# Rare decay $D^0 \to \gamma\gamma$

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We present a calculation of the rare decay mode $D^0 \to \gamma\gamma$, in which the long distance contributions are expected to be dominant. Using the heavy quark chiral Lagrangian with a strong $g$ coupling as recently determined by CLEO from the $D^* \to D\pi$ width, we consider both the anomaly contribution that relates to the annihilation part of the weak Lagrangian and the one-loop $\pi,K$ diagrams. The loop contributions, which are proportional to $g$ and contain the $a_1$ Wilson coefficient, are found to dominate the decay amplitude, which turns out to be mainly parity violating. The branching ratio is then calculated to be $(1.0 \pm 0.5) \times 10^{-8}$. The observation of an order of magnitude larger branching ratio could be indicative of new physics.

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## I. INTRODUCTION

With the new data coming and expected from the $B$ factories, there is very strong emphasis and activity in the field of $B$ physics in all its aspects. This includes the rare decays of $B$ mesons, which are considered as an attractive source for possible signals of new physics. In contrast to the growing efforts to understand the mechanisms of rare $B$ decays, rare $D$ decays have received less attention in recent years. Partly this is because theoretical investigations of $D$ weak decays are rather difficult, due to the presence of many resonances close to this energy region. The penguin effects on the other hand, which are very important in $B$ and also in $K$ decays, are usually suppressed in the case of charm mesons due to the presence of $d$, $s$, and $b$ quarks in the loop with the respective values of Cabibbo-Kobayashi-Maskawa (CKM) elements.

Nevertheless, $D$ meson physics has produced some interesting results in the past year. Experimental results on time dependent decay rates of $D^0 \to K^+\pi^-$ by CLEO [1] and $D^0 \to K^+K^-$ and $D^0 \to K^-\pi^+$ by FOCUS [2] have stimulated several studies on the $D^0$-$\bar{D}^0$ oscillations [3]. The $D^*$ decay width recently measured by CLEO [4] has provided the long expected information on the value of $D^*D\pi$ coupling. Among the rare $D$ decays, the decays $D \to V\gamma$ and $D \to V(P)l^+l^-$ are the subjects of CLEO and Fermilab searches [5]. On the theoretical side, these rare decays of charm mesons into a light vector meson and a photon or lepton pair have been considered recently by several authors (see, e.g., [6–11], and for radiative leptonic $D$ meson decay see [12]). The investigations of $D \to V\gamma$ showed that certain branching ratios can be as large as $10^{-5}$, as for $D^0 \to \bar{K}^{*0}\gamma$, $D_s^+ \to \rho^+\gamma$ [6,11]. However, the decays that are of some relevance to the $D^0 \to 2\gamma$ mode studied here, such as $D^0 \to \rho^0\gamma$, $D^0 \to \omega\gamma$, are expected to have branching ratios in the $10^{-6}$ range [13]. Thus, it is hard to believe that the branching ratio of the $D^0 \to 2\gamma$ decay mode can be as high as $10^{-5}$ in the standard model (SM), as found by [14]. Apart from this estimate, there is no other detailed work on $D^0 \to 2\gamma$ in the literature, to the best of our knowledge.

On the other hand, in the $B$ and $K$ meson systems there are numerous studies of the decays to two photons. For example, the $B_s \to \gamma\gamma$ decay has been studied using various approaches within the SM and beyond. In the SM, the short distance (SD) contribution [15] leads to a branching ratio $B(B_s \to \gamma\gamma) \simeq 3.8 \times 10^{-7}$. The QCD corrections enhance this rate to $5 \times 10^{-7}$ [16]. On the other hand, in some of the SM extensions the branching ratio can be considerably larger. The two-Higgs-doublet scenario, for example, could enhance this branching ratio by an order of magnitude [17]. Such "new physics" effects could at least in principle be dwarfed by long distance (LD) effects. However, existing calculations show that these are not larger than the SD contribution [18], which is typical of the situation in radiative $B$ decays [19]. In the $K^0$ system the situation is rather different. Here, the SD contribution is too small to account for the observed rates of $K_S \to 2\gamma$, $K_L \to 2\gamma$ by factors of $\sim 3-5$ [20], although it could be of relevance in the mechanism of $CP$ violation. Many detailed calculations of these processes have been performed over the years (see recent Refs. [20–23] and references therein), especially using the chiral approach to account for the pole diagrams and the loops. These LD contributions lead to rates that are compatible with existing measurements.

Motivated by the experimental efforts to observe rare $D$ meson decays, as well as by the lack of detailed theoretical treatments, we undertook an investigation of the $D^0 \to \gamma\gamma$

 

decay. The short distance contribution is expected to be rather small, as already encountered in the one-photon decays [6,7]; hence the main contribution should come from long distance interactions. In order to treat the long distance contributions, we use the heavy quark effective theory combined with chiral perturbation theory (HQ$\chi$PT) [24]. This approach was used before for treating $D^*$ strong and electromagnetic decays [25–27]. The leptonic and semileptonic decays of $D$ mesons were also treated within the same framework (see [25] and references therein).

The approach of HQ$\chi$PT introduces several coupling constants that have to be determined from experiment. The recent measurement of the $D^*$ decay width [4] has determined the $D^*D\pi$ coupling, which is related to $g$, the basic strong coupling of the Lagrangian. There is more ambiguity, however, concerning the value of the anomalous electromagnetic coupling, which is responsible for the $D^*D\gamma$ decays [26,27], as we shall discuss later.

Let us address now some issues concerning the theoretical framework used in our treatment. For the weak vertex we shall use the factorization of weak currents with nonfactorizable contributions coming from chiral loops. The typical energy of intermediate pseudoscalar mesons is of order $m_D/2$, so that the chiral expansion $p/\Lambda_\chi$ (for $\Lambda_\chi \gtrsim 1$ GeV) is rather close to unity. Thus, for the decay under study here we extend the possible range of applicability of the chiral expansion of HQ$\chi$PT, compared to previous treatments like $D^* \to D\pi$, $D^* \to D\gamma$ [26], or $D^* \to D\gamma\gamma$ [27], in which a heavy meson appears in the final state, making the use of chiral perturbation theory rather natural. The suitability of our undertaking here must be confronted with experiment, and possibly other theoretical approaches.

At this point we also remark that the contribution of order $\mathcal{O}(p)$ does not exist in the $D^0 \to \gamma\gamma$ decay, and the amplitude starts with a contribution of order $\mathcal{O}(p^3)$. At this order the amplitude receives an annihilation type contribution proportional to the $a_2$ Wilson coefficient, with the Wess-Zumino anomalous term coupling light pseudoscalars to two photons. As we will show, the total amplitude is dominated by terms proportional to $a_1$ that contribute only through loops with Goldstone bosons. Loop contributions proportional to $a_2$ vanish at this order. We point out that any other model that does not involve intermediate charged states cannot give this kind of contribution. Therefore, the chiral loops naturally include effects of intermediate meson exchange.

The chiral loops of order $\mathcal{O}(p^3)$ are finite, as they are in the similar case of $K \to \gamma\gamma$ decays [20–23]. The next to leading terms might be almost of the same order of magnitude compared to the leading $\mathcal{O}(p^3)$ term, the expected suppression being approximately $p^2/\Lambda_\chi^2$. The inclusion of next order terms in the chiral expansion is not straightforward in the present approach. We include, however, terms that contain the anomalous electromagnetic coupling, and appear as next to leading order terms in the chiral expansion, in view of their potentially large contribution [as in the $B^*(D^*) \to B(D)\gamma\gamma$ decays considered in [27]]. As it turns out, these terms are suppressed compared to the leading loop effects, which at least partially justifies the use of HQ$\chi$PT for the decay under consideration. Contributions of the same order could arise from light resonances like $\rho$, and $K^*$, $a_0(980)$, $f_0(975)$. Such resonances are sometimes treated with hidden gauge symmetry (see, e.g., [25]), which is not compatible with chiral perturbation symmetry. Therefore, a consistent calculation of these terms is beyond our scheme and we disregard their possible effect.

Our paper is organized as follows. In Sec. II we present the basic features of the model. We give the results and their discussion in Sec. III and conclude with a summary in Sec. IV.

## II. THE THEORETICAL FRAMEWORK

The invariant amplitude for $D^0 \to \gamma\gamma$ decay can be written using gauge and Lorentz invariance in the following form:

$$M = \left[ iM^{(-)}\left( g^{\mu\nu} - \frac{k_2^\mu k_1^\nu}{k_1 \cdot k_2} \right) + M^{(+)} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \right] \epsilon_{1\mu} \epsilon_{2\nu}, \tag{1}$$

where $M^{(-)}$ is the parity violating and $M^{(+)}$ the parity conserving part of the amplitude, while $k_{1(2)}$ and $\epsilon_{1(2)}$ are respectively the four-momenta and the polarization vectors of the outgoing photons.

In the discussion of weak radiative $q' \to q\gamma\gamma$ or $q' \to q\gamma$ decays, usually the short and long distance contribution are separated. The SD contribution in these transitions is a result of penguinlike transitions, while the long distance contribution arises in particular pseudoscalar meson decays as a result of the nonleptonic four-quark weak Lagrangian, when the photon is emitted from the quark legs. Here we follow this classification. In the case of $b \to s\gamma\gamma$ decay [28] it was noticed that without QCD corrections the rate $\Gamma(b \to s\gamma\gamma)/\Gamma(b \to s\gamma)$ is about $10^{-3}$. One expects that a similar effect will show up in the case of $c \to u\gamma\gamma$ decays. That is, according to the result of [28] the largest contribution to the $c \to u\gamma\gamma$ amplitude would arise from the photon emitted from either $c$ or $u$ quark legs in the case of the penguinlike transition $c \to u\gamma$. Without QCD corrections the branching ratio for $c \to u\gamma$ is rather suppressed, being of the order of $10^{-17}$ [7,8]. The QCD corrections [29] enhance it up to the order of $10^{-8}$.

In our approach we include the $c \to u\gamma$ short distance contribution by using the Lagrangian

$$\mathcal{L} = -\frac{G_f}{\sqrt{2}} V_{us} V_{cs}^* C_{7\gamma}^{eff} \frac{e}{4\pi^2} F_{\mu\nu} m_c \left[ \overline{u} \sigma^{\mu\nu} \frac{1}{2} (1+\gamma_5) c \right], \tag{2}$$

where $m_c$ is the charm quark mass. In our analysis we follow [29,30] and we take $C_{7\gamma}^{eff} = (-0.7 + 2i) \times 10^{-2}$.

The main LD contribution will arise from the effective four-quark nonleptonic $\Delta C = 1$ weak Lagrangian given by

$$\mathcal{L} = -\frac{G_f}{\sqrt{2}} \sum_{q=d,s} V_{uq} V^*_{cq} [a_1 (\bar{q}\Gamma^\mu c)(\bar{u}\Gamma_\mu q) + a_2 (\bar{u}\Gamma^\mu c)(\bar{q}\Gamma_\mu q)], \tag{3}$$

where $\Gamma^\mu = \gamma^\mu(1-\gamma_5)$, $a_i$ are effective Wilson coefficients [31], and $V_{q_i q_j}$ are CKM matrix elements. At this point it is worth pointing out that long distance interactions will contribute only if the SU(3) flavor symmetry is broken, i.e., if $m_s \neq m_d$. That is, because $V_{ud}V^*_{cd} \simeq -V_{us}V^*_{cs}$, if $m_d = m_s$ the contributions arising from the weak Lagrangian (3) cancel.

Now, we turn to describing some of the basic features of the HQ$\chi$PT. This model will serve us as a hadronized counterpart of the quark effective weak Lagrangian. One has the usual $\mathcal{O}(p^2)$ chiral Lagrangian for the light pseudoscalar mesons,

$$\mathcal{L}^{(2)}_{str} = \frac{f^2}{8} \mathrm{tr}(\partial^\mu \Sigma \partial_\mu \Sigma^\dagger) + \frac{f^2 B_0}{4} \mathrm{tr}(\mathcal{M}_q \Sigma + \mathcal{M}_q \Sigma^\dagger), \tag{4}$$

where $\Sigma = \exp(2i\Pi/f)$ with $\Pi = \Sigma_j \lambda^j \pi^j/\sqrt{2}$ containing the Goldstone bosons $\pi, K, \eta$, and $f$ is the pion constant, while the trace tr runs over flavor indices and $\mathcal{M}_q = \mathrm{diag}(m_u, m_d, m_s)$ is the current quark mass matrix. From this Lagrangian, we can deduce the light weak current of order $\mathcal{O}(p)$,

$$j^X_\mu = -i \frac{f^2}{4} \mathrm{tr}(\Sigma \partial_\mu \Sigma^\dagger \lambda^X), \tag{5}$$

corresponding to the quark current $j^X_\mu = \bar{q}_L \gamma_\mu \lambda^X q_L$. [($\lambda^X$ is an SU(3) flavor matrix.)]

In the heavy meson sector interacting with light mesons we have the following lowest order $\mathcal{O}(p)$ chiral Lagrangian:

$$\mathcal{L}^{(1)}_{str} = -\mathrm{Tr}(\bar{H}_a i v \cdot D_{ab} H_b) + g\, \mathrm{Tr}(\bar{H}_a H_b \gamma_\mu \mathcal{A}^\mu_{ba} \gamma_5), \tag{6}$$

where $D^\mu_{ab} H_b = \partial^\mu H_a - H_b \mathcal{V}^\mu_{ba}$, while the trace Tr runs over Dirac indices. Note that in Eq. (6) and the rest of this section $a$ and $b$ are *flavor* indices.

The vector and axial vector fields $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ in Eq. (6) are given by

$$\mathcal{V}_\mu = \frac{1}{2}(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi), \quad \mathcal{A}_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), \tag{7}$$

where $\xi = \exp(i\Pi/f)$. The heavy meson field $H_a$ contains a spin zero and a spin 1 boson,

$$H_a = P_+ (P_{\mu a} \gamma^\mu - P_{5a} \gamma_5), \tag{8}$$

$$\bar{H}_a = \gamma^0 (H_a)^\dagger \gamma^0 = [P^\dagger_{\mu a} \gamma^\mu + P^\dagger_{5a} \gamma_5] P_+, \tag{9}$$

with $P_\pm = (1 \pm \gamma^\mu v_\mu)/2$ being the projection operators. The field $P_5$ ($P^\dagger_5$) annihilates (creates) a pseudoscalar meson with a heavy quark having velocity $v$, and similarly for spin 1 mesons.

For a decaying heavy quark, the weak current is given by

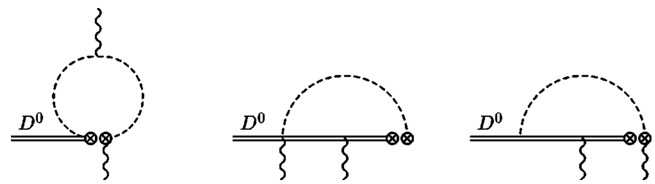

FIG. 1. One-loop diagrams [not containing $\beta$-like terms (14)] that give vanishing contributions. The dashed line represents charged Goldstone bosons flowing in the loop ($K^+, \pi^+$), while the double line represents heavy mesons $D$ and $D^*$.

$$J^\lambda_a = \bar{q}_a \gamma^\lambda L Q, \tag{10}$$

where $L = (1-\gamma_5)/2$, $Q$ is the heavy quark field in the full theory, in our case a $c$ quark, and $q$ is the light quark field.

On symmetry grounds, the heavy-light weak current is bosonized in the following way [24]:

$$J^\lambda_a = \frac{i\alpha}{2} \mathrm{Tr}[\gamma^\lambda L H_b \xi^\dagger_{ba}], \tag{11}$$

where $\alpha$ is related to the physical decay constant $f_D$ through the well known matrix element

$$\langle 0 | \bar{u} \gamma^\lambda \gamma_5 c | D^0 \rangle = -2 \langle 0 | J^\lambda_a | D^0 \rangle = i m_D v^\lambda f_D. \tag{12}$$

Note that the current (11) is $\mathcal{O}(p^0)$ in the chiral counting.

In the calculation of the short distance contribution (2) there appears the operator $[\bar{u}\sigma_{\mu\nu}\frac{1}{2}(1+\gamma_5)c]$. Using heavy quark symmetry this operator can be translated into an operator containing meson fields only [32]:

$$\left[\bar{u}\sigma_{\mu\nu}\frac{1}{2}(1+\gamma_5)c\right] \to \frac{i\alpha}{2} \mathrm{Tr}\left[\sigma_{\mu\nu}\frac{1}{2}(1+\gamma_5) H_b \xi^\dagger_{ba}\right]. \tag{13}$$

The photon couplings are obtained by gauging the Lagrangians (4) and (6) and the light current (5) with the U(1) photon field $B_\mu$. The covariant derivatives are then $\mathcal{D}^\mu_{ab} H_b = \partial^\mu H_a + ieB^\mu (Q'H - HQ)_a - H_b \mathcal{V}^\mu_{ba}$ and $\mathcal{D}_\mu = \partial_\mu \xi + ieB_\mu [Q, \xi]$ with $Q = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ and $Q' = \frac{2}{3}$ (for our case of the $c$ quark). The vector and axial vector fields (7) change after gauging and now they read $\mathcal{V}_\mu = 1/2(\xi \mathcal{D}_\mu \xi^\dagger + \xi^\dagger \mathcal{D}_\mu \xi)$ and $\mathcal{A}_\mu = i/2(\xi^\dagger \mathcal{D}_\mu \xi - \xi \mathcal{D}_\mu \xi^\dagger)$. The light weak current (5) contains after gauging the covariant derivative $\mathcal{D}_\mu$ instead of $\partial_\mu$. However, the gauging procedure alone does not introduce a coupling between heavy vector and pseudoscalar meson fields and the photon without emission or an absorption of additional Goldstone boson, which is needed to account, for example, for $D^* \to D\gamma$. To describe this electromagnetic interaction we follow [26], introducing an additional gauge invariant contact term with an unknown coupling $\beta$ of dimension $-1$:

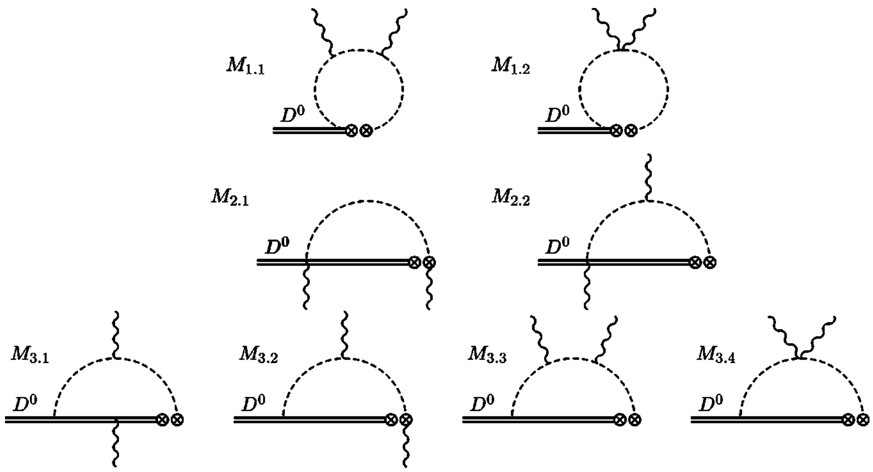


FIG. 2. One-loop diagrams, not containing beta-like terms (14), that give nonvanishing contributions to the $D^0 \to \gamma\gamma$ decay amplitude. Each sum of the amplitudes on the diagrams in one row $M_i = \Sigma_j M_{i.j}$ is gauge invariant and finite. Numerical values are listed in Table I below.

$$\mathcal{L}_\beta = -\frac{\beta e}{4} \mathrm{Tr}\, \overline{H}_a H_b \sigma^{\mu\nu} F_{\mu\nu} Q^\xi_{ba} - \frac{e}{4 m_Q} Q' \,\mathrm{Tr}\, \overline{H}_a \sigma^{\mu\nu} H_a F_{\mu\nu}, \tag{14}$$

where $Q^\xi = \frac{1}{2}(\xi^\dagger Q \xi + \xi Q \xi^\dagger)$ and $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The first term concerns the contribution of the light quarks in the heavy meson and the second term describes emission of a photon from the heavy quark. Its coefficient is fixed by heavy quark symmetry. From this "anomalous" interaction, both $H^* H \gamma$ and $H^* H^* \gamma$ interaction terms arise. Even though the Lagrangian (14) is formally $1/m_Q \sim m_q$ suppressed, we do not neglect it, as it has been found that it gives a sizable contribution to $D^*(B^*) \to D(B)\gamma\gamma$ decays [27]. In the case of $D^0 \to \gamma\gamma$ it gives the largest contribution to the parity conserving part of the amplitude; however, it does not contribute to the decay rate by more than 10%, as will be shown later. The Lagrangian (14) in principle receives a number of other contributions at the order of $1/m_Q$; however, these can be absorbed in the definition of $\beta$ for the processes considered [26].

## III. RESULTS

The decay width for the $D^0 \to \gamma\gamma$ decay can be obtained using the amplitude decomposition in Eq. (1):

$$\Gamma_{D^0 \to \gamma\gamma} = \frac{1}{16 \pi m_D} \left( |M^{(-)}|^2 + \tfrac{1}{4} |M^{(+)}|^2 m_D^4 \right). \tag{15}$$

The short distance contribution to the $D^0 \to \gamma\gamma$ decay width is estimated using the $c \to u\gamma$ effective Lagrangian (2), (13) with one photon emitted from the $D^0$ leg via the $\mathcal{L}_\beta$ term (14). The parity violating part of the short distance amplitude is

$$M_{SD}^{(-)} = \frac{m_D^{3/2}}{12\pi^2} \frac{G_f}{\sqrt{2}} V_{us} V_{cs}^* C_{7\gamma}^{eff} e^2 (\beta m_c + 1) \alpha \frac{1}{1 + 2\Delta^*/m_D}, \tag{16}$$

while the parity conserving part of the amplitude is

$$M_{SD}^{(+)} = \frac{\sqrt{m_D}}{12\pi^2} \frac{G_f}{\sqrt{2}} V_{us} V_{cs}^* C_{7\gamma}^{eff} e^2 (\beta m_c + 1) \alpha \frac{2}{m_D + 2\Delta^*}, \tag{17}$$

where $\Delta^* = m_{D^{0*}} - m_{D^0}$. Turning now to the long distance contributions, we depict in Figs. 1 and 2 the loop diagrams arising to leading order $\mathcal{O}(p^3)$ by using Eqs. (4)–(6) and (11). The circled cross indicates the weak interaction. In Fig. 1 we grouped all diagrams that vanish from symmetry considerations. All nonvanishing contributions are assembled in Fig. 2. We denote the gauge invariant sums corresponding to nonvanishing diagrams of Fig. 2 by $M_i^{(\pm)} = \Sigma_j M_{i.j}^{(\pm)}$ (the gauge invariant sums are sums of diagrams in each row of Fig. 2), where $+$ $(-)$ denotes the parity conserving (violating) part of the amplitude, as in Eq. (15). The parity violating sums, which arise from the first term in Eq. (3), are

$$M_1^{(-)} = -\frac{(m_D)^{3/2}}{4\pi^2} \frac{G_f}{\sqrt{2}} a_1 \alpha e^2 [V_{us} V_{cs}^* M_4(m_K, -m_D^2/2) + V_{ud} V_{cd}^* M_4(m_\pi, -m_D^2/2)], \tag{18}$$

$$\begin{aligned} M_2^{(-)} = {} & \sqrt{m_D} \frac{G_f}{\sqrt{2}} a_1 e^2 g \alpha \frac{1}{8\pi^2} \Bigg[ V_{us} V_{cs}^* \Bigg( \frac{1}{m_D/2 + \Delta_s^*} \\ & \times I_2(m_K, m_D/2 + \Delta_s^*) - 2 G_3(m_K, m_D + \Delta_s^* - m_D/2) \Bigg) \\ & + V_{ud} V_{cd}^* \Bigg( \frac{1}{m_D/2 + \Delta_d^*} I_2(m_\pi, m_D/2 + \Delta_d^*) \\ & - 2 G_3(m_\pi, m_D + \Delta_d^*, -m_D/2) \Bigg) \Bigg], \end{aligned} \tag{19}$$

$$M_3^{(-)} = \sqrt{m_D} \frac{G_f}{\sqrt{2}} a_1 g e^2 \alpha \frac{1}{2\pi^2} [V_{us} V_{cs}^* f(m_K, \Delta_s^*, m_D) + V_{ud} V_{cd}^* f(m_\pi, \Delta_d^*, m_D)], \tag{20}$$

with

$$f(m,\Delta,m_D)=\frac{m^2}{m_D}\left[G_0(m,\Delta+m_D/2,m_D/2)-\frac{1}{2}G_0(m,\Delta,m_D/2)\right]+\frac{5m_D}{8}+\frac{\Delta}{2}+\frac{(m^2-\Delta^2)}{2}\left[\frac{1}{m_D}N_0(m,m_D^2)+\left(\frac{1}{2}+\frac{\Delta}{m_D}\right)\right.$$
$$\left.\times\bar{G}_0(m,\Delta+m_D/2,m_D)+m^2\bar{M}_0(m,\Delta+m_D/2,m_D)\right]+\left(\Delta-\frac{m_D}{2}\right)M_2(m,-m_D^2/2)$$
$$-\frac{1}{4}\left(\frac{m_D}{2}-\Delta\right)N_0(m,m_D^2)+\frac{(2\Delta+m_D)}{4m_D(\Delta+m_D)}I_2(m,\Delta+m_D)$$
$$-\frac{(3m_D^2/2+3\Delta m_D+2\Delta^2-2m^2)}{2m_D^2(m_D+2\Delta)}I_2(m,\Delta+m_D/2)+\frac{(m_D\Delta-2m^2+2\Delta^2)}{4m_D^2\Delta}I_2(m,\Delta). \tag{21}$$

The parity conserving parts of the amplitude $M_i^{(+)}$ vanish for the diagrams in Fig. 2. We denoted $\Delta_q^{(*)}=m_{D_q^{(*)}}-m_{D^0}$, while functions $I_2(m,\Delta)$, $G_0(m,\Delta,v\cdot k)$, $G_3(m,\Delta,v\cdot k)$, $N_0(m,k^2)$, $\bar{G}_0(m,\Delta,v\cdot k)$, $\bar{M}_0(m,\Delta,v\cdot k)$, $M_2(m,k_1\cdot k_2)$, and $M_4(m,k_1\cdot k_2)$ are presented in the Appendix. Note that the sums of amplitudes (18)–(20) are gauge invariant and finite. This is expected, since one cannot generate counterterms at this order. There is no $\mu$ dependence apart from the one hidden in $a_1$; even though $\mu$ appears in the above functions it cancels out completely. Note also that the one-loop chiral corrections vanish in the exact SU(3) limit, i.e., when $m_K \to m_\pi$, as expected. One should note that taking the chiral limit (i.e., $m_s, m_d \to 0$) is not unambiguous. That is, in the combined heavy quark effective theory and the chiral perturbation theory, as well as chiral logarithms there are also functions of the form $F(m_q/\Delta)$ whose value depends on the way one takes the limit (see, e.g., Ref. [33]).

We remark that there exist additional diagrams of the same order in the chiral expansion as the ones given in Fig. 2, but proportional to the $a_2$ part of the effective weak Lagrangian (3). In these additional diagrams the chiral loop is attached to the light current in the factorized vertex, while the photons are emitted from the pseudoscalars in the loop, or they come from the weak vertex. However, the amplitudes of these diagrams vanish due to Lorentz symmetry.

The contribution coming from the anomalous coupling $\pi^0\gamma\gamma$, $\eta\gamma\gamma$, $\eta'\gamma\gamma$ (Fig. 3) is

$$M_{Anom.}^{(+)}=-\sqrt{m_D}\frac{G_f}{\sqrt{2}}a_2\alpha\frac{e^2}{4\pi^2}\sum_{P=\pi^0,\eta,\eta'}\frac{m_D}{m_D^2-m_P^2}K_P,$$

$$K_{\pi^0}=V_{ud}V_{cd}^*,$$

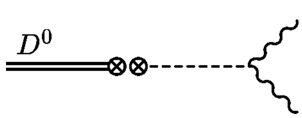


FIG. 3. Anomalous contributions to $D^0 \to \gamma\gamma$ decay. The intermediate pseudoscalar mesons propagating from the weak vertex are $\pi^0, \eta, \eta'$.

$$K_\eta=\left[V_{ud}V_{cd}^*\left(\frac{\sin\Theta}{\sqrt{3}}-\frac{\cos\Theta}{\sqrt{6}}\right)+V_{us}V_{cs}^*\left(\frac{\sin\Theta}{\sqrt{3}}+\frac{\sqrt{2}\cos\Theta}{\sqrt{3}}\right)\right]\times\left[\frac{\sqrt{2}\cos\Theta}{\sqrt{3}}-\frac{4\sin\Theta}{\sqrt{3}}\right]$$

$$K'_\eta=\left[-V_{ud}V_{cd}^*\left(\frac{\sin\Theta}{\sqrt{6}}+\frac{\cos\Theta}{\sqrt{3}}\right)+V_{us}V_{cs}^*\left(\frac{\sqrt{2}\sin\Theta}{\sqrt{3}}-\frac{\cos\Theta}{\sqrt{3}}\right)\right]\times\left[\frac{\sqrt{2}\sin\Theta}{\sqrt{3}}+\frac{4\cos\Theta}{\sqrt{3}}\right], \tag{22}$$

where $\theta=-20°\pm5°$ is the $\eta$-$\eta'$ mixing angle and we have set $f_\pi=f_{\eta_8}=f_{\eta_0}$. This choice of parameters reproduces the experimental results for the $\pi^0\to\gamma\gamma$, $\eta\to\gamma\gamma$, and $\eta'\to\gamma\gamma$ decay widths [34]. In the numerical evaluation we use the values of $\alpha$ and $g$ obtained within the same framework as in [25,26,32,33,35]. The coupling $g$ is extracted from existing

TABLE I. Table of the nonvanishing finite amplitudes. The amplitudes coming from the anomalous and short distance ($C_{7\gamma}^{eff}$) Lagrangians are presented. The finite and gauge invariant sums of one-loop amplitudes are listed in the next three lines ($M_i^{(\pm)}=\Sigma_j M_{i,j}^{(\pm)}$). The numbers 1,2,3 denote the corresponding row of diagrams in Fig. 2. In the last line the sum of all amplitudes is given.

| | $M_i^{(-)}$ ($10^{-10}$ GeV) | | $M_i^{(+)}$ ($10^{-10}$ GeV$^{-1}$) | |
|---|---|---|---|---|
| Anom. | 0 | | $-0.53$ | |
| SD | $-0.27$ | $-0.81i$ | $-0.16$ | $-0.47i$ |
| 1 | 3.55 | $+9.36i$ | 0 | |
| 2 | 1.67 | | 0 | |
| 3 | $-0.54$ | $+2.84i$ | 0 | |
| $\Sigma_i M_i^{(\pm)}$ | 4.41 | $+11.39i$ | $-0.69$ | $-0.47i$ |

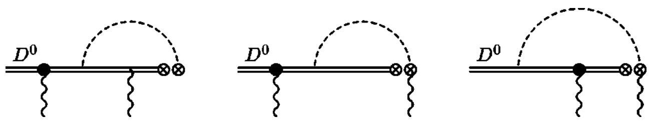

FIG. 4. The diagrams with one $\beta$-like (14) coupling (described by ●), which give vanishing amplitudes.

experimental data on $D^*\to D\pi$. Recently the CLEO Collaboration obtained the first measurement of the $D^{*+}$ decay width $\Gamma(D^{*+})=96\pm4\pm22$ keV [4] by studying $D^*\to D^0\pi^+$. Using this value of the decay width together with the branching ratio $B(D^{*+}\to D^0\pi^+)=(67.7\pm0.5)\%$ one immediately finds at tree level that $g=0.59\pm0.08$. The chiral corrections to this coupling were found to contribute about 10% [25,26]. In order to obtain the $\alpha$ coupling, we use present experimental data on $D_s$ leptonic decays, namely, at the tree level there is the relation $f_D=f_{D_s}=\alpha/\sqrt{m_D}$. From the experimental branching ratio $D_s\to\mu\nu_\mu$ and the $D_s$ decay width [34] one gets $f_{D_s}=0.23\pm0.05$ GeV and $\alpha=0.31\pm0.04$ GeV$^{3/2}$. The SU(3) breaking effects in the form of chiral loops and the counterterms can change the extracted value of $\alpha$. One-chiral-loop corrections can amount to about 40% when $g$ is taken to be 0.59. This value might be changed by the finite parts of the counterterms. However, the contributions coming from counterterms are not known and due to the lack of experimental data they cannot be fixed yet. In our calculation we take $\alpha=0.31$ GeV$^{3/2}$, keeping in mind that the chiral corrections might be important (for instance, setting counterterms to zero in a one-loop calculation one gets $\alpha=0.21\pm0.04$ GeV$^{3/2}$ using $g=0.59$; for details see Appendix B of [36]). For the Wilson coefficients $a_1$ we take 1.26 and $a_2=-0.47$ [31]. We present the numerical results for the one loop amplitudes in Table I.

In the determination of $D^*\to D\gamma\gamma$ and $B^*\to B\gamma\gamma$ a sizable contribution from $\beta$-like electromagnetic terms (14) has been found [27]. Therefore we have to investigate their effect in the $D^0\to\gamma\gamma$ decay amplitude. The considerations of Eq. (14) give us additional diagrams which are given in Figs. 4 and 5, where the $\beta$ vertex is indicated by ●. The nonzero parity violating parts of the one-loop diagrams containing $\beta$ coupling are (Fig. 5)

$$M^{(-)}_{\beta.4}=\sqrt{m_D}\frac{G_f}{\sqrt{2}}a_1e^2g\alpha\left(\beta+\frac{1}{m_c}\right)\frac{1}{(m_D+2\Delta^*)}\frac{1}{16\pi^2}\frac{m_D^2}{3}\times[V_{us}V^*_{cs}G_3(m_K,m_D+\Delta^*_s,-m_D/2)+V_{ud}V^*_{cd}G_3(m_\pi,m_D+\Delta^*_d,-m_D/2)], \tag{23}$$

$$M^{(-)}_{\beta.6}=\frac{G_f}{\sqrt{2}}a_1e^2g\alpha\left(\beta-\frac{2}{m_c}\right)\frac{(m_D)^{3/2}}{48\pi^2}\times\{V_{us}V^*_{cs}[G_3(m_Km_D/2+\Delta^*_s,m_D/2)-G_3(m_K,\Delta^*_s,m_D/2)]+V_{ud}V^*_{cd}[G_3(m_\pi,m_D/2+\Delta^*_d,m_D/2)-G_3(m_\pi,\Delta^*_d,m_D/2)]\}, \tag{24}$$

while the parity conserving parts of the amplitudes arising from the one-loop diagrams with $\beta$ coupling are

$$M^{(+)}_{\beta.1}=\frac{G_f}{\sqrt{2m_D}}a_1e^2\alpha\left(\beta+\frac{1}{m_c}\right)\frac{1}{m_D+2\Delta^*}\frac{1}{12\pi^2}\times[V_{us}V^*_{cs}I_1(m_K)+V_{ud}V^*_{cd}I_1(m_\pi)], \tag{25}$$

$$M^{(+)}_{\beta.2}=-\frac{G_f}{\sqrt{2m_D}}a_1e^2\alpha\left(\beta+\frac{1}{m_c}\right)\frac{1}{(m_D+2\Delta^*)}\frac{1}{12\pi^2}\times[V_{us}V^*_{cs}I_1(m_K)+V_{ud}V^*_{cd}I_1(m_\pi)]=-M^{(+)}_{\beta.1}, \tag{26}$$

$$M^{(+)}_{\beta.3}=-\frac{G_f}{\sqrt{2m_D}}a_1e^2g\alpha\left(\beta+\frac{1}{m_c}\right)\frac{1}{m_D+2\Delta^*}\frac{1}{12\pi^2}\times\{V_{us}V^*_{cs}[I_2(m_K,m_D+\Delta_s)+I_1(m_K)]+V_{ud}V^*_{cd}[I_2(m_\pi,m_D+\Delta_d)+I_1(m_\pi)]\}, \tag{27}$$

$$M^{(+)}_{\beta.4}=\frac{G_f}{\sqrt{2m_D}}a_1e^2g\alpha\left(\beta+\frac{1}{m_c}\right)\frac{1}{m_D+2\Delta^*}\frac{1}{6\pi^2}\times\left\{V_{us}V^*_{cs}\left[\frac{1}{2}I_1(m_K)+(m_D+\Delta_s)G_3\Big(m_K,m_D+\Delta_s,-m_D/2\Big)\right]+V_{ud}V^*_{cd}\left[\frac{1}{2}I_1(m_\pi)+(m_D+\Delta_d)\times G_3(m_\pi,m_D+\Delta_d,-m_D/2)\right]\right\}, \tag{28}$$

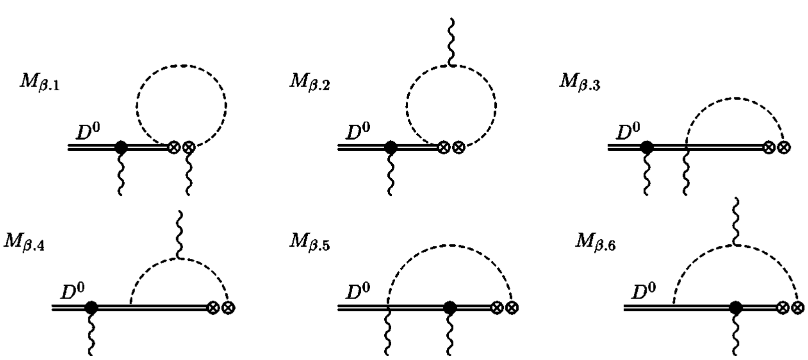

FIG. 5. The diagrams that give nonzero amplitudes with one $\beta$-like coupling.

TABLE II. Table of nonzero contributions of the amplitudes coming from the diagrams with $\beta$ coupling (Fig. 5). In the last line the sums of the contributions are presented. We use $\beta = 2.3$ GeV$^{-1}$, $m_c = 1.4$ GeV.

| Diag. | $M_i^{(-)}(10^{-10}$ GeV) | $M_i^{(+)}(10^{-10}$ GeV$^{-1}$) |
|---|---|---|
| $\beta$.1 | 0 | $-2.69$ |
| $\beta$.2 | 0 | 2.69 |
| $\beta$.3 | 0 | 2.11 |
| $\beta$.4 | 0.88 | $-0.007$ |
| $\beta$.5 | 0 | 0.51 |
| $\beta$.6 | $-2.88$ | $-0.52$ |
| $\Sigma_i M_i^{(\pm)}$ | $-2.00$ | 2.09 |

$$M_{\beta.5}^{(+)} = \frac{G_f}{\sqrt{2m_D}} a_1 e^2 g \alpha \left( -\beta + \frac{2}{m_c} \right) \frac{1}{24\pi^2} \times \Bigg\{ V_{us} V_{cs}^* \frac{1}{m_D + 2(\Delta_s - \Delta_s^*)} \Big[ I_2(m_K, m_D + \Delta_s) - I_2\Big( m_K, m_D/2 + \Delta_s^* \Big) \Big] + V_{ud} V_{cd}^* \frac{1}{m_D + 2(\Delta_d - \Delta_d^*)} \times \Big[ I_2(m_\pi, m_D + \Delta_d) - I_2\Big( m_\pi, m_D/2 + \Delta_d^* \Big) \Big] \Bigg\}, \tag{29}$$

$$M_{\beta.6}^{(+)} = \frac{G_f}{\sqrt{2m_D}} a_1 e^2 g \alpha \left( \beta - \frac{2}{m_c} \right) \frac{1}{24\pi^2} \times \Bigg\{ V_{us} V_{cs}^* \Bigg[ -\frac{(m_D/2 + \Delta_s^*)}{(m_D/2 + \Delta_s - \Delta_s^*)} \times G_3\Big( m_K, m_D/2 + \Delta_s^*, -m_D/2 \Big) + \frac{(m_D + \Delta_s)}{(m_D/2 + \Delta_s - \Delta_s^*)} G_3(m_K, m_D + \Delta_s, -m_D/2) \Bigg] + V_{ud} V_{cd}^* \Bigg[ -\frac{(m_D/2 + \Delta_d^*)}{(m_D/2 + \Delta_d - \Delta_d^*)} \times G_3(m_\pi, m_D/2 + \Delta_d^*, -m_D/2) + \frac{(m_D + \Delta_d)}{(m_D/2 + \Delta_d - \Delta_d^*)} G_3(m_\pi, m_D + \Delta_d, -m_D/2) \Bigg] \Bigg\}. \tag{30}$$

The amplitudes with $\beta$ coupling are not finite and have to be regularized. We use the modified minimal subtraction ($\overline{\rm MS}$) prescription $\bar{\Delta} = 1$ as in [26] (note that in [25] $\bar{\Delta} = 0$ was used). We take $\mu = 1$ GeV$\simeq \Lambda_\chi$ as in [26].

In order to obtain the value of $\beta$ we use the available experimental data from $D^{*+} \to D^+ \gamma$ and $D^{*0} \to D^0 \gamma$ decays. For instance, one can use the recently determined $D^{*+}$ decay width $\Gamma(D^{*+}) = 96 \pm 4 \pm 22$ keV [37] together with the branching ratio $B(D^{*+} \to D^+ \gamma) = (1.6 \pm 0.4)\%$ [34]. At tree level one has

$$\Gamma(D^{*+} \to D^+ \gamma) = \frac{e^2}{12\pi} \left( \frac{2}{3} \frac{1}{m_c} - \frac{1}{3} \beta \right)^2 k_\gamma^3, \tag{31}$$

with $k_\gamma = (m_{D^*}/2)(1 - m_D^2/m_{D^*}^2)$ the momentum of the outgoing photon. Using the experimental data and $m_c = 1.4$ GeV one arrives at[1] $\beta = 2.9 \pm 0.4$ GeV$^{-1}$, where the errors reflect the experimental errors.

On the other hand one can also use the ratio of partial decay widths in the $D^{*0}$ system $\Gamma(D^{*0} \to D^0 \gamma) : \Gamma(D^{*0} \to D^0 \pi^0) = (38.1 \pm 2.9) : (61.9 \pm 2.9)$, where the experimental errors are considerably smaller than in the previous case. At tree level one has

$$\frac{\Gamma(D^{*0} \to D^0 \gamma)}{\Gamma(D^{*0} \to D^0 \pi^0)} = \frac{e^2}{12\pi} \frac{k_\gamma^3}{k_\pi^3} \frac{12\pi f^2}{g^2} \left( \frac{2}{3}\beta + \frac{2}{3} \frac{1}{m_c} \right)^2, \tag{32}$$

with $k_\gamma$ and $k_\pi$ the momenta of the outgoing photon and the pion, respectively. Using $m_c = 1.4$ GeV, $g = 0.59$, $f = f_\pi = 132$ MeV, one arrives at[2] $\beta = 2.3 \pm 0.2$ GeV$^{-1}$, where errors quoted again reflect experimental errors only. The $\beta$ couplings coming from from $D^{*+}$ [Eq. (31)] and $D^{*0}$ [Eq. (32)] are in fair agreement, but not equal. This signals that other contributions coming from chiral loops and higher order terms that would alter our determination of $\beta$ might be important. Since the contribution of chiral loops to $\Gamma(D^{*+} \to D^+ \gamma)$ is approximately 50%, while for $D^{*0} \to D^0 \gamma$ it is about 20% [26], we use in our numerical calculations the value of $\beta = 2.3$ GeV$^{-1}$ obtained from $\Gamma(D^{*0} \to D^0 \gamma)$. Results are shown in Table II using $\beta = 2.3$ GeV$^{-1}$ and $m_c = 1.4$ GeV. Inspection of Tables I and II reveals that for the real parts of the amplitudes the contributions of Figs. 2,3 and of Fig. 5 are comparable in size. However, the decay rate is dominated by the contribution of the imaginary part of the parity violating amplitude, which arises from the one-loop diagrams of Fig. 2. For the parity conserving amplitude, the contributions of SD, anomaly, and $\beta$-like terms are comparable in magnitude.

Due to the suppression of $a_2$ in comparison to $a_1$, we do not include diagrams proportional to $a_2$ in the calculation of terms with $\beta$.

Using short distance contributions, the finite one-loop diagrams and the anomaly parts of the amplitudes (shown in Figs. 2 and 3 and with numerical values of the amplitudes as listed in Table I), one obtains

[1]There is also a solution of Eq. (31) $\beta = 0.09 \pm 04$ GeV$^{-1}$ which, however, does not agree with the determination of $\beta$ from $D^{*0}$ decay.

[2]The other solution is $\beta = -3.6 \pm 0.2$ GeV$^{-1}$ which does not agree with $D^{*+}$ data.

$$B(D^0 \to \gamma\gamma) = 1.0 \times 10^{-8}. \tag{33}$$

This result is slightly changed when one takes into account the terms dependent on $\beta$ [Eq. (14)]. The branching ratio obtained when we sum all contributions is

$$B(D^0 \to \gamma\gamma) = 0.95 \times 10^{-8}. \tag{34}$$

By varying $\beta$ within 1 GeV$^{-1} \leqslant \beta \leqslant 5$ GeV$^{-1}$ and keeping $g = 0.59 \pm 0.08$, the branching ratio is changed by at most 10%. On the other hand, one has to keep in mind that the loop contributions involving beta are not finite and have to be regulated. We have used the $\overline{\text{MS}}$ scheme, with the divergent parts being absorbed by counterterms. The size of these is not known, so they might influence the error in our prediction of the branching ratio. Note also that changing $\alpha$ would affect the predicted branching ratio. For instance, if the chiral corrections do decrease the value of $\alpha$ by 30% this would decrease the predicted branching ratio down to 0.5 $\times 10^{-8}$.

## IV. SUMMARY

We have presented here a detailed calculation of the decay amplitude $D^0 \to \gamma\gamma$, which includes short distance and long distance contributions, by making use of the theoretical tool of the heavy quark chiral perturbation theory Lagrangian. Within this framework, the leading contributions are found to arise from the charged $\pi$ and $K$ mesons running in the chiral loops, and are of order $\mathcal{O}(p^3)$. These terms are finite and contribute only to the parity violating part of the amplitude. The inclusion of terms of higher order in the chiral expansion is unfortunately plagued by the uncertainty caused by the lack of knowledge of the counterterms. As to the parity conserving part of the decay, it is given by terms coming from the short distance contribution, the anomaly, and from loop terms containing the beta coupling, the latter giving most of the amplitude. The size of this part of the amplitude is approximately one order of magnitude smaller than the parity violating amplitude, thus contributing less than 20% to the decay rate. Therefore, our calculation predicts that the $D \to 2\gamma$ decay is mostly a parity violating transition.

In addition to the uncertainties we have mentioned, there is the question of the suitability of the chiral expansion for the energy involved in this process; the size of the uncertainty related to this is difficult to estimate. Altogether, our estimate is that the total uncertainty is not larger than 50%. Accordingly, we conclude that the predicted branching ratio is

$$B(D^0 \to \gamma\gamma) = (1.0 \pm 0.5) \times 10^{-8}. \tag{35}$$

That this result is reasonable can be deduced also from a comparison with the calculated decay rates for $D^0 \to \rho(\omega)\gamma$, which are found to be expected with a branching ratio of approximately $10^{-6}$ [6,7,13].

We look forward to experimental attempts at detecting this decay. Our result suggests that the observation of $D \to 2\gamma$ at a rate that is an order of magnitude larger than Eq. (35) could be a signal for the type of new physics that leads to sizable enhancement [30] of the short distance $c \to u\gamma$ transition.

## ACKNOWLEDGMENTS

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## APPENDIX: LIST OF CHIRAL LOOP INTEGRALS

Here we list dimensionally regularized integrals needed in evaluation of $\chi$PT and HQ$\chi$PT one-loop graphs shown in Fig. 5:

$$i\mu^{\epsilon} \int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}} \frac{1}{q^2 - m^2} = \frac{1}{16\pi^2} I_1(m), \tag{A1}$$

$$i\mu^{\epsilon} \int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}} \frac{1}{(q^2 - m^2)(q \cdot v - \Delta)} = \frac{1}{16\pi^2} \frac{1}{\Delta} I_2(m,\Delta), \tag{A2}$$

with

$$I_1(m) = m^2 \ln\left(\frac{m^2}{\mu^2}\right) - m^2 \bar{\Delta}, \tag{A3}$$

$$I_2(m,\Delta) = -2\Delta^2 \ln\left(\frac{m^2}{\mu^2}\right) - 4\Delta^2 F\left(\frac{m}{\Delta}\right) + 2\Delta^2(1 + \bar{\Delta}), \tag{A4}$$

where $\bar{\Delta} = 2/\epsilon - \gamma + \ln(4\pi) + 1$ (in calculation $\bar{\Delta} = 1$), while $F(x)$ is the function calculated by Stewart in [26], valid for negative and positive values of the argument

$$F\left(\frac{1}{x}\right) = \begin{cases} -\dfrac{\sqrt{1-x^2}}{x}\left[\dfrac{\pi}{2} - \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)\right], & |x| \leqslant 1, \\ \dfrac{\sqrt{x^2-1}}{x} \ln(x + \sqrt{x^2-1}), & |x| \geqslant 1. \end{cases} \tag{A5}$$

The other integrals needed are (for $k^2 = 0$)

$$i\int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - m^2)[(q+k)^2 - m^2](q \cdot v - \Delta)} = \frac{1}{16\pi^2} \frac{1}{v \cdot k} G_0(m,\Delta,v \cdot k), \tag{A6}$$

$$G_0(m,\Delta,v \cdot k) = h^2(m,\Delta) - h^2(m,\Delta + v \cdot k) - \pi[h(m,\Delta) - h(m,\Delta + v \cdot k)],$$

where

$$h(m,\Delta)=\begin{cases}\arctan\left(\dfrac{\Delta}{\sqrt{m^2-\Delta^2}}\right), & |m|>|\Delta|,\\ i\ln\left|\dfrac{m}{\Delta-\sqrt{\Delta^2-m^2}}\right|+\mathrm{sgn}(\Delta)\dfrac{\pi}{2}, & |m|<|\Delta|,\end{cases} \tag{A7}$$

and

$$i\mu^{\epsilon}\int\frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{q^{\mu}q^{\nu}}{(q^2-m^2)[(q+k)^2-m^2](q\cdot v-\Delta)}$$
$$=\frac{1}{16\pi^2}[g^{\mu\nu}G_3(m,\Delta,v\cdot k)+(v^{\mu}k^{\nu}+k^{\mu}v^{\nu})\times G_4(m,\Delta,v\cdot k)+k^{\mu}k^{\nu}G_5(m,\Delta,v\cdot k)+v^{\mu}v^{\nu}G_6(m,\Delta,v\cdot k)], \tag{A8}$$

with

$$G_3(m,\Delta,v\cdot k)=\frac{m^2}{2v\cdot k}G_0(m,\Delta,v\cdot k)-\frac{1}{4v\cdot k}[I_2(m,\Delta)-I_2(m,\Delta+v\cdot k)]+\Delta+\frac{v\cdot k}{2}, \tag{A9a}$$

$$G_4(m,\Delta,v\cdot k)=\frac{1}{v\cdot k}\left[\frac{1}{2(\Delta+v\cdot k)}I_2(m,\Delta+v\cdot k)-G_3(m,\Delta,v\cdot k)\right], \tag{A9b}$$

$$G_5(m,\Delta,v\cdot k)=\frac{1}{v\cdot k}\left[-\frac{1}{2}N_0(m,0)+\Delta G_2(m,\Delta,v\cdot k)-G_4(m,\Delta+v\cdot k)\right], \tag{A9c}$$

$$G_6(m,\Delta,v\cdot k)=\frac{1}{2v\cdot k}[I_2(m,\Delta)-I_2(m,\Delta+v\cdot k)], \tag{A9d}$$

where $I_2(m,\Delta)$ is defined in Eq. (A2) and $N_0(m,k^2)$ in Eq. (A14).

In calculation we also need several other integrals that have been calculated for the case $k_1^{\mu}+k_2^{\mu}=m_D v^{\mu}$, $k_1^2=k_2^2=0$, $v\cdot k_1=v\cdot k_2=(m_D/2)$:

$$i\int\frac{d^4q}{(2\pi)^4}\frac{1}{[(q+k_1)^2-m^2][(q-k_2)^2-m^2](q\cdot v-\Delta)}=\frac{1}{16\pi^2}\bar{G}_0(m,\Delta,m_D), \tag{A10}$$

$$i\int\frac{d^4q}{(2\pi)^4}\frac{1}{[(q+k_1)^2-m^2][(q-k_2)^2-m^2](q^2-m^2)(q\cdot v-\Delta)}=\frac{1}{16\pi^2}\bar{M}_0(m,\Delta,m_D), \tag{A11}$$

where

$$\bar{G}_0(m,\Delta,m_D)=\frac{2}{\Delta m_D}\left\{\left[\frac{\pi}{2}-h(m,\Delta-m_D/2)\right]\sqrt{m^2-\left(\Delta-\frac{m_D}{2}\right)^2-i\delta}-\left[\frac{\pi}{2}-h(m,\Delta+m_D/2)\right]\sqrt{m^2-\left(\Delta+\frac{m_D}{2}\right)^2-i\delta}-2h(m,m_D/2)\sqrt{m^2-\frac{m_D^2}{4}-i\delta}\right\}, \tag{A12}$$

$$\bar{M}_0(m,\Delta,m_D)=\frac{1}{\Delta m_D^2}\left\{-2h^2(m,m_D/2)-2h^2(m,\Delta)+h^2(m,\Delta-m_D/2)+h^2(m,\Delta+m_D/2)+i\pi\ln\left[\frac{\Delta-m_D/2-i\sqrt{m^2-(\Delta-m_D/2)^2-i\delta}}{-\Delta-m_D/2-i\sqrt{m^2-(\Delta+m_D/2)^2-i\delta}}\right]+i\pi\ln\left[\frac{\Delta+i\sqrt{m^2-\Delta^2-i\delta}}{-\Delta+i\sqrt{m^2-\Delta^2-i\delta}}\right]\right\}, \tag{A13}$$

with $\delta>0$ an infinitesimal positive parameter.

The chiral loops needed are

$$i\mu^{\epsilon}\int\frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{1}{(q^2-m^2)[(q+k)^2-m^2]}=\frac{1}{16\pi^2}N_0(m,k^2),$$

$$N_0(m,k^2)=-\bar{\Delta}+1-H(k^2/m^2)+\ln|m^2/\mu^2|, \tag{A14}$$

where

$$H(a)=\begin{cases} 2\left(1-\sqrt{4/a-1}\arctan\left(\frac{1}{\sqrt{4/a-1}}\right)\right), & 0<a<4, \\ 2\left(1-\sqrt{1-4/a}\,\frac{1}{2}\left\{\ln\left|\frac{\sqrt{1-4/a}+1}{\sqrt{1-4/a}-1}\right|-i\pi\Theta(a-4)\right\}\right) & \text{otherwise,}\end{cases} \tag{A15}$$

and for $k_1^2=k_2^2=0$

$$i\mu^{\epsilon}\int\frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{q^{\mu}q^{\nu}}{(q^2-m^2)[(q+k_1)^2-m^2][(q+k_2)^2-m^2]}=-\frac{1}{16\pi^2}\Bigg[g^{\mu\nu}M_2(m,k_1\cdot k_2)-\frac{k_1^{\mu}k_1^{\nu}+k_2^{\mu}k_2^{\nu}}{k_1\cdot k_2}\times M_3(m,k_1\cdot k_2)-\frac{k_1^{\mu}k_2^{\nu}+k_1^{\nu}k_2^{\mu}}{k_1\cdot k_2}M_4(m,k_1\cdot k_2)\Bigg], \tag{A16}$$

with

$$M_2(m,k_1\cdot k_2)=\frac{1}{2}\left\{\frac{1}{2}\left[\bar{\Delta}-\ln\left(\frac{m^2}{\mu^2}\right)\right]+\frac{1}{a}\left[\mathrm{Li}_2\left(\frac{2}{1+\sqrt{\phantom{x}}}\right)+\mathrm{Li}_2\left(\frac{2}{1-\sqrt{\phantom{x}}}\right)\right]+1-\sqrt{\phantom{x}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{\phantom{x}}}\right)\right\}, \tag{A17a}$$

$$M_3(m,k_1\cdot k_2)=\frac{1}{2}\left(\sqrt{\phantom{x}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{\phantom{x}}}\right)-1\right), \tag{A17b}$$

$$M_4(m,k_1\cdot k_2)=\frac{1}{4}+\frac{1}{2a}\left[\mathrm{Li}_2\left(\frac{2}{1+\sqrt{\phantom{x}}}\right)+\mathrm{Li}_2\left(\frac{2}{1-\sqrt{\phantom{x}}}\right)\right], \tag{A17c}$$

where we have abbreviated $a=2k_1\cdot k_2/m^2$ and $\sqrt{\phantom{x}}=\sqrt{1+2m^2/k_1\cdot k_2}$, while $\mathrm{Li}_2(x)$ is a polylogarithmic function.


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