Analysis of $B \to \phi K$ decays in QCD factorization

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We analyze the decay $B \to \phi K$ within the framework of QCD-improved factorization. We find that although the twist-3 kaon distribution amplitude dominates the spectator interactions, it will suppress the decay rates slightly. The weak annihilation diagrams induced by $(S-P)(S+P)$ penguin operators, which are formally power suppressed by order $(\Lambda_{\rm QCD}/m_b)^2$, are chirally and logarithmically enhanced. Therefore, these annihilation contributions are not subject to helicity suppression and can be sizable. The predicted branching ratio of $B^- \to \phi K^-$ is $(3.8 \pm 0.6) \times 10^{-6}$ in the absence of annihilation contributions, and it becomes $(4.3^{+3.0}_{-1.4}) \times 10^{-6}$ when annihilation effects are taken into account. The prediction is consistent with the experimental data.

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I. INTRODUCTION

Previously CLEO put an upper limit on the decay mode $B \to \phi K$ [1]:

$$\mathcal{B}(B^\pm \to \phi K^\pm) < 5.9 \times 10^{-6}. \tag{1.1}$$

However, CLEO [2], BELLE [3] and BaBar [4] recently reported the results

$$\mathcal{B}(B^\pm \to \phi K^\pm) = \begin{cases} (5.5^{+2.1}_{-1.8} \pm 0.6) \times 10^{-6} & \text{CLEO} \\ (7.7^{+1.6}_{-1.4} \pm 0.8) \times 10^{-6} & \text{BaBar} \\ (10.6^{+2.1}_{-1.9} \pm 2.2) \times 10^{-6} & \text{BELLE} \end{cases} \tag{1.2}$$

and

$$\mathcal{B}(B^0 \to \phi K^0) = \begin{cases} (5.4^{+3.7}_{-2.7} \pm 0.7) \times 10^{-6} < 12.3 \times 10^{-6} & \text{CLEO} \\ (8.1^{+3.1}_{-2.5} \pm 0.8) \times 10^{-6} & \text{BaBar} \\ (8.7^{+3.8}_{-3.0} \pm 1.5) \times 10^{-6} & \text{BELLE.} \end{cases} \tag{1.3}$$

It is known that the neutral mode $B^0 \to \phi K^0$ is a pure penguin process, while the charged mode ϕK^- receives an additional (though very small) contribution from the tree diagram. The predicted branching ratio is very sensitive to the nonfactorizable effects which are sometimes parametrized in terms of the effective number of colors $N_c^{\rm eff}$; it falls into a broad range $(13–0.4) \times 10^{-6}$ for $N_c^{\rm eff} = 2 \sim \infty$ [5]. Therefore, a theory calculation of the nonfactorizable corrections is urgently needed in order to have a reliable prediction which can be used to compare with experiment.

A calculation of $B \to \phi K$ within the framework of QCD-improved factorization was carried out recently in Ref. [6]. However, the analysis of Ref. [6] is limited to the leading order in $1/m_b$, and hence the potentially important annihilation contributions which are power suppressed in the heavy quark limit are not included.

In the present paper we will analyze the decay $B \to \phi K$ within the framework of QCD-improved factorization. We will study the important twist-3 effects on spectator interactions and also focus on the annihilation diagrams which are customarily assumed to be negligible based on the helicity suppression argument. However, weak annihilations induced by the $(S-P)(S+P)$ penguin operators are no longer subject to helicity suppression, and hence can be sizable. This is indeed what we found in this work.

II. GENERALIZED FACTORIZATION

The effective Hamiltonian relevant for $B \to \phi K$ has the form

$$\mathcal{H}_{\rm eff}(\Delta B=1) = \frac{G_F}{\sqrt{2}} \Bigg\{ V_{ub} V_{us}^* [c_1(\mu) O_1(\mu) + c_2(\mu) O_2(\mu)] - V_{tb} V_{ts}^* \left(\sum_{i=3}^{10} c_i(\mu) O_i(\mu) + c_g(\mu) O_g(\mu) \right) \Bigg\} + \text{H.c.}, \tag{2.1}$$

where

$$O_1 = (\bar{u}b)_{V-A} (\bar{s}u)_{V-A}, \quad O_2 = (\bar{u}_\alpha b_\beta)_{V-A} (\bar{s}_\beta u_\alpha)_{V-A},$$

$$O_{3(5)} = (\bar{s}b)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A(V+A)},$$

$$O_{4(6)} = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A(V+A)}, \tag{2.2}$$

$$O_{7(9)}=\frac{3}{2}(\bar{s}b)_{V-A}\sum_{q'} e_{q'}(\bar{q}'q')_{V+A(V-A)},$$

$$O_{8(10)}=\frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A(V-A)},$$

$$O_g=\frac{g_s}{8\pi^2}m_b\bar{s}\sigma^{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}(1+\gamma_5)b,$$

with $(\bar{q}_1q_2)_{V\pm A}\equiv\bar{q}_1\gamma_\mu(1\pm\gamma_5)q_2$, O_3–O_6 being the QCD penguin operators, O_7–O_{10} the electroweak penguin operators, and O_g the chromomagnetic dipole operator.

In the generalized factorization approach for hadronic weak decays, the decay amplitudes of $B\to\phi K$ read (in units of $G_F/\sqrt{2}$) [7,8]

$$\begin{aligned}A(B^-\to K^-\phi)&=-V_{tb}V_{ts}^*\Bigg\{\left[a_3+a_4+a_5-\frac{1}{2}(a_7+a_9+a_{10})\right]X^{(B^-K^-,\phi)}\\&+\left[a_4+a_{10}-2(a_6+a_8)\frac{m^2_{B^-}}{(m_s+m_u)(m_b+m_u)}\right]\\&\times X^{(B^-,\phi K^-)}\Bigg\}+V_{ub}V_{us}^*a_1X^{(B^-,\phi K^-)},\end{aligned}\tag{2.3}$$

$$\begin{aligned}A(\bar{B}^0\to\bar{K}^0\phi)&=-V_{tb}V_{ts}^*\Bigg\{\left[a_3+a_4+a_5-\frac{1}{2}(a_7+a_9+a_{10})\right]\\&\times X^{(\bar{B}^0\bar{K}^0,\phi)}+\Bigg[a_4-\frac{1}{2}a_{10}-(2a_6-a_8)\\&\times\frac{m^2_{\bar{B}^0}}{(m_s+m_d)(m_b+m_d)}\Bigg]X^{(\bar{B}^0,\phi\bar{K}^0)}\Bigg\},\end{aligned}$$

where the factorized terms

$$\begin{aligned}X^{(BK,\phi)}&\equiv\langle\phi|(\bar{s}s)_{V-A}|0\rangle\langle K|(\bar{q}b)_{V-A}|\bar{B}\rangle\\&=2f_\phi m_\phi F_1^{BK}(m^2_\phi)(\varepsilon^*\cdot p_B),\\X^{(B,K\phi)}&\equiv\langle\phi K|(\bar{s}q)_{V-A}|0\rangle\langle 0|(\bar{q}b)_{V-A}|B\rangle\\&=2f_Bm_\phi A_0^{\phi K}(m_B^2)(\varepsilon^*\cdot p_B)\end{aligned}\tag{2.4}$$

can be expressed in terms of the form factors F_1^{BK} and $A_0^{\phi K}$ (for a definition of the form factors, see Ref. [9]) and the decay constants f_ϕ and f_B, and the nonfactorized contributions parametrized in terms of χ_i are lumped into the effective number of colors $N_c^{\rm eff}$:

$$\left(\frac{1}{N_c^{\rm eff}}\right)_i\equiv\frac{1}{N_c}+\chi_i;\tag{2.5}$$

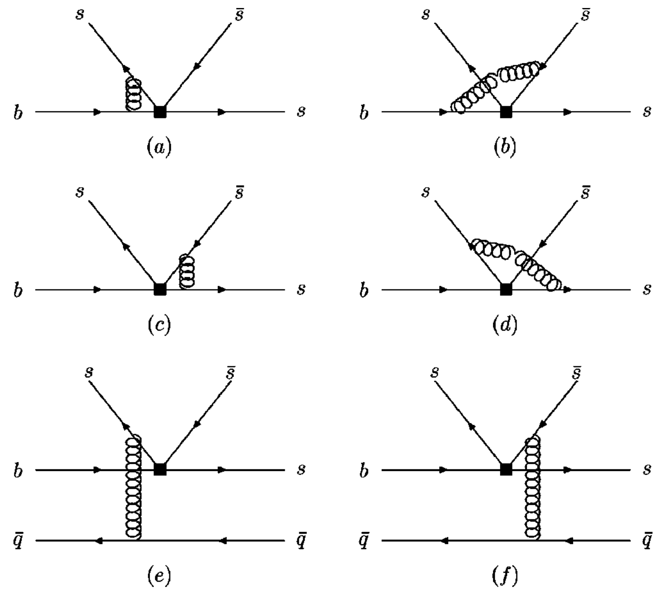

FIG. 1. Vertex and spectator corrections to $B\to\phi K$.

thus the effective parameters a_i appearing in Eq. (2.3) read

$$a_{2i}=c_{2i}+\frac{1}{(N_c^{\rm eff})_{2i}}c_{2i-1},\quad a_{2i-1}=c_{2i-1}+\frac{1}{(N_c^{\rm eff})_{2i-1}}c_{2i}.\tag{2.6}$$

It is known that the parameters a_3 and a_5 depend strongly on $N_c^{\rm eff}$, while a_4 is $N_c^{\rm eff}$ stable (see, for example, Refs. [7,8]). Therefore, the prediction of $B\to\phi K$ rates is sensitive to $N_c^{\rm eff}$, and hence to the nonfactorizable terms χ_i; it varies from 13×10^{-6} to 0.4×10^{-6} for $N_c^{\rm eff}$ ranging from 2 to ∞ [5].

Owing to the unknown form factor $A_0^{\phi K}(m_B^2)$ at large q^2, it is conventional to neglect the annihilation contribution based on the argument of helicity suppression, which amounts to having a vanishing form factor $A_0^{\phi K}(m_B^2)$. However, this argument is valid only for $(V-A)(V-A)$ interactions but not for $(S-P)(S+P)$ ones. This explains the large enhancement factor of $m_B^2/(m_bm_s)$ for the penguin contributions [see Eq. (2.3)]. Therefore, it is conceivable that the annihilation contribution could be sizable and significant.

III. NONFACTORIZBALE EFFECTS IN PENGUIN AMPLITUDES

We next proceed to compute the nonfactorizable effects in the QCD-improved factorization approach. For simplicity we will neglect the light quark masses. In the chiral limit, the kaon is massless, but the ϕ meson has a finite mass. We consider the vertex corrections and hard spectator interactions depicted in Fig. 1 as well as the annihilation diagrams shown in Fig. 2. Recently we analyzed $B\to J/\psi K$ decays within the framework of QCD factorization [10]. The study of $B\to\phi K$ is quite similar to the $J/\psi K$ mode, except for the absence of weak annihilations in the latter. The reader is referred to Ref. [10] for details. The resultant amplitudes are

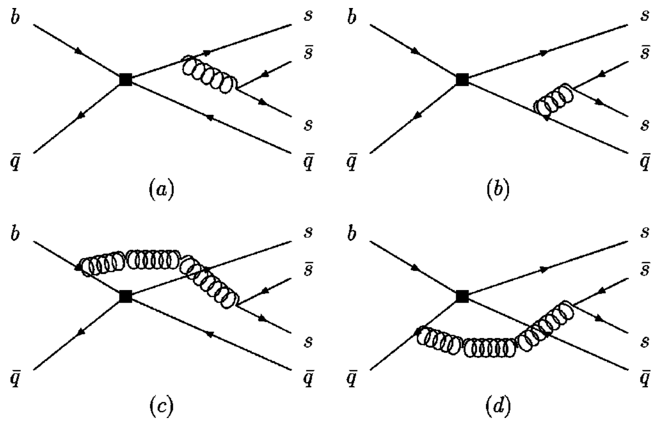

FIG. 2. Annihilation diagrams for $B\to\phi K$ decays.

$$A(B^-\to K^-\phi)$$

$$=-V_{tb}V_{ts}^*\Bigg\{\Bigg[a_3+a_4+a_5-\frac{1}{2}(a_7+a_9+a_{10})\Bigg]X^{(B^-K^-,\phi)}+\Bigg[(c_3+c_9)\mathcal{A}_{nf}^1+(c_5+c_7)\mathcal{A}_{nf}^2+\Bigg(c_6+c_8+\frac{1}{3}(c_5+c_7)\Bigg)\mathcal{A}_f\Bigg]\Bigg\}+V_{ub}V_{us}^*c_2\mathcal{A}_{nf}^1, \tag{3.1}$$

$$A(\overline{B}^0\to\overline{K}^0\phi)$$

$$=-V_{tb}V_{ts}^*\Bigg\{\Bigg[a_3+a_4+a_5-\frac{1}{2}(a_7+a_9+a_{10})\Bigg]\times X^{(\overline{B}^0\overline{K}^0,\phi)}+\Bigg[\Bigg(c_3-\frac{1}{2}c_9\Bigg)\mathcal{A}_{nf}^1+\Bigg(c_5-\frac{1}{2}c_7\Bigg)\mathcal{A}_{nf}^2+\Bigg(c_6+\frac{1}{3}c_5-\frac{1}{2}c_8-\frac{1}{6}c_7\Bigg)\mathcal{A}_f\Bigg]\Bigg\},$$

where

$$a_3=c_3+\frac{c_4}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_4\left[-\begin{pmatrix}18\\14\end{pmatrix}-12\ln\frac{\mu}{m_b}+f_I+f_{II}\right],$$

$$a_4=c_4+\frac{c_3}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}\Bigg\{c_3\left[-\begin{pmatrix}18\\14\end{pmatrix}-12\ln\frac{\mu}{m_b}+f_I+f_{II}\right]+\left(c_3-\frac{c_9}{2}\right)(G(s_s)+G(s_b))-c_1\left(\frac{\lambda_u}{\lambda_t}G(s_u)+\frac{\lambda_c}{\lambda_t}G(s_c)\right)+\sum_{q=u,d,s,c,b}\left(c_4+c_6+\frac{3}{2}e_q(c_8+c_{10})\right)G(s_q)+c_gG_g\Bigg\},$$

$$a_5=c_5+\frac{c_6}{N_c}-\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_6\left[-\begin{pmatrix}6\\18\end{pmatrix}-12\ln\frac{\mu}{m_b}+f_I+f_{II}\right], \tag{3.2}$$

$$a_7=c_7+\frac{c_8}{N_c}-\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_8\left[-\begin{pmatrix}6\\18\end{pmatrix}-12\ln\frac{\mu}{m_b}+f_I+f_{II}\right]-\frac{\alpha}{9\pi}N_cC_e,$$

$$a_9=c_9+\frac{c_{10}}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_{10}\left[-\begin{pmatrix}18\\14\end{pmatrix}-12\ln\frac{\mu}{m_b}+f_I+f_{II}\right]-\frac{\alpha}{9\pi}N_cC_e,$$

$$a_{10}=c_{10}+\frac{c_9}{N_c}+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}c_9\left[-\begin{pmatrix}18\\14\end{pmatrix}-12\ln\frac{\mu}{m_b}+f_I+f_{II}\right]-\frac{\alpha}{9\pi}C_e.$$

In Eq. (3.2), the upper entry of the matrix is evaluated in the naive dimensional regularization (NDR) scheme for γ_5 and the lower entry in the 't Hooft–Veltman (HV) renormalization scheme, $C_F=(N_c^2-1)/(2N_c)$, and $s_q=m_q^2/m_b^2$, $\lambda_{q'}=V_{q'b}V_{q's}^*$, and α is the electromagnetic fine-structure coupling constant. The other terms in Eqs. (3.2) are

$$G(s)=\frac{2}{3}-\frac{4}{3}\ln\frac{\mu}{m_b}+4\int_0^1 d\xi\Phi^{\phi}(\xi)\int_0^1 du u(1-u)\times\ln[s-u(1-u)(1-\xi)],$$

$$G_g=-\int_0^1 d\xi\Phi^{\phi}(\xi)\frac{2}{1-\xi}, \tag{3.3}$$

$$C_e=\left(\frac{\lambda_u}{\lambda_t}G(s_u)+\frac{\lambda_c}{\lambda_t}G(s_c)\right)\left(c_2+\frac{c_1}{N_c}\right),$$

where $\Phi^{\phi}(\xi)$ is the light-cone distribution amplitude (LCDA) of the ϕ meson, which will be discussed shortly. $B\to\phi K$ do not receive factorizable contributions from a_6 and a_8 except for the annihilation topologies. The nonfactorizable annihilation contributions $\mathcal{A}_{nf}^{1,2}$ and the factorizable annihilation amplitude $\mathcal{A}_f$ will also be elucidated below.

Note that the effective parameters a_i appearing in Eqs. (3.2) are renormalization scale and γ_5-scheme independent. Since only one gluon exchange is considered in the annihilation diagrams (see Fig. 2), one may wonder the scale and scheme dependence of the annihilation amplitude given in Eq. (3.1). Fortunately, as we shall see below, the annihilation contribution is predominated by penguin effects characterized by the parameters $a_6=c_6+\frac{1}{3}c_5$ and $a_8=c_8+\frac{1}{3}c_7$ multiplied by μ_χ. It turns out that the scale dependence of a_6 and a_8 is canceled by the corresponding dependence in μ_χ

owing to the running quark masses. Consequently, the annihilation amplitude is essentially scale independent.

The hard scattering kernel f_I appearing in Eq. (3.2) reads

$$f_I=\int_0^1 d\xi\Phi^\phi(\xi)\left\{\frac{3(1-2\xi)}{1-\xi}\ln\xi-3i\pi+\frac{2z(1-\xi)}{1-z\xi}+3\ln(1-z)+\left(\frac{1-\xi}{(1-z\xi)^2}-\frac{\xi}{[1-z(1-x)]^2}\right)z^2\xi\ln z\xi+\frac{z^2\xi^2[\ln(1-z)-i\pi]}{[1-z(1-\xi)]^2}\right\}, \tag{3.4}$$

where $z\equiv m_\phi^2/m_B^2$. For completeness, we have included the ϕ mass corrections to[1] f_I, though such corrections are very small. In the $m_\phi\to 0$ limit, f_I has the same expression as that in $B\to\pi\pi$ decay [11], as it should be. The hard scattering kernel f_{II} arises from the hard spectator diagrams [Figs. 1(e) and 1(f)], and has the form [10]

$$f_{II}=\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\frac{4\pi^2}{N_c}\frac{f_Kf_B}{F_1^{BK}(m_\phi^2)m_B^2}\frac{1}{1-z}\times\int_0^1\frac{d\bar\rho}{\bar\rho}\Phi_1^B(\bar\rho)\int_0^1\frac{d\xi}{\xi}\Phi^\phi(\xi)\int_0^1\frac{d\bar\eta}{\bar\eta}\times\left(\Phi^K(\bar\eta)+\frac{2\mu_\chi(\mu_h)}{m_B}\frac{1}{(1-z)^2}\frac{\Phi_\sigma^K(\bar\eta)}{6\bar\eta}\right), \tag{3.5}$$

where

$$2\mu_\chi(\mu)=\frac{2m_K^2}{m_s(\mu)+m_u(\mu)}=\frac{-4\langle\bar qq\rangle}{f_K^2} \tag{3.6}$$

is proportional to the quark condensate; the B meson wave function Φ_1^B is defined by [11]

$$\langle 0|\bar q_\alpha(x)b_\beta(0)|\bar B(p)\rangle|_{x_+=x_\perp=0}=-\frac{if_B}{4}[(\not p+m_B)\gamma_5]_{\beta\gamma}\int_0^1 d\bar\rho e^{-i\bar\rho p_+x_-}\times[\Phi_1^B(\bar\rho)+\not n_-\Phi_2^B(\bar\rho)]_{\gamma\alpha}, \tag{3.7}$$

with $n_-=(1,0,0,-1)$, and Φ_σ^K is a twist-3 kaon LCDA defined in the tensor matrix element [13]:

$$\langle K^-(P)|\bar s(0)\sigma_{\mu\nu}\gamma_5u(x)|0\rangle=-\frac{i}{6}\frac{f_Km_K^2}{m_s+m_u}\left[1-\left(\frac{m_s+m_u}{m_K}\right)^2\right]\times(P_\mu x_\nu-P_\nu x_\mu)\int_0^1 d\bar\eta e^{i\bar\eta P\cdot x}\Phi_\sigma^K(\bar\eta). \tag{3.8}$$

Since, asymptotically, $\Phi_\sigma^K(\bar\eta)=6\bar\eta(1-\bar\eta)$, the logarithmic divergence of the $\bar\eta$ integral in Eq. (3.5) implies that the spectator interaction is dominated by soft gluon exchanges between the spectator quark and the strange or antistrange quark of ϕ. Hence QCD factorization breaks down at twist-3 order. Note that the hard gluon exchange in the spectator diagrams is not as hard as in the vertex diagrams. Since the virtual gluon's momentum squared there is $k^2=(-\bar\rho p_B+\bar\eta p_K)^2\approx-\bar\rho\bar\eta m_B^2\sim-\mu_h m_b$, where μ_h is the hadronic scale $\sim$500 MeV, we will set $\alpha_s\approx\alpha_s(\sqrt{\mu_h m_b})$ in the spectator diagrams. For the second term in Eq. (3.5), due to the end point divergence, the scale may correspond to a softer scale μ_s. However, since $\alpha_s\mu_\chi$ is weakly scale dependent, we can treat it at the $\sqrt{\mu_h m_b}$ scale. The corresponding Wilson coefficients in the spectator diagrams are also evaluated at the μ_h scale.

The infrared divergence is manifested in the integral $\int_0^1 d\bar\eta/\bar\eta$. However, it is known that the collinear expansion cannot be correct in the end point region owing to the transverse momentum $\langle k_T\rangle$ of the quark, which is, on average, about 300 MeV, the order of the meson's size. Thus the lower limit of $\int_0^1 d\bar\eta/\bar\eta$ should be approximately proportional to $2\langle k_T\rangle/m_b$; or, equivalently, $\int_0^1 d\bar\eta/\bar\eta$ can be approximately replaced by $\int_0^1 d\bar\eta/(\bar\eta+\langle 2k_T\rangle/m_b)$. A consistent treatment of k_T in the calculation is still an issue since k_T itself is a higher twist effect in the QCD factorization approach. Thus we will treat the divergent integral as an unknown "model" parameter, and write

$$Y\equiv\int_0^1\frac{d\bar\eta}{\bar\eta}=\ln\left(\frac{m_B}{\mu_h}\right)(1+\rho_H), \tag{3.9}$$

with ρ_H being a complex number whose phase may be caused by soft rescattering [14]. We see that although the scattering kernel induced by the twist-3 LCDA of the kaon is formally power suppressed in the heavy quark limit, it is chirally enhanced by a factor of $(2\mu_\chi/\Lambda_{\rm QCD})\sim O(10)$, and logarithmically enhanced by the infrared logarithms.

Finally, we wish to remark that the leading-twist LCDA's of the ϕ meson are given by [15]

$$\langle\phi(P,\lambda)|\bar s(x)\gamma_\mu s(0)|0\rangle=f_\phi m_\phi\frac{\varepsilon^{*(\lambda)}\cdot x}{P\cdot x}P_\mu\int_0^1 d\xi e^{i\xi P\cdot x}\Phi_\parallel^\phi(\xi),$$

[1] Equation (3.4) can be obtained from Eq. (19) of Ref. [12] or from Eqs. (2.22) and (2.23) of Ref. [10] by neglecting the ξ^2 terms arising from the transverse wave function $\Phi_\perp^\phi$ and applying the relation $F_0^{BK}(m_\phi^2)/F_1^{BK}(m_\phi^2)=(m_B^2-m_\phi^2)/m_B^2$ for form factors.

$$\langle\phi(P,\lambda)|\overline{s(x)}\sigma_{\mu\nu}s(0)|0\rangle = -if_\phi^T(\varepsilon_\mu^{*(\lambda)}P_\nu-\varepsilon_\nu^{*(\lambda)}P_\mu)\int_0^1 d\xi e^{i\xi P\cdot x}\Phi_\perp^\phi(\xi), \quad (3.10)$$

where ε^* is the polarization vector of ϕ, ξ is the light-cone momentum fraction of the strange quark in ϕ, and f_ϕ and f_ϕ^T are vector and tensor decay constants, respectively, the latter of which is scale dependent. Although $\Phi_\parallel^\phi$ and $\Phi_\perp^\phi$ have the same asymptotic form, it is found that the transverse distribution amplitude (DA) does not contribute to f_I and f_{II} if the light quarks are massless. The contribution of $\Phi_\perp^\phi$ to vertex corrections is suppressed by a factor of m_ϕ/m_B, and hence can be neglected.

IV. ANNIHILATION AMPLITUDES

As shown in Ref. [11], the annihilation amplitude is formally power suppressed by order $\Lambda_{\rm QCD}/m_b$. Nevertheless, it was stressed in the PQCD approach that annihilation contributions in hadronic charmless B decays are not negligible [16]. There are four weak annihilation diagrams depicted in Fig. 2. We first consider the annihilation amplitudes induced by $(V-A)(V-A)$ operators. The first two diagrams, [Figs. 2(a) and 2(b)], are factorizable diagrams, and their contributions are of order m_ϕ^2/m_B^2 and hence can be neglected. Indeed, the factorizable annihilation amplitude should vanish in $m_\phi\to 0$ limit owing to current conservation. It is easily seen that only $O_{\rm odd}$ operators contribute to the nonfactorizable annihilation diagrams [Figs. 2(c) and 2(d)]. It turns out that the nonfactorizable annihilations are dominated by Fig. 2(d) owing to an endpoint contribution. Explicit calculations yield [see Eq. (3.1)]

$$\begin{aligned}\mathcal{A}_{nf}^1 &= -2H\left\{\int_0^1 d\bar\rho d\bar\xi d\eta\frac{\Phi_1^B(\bar\rho)\Phi^\phi(\bar\xi)\Phi^K(\eta)}{(\bar\rho-\bar\xi)\bar\xi\eta} + \int_0^1 d\rho d\bar\xi d\eta\frac{\Phi_1^B(\rho)\Phi^\phi(\bar\xi)\Phi^K(\eta)}{\eta[(\rho-\bar\xi)(\rho-\eta)-1]}\right\}\\ &\cong 2H\left\{6(Y'-1)\int_0^1 d\eta\frac{\Phi^K(\eta)}{\eta} - \int_0^1 d\bar\xi d\eta\frac{\Phi^\phi(\bar\xi)\Phi^K(\eta)}{\eta(\bar\xi\eta-\bar\xi-\eta)}\right\},\end{aligned} \quad (4.1)$$

where

$$H=\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}\frac{4\pi^2}{N_c}\frac{f_Bf_Kf_\phi m_\phi}{m_B^2}(\varepsilon^*\cdot p_B), \quad (4.2)$$

and we have applied the approximation $\rho\approx 1$ and $\bar\rho=1-\rho\approx 0$. In Eq. (4.1) the first term in brackets comes from Fig. 2(d), and the second term from Fig. 2(c). Since the soft phase of annihilation diagrams is not necessarily the same as that of the spectator diagram, we write

$$Y'\equiv\int_0^1\frac{d\bar\eta}{\bar\eta}=\ln\left(\frac{m_B}{\mu_h}\right)(1+\rho_A), \quad (4.3)$$

where the phase is characterized by the complex parameter ρ_A.

For $(V-A)(V+A)$ operators O_5-O_8, the twist-2 kaon DA makes no contribution. Therefore, we need to consider the twist-3 kaon LCDA's Φ_p^K and Φ_σ^K, with the former being defined in the pseudoscalar matrix element [13]

$$\langle K^-(P)|\overline{s(0)}i\gamma_5u(x)|0\rangle=\frac{f_Km_K^2}{m_s+m_u}\int_0^1 d\bar\eta e^{i\bar\eta P\cdot x}\Phi_p^K(\bar\eta). \quad (4.4)$$

The factorizable annihilation amplitude has the expression

$$\begin{aligned}\mathcal{A}_f &= 4N_cH\left(\frac{2\mu_\chi}{m_B}\right)\int_0^1 d\bar\xi\int_0^1 d\eta\Phi^\phi(\bar\xi)\left[\Phi_p^K(\eta)\left(\frac{1}{\bar\xi^2}-\frac{1}{2\bar\xi}\right)\frac{1}{\eta} + \Phi_\sigma^K(\eta)\frac{1}{4\bar\xi\eta^2}\right]\\ &= 24N_cH\left(\frac{2\mu_\chi}{m_B}\right)Y'\left(Y'-\frac{1}{2}\right),\end{aligned} \quad (4.5)$$

where we have applied the LCDA's $\Phi^\phi(\bar\xi)=6\bar\xi(1-\bar\xi)$, $\Phi_p^K(\eta)=1$, and $\Phi_\sigma^K(\eta)=6\eta(1-\eta)$. Likewise, the nonfactorizable annihilations induced by the penguin operators O_5 and O_7 read

$$\mathcal{A}_{nf}^2=-H\left(\frac{2\mu_\chi}{m_B}\right)\left\{6Y'(Y'-1) + \int_0^1 d\bar\xi\int_{\Lambda_{\rm QCD}/m_b}^1 d\eta\Phi^\phi(\bar\xi)\Phi_p^K(\eta)\frac{2-\eta}{\bar\xi\eta(\bar\xi\eta-\bar\xi-\eta)}\right\}, \quad (4.6)$$

where the dominating first term in brackets stems from Fig. 2(d). Note that we have introduced a cutoff $\Lambda_{\rm QCD}/m_b$ to regulate the infrared divergence that occurs in the second term.

Although the annihilation amplitudes $\mathcal{A}_f$ and $\mathcal{A}_{nf}^2$ are formally of order $(\Lambda_{\rm QCD}/m_b)^2$, they receive two large enhancements: one from the chiral enhancement $(2\mu_\chi/\Lambda_{\rm QCD})\sim\mathcal{O}(10)$ and the other from the logarithmic end point divergence of the infrared divergent integral Y'. Consequently, the annihilation effects can be sizable. Physically, this is because the penguin-induced annihilation contributions are not subject to helicity suppression.

V. RESULTS AND DISCUSSIONS

To proceed for numerical calculations, we employ the meson LCDA's as follows:

$$\Phi^K(\bar\eta,\mu^2)=6\bar\eta(1-\bar\eta)\left(1+\sum_{n=1}^\infty a_{2n}^K(\mu^2)C_{2n}^{3/2}(2\bar\eta-1)\right),$$

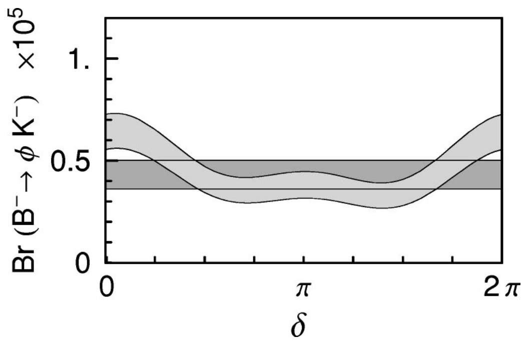

FIG. 3. Branching ratio of $B^-\to\phi K^-$ vs the phase of the complex parameter ρ_A [see Eq. (4.3)], where the dark (horizontal) and light bands correspond to $|\rho_A|=0,\ 1$, respectively, with a variation of $|\rho_H|$ from 0 to 1.

$$\Phi_1^B(\bar\rho)=N_B\bar\rho^2(1-\bar\rho)^2\exp\left[-\frac{1}{2}\left(\frac{\bar\rho m_B}{\omega_B}\right)^2\right], \tag{5.1}$$

$$\Phi^\phi(\bar\xi)=\Phi_\parallel^\phi(\bar\xi)=6\bar\xi(1-\bar\xi),$$

where $C_n^{3/2}$ are Gegenbauer polynomials, and the values of the Gegenbauer moments a_n^K are available in Ref. [13], $\omega_B=0.25$ GeV, and N_B is a normalization constant. We use the decay constants $f_K=0.16$ GeV, $f_B=0.19$ GeV, and $f_\phi=0.237$ GeV, and the running quark masses $m_b(m_b)=4.40$ GeV, $m_s(m_b)=90$ MeV, $m_d(m_b)=4.6$ MeV, and $m_u(m_b)=2.3$ MeV. The next-to-leading-order Wilson coefficients $c_i(\mu)$ in NDR and HV γ_5 schemes are taken from Table XXII of Ref. [17]; they are evaluated at $\mu=m_b(m_b)=4.40$ GeV and $\Lambda_{\overline{\rm MS}}^{(5)}=225$ MeV. For form factors we use $F_1^{BK}(m_\phi^2)=0.38$ as a benchmarked value. Note that $F_1^{BK}(m_\phi^2)=0.407$ in the Bauer-Stech-Wirbel model [9], while it is 0.37 in a QCD sum rule calculation [18].

For the parameters ρ_H in Eq. (3.9) and ρ_A in Eq. (4.3), in principle they may be complex due to final-state soft rescattering. We find that the decay rate is much more sensitive to ρ_A than to ρ_H. Presumably, some information on the parameter ρ can be extracted from the study of $B\to K\pi$ modes. It was shown recently in Ref. [14] that increasing the parameter $|\rho_A|$ from 1 to 2 would increase the corresponding error on the $K\pi$ branching ratios, in which case it would require a considerable fine tuning of the strong interaction phase of Y' in annihilation diagrams to reproduce the experimental value of the branching ratio. Hence it is reasonable to assume that the model parameters are in the range $|\rho|\le 1$. Writing $\rho_A=|\rho_A|\exp(i\delta)$, the branching ratio of $B\to\phi K$ vs the phase δ is plotted in Fig. 3. We obtain

$$\begin{aligned}\mathcal{B}(B^-\to\phi K^-)&=(4.3^{+3.0}_{-1.4})\times10^{-6},\\ \mathcal{B}(B^0\to\phi K^0)&=(4.0^{+2.9}_{-1.4})\times10^{-6},\end{aligned} \tag{5.2}$$

where the central value corresponds to the default values $\rho_A=\rho_H=0$ and the errors come from the variation of $|\rho_H|$ and $|\rho_A|$ from 0 to 1; that is, the theoretical uncertainties come from power corrections of twist-3 spectator interactions and annihilation contributions. Therefore, the predicted branching ratio is consistent with the experimental data [see Eq. (1.2) and (1.3)]. The corresponding absolute ratio of the annihilation to penguin amplitudes depends on the annihilation phase, and is at most of order 0.25. In the absence of annihilation effects, the branching ratios are given by

$$\begin{aligned}\mathcal{B}(B^\pm\to\phi K^\pm)&=\left(\frac{F_1^{BK}(m_\phi^2)}{0.38}\right)(3.8\pm0.6)\times10^{-6},\\ \mathcal{B}(B^0\to\phi K^0)&=\left(\frac{F_1^{BK}(m_\phi^2)}{0.38}\right)(3.6\pm0.6)\times10^{-6},\end{aligned} \tag{5.3}$$

here the error arises from the variation of ρ_H from 0 to 1.

Needless to say, the major theoretical uncertainty stems from the unknown model parameters ρ_H and ρ_A. It should be stressed that the infrared divergence here is always of the logarithmic type, and other possible linear divergence that occur in annihilation diagrams with twist-3 wave functions are explicitly canceled out. As stressed in passing, the infrared divergence stems from a misuse of the collinear expansion in the end point region where the effect of the quark's transverse momentum is important. Since k_T is a higher-twist effect in QCD factorization, at present we treat the infrared divergent integral in a model manner.

Several remarks are in order. (i) The calculations are rather insensitive to the unitarity angle γ as the Cabibbo-Kobayashi-Maskawa matrix element $V_{ub}V_{us}^*$ is considerably suppressed. (ii) The scattering kernel f_{II} is dominated by the twist-3 effect. However, since the Wilson coefficients c_{2i} and c_{2i-1} have opposite signs, it turns out that the magnitudes of a_{3-10} [see Eq. (3.2)] are slightly reduced by the twist-3 terms, and therefore the branching ratio is suppressed by about 10% in the presence of twist-3 effects in spectator interactions. (iii) In the QCD factorization approach, the strong phase of annihilation amplitude is of order α_s^2 since it comes from the annihilation diagrams in which the gluon line is inset with an enclosed quark loop, resembling the vacuum-polarization bubble. Consequently, the phase of the annihilation contribution is likely dominated by the soft contribution induced by soft scattering as characterized by the parameter ρ_A. This is in contrast to the perturbative QCD approach where the annihilation contributions have large strong phases [16].

VI. CONCLUSIONS

We have analyzed the decay $B\to\phi K$ within the framework of QCD-improved factorization, and taken into account some power-suppressed corrections. Our conclusions are the following:

(1) Although the twist-3 kaon distribution amplitude dominates the spectator interactions, it will suppress the decay rates of $B\to\phi K$ slightly by about 10%. In the absence of annihilation contributions, the branching ratio is $(3.8\pm0.6)\times10^{-6}$ for ϕK^- and $(3.6\pm0.6)\times10^{-6}$ for ϕK^0.

(2) The weak annihilation diagrams induced by $(S-P)(S+P)$ penguin operators, which are formally power

suppressed by order $(\Lambda_{\rm QCD}/m_b)^2$, are chirally and logarithmically enhanced. Therefore, these annihilation contributions are not subject to helicity suppression, and in principle can be sizable.

(3) The branching ratio is predicted to be $(4.3^{+3.0}_{-1.4})\times 10^{-6}$ for $B^-\to\phi K^-$ and $(4.0^{+2.9}_{-1.4})\times 10^{-6}$ for $B^0\to\phi K^0$, where theoretical uncertainties come from power corrections of twist-3 spectator interactions and annihilation contributions. The corresponding absolute ratio of annihilation to penguin amplitudes depends on the annihilation phase and is at most 25%.

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