally, we consider some physical implications of this bound, including the effect on the top squark mass spectrum and the one-loop upper bound on the lightest CP-even Higgs boson mass.

II. THE CCB VACUUM IN THE PLANE $(H_2,\tilde{t}_L,\tilde{t}_R)$

At the tree level, the effective potential in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$ reads [1–3]

$$V_3=m_2^2H_2^2+m_{\tilde{t}_L}^2\tilde{t}_L^2+m_{\tilde{t}_R}^2\tilde{t}_R^2-2Y_tA_tH_2\tilde{t}_L\tilde{t}_R+Y_t^2(H_2^2\tilde{t}_L^2+H_2^2\tilde{t}_R^2+\tilde{t}_L^2\tilde{t}_R^2)+\frac{g_1^2}{8}\left(H_2^2+\frac{\tilde{t}_L^2}{3}-\frac{4\tilde{t}_R^2}{3}\right)^2+\frac{g_2^2}{8}(H_2^2-\tilde{t}_L^2)^2+\frac{g_3^2}{6}(\tilde{t}_L^2-\tilde{t}_R^2)^2, \quad (1)$$

where H_2 denotes the neutral component of the corresponding Higgs scalar $SU(2)_L$ doublet, and $\tilde{t}_L$, $\tilde{t}_R$ are, respectively, the left and right top squark fields. All fields are supposed to be real. $H_2,\tilde{t}_L$, the top quark Yukawa coupling Y_t and the trilinear soft term A_t are also assumed to be positive, which can be arranged by a phase redefinition of the fields. Finally, positivity for the squared soft squark masses

*Electronic address: lemouel@physics.auth.gr

$m^2_{\tilde{t}_L}, m^2_{\tilde{t}_R}$ is assumed to avoid an obvious instability of the potential at the origin of the fields.

As is well known, this potential becomes negative in the D-flat direction $|H_2|=|\tilde{t}_L|=|\tilde{t}_R|$, unless the well-known condition [1,2],

$$A_t^2 \leqslant (A_t^D)^2 \equiv 3(m^2_{\tilde{t}_L}+m^2_{\tilde{t}_R}+m_2^2) \tag{2}$$

is verified. If the top quark Yukawa coupling were as small as the Yukawa couplings of the first two generations of quarks, this relation would provide an accurate necessary and sufficient condition to avoid CCB in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$ [1–3]. But this is not the case and large deviations of the CCB vacuum from all D-flat directions are typically observed, making more restrictive the critical CCB bound on A_t. In fact, looking at the extremal equations associated with the potential V_3, Eq. (1), one finds that the minimum of the potential lies in the D-flat direction only for

$$m^2_{\tilde{t}_L}=m^2_{\tilde{t}_R}=m_2^2. \tag{3}$$

Violations of these relations trigger deviations from the D-flat directions [3]. In particular, the first relation in Eq. (3) is intimately related to the deviation from the $SU(3)_c$ D-flat direction [3], previously disregarded in [2]. We can conveniently keep track of this feature by defining $f\equiv \tilde{t}_R/\tilde{t}_L$. Alignment of the CCB vacuum in the $SU(3)_c$ D-flat direction will correspond to $\langle f\rangle = \pm 1$.

Replacing $\tilde{t}_R \to f\tilde{t}_L$ in V_3, Eq. (1), which is unambiguous provided by $\tilde{t}_L \neq 0$, the extremal equation associated with a nontrivial CCB VEV $\langle \tilde{t}_L\rangle$ is straightforwardly solved, and gives [3]

$$\langle \tilde{t}_L\rangle^2 = -2\frac{B_3}{A_3}, \tag{4}$$

with

$$B_3 = H_2^2\frac{(12Y_t^2-4g_1^2)f^2+12Y_t^2+g_1^2-3g_2^2}{12} - 2A_tY_tfH_2 + m^2_{\tilde{t}_L} + f^2 m^2_{\tilde{t}_R}, \tag{5}$$

$$A_3 = 4Y_t^2f^2 + \frac{g_1^2(4f^2-1)^2}{18} + \frac{g_2^2f^4}{2} + \frac{2g_3^2(f^2-1)^2}{3}, \tag{6}$$

where the field parameters H_2, f take their vacuum expectation values $H_2=\langle H_2\rangle$, $f=\langle f\rangle$. Consistency of this solution requires that $B_3\leqslant 0$, implying on one hand $\langle f\rangle \geqslant 0$ and therefore $\langle H_2\rangle, \langle \tilde{t}_{L/R}\rangle \geqslant 0$ [3]. On the other, it is easily shown that for $A_t \leqslant A_t^{(0)}$, where

$$A_t^{(0)} \equiv m_{\tilde{t}_L}\sqrt{1-\frac{g_1^2}{3Y_t^2}} + m_{\tilde{t}_R}\sqrt{1-\frac{(3g_2^2-g_1^2)}{12Y_t^2}} \simeq m_{\tilde{t}_L}+m_{\tilde{t}_R} \tag{7}$$

the global minimum of the potential V_3, Eq. (1), is automatically trapped in the plane $\tilde{t}_R=\tilde{t}_L=0$ and cannot be lower than the EW vacuum. (In this situation, we have always $B_3 \geqslant 0$, whatever $\langle f\rangle$ is.) The bound $A_t^{(0)}$ therefore provides a very simple sufficient condition on A_t to avoid CCB in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$ [3]. It is also restrictive enough to secure some interesting model-dependent scenarios. This includes the infrared quasifixed point scenario in an MSUGRA context, for both low and large $\tan\beta$ [3]. In a quite different context, we also mention the effective MSSM recently proposed, coming from an underlying model where the top quark or top squark sector is living in the bulk of an extra dimension [9]. In this case, the trilinear soft term $|A_t|$ is related to the soft squark mass $m_{\tilde{t}_R}$, with $|A_t| = m_{\tilde{t}_R}$. A quick look at Eq. (7) shows that this model is also exempt from a CCB vacuum in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$.

The previous sufficient bound to avoid CCB in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$ can be improved. This requires some information on the remaining extremal equations associated with H_2 and f. Taking $\tilde{t}_L=\langle\tilde{t}_L\rangle$ in the potential V_3, Eq. (1), these equations are straightforwardly obtained. The derivative with respect to f provides an equation quadratic in H_2 and quartic in f

$$a_3fH_2^2 + b_3H_2 + c_3f = 0, \tag{8}$$

where $a_3 = 36Y_t^4(-1+f^2)+O(g_i^2)$, $b_3 = O(g_i^2)$, $c_3 = -36Y_t^2(m^2_{\tilde{t}_L} - f^2m^2_{\tilde{t}_R}) + O(g_i^2)$ (the exact value of the coefficients a_3, b_3, c_3 can be found in [3]). Numerical investigation shows that, at the CCB vacuum, the gauge contributions $\sim O(g_i^2)$ to the coefficients a_3, b_3, c_3 can be safely neglected. Doing so, the extremal equation, Eq. (8), is solved exactly and gives

$$\langle f\rangle \simeq \sqrt{\frac{m^2_{\tilde{t}_L}+Y_t^2\langle H_2\rangle^2}{m^2_{\tilde{t}_R}+Y_t^2\langle H_2\rangle^2}}. \tag{9}$$

A more transparent approximation of the deviation parameter $\langle f\rangle$, independent of the CCB VEV $\langle H_2\rangle$, can be subsequently derived [3]:

$$\langle f\rangle \simeq f_3^{(0)} \equiv \sqrt{\frac{A_t^2+2m^2_{\tilde{t}_L}-m^2_{\tilde{t}_R}}{A_t^2+2m^2_{\tilde{t}_R}-m^2_{\tilde{t}_L}}}. \tag{10}$$

The accuracy of this approximated value proves to be excellent, whatever the values of the soft parameters $A_t, m_{\tilde{t}_L}, m_{\tilde{t}_R}$ are: it fits the exact result $\langle f\rangle$ within 5%, or even less for not a too large splitting between the soft squark masses $m_{\tilde{t}_L}, m_{\tilde{t}_R}$ [3]. It also clearly indicates that alignment of the CCB vacuum in the $SU(3)_c$ D-flat direction is a model-dependent feature which occurs in two different cases:

For $A_t \geqslant 2\,\mathrm{Max}[m_{\tilde{t}_L}, m_{\tilde{t}_R}]$. However, A_t is also very large compared to the critical CCB bound on A_t above which CCB occurs.

For $m^2_{\tilde{t}_L}=m^2_{\tilde{t}_R}$ [the first mass relation in Eq. (3)]. Then, the potential V_3, Eq. (1), has an underlying approximate symmetry $\tilde{t}_L \leftrightarrow \tilde{t}_R$ broken by tiny $O(g_1^2,g_2^2)$ contributions. Therefore, any nontrivial CCB extremum is necessarily nearly aligned in the $\mathrm{SU}(3)_c$ D-flat direction. Model building may favor the latter situation. This occurs approximately, e.g., in MSUGRA models [10]. In Ref. [2], such models were investigated, neglecting in a first approximation the effect of the deviation from the $\mathrm{SU}(3)_c$ D-flat direction and CCB conditions [evaluated in the extended plane $(H_1,H_2,\tilde{t}_L,\tilde{t}_R,\tilde{\nu}_L)$] were given in terms of the universal soft SUSY breaking parameters at the grand unified theory (GUT) scale. However, in a model-independent way, the possibility of a large splitting between the soft squark masses is not ruled out and is even favored in some interesting alternatives to the MSUGRA scenarios. Some of them incorporate a substantial amount of nonuniversality between the soft SUSY breaking terms [4,5,11,12] and possibly show large violations of the relation $m_{\tilde{t}_L}=m_{\tilde{t}_R}$ at the EW scale. This occurs, e.g., in some anomaly mediated scenarios [4,5]. In this case, the effect of the deviation from the $\mathrm{SU}(3)_c$ D-flat direction on the critical CCB bound cannot be neglected, as will be illustrated in the following, and must be properly taken into account.

The extremal equation associated with H_2 is cubic in H_2 and quartic in f. It reads [3]

$$\alpha_3 H_2^3+\beta_3 H_2^2+\gamma_3 H_2+\delta_3=0, \tag{11}$$

where

$$\alpha_3=-36Y_t^4(f^2+1)^2+[3g_3^2(g_1^2+g_2^2)+4g_1^2g_2^2](f^2-1)^2 + 6Y_t^2g_1^2(4f^4+6f^2-1)+18Y_t^2g_2^2(2f^2+1), \tag{12}$$

$$\beta_3=9A_tY_tf[(12Y_t^2-4g_1^2)f^2+12Y_t^2+g_1^2-3g_2^2], \tag{13}$$

$$\gamma_3=-72A_t^2f^2Y_t^2-3(m^2_{\tilde{t}_L}+f^2m^2_{\tilde{t}_R})[(12Y_t^2-4g_1^2)f^2+12Y_t^2 + g_1^2-3g_2^2]+m_2^2[72Y_t^2f^2+g_1^2(4f^2-1)^2+9g_2^2 + 12g_3^2(f^2-1)^2], \tag{14}$$

$$\delta_3=36A_tY_tf(m^2_{\tilde{t}_L}+f^2m^2_{\tilde{t}_R}). \tag{15}$$

This equation, considered as a cubic polynomial in H_2, may be solved exactly. In a compact trigonomical form, the CCB VEV $\langle H_2\rangle$ reads

$$\langle H_2\rangle=-\frac{\beta_3}{3\alpha_3}\left(1-2\frac{\sqrt{-\mathcal{N}}}{\beta_3}\cos\left[\frac{\phi+4\pi}{3}\right]\right), \tag{16}$$

$$\cos\phi=-\frac{\mathcal{M}}{2\sqrt{-\mathcal{N}^3}}, \quad \phi\in[0,\pi], \tag{17}$$

where

$$\mathcal{M}\equiv 2\beta_3^3-9\alpha_3\beta_3\gamma_3+27\alpha_3^2\delta_3 \tag{18}$$

$$\mathcal{N}=-\beta_3^2+3\alpha_3\gamma_3, \tag{19}$$

where the coefficient $\alpha_3,\beta_3,\gamma_3,\delta_3$, Eqs. (12)–(15), should be evaluated with $f=\langle f\rangle$. For clarity, let us add some comments concerning this solution.

It can be shown that a CCB vacuum may develop in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$ only if the extremal equation, Eq. (11), has three real roots in H_2. Then, these roots are found to be positive and the CCB VEV $\langle H_2\rangle$ is given by the intermediate one [3], which is written in its trigonomical analytic form in Eqs. (16)–(19). $\langle H_2\rangle$ depends on $\langle f\rangle$, for which we have an accurate approximation $\langle f\rangle\simeq f_3^{(0)}$, Eq. (10). This way, we obtain in turn an accurate approximation for $\langle H_2\rangle$, which fits the exact result within less than 1%. This accuracy is enough in particular to evaluate the critical CCB bound on A_t with a precision of order ~ 1 GeV.[1]

The extremal equation, Eq. (11), has three real roots in H_2 if (and only if) the following condition is verified:

$$\mathcal{C}_3\equiv\mathcal{M}^2+4\mathcal{N}^3\leqslant 0. \tag{20}$$

As noted above, no CCB vacuum may develop for $\mathcal{C}_3\geqslant 0$. Therefore, the quantity $\mathcal{C}_3$ provides an additional criterion to avoid CCB in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$, which enables us to improve the sufficient bound $A_t^{(0)}$, Eq. (7). Taking $f=f_3^{(0)}$, and giving values to all parameters except A_t, the equation $\mathcal{C}_3=0$ can be solved numerically as a function of A_t. Below the largest positive solution thus obtained, denoted $A_t^{(1)}$, the relation $\mathcal{C}_3\geqslant 0$ is always found. Typically, the hierarchy $A_t^{(1)}\geqslant A_t^{(0)}$ holds, except in the unphysical regime where the lightest stop mass is vanishing $m_{\tilde{t}_1}\simeq 0$ [3]. Nevertheless, defining

$$A_t^{\mathrm{suf}}\equiv\mathrm{Max}[A_t^{(0)},A_t^{(1)}], \tag{21}$$

this quantity may be identified as the optimal sufficient bound on A_t below which no CCB vacuum may develop in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$. For $A_t=A_t^{\mathrm{suf}}$, a consistent local CCB vacuum with real and positive VEVs $(\langle H_2\rangle,\langle\tilde{t}_L\rangle,\langle f\rangle)$ begins to develop and soon becomes global with increasing A_t [3].

The evaluation of the critical CCB bound on A_t, above which CCB occurs, requires some additional information on the EW vacuum. At the tree level, this vacuum is determined by the extremal equations

$$\frac{m_1^2-m_2^2\tan^2\beta}{\tan^2\beta-1}-\frac{m^2_{Z^0}}{2}=0, \tag{22}$$

$$(m_1^2+m_2^2)\tan\beta-m_3^2(1+\tan^2\beta)=0, \tag{23}$$

where $\tan\beta\equiv v_2/v_1$ is the ratio of the VEV's v_1,v_2 of the EW vacuum (with $v\equiv\sqrt{v_1^2+v_2^2}=174$ GeV). For $\tan\beta\geqslant 1$, the minimal value of the EW potential reads

[1]This precision can be improved at will with the iterative procedure proposed in [3].

$$\langle V\rangle|_{\rm EW}=-\frac{[m_2^2-m_1^2+\sqrt{(m_1^2+m_2^2)^2-4m_3^4}]^2}{2(g_1^2+g_2^2)}. \quad (24)$$

At a consistent EW vacuum, this quantity depends only on two free parameters that may conveniently be chosen to be $\tan\beta$ and the pseudoscalar mass $m_{A^0}=\sqrt{m_1^2+m_2^2}$. The critical CCB bound $A_t^{\rm CCB}$ is found by comparing the depth of the potential at the EW vacuum and at the CCB vacuum:

$$\mathrm{CCB}\Leftrightarrow\langle V_3\rangle<\langle V\rangle|_{\rm EW} \quad (25)$$

$$\Leftrightarrow A_t>A_t^{\rm CCB}[m_{\tilde t_L},m_{\tilde t_R};m_1,m_2,m_3,Y_t,g_1,g_2,g_3]. \quad (26)$$

The maximal depth of the potential V_3, Eq. (1), can be given an analytical (though complicated) expression by taking the excellent approximations of the CCB VEV's given in Eqs. (4), (10), and (16). This way, we incorporate all possible deviations of the CCB vacuum from all D-flat directions, including the $SU(3)_c$ D-flat one previously disregarded [2]. When all parameters are chosen [except A_t], the critical CCB bound $A_t^{\rm CCB}$ is straightforwardly obtained by a numerical scan of the region $A_t\geqslant A_t^{\rm suf}$. We have $A_t^{\rm CCB}\simeq A_t^{\rm suf}$, because the EW potential is not very deep $\langle V\rangle|_{\rm EW}\sim-m_{Z^0}^4/(g_1^2+g_2^2)$, whereas the depth of the CCB potential increases rapidly with A_t. This simple procedure provides an excellent approximated value for the critical CCB bound which fits the exact result $A_t^{\rm CCB}$ with an accuracy of order 1 GeV.

Let us add a few words concerning the impact of radiative corrections on $A_t^{\rm CCB}$. As is well known, the results obtained with the tree-level approximation of the potential may incorporate leading one-loop corrections, provided all quantities are evaluated at an appropriate field-dependent scale [13,2]. This numerical observation was, in fact, intensively used in the context of CCB studies in order to use the relative simplicity of the tree-level potential [2]. For the EW potential at the EW vacuum, the appropriate scale is the SUSY scale $Q_{\rm SUSY}$, with $Q_{\rm SUSY}\sim M_{\rm SUSY}\equiv\sqrt{(m_{\tilde t_L}^2+m_{\tilde t_R}^2)/2}$ for $M_{\rm SUSY}\geqslant m_t$, whereas for $M_{\rm SUSY}\lesssim m_t$, $Q_{\rm SUSY}$ should be taken at a more significant SUSY mass. In the vicinity of $A_t\simeq A_t^{\rm suf}$ [$\simeq A_t^{\rm CCB}$], it can be shown that the scale adapted to the CCB potential V_3, Eq. (1), at the CCB vacuum, is also the SUSY scale $Q_{\rm SUSY}$. This result, which is in agreement with [2], is due essentially to the fact that the CCB vacuum proves to be rather close to the EW vacuum [3]. Therefore, provided the tree-level comparison in Eq. (25) is performed at the SUSY scale $Q_{\rm SUSY}$, the critical bound $A_t^{\rm CCB}$ thus obtained should also incorporate leading one-loop corrections.

III. THE CCB CONDITIONS ON A_t FOR LARGE $\tan\beta$

In this section, we investigate the effect of a mass splitting between the soft squark masses $m_{\tilde t_L},m_{\tilde t_R}$ on the CCB bounds presented in Sec. II. We focus on the asymptotic regime $\tan\beta=+\infty$. This choice is motivated by the nice properties of the critical CCB condition on A_t in this regime. Before turning to the numerical analysis, let us first briefly comment on the latter.

In the plane $(H_2,\tilde t_L,\tilde t_R)$, the critical CCB bound $A_t^{\rm CCB}$, Eq. (26), proves to be decreasing for increasing $\tan\beta$, and increasing for increasing m_{A^0}. Moreover, in the limit $\tan\beta=+\infty$, it can be shown that the most restrictive CCB bound in the plane $(H_2,\tilde t_L,\tilde t_R)$ is obtained, whatever the values m_{A^0} and $\tan\beta(\geqslant 1)$ are. Therefore, the benchmark value $A_t^{\rm CCB}|_{\tan\beta=+\infty}$ can be considered as an optimal sufficient condition to avoid a dangerous CCB vacuum in this plane:

$$A_t\leqslant A_t^{\rm CCB}|_{\tan\beta=+\infty}\Rightarrow \text{No CCB in the plane } (H_2,\tilde t_L,\tilde t_R). \quad (27)$$

We stress however that this interesting property does not prevent a CCB situation from outside the plane $(H_2,\tilde t_L,\tilde t_R)$. In particular, CCB may still occur in the extended plane $(H_1,H_2,\tilde t_L,\tilde t_R)$, as a full investigation of the optimal CCB conditions in this plane indeed shows. This will be presented elsewhere [8]. To enlighten the importance of the value $A_t^{\rm CCB}$ for $\tan\beta=+\infty$, we borrow an interesting property of the CCB bound on the stop mixing term $\tilde A_t\equiv A_t+\mu/\tan\beta$ from this study. As is well known, this quantity plays a central role in Higgs phenomenology [6,7,14]. It can be shown that if $|\tilde A_t|$ exceeds some critical value, which depends on $m_{A^0},\tan\beta,\mu$ and also $m_{\tilde t_L},m_{\tilde t_R}$, CCB occurs in the plane $(H_1,H_2,\tilde t_L,\tilde t_R)$. In the interesting phenomenological region $m_{\tilde t_L},m_{\tilde t_R}\gtrsim m_t$, one finds in addition that this critical value is maximal for $\tan\beta=+\infty$ and $\mu=0$ and that this maximal value moreover coincides with the CCB bound $A_t^{\rm CCB}|_{\tan\beta=+\infty}$ obtained in the plane $(H_2,\tilde t_L,\tilde t_R)$. To summarize, $A_t^{\rm CCB}|_{\tan\beta=+\infty}$ also provides a CCB maximal mixing for the stop fields, above which CCB unavoidably occurs in the plane $(H_1,H_2,\tilde t_L,\tilde t_R)$:

$$|\tilde A_t|\geqslant A_t^{\rm CCB}|_{\tan\beta=+\infty}\Rightarrow \text{CCB in the plane } (H_1,H_2,\tilde t_L,\tilde t_R). \quad (28)$$

This important property is our main motivation to study in detail the numerical behavior of this benchmark value. In the following, we shall also give an accurate analytic approximation for it, which should be quite useful for phenomenological applications and to constrain model building.

For $\tan\beta\to+\infty$, the EW vacuum is driven in the plane $(H_2,\tilde t_L,\tilde t_R)$ and appears as an additional minimum of the potential V_3, Eq. (1), with VEV's $v_2^2=2m_{Z^0}^2/(g_1^2+g_2^2)$, $\langle\tilde t_{L/R}\rangle=0$, giving $\langle V\rangle|_{\rm EW}=-m_{Z^0}^4/2(g_1^2+g_2^2)$. Stability of the EW vacuum in the plane $(\tilde t_L,\tilde t_R)$ is required, otherwise an obvious CCB situation will occur. Therefore, we may write a new CCB condition on A_t, which merely encodes the physical requirement of avoiding a tachyonic lightest stop mass [3]:

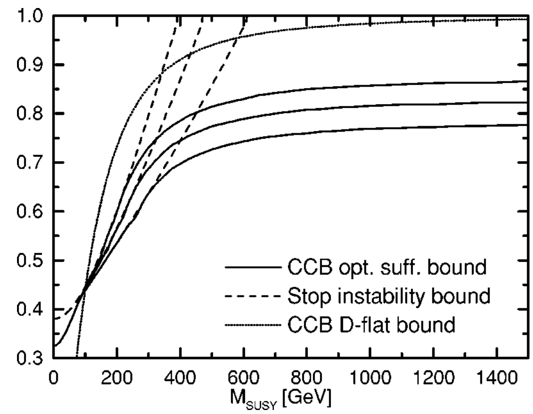

FIG. 1. The CCB optimal sufficient bound $A_t^{\rm suf}$, the D-flat bound A_t^D, and the instability bound $A_t^{\rm inst}$ vs $M_{\rm SUSY}$. All bounds are normalized to $\sqrt{6}m_{\tilde{t}}$. The higher, intermediate, and lower lines correspond, respectively, to $r=1,\ 2$, and 3.

$$A_t \leqslant A_t^{\rm inst} \equiv \frac{\sqrt{(6m_{\tilde{t}_L}^2+6m_t^2+m_{Z^0}^2-4m_{W^\pm}^2)(3m_{\tilde{t}_R}^2+3m_t^2+2m_{W^\pm}^2)}}{3\sqrt{2}m_t}. \quad (29)$$

As we will see, unlike the D-flat bound A_t^D, Eq. (2), the sufficient bound $A_t^{\rm suf}$, Eq. (21), and the CCB maximal mixing $A_t^{\rm CCB}$, Eq. (26), automatically fulfill this requirement. Notice that the instability bound $A_t^{\rm inst}$ is only a function of the soft squark masses $m_{\tilde{t}_L}, m_{\tilde{t}_R}$, the top mass, and the gauge boson masses. This result actually extends to the CCB bounds $A_t^{\rm suf}$ and $A_t^{\rm CCB}$ [for $\tan\beta\to+\infty$]. In our numerical analysis, we take $m_t=175$ GeV and display the CCB bounds as a function of an average of the soft squark masses $M_{\rm SUSY}\equiv\sqrt{(m_{\tilde{t}_L}^2+m_{\tilde{t}_R}^2)/2}$, for three different values of the splitting parameter $r\equiv m_{\tilde{t}_L}/m_{\tilde{t}_R}=1,2,3$. The latter parameter will enable us to conveniently survey the effect of the CCB vacuum deviation from the $SU(3)_c$ D-flat direction. We remark that splitting parameters as large as $r=2$ can be found, for instance, in anomaly mediated scenarios [5]. To be exhaustive, we also consider the possibility of a very large splitting term $r=3$.

In Fig. 1, we display the optimal sufficient bound $A_t^{\rm suf}$, Eq. (21), the traditional bound in the D-flat direction A_t^D, Eq. (2), and the instability bound $A_t^{\rm inst}$, Eq. (29), as a function of $M_{\rm SUSY}$. All bounds are normalized to $\sqrt{6}m_{\tilde{t}}$, where $m_{\tilde{t}}\equiv\sqrt{M_{\rm SUSY}^2+m_t^2}$. We note first that the sufficient bound $A_t^{\rm suf}$ automatically fulfills the important requirement of avoiding a tachyonic lightest stop. We always have $A_t^{\rm inst}\geqslant A_t^{\rm suf}$. A large region (which depends on r) is found where the relation $A_t^{\rm inst}=A_t^{\rm suf}$ holds, implying that no dangerous CCB vacuum may exist unless the EW vacuum is unstable. In this interference regime, the EW and the CCB vacua actually overlap and the CCB VEV $\langle f\rangle$ proves to be connected quite simply to the stop mixing angle $\tilde{\theta}$, with the relation $\langle f\rangle=\tan\tilde{\theta}$ [3]. Here, the slightest deviation of the CCB vacuum from the $SU(3)_c$ D-flat direction must be taken into account in the evaluation of the CCB condition, to avoid a tachyonic stop mass. In contrast, the D-flat bound A_t^D has rather bad behavior. In particular, for low $M_{\rm SUSY}$, notice that it is not restrictive enough to avoid a tachyonic lightest stop.

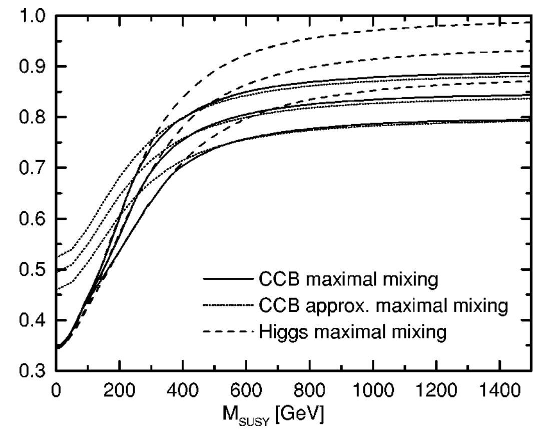

FIG. 2. The CCB maximal mixing $A_t^{\rm CCB}$, its approximation $A_t^{\rm CCB}|_{\rm app.}$ and the one-loop Higgs maximal mixing A_t^H, vs $M_{\rm SUSY}$. All bounds are normalized to $\sqrt{6}m_{\tilde{t}}$. The higher, intermediate and lower lines correspond, respectively, to $r=1,\ 2$, and 3.

Comparing the D-flat bound A_t^D with the sufficient bound $A_t^{\rm suf}$ for $r=1,2,3$, a precise indication of the effect of the CCB vacuum deviation from all D-flat directions can be obtained. For large $M_{\rm SUSY}$, we observe first that these bounds enter an asymptotic regime, with A_t^D larger than $A_t^{\rm suf}$. For $r=1$, the CCB vacuum is aligned in the $SU(3)_c$ D-flat direction. Therefore, the discrepancy between A_t^D and $A_t^{\rm suf}|_{r=1}$ is essentially due to the deviation from the $SU(2)_c\times U(1)_Y$ D-flat direction, triggered by the large violation of the relation $M_{\rm SUSY}^2=m_2^2$, Eq. (3). For $M_{\rm SUSY}=1$ TeV, we have, e.g., $A_t^D-A_t^{\rm suf}|_{r=1}\simeq365$ GeV. For $r=2,3$, large deviations of the CCB vacuum from the $SU(3)_c$ D-flat direction now occur and the sufficient bound $A_t^{\rm suf}$ is lowered. We have, e.g., for $M_{\rm SUSY}=1$ TeV, $A_t^D-A_t^{\rm suf}|_{r=2}\simeq475$ GeV and $A_t^D-A_t^{\rm suf}|_{r=3}\simeq585$ GeV, and the fraction due to the deviation from the $SU(3)_c$ D-flat direction represents 23% for $r=2$ (37.5% for $r=3$) of the total effect. This clearly illustrates that this additional contribution to the CCB condition may be important and should not be neglected for a large splitting between $m_{\tilde{t}_L}, m_{\tilde{t}_R}$.

In Fig. 2, we now display the CCB maximal mixing $A_t^{\rm CCB}$, Eq. (26), as a function of $M_{\rm SUSY}$, for $r=1,2,3$. For comparison, we remark that $A_t^{\rm CCB}$ closely follows the sufficient bound $A_t^{\rm suf}$ displayed in Fig. 1, with $A_t^{\rm CCB}\geqslant A_t^{\rm suf}$ for all values of $M_{\rm SUSY}$. In the interference regime mentioned previously, this inequality is saturated and furthermore we have $A_t^{\rm CCB}=A_t^{\rm inst}$. For $M_{\rm SUSY}\geqslant m_t$, we find $A_t^{\rm suf}\leqslant A_t^{\rm CCB}\leqslant1.025A_t^{\rm suf}$ for $r=1,2,3$, the lower value being reached for $M_{\rm SUSY}\sim m_t$ and the larger for $M_{\rm SUSY}\sim1.5$ TeV. For large $M_{\rm SUSY}\gtrsim700$ GeV, we observe also that the CCB bound $A_t^{\rm CCB}$ enters an asymptotic regime in which only tiny variations still occur.

In Fig. 2, an approximation of the critical CCB bound

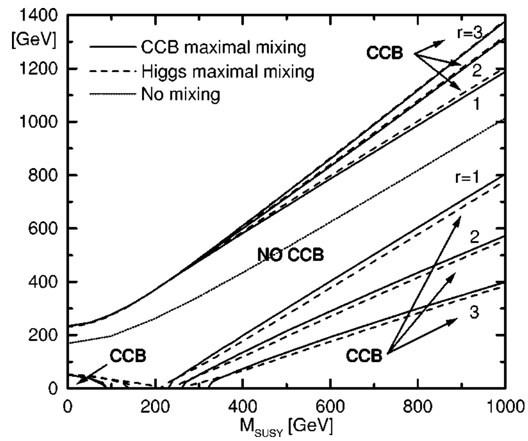

FIG. 3. The bounds on the stop masses for the CCB maximal mixing and the Higgs maximal mixing, versus $M_{\rm SUSY}$, for $r=1$, 2, and 3. Lines below (respectively, above) the no mixing case give lower (respectively, upper) bounds on the lightest (respectively, heaviest) top squark mass.

$A_t^{\rm CCB}$ is also displayed. It reads analytically

$$A_t^{\rm CCB}|_{\rm app.}=\frac{2}{15}\sqrt{\frac{2}{3}}(21-r)\times\frac{m_{\tilde t}^3}{m_{\tilde t}^2-\frac{(1+r)^2(45-42\sqrt{3}+2\sqrt{3}r)}{4\sqrt{3}(21-r)r}m_t^2}. \tag{30}$$

This approximation fulfills two important requirements: $A_t^{\rm CCB}|_{\rm app.}\simeq\frac{2}{15}\sqrt{\frac{2}{3}}(21-r)m_{\tilde t}$ for large $m_{\tilde t}$, which is consistent with the numerical behavior observed; at first order in $m_t/m_{\tilde t}$, $A_t^{\rm CCB}|_{\rm app.}\simeq m_{\tilde t_L}+m_{\tilde t_R}$ for $m_{\tilde t_L}m_{\tilde t_R}\simeq m_t^2$, as required at the center of the interference regime [See Eq. (43) in Ref. [3]]. We stress that this approximation holds only for $r\geqslant 1$. However, numerical investigation shows that no significant variation of $A_t^{\rm CCB}$ occurs under the transformation $r\to 1/r$ [3]. Accordingly, for $m_{\tilde t_L}\leqslant m_{\tilde t_R}$, the analytical expression Eq. (30) for $A_t^{\rm CCB}|_{\rm app.}$ should be adapted by redefining $r\equiv m_{\tilde t_R}/m_{\tilde t_L}$. For $M_{\rm SUSY}$ large enough, Fig. 2 shows the excellent accuracy of the approximation $A_t^{\rm CCB}|_{\rm app.}$. For all values of r, it fits the exact result $A_t^{\rm CCB}$ within less than 1%. In contrast, for low $M_{\rm SUSY}$, it behaves rather badly. However, this feature occurs only in a region of the parameter space where the lightest stop mass is small, i.e., $m_{\tilde t_1}\lesssim 100$ GeV (see Fig. 3). Such a region is nearly completely excluded by experimental data [15].

Finally, in Fig. 2 we display the one-loop Higgs maximal mixing for the stop masses, denoted A_t^H in the following. As is well known, the tree-level lightest CP-even Higgs boson mass receives large one-loop corrections from loops of top and stop quark fields, which are essential to overcome the tree-level upper bound $m_h\leqslant m_{Z^0}$ [6]. For $\tan\beta=+\infty$ and $m_{A^0}\gg m_t$, these corrections are maximized and we have in the top-stop approximation [6]:

$$m_h^2=m_{Z^0}^2+\frac{3m_t^4}{8\pi v^2}\left[\text{Log}\frac{m_{\tilde t_1}m_{\tilde t_2}}{m_t^2}+\frac{4A_t^2}{m_{\tilde t_2}^2-m_{\tilde t_1}^2}\text{Log}\frac{m_{\tilde t_2}}{m_{\tilde t_1}}+\frac{2A_t^4}{(m_{\tilde t_2}^2-m_{\tilde t_1}^2)^2}\left(1-\frac{m_{\tilde t_2}^2+m_{\tilde t_1}^2}{m_{\tilde t_2}^2-m_{\tilde t_1}^2}\text{Log}\frac{m_{\tilde t_2}}{m_{\tilde t_1}}\right)\right],$$
$$v=174\ \text{GeV}, \tag{31}$$

where the stop masses read

$$m_{\tilde t_1,\tilde t_2}^2=\frac{m_{\tilde t_L}^2+m_{\tilde t_R}^2}{2}+m_t^2-\frac{1}{4}m_{Z^0}^2\mp\sqrt{m_t^2A_t^2+\frac{[6(m_{\tilde t_L}^2-m_{\tilde t_R}^2)-8m_{W^\pm}^2+5m_{Z^0}^2]^2}{144}}. \tag{32}$$

The Higgs maximal mixing value A_t^H, which maximizes m_h in Eq. (31), can be obtained numerically as a function of $M_{\rm SUSY}$ and r. As is well known, for large $M_{\rm SUSY}=m_{\tilde t_L}=m_{\tilde t_R}\gg m_t$, A_t^H takes the simple expression: $A_t^H=\sqrt{6}m_{\tilde t}$ [6,7] (this value is actually used as a normalization factor in Figs. 1 and 2). Indeed, this asymptotic behavior is observed in Fig. 2 for $r=1$. In addition, Fig. 2 shows that A_t^H is decreasing for an increasing splitting between the soft squark masses [6].

For low $M_{\rm SUSY}$, the CCB maximal mixing $A_t^{\rm CCB}$ and the Higgs maximal mixing A_t^H follow each other closely, showing that m_h is maximal for $A_t\simeq A_t^{\rm CCB}$ (we have furthermore $A_t\simeq A_t^{\rm inst}$). Notice that A_t^H can be lower than $A_t^{\rm CCB}$ for $M_{\rm SUSY}\lesssim m_t$, though just slightly and, moreover, in an unphysical region where the lightest top squark mass is vanishing $m_{\tilde t_1}\simeq 0$ GeV (see Fig. 3).[2] For larger values of $M_{\rm SUSY}$, the CCB maximal mixing clearly rules out the Higgs maximal mixing. For $M_{\rm SUSY}=1$ TeV, $A_t^{\rm CCB}$ is about 10% below A_t^H, for $r=1,2,3$. Thus, the large exclusion already observed in Ref. [3] for equal soft squark masses is also found in the presence of a large mass splitting.

In Fig. 3, we compare the bounds on the top squark mass spectrum for the CCB and the Higgs maximal mixing values. In both cases, we display below (respectively, above) the no-mixing line, i.e., $m_{\tilde t_1,\tilde t_2}^2=M_{\rm SUSY}^2+m_t^2-m_{Z^0}^2/4$, the corresponding lower (respectively, upper) bounds on the lightest stop mass $m_{\tilde t_1}$ (respectively, the heaviest stop mass $m_{\tilde t_2}$). For

[2]Some residual gauge contributions are actually neglected in the writing of Eq. (31) in order to gain approximate independence for m_h with respect to the renormalization scale Q [6]. For low $M_{\rm SUSY}$, such contributions may presumably restore the hierarchy $A_t^H\geqslant A_t^{\rm CCB}$.

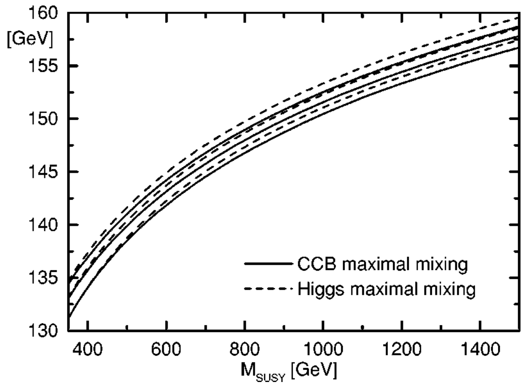

FIG. 4. The one-loop upper bounds on m_h for the CCB maximal mixing and the Higgs maximal mixing vs $M_{\rm SUSY}$. The higher, intermediate, and lower lines correspond, respectively, to $r=1$, 2, and 3.

large $M_{\rm SUSY}$, the allowed range for the top squark mass spectrum is enlarged for an increasing splitting between the soft squark masses, despite the decrease of the CCB and the Higgs maximal mixing values for increasing r (see Fig. 2). Obviously, the CCB bounds on the top squark mass spectrum are more restrictive than the Higgs maximal mixing ones. This effect is, however, not very large, although not negligible for $r=1$. For instance, for $M_{\rm SUSY}=1$ TeV we have, respectively, $\Delta m_{\tilde t_1}\simeq(25.5,16.5,14.5)$ GeV and $\Delta m_{\tilde t_2}\simeq(17,7,4)$ GeV, for $r=1,2,3$.

Figure 3 exhibits another interesting feature. Taking conservatively $m_{\tilde t_1}\gtrsim 100$ GeV as an experimental limit on the lightest stop mass [15], we find that a stop mixing value as large as the CCB maximal mixing is excluded in a large part of the parameter space, i.e., $M_{\rm SUSY}\lesssim(310,360,440)$ GeV for $r=1,2,3$. In the respective domains, the EW vacuum is not threatened by the CCB vacuum in the plane $(H_2,\tilde t_L,\tilde t_R)$, and is automatically stable. This result illustrates how a precise study of CCB conditions can produce refined statements concerning metastability of the EW vacuum [16].[3]

In Fig. 4, we finally compare the one-loop upper bound on the CP-even Higgs boson m_h in the top-stop approximation, Eq. (31), for the CCB and the Higgs maximal mixing values. In both cases, this bound is decreasing with increasing r, but the effect is rather small, at most $\sim 2-3$ GeV. The mass discrepancy between the CCB and the Higgs maximal mixing cases is negligible for $M_{\rm SUSY}\sim 500$ GeV. It is slowly increasing with $M_{\rm SUSY}$, but is still small for $M_{\rm SUSY}=1500$ GeV, where it is ~ 1 GeV for $r=1,2,3$. Let us note that in the large $\tan\beta$ regime investigated here, m_h may also receive at one-loop level important additional contributions, coming in particular from the bottom/sbottom sector [6,7]. Such contributions would modify the numerical upper bound on m_h, but not the discrepancy between the CCB and the Higgs maximal mixing values presented here, which depend essentially on the top-stop contribution.

Thus, the numerical benefit of taking the CCB maximal mixing rather than the exact one-loop Higgs maximal mixing to constrain the top squark mass spectrum and the one-loop upper bound on m_h, is not very large (although not always negligible). However, on theoretical ground, this statement must be completed by stressing that the requirement of avoiding a dangerous CCB vacuum provides a strong and independent physical motivation to consider stop mixing terms smaller than the Higgs maximal mixing. The latter in contrast represents a benchmark mixing, useful essentially in keeping track of the value that maximizes the lightest CP-even Higgs boson mass. Moreover, outside this context, the CCB maximal mixing can have more drastic phenomenological implications. For instance, at the tree level, it was shown that the cross section of production of the lightest CP-even Higgs boson h in association with a lightest stop pair is strongly enhanced for a large stop mixing term $|\tilde A_t|\sim\sqrt{6}m_{\tilde t}$ and can even exceed the production cross section in association with a top quark pair at the CERN Large Hadron Collider [14]. However, this interesting window for the discovery of supersymmetric particles opens for a lightest stop mass light enough at $m_{\tilde t_1}\sim m_t$, therefore in a region of the parameter space where the optimal CCB conditions are very restrictive. Clearly, Figs. 2 and 3 show that the two requirements, a light stop mass $m_{\tilde t_1}\sim m_t$ and a large stop mixing $|\tilde A_t|\sim\sqrt{6}m_{\tilde t}$, are in conflict [we note that CCB occurs in the plane $(H_1,H_2,\tilde t_L,\tilde t_R)$ for $|\tilde A_t|\geqslant A_t^{\rm CCB}|_{\tan\beta=+\infty}$, Eq. (28)]. Hence, we expect a dramatic reduction of the cross section of such a process in the CCB allowed region of the parameter space.[4] This example illustrates the phenomenological usefulness of the CCB maximal mixing $A_t^{\rm CCB}$ for $\tan\beta=+\infty$, considered in this paper. As noted before, this benchmark mixing can also be used to avoid metastability of the EW vacuum in model-dependent scenarios, which unavoidably occurs if $|\tilde A_t|\geqslant A_t^{\rm CCB}|_{\tan\beta=+\infty}$ at the SUSY scale. In these contexts, the simple approximation $A_t^{\rm CCB}|_{\rm app.}$, Eq. (30), should be of particular interest. Moreover, it is definitely more reliable than the traditional CCB bound in the D-flat direction A_t^D, Eq. (2), often considered as a first guess of the impact of CCB conditions, but which largely underestimates the restrictive power of the latter.

Finally, we remark that two-loop contributions provide important contributions to m_h and induce a displacement of the Higgs maximal mixing, which may become more restrictive than the CCB maximal mixing $A_t^{\rm CCB}$: for $m_{A^0},M_{\rm SUSY}\geqslant m_{Z^0}$, $\tan\beta=+\infty$ and $r=1$, $A_t^H|_{2-\rm loop}\simeq 2m_{\tilde t}$ [7], whereas $A_t^{\rm CCB}\simeq 2.17m_{\tilde t}$. However, at the two-loop level, a precise investigation of the effect of CCB conditions on the Higgs

[3] In the metastability domain, we further remark that the analytic expressions for the VEV's of the CCB vacuum presented in the last section should also be very useful in precisely evaluating the CCB metastability condition on A_t [16], for large $\tan\beta$.

[4] It remains to be determined if in some regions of the parameter space, this process is still favored compared to the production rate in association with a top quark pair. This will be the subject of future investigations.

boson mass m_h requires the evaluation of the one-loop CCB bound $A_t^{\rm CCB}|_{1-\rm loop}$. This value incorporates in particular contributions which escape our tree-level improved CCB bound $A_t^{\rm CCB}$, evaluated at the SUSY scale. Therefore, the previous comparison seems somewhat misleading. However it raises the important question of the hierarchy between $A_t^{\rm CCB}|_{1-\rm loop}$ and $A_t^H|_{2-\rm loop}$. For low $M_{\rm SUSY}\lesssim m_t$, we remark that the relation $A_t^{\rm CCB}\simeq A_t^{\rm inst}\simeq A_t^H$ (see Figs. 1 and 2) should persist at the next loop level. For large $\tan\beta$, it is due essentially to the presence of an interference regime where the CCB vacuum and the EW vacuum overlap. At the one-loop level, the EW vacuum is still driven in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$ for large $\tan\beta$, and such a regime should therefore also be found. For $M_{\rm SUSY}\gg m_t$, things are not so clear. However, we may reasonably expect that one-loop corrections will also lower the CCB maximal mixing, as occurs for the Higgs maximal mixing, implying presumably the hierarchy $A_t^{\rm CCB}|_{1-\rm loop}\leqslant A_t^H|_{2-\rm loop}$. This expectation can be checked only by a complete one-loop investigation of the CCB conditions in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$, for large $\tan\beta$, which we plan to do in the future.

IV. CONCLUSION

In this paper, we investigated the optimal CCB condition on A_t in the plane $(H_2,\tilde{t}_L,\tilde{t}_R)$, taking into account the possibility of a large mass splitting between the soft squark masses $m_{\tilde{t}_L},m_{\tilde{t}_R}$, as occurs in interesting models [4,5]. We essentially focused on the asymptotic regime $\tan\beta=+\infty$, motivated by the nice features of the CCB bound in this regime. In particular, a complete investigation of CCB conditions in the extended plane $(H_1,H_2,\tilde{t}_L,\tilde{t}_R)$, which will be presented elsewhere [8], shows that the stop mixing term $\tilde{A}_t$, in absolute value, should not exceed this benchmark value, otherwise CCB unavoidably occurs. This CCB bound should therefore be useful for phenomenological applications. For this reason, we presented an accurate analytic approximation for it, which fits the exact result within less than 1% in the interesting phenomenological region where $m_{\tilde{t}_1}\gtrsim 100$ GeV.

For $M_{\rm SUSY}\gg m_t$, we showed that the one-loop Higgs maximal mixing is ruled out by more than 10%, whatever the splitting between the soft squark masses $m_{\tilde{t}_L},m_{\tilde{t}_R}$ is. Compared to the Higgs maximal mixing, the effect of the CCB maximal mixing on the top squark mass spectrum and on the one-loop upper bound on m_h is not very large, though not always negligible. We pointed out however that larger effects can be expected in Higgs phenomenology, which can be more sensitive to such an exclusion. Further investigations are in progress in this direction.

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