Realization of the large mixing angle solar neutrino solution in an $SO(10)$ supersymmetric grand unified model

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An $SO(10)$ supersymmetric grand unified model proposed earlier leading to the solar solution involving ''just-so'' vacuum oscillations is reexamined to study its ability to obtain the other possible solar solutions. It is found that all four viable solar neutrino oscillation solutions can be achieved in the model simply by modification of the right-handed Majorana neutrino mass matrix M_R. Whereas the small mixing and vacuum solutions are easily obtained with several texture zeros in M_R, the currently favored large mixing angle solution requires a nearly geometric hierarchical form for M_R that leads by the seesaw formula to a light neutrino mass matrix which has two or three texture zeros. The form of the matrix which provides the ''fine-tuning'' necessary to achieve the large mixing angle solution can be understood in terms of Froggatt-Nielsen diagrams for the Dirac and right-handed Majorana neutrino mass matrices. The solution satisfies several leptogenesis requirements which in turn can be responsible for the baryon asymmetry in the universe.

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I. INTRODUCTION

Recent results from the Super-Kamiokande Collaboration [1] involving atmospheric neutrinos have rather convincingly demonstrated the partial disappearance of muon-neutrinos and favor the oscillation of muon-neutrinos into tau-neutrinos, rather than into sterile neutrinos at the 99% confidence level. With regard to solar neutrinos, the situation is somewhat more ambiguous. On the basis of the recently announced 1258 day sample results from Super-Kamiokande [2], together with the flux data from the chlorine [3] and gallium [4] experiments, the partial disappearance of electron-neutrinos through oscillations into the active flavors of muon- or tau-neutrinos is favored over oscillations into purely sterile neutrinos, with the large mixing angle (LMA) solution strongly preferred over the small mixing angle (SMA), the low probability, low mass (LOW), and the quasivacuum (QVO) solutions. Several recent analyses [5] based on the smaller 1117 day sample are basically in agreement with this conclusion by Super-Kamiokande but assign slightly higher probabilities to the other three solutions than does Ref. [2].

Whereas the data at present prefer the LMA solution to the solar neutrino problem, from a model building point of view the LMA solution seems by far the most difficult solution to obtain [6]. Many published models of neutrino masses and mixings either cannot obtain the LMA solution, or can only obtain it by fine tuning parameters. It is thus of importance to reexamine various approaches to see whether they have sufficient flexibility to accommodate the LMA solution in a natural way.

One approach that is particularly flexible is the so-called ''lopsided mass matrix'' approach. The idea here is that the large atmospheric neutrino mixing angle arises from the form of the *charged lepton* mass matrix. In other words, in this approach $U_{\mu 3}$ is more naturally thought of as a mixing of μ and τ rather than of ν_μ and ν_τ. On the other hand, the solar neutrino mixing can come from the *neutrino* mass matrix. In this way the atmospheric neutrino problem and the solar neutrino problem can be decoupled from each other. This is one feature that allows the lopsided mass matrix models to be more flexible in dealing with the solar neutrino problem. In this paper we study an especially simple but very predictive example of a lopsided mass matrix model to see whether it can accommodate the LMA solution in a natural way, that is, without fine-tuning.

The model we shall discuss was developed in a series of papers [7–9] by the present authors, together with Babu earlier in the collaboration. The model is based on supersymmetric $SO(10)$ grand unification. As is well known, $SO(10)$ symmetry typically relates the forms of the Dirac mass matrices of the up quarks, down quarks, charged leptons and neutrinos (which we denote by U, D, L, and N, respectively) very closely to each other. In this model, the lopsidedness of the charged lepton mass matrix, L, and of the down quark mass matrix, D, allow an elegant explanation of many of the features of the quark and lepton masses and mixings; in particular, the fact that $U_{\mu 3}$ is large whereas V_{cb} is small. An interesting point is that in this model the largeness of the atmospheric mixing angle $U_{\mu 3}$ is forced upon one by the structure of L, which in turn is tied by $SO(10)$ symmetry to the forms of the other Dirac mass matrices. On the other hand, as is again typical of $SO(10)$ unification, the Majorana mass matrix of the right-handed neutrinos, M_R, is only indirectly related to the Dirac matrices, and is therefore much

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less constrained. This allows various possibilities for solar neutrino mixing.

In the first papers describing this model [7], it was found that the SMA solution is very easily obtained if one assumes certain simple forms for M_R, specifically ones which have zeros in the 12, 13, 21 and 31 elements. Later it was realized that the QVO solution is also easily obtained [8] by assuming certain other simple forms for M_R. However, it was found that the simplest looking forms for M_R, namely those with many texture zeros, cannot give the LMA solution [9]. In light of the recent claim that the LMA solution is strongly favored, we re-examine this model to see whether the LMA can be obtained in a natural way. In fact, we look at all four solar solutions.

In Sec. II we specify the conditions for each of the four solar solutions. The Dirac mass matrices and parameters obtained earlier for the $SO(10)$ model in question are presented in Sec. III, where we also numerically determine the structures of the right-handed Majorana matrix needed to reproduce all four solutions. A survey of these numerical results in Sec. IV reveals that M_R for the LMA solution, in particular, has a remarkably simple texture which can be easily related to the Dirac neutrino matrix. For this case, the seesaw mechanism then leads to a light neutrino mass matrix which has two or three texture zeros. The implications of this solution for leptogenesis are briefly discussed.

II. PREFERRED REGIONS IN THE NEUTRINO MIXING PLANE

Here we summarize the preferred points in the neutrino mixing plane for the atmospheric neutrino and the four viable solar neutrino oscillation solutions. We use this information to reconstruct the Maki-Nakagawa-Sakata (MNS) [10] neutrino mixing matrix for each of the four solutions.

For the atmospheric neutrino oscillations, the best fit values obtained are [1]

$$\Delta m^2_{32} = 3.2\times 10^{-3}\ \text{eV}^2,$$
$$\sin^2 2\theta_{23} = 1.000, \tag{1}$$

in terms of $\Delta m^2_{ij} \equiv m^2_i - m^2_j$ with $\sin^2\theta_{atm} = 4|U_{\mu 3}|^2|U_{\tau 3}|^2$ expressed in terms of the MNS leptonic mixing matrix elements. Note that to a high degree, the atmospheric neutrino mixing is observed to be maximal. The best fit values for the four solar neutrino solutions according to an earlier analysis by Gonzalez-Garcia [11] are

$$\text{SMA:}\quad \Delta m^2_{21} = 5.0\times 10^{-6}\ \text{eV}^2,$$
$$\sin^2 2\theta_{12} = 0.0024, \quad \tan^2\theta_{12} = 0.0006,$$
$$\text{LMA:}\quad \Delta m^2_{21} = 3.2\times 10^{-5}\ \text{eV}^2,$$
$$\sin^2 2\theta_{12} = 0.75, \quad \tan^2\theta_{12} = 0.33,$$
$$\text{LOW:}\quad \Delta m^2_{21} = 1.0\times 10^{-7}\ \text{eV}^2,$$
$$\sin^2 2\theta_{12} = 0.96, \quad \tan^2\theta_{12} = 0.67,$$
$$\text{QVO:}\quad \Delta m^2_{21} = 8.6\times 10^{-10}\ \text{eV}^2,$$
$$\sin^2 2\theta_{12} = 0.96, \quad \tan^2\theta_{12} = 1.5. \tag{2}$$

In general the MNS mixing matrix, analogous to the CKM quark mixing matrix, can be written as

$$U_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{3}$$

in terms of $c_{12} = \cos\theta_{12}$, $s_{12} = \sin\theta_{12}$, etc. With the oscillation parameters relevant to the scenarios indicated above, to a very good approximation $\theta_{13} = 0°$ and $\theta_{23} = 45°$ whereby Eq. (3) becomes essentially real and of the form

$$U_{MNS} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \tag{4}$$

where the light neutrino mass eigenstates are given in terms of the flavor states by

$$\nu_3 = \frac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau),$$
$$\nu_2 = \nu_e \sin\theta_{12} + \frac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)\cos\theta_{12},$$
$$\nu_1 = \nu_e \cos\theta_{12} - \frac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)\sin\theta_{12}. \tag{5}$$

For the SMA solution, $\theta_{12} = 1.4°$, while the three large mixing solar solutions differ from maximal in that the angle is

approximately 30° for the LMA, 39° for the LOW, and 51° for the QVO solutions. Numerically we find, for each case,

$$U_{MNS}^{(SMA)}=\begin{pmatrix} 0.9997 & 0.0241 & 0 \\ -0.0170 & 0.7069 & 0.7071 \\ 0.0170 & -0.7069 & 0.7071 \end{pmatrix},$$

$$U_{MNS}^{(LMA)}=\begin{pmatrix} 0.866 & 0.500 & 0 \\ -0.354 & 0.612 & 0.707 \\ 0.354 & -0.612 & 0.707 \end{pmatrix},$$

$$U_{MNS}^{(LOW)}=\begin{pmatrix} 0.774 & 0.633 & 0 \\ -0.448 & 0.547 & 0.707 \\ 0.448 & -0.547 & 0.707 \end{pmatrix},$$

$$U_{MNS}^{(QVO)}=\begin{pmatrix} 0.633 & 0.775 & 0 \\ -0.548 & 0.447 & 0.707 \\ 0.548 & -0.447 & 0.707 \end{pmatrix}. \quad (6)$$

III. MODEL MASS MATRICES AND NUMERICAL DETERMINATIONS OF M_R

The model we are studying here is an $SO(10)$ grand unified model. For details of its field content, the flavor symmetry $U(1)\times Z_2\times Z_2$, couplings, and so forth, the reader is referred to the series of papers in which the model was developed [7,9]. Here we will only mention a few of the features of the model important for the present considerations.

This model arose from an attempt to construct a realistic $SO(10)$ model with the simplest possible, or "minimal," Higgs content [12]. This attempt led very naturally to the following structures at the grand unified theory (GUT) scale for the Dirac mass matrices of the up quarks, down quarks, neutrinos, and charged leptons, labeled U, D, N, and L, respectively:

$$U=\begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U,$$

$$D=\begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma+\epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} M_D,$$

$$N=\begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U,$$

$$L=\begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma+\epsilon & 1 \end{pmatrix} M_D. \quad (7)$$

A crucial point is that the four Dirac matrices are closely related to each other by the group theory of $SO(10)$ and that their forms are definitely fixed in terms of a few parameters. As a result the model is very predictive, and in fact gives excellent agreement with all the known facts about the Cabibbo-Kobayashi-Maskawa (CKM) mixings, the quark masses, and the charged lepton masses. By fitting these data, taking into account the renormalization effects from the GUT scale to low energies, the following values of the parameters were obtained:

$$M_U\simeq 113 \text{ GeV}, \quad M_D\simeq 1 \text{ GeV},$$

$$\sigma=1.78, \quad \epsilon=0.145,$$

$$\delta=0.0086, \quad \delta'=0.0079,$$

$$\phi=54°, \quad \eta=8\times 10^{-6}. \quad (8)$$

A critical feature of the model is that the parameter σ is of order unity, and appears in an asymmetrical or "lopsided" way in L and D. This fact plays many roles in the model and is indeed the key to its economy and success in fitting the data. It explains (a) why $m_c/m_t \ll m_s/m_b$, since $m_c/m_t \sim \epsilon^2$, while $m_s/m_b\sim\epsilon\sigma$, (b) why the Georgi-Jarlskog relation $m_s/m_b\cong\frac{1}{3}m_\mu/m_\tau$ holds, since without the σ term a factor of $\frac{1}{9}$ rather than $\frac{1}{3}$ would result, and (c) why $V_{cb} \ll U_{\mu 3}$. The reason for the last is that σ appears in the 32 element of L, where it causes a large mixing of left-handed muon and tau leptons, i.e. large $U_{\mu 3}$, whereas it appears in the 23 element of D, where it causes a large mixing of right-handed quarks, which is not relevant to V_{cb}. The mixing V_{cb} is instead controlled by the 32 element of D, which is the small parameter $\epsilon/3$. The fact that σ appears transposed between D and L has to do with the $SU(5)$ structure of the fields involved.

For present purposes the most important fact is that the largeness of the atmospheric neutrino mixing angle comes from the parameter σ in the charged lepton mass matrix L. The contribution of the neutrino mass matrix to this mixing is formally of order ϵ, as can be seen from the form of N, and is therefore numerically small for generic choices of M_R. On the other hand, one sees that the solar neutrino mixing angle receives only a small contribution from the charged lepton sector, since the 12 and 21 elements of L are small. Therefore, whether the solar angle is large or small is controlled by the neutrino mass matrix $M_\nu=-N^T M_R^{-1} N$, or in other words by M_R, since N is fixed. The form of M_R is rather independent of the forms of the Dirac matrices given in Eq. (7) because it comes from completely different operators. That is why in this model—and indeed in the general framework [6] of "lopsided mass matrix models" in which the atmospheric angle arises from large lopsided entries in L—there is great flexibility in how the solar neutrino problem is solved. Different solar oscillation solutions can be obtained by changing the form of M_R without affecting in any way the fits to the CKM parameters, the masses of the quarks and charged leptons, or the fact that the atmospheric neutrino angle is large.

In our first papers where this model was discussed, forms of M_R were assumed in which the SMA solar solution was naturally obtained. Indeed, one sees immediately that if M_R has vanishing 12, 21, 13, and 31 elements, M_ν does not contribute to the solar neutrino angle, which then comes entirely from L and is therefore small.

The QVO solution can also be very easily obtained. In [9] the following simple form of M_R was constructed:

$$M_R = \begin{pmatrix} 0 & A\epsilon^3 & 0 \\ A\epsilon^3 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R . \tag{9}$$

With this form the seesaw formula [13] gives the light neutrino mass matrix to be

$$M_\nu = N^T M_R^{-1} N = \begin{pmatrix} 0 & 0 & -\eta/(A\epsilon^2) \\ 0 & \epsilon^2 & \epsilon \\ -\eta/(A\epsilon^2) & \epsilon & 1 \end{pmatrix} M_U^2/\Lambda_R . \tag{10}$$

With $\Lambda_R = 2.4\times 10^{14}$ GeV and $A = 0.05$, a fairly reasonable fit to the quasi-vacuum solution then emerged with

$$m_3 = 54.3 \text{ meV}, \quad m_2 = 59.6 \ \mu\text{eV}, \quad m_1 = 56.5 \ \mu\text{eV},$$

$$M_3 = 2.4\times 10^{14} \text{ GeV}, \quad M_2 = M_1 = 3.66\times 10^{10} \text{ GeV},$$

$$U_{e2} = 0.733, \quad U_{e3} = 0.047, \quad U_{\mu 3} = -0.818,$$

$$\delta'_{CP} = -0.2°,$$

$$\Delta m^2_{32} = 3.0\times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{atm} = 0.89,$$

$$\Delta m^2_{21} = 3.6\times 10^{-10} \text{ eV}^2, \quad \sin^2 2\theta_{solar} = 0.99,$$

$$\Delta m^2_{31} = 3.0\times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{reac} = 0.009. \tag{11}$$

We now wish to search for right-handed Majorana mass matrix textures which fit more accurately each of the four solar neutrino solutions. We first note that the MNS mixing matrix corresponds to the product of two unitary transformations,

$$U_{MNS} = U_L^\dagger U_\nu , \tag{12}$$

where U_L diagonalizes the Hermitian lepton matrix $L^\dagger L$, and U_ν diagonalizes the light neutrino mass matrix which we assume to be real and symmetric for simplicity:

$$L^{\text{diag}\dagger} L^{\text{diag}} = U_L^\dagger L^\dagger L U_L , \quad M_\nu^{\text{diag}} = U_\nu^T M_\nu U_\nu . \tag{13}$$

It is easy to see that, given a specific pattern of neutrino masses and mixings, one can invert to find a form of M_R that will give that pattern. To be given a pattern of neutrino masses and mixings means that one is given the MNS mixing matrix U_{MNS} and the neutrino mass eigenvalues m_1, m_2, and m_3. On the other hand, the model itself specifies the charged lepton matrix, L, and the neutrino Dirac mass matrix, N; cf. Eq. (7). Thus M_R can be inferred as follows. First, U_L can be directly obtained from diagonalization of $L^\dagger L$. Then U_L together with the given U_{MNS} determine U_ν through Eq. (12). Although L and hence U_L are complex, we can obtain a real U_ν by making use of the freedom to perform a phase rotation on U_{MNS}, so that

$$U_\nu = U_L \, diag(1,1,e^{-i\phi}) U_{MNS} . \tag{14}$$

Then, defining

$$M_\nu^{\text{diag}} = diag(m_1, -m_2, m_3), \tag{15}$$

with hierarchical masses chosen which are related to the Δm^2_{ij}'s, one can use this and the matrix U_ν already found to determine M_ν by using the second of Eq. (13). Finally, one can use the N known from the model and M_ν to find M_R by inverting the see-saw formula

$$M_R = N M_\nu^{-1} N^T . \tag{16}$$

We present the numerical results for each of the four solar solutions as follows:

$$M_R^{(SMA)} = \begin{pmatrix} 0.156\times 10^{-7} & -0.190\times 10^{-4} & 0.116\times 10^{-3} \\ -0.190\times 10^{-4} & 0.0105 & -0.123 \\ 0.116\times 10^{-3} & -0.123 & 1.000 \end{pmatrix} \times 5.2\times 10^{14} \text{ GeV},$$

$$\text{with} \quad M_1 = 3.7\times 10^6, \quad M_2 = 2.3\times 10^{12}, \quad M_3 = 5.3\times 10^{14} \text{ GeV},$$

$$M_R^{(LMA)} = \begin{pmatrix} 8.30\times 10^{-10} & -0.511\times 10^{-5} & 2.13\times 10^{-5} \\ -0.511\times 10^{-5} & 0.0244 & -0.155 \\ 2.13\times 10^{-5} & -0.155 & 1.000 \end{pmatrix} \times 3.0\times 10^{14} \text{ GeV},$$

$$\text{with} \quad M_1 = 4.2\times 10^6, \quad M_2 = 6.7\times 10^{10}, \quad M_3 = 3.1\times 10^{14} \text{ GeV},$$

$$M_R^{(LOW)} = \begin{pmatrix} 5.15\times10^{-10} & -1.43\times10^{-5} & 5.46\times10^{-5} \\ -1.43\times10^{-5} & 0.0292 & -0.176 \\ 5.46\times10^{-5} & -0.176 & 1.000 \end{pmatrix} \times 5.8\times10^{14}\ \text{GeV},$$

$$\text{with} \quad M_1 = 6.0\times10^{6}, \quad M_2 = 9.7\times10^{11}, \quad M_3 = 6.0\times10^{14}\ \text{GeV},$$

$$M_R^{(QVO)} = \begin{pmatrix} -6.98\times10^{-10} & -1.33\times10^{-5} & 4.75\times10^{-5} \\ -1.33\times10^{-5} & 0.0481 & -0.222 \\ 4.75\times10^{-5} & -0.222 & 1.000 \end{pmatrix} \times 3.2\times10^{15}\ \text{GeV},$$

$$\text{with} \quad M_1 = 8.8\times10^{6}, \quad M_2 = 3.8\times10^{12}, \quad M_3 = 3.3\times10^{15}\ \text{GeV}. \tag{17}$$

Strictly speaking, the above results were obtained at the GUT scale, but with the moderate value of $\tan\beta \sim 5$ preferred by the model [9], and for the hierarchical and sign choices given in Eq. (15) above, the evolutions in masses and mixings from the GUT scale to the low scales are extremely small and can be neglected [14].

That one can find forms for M_R that reproduce the various solar neutrino solutions is in itself not very significant, for as we have just seen, this is guaranteed as long as the relevant matrices are invertible. The significant question is whether the matrix M_R that gives a certain solar solution is obtainable in the model under discussion in a simple way without fine-tuning. The forms for $M_R^{(SMA)}$ and $M_R^{(QVO)}$ given in Eq. (17) are complicated-looking. However, these are the forms that reproduce the present *best-fit* SMA and QVO solutions according to [11]. One already knows from our previous work, as has already been mentioned, that much simpler forms for M_R, having several texture zeros, give perfectly satisfactory SMA and QVO solutions; moreover, those simpler forms are obtainable straightforwardly without fine-tuning. But that same earlier work shows that forms for M_R having several texture zeros do not yield a satisfactory LMA solution in this model. The question is then whether the form $M_R^{(LMA)}$ given in Eq. (17), or something sufficiently close to it, can be obtained simply and naturally in the model. We now turn to this question.

IV. SIMPLE ANALYTIC FORM FOR M_R INVOLVING THE LMA SOLUTION

At first glance the form of $M_R^{(LMA)}$ in Eq. (17) looks very complicated. However, it has some significant features that suggest that it may be obtainable in a simple way. First of all, one sees that $(M_R)_{23} = (M_R)_{32} \simeq -\epsilon$ and $(M_R)_{22} \simeq \epsilon^2$, where ϵ is the parameter that appears in the Dirac matrix, N, of Eq. (7). To a good approximation we can therefore introduce the analytic form

$$M_R^{(LMA)} = \begin{pmatrix} c^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R, \tag{18}$$

written in terms of parameters appearing in the Dirac neutrino matrix, where $\epsilon = 0.145$ and $\eta = 0.8\times10^{-5}$ as before, cf. Eq. (8), and $\Lambda_R = 2.5\times10^{14}$ GeV. It will turn out that the new parameters a, b and c are all of order unity in order to obtain the LMA solar solution. Making use of the seesaw formula, we then find

$$M_\nu^{(LMA)} \sim \begin{pmatrix} 0 & \epsilon/(a-b) & 0 \\ \epsilon/(a-b) & -\epsilon^2(c^2-b^2)/(a-b)^2 & -b\epsilon/(a-b) \\ 0 & -b\epsilon/(a-b) & 1 \end{pmatrix} M_U^2/\Lambda_R. \tag{19}$$

It is interesting, and we shall see, relevant to leptogenesis, that this form has some texture zeros. These texture zeros follow directly from the form of the 23 block of Eq. (18). That this 23 block has rank 1 immediately suggests that it can arise from diagrams of the Froggatt-Nielson type [15]. Moreover, the fact that the same parameter ϵ appears in both M_R and N suggests the possibility that the hierarchies in the 23 blocks of both matrices may have the same origin. These suggestions can be realized as we now show.

In Fig. 1 we repeat for clarity the diagrams in our model which contributed to the Dirac matrices in the 2-3 sector. The dominant 33 elements arise from the $\mathbf{10}_H$ Higgs electroweak doublet contributions. For the 23 and/or 32 elements, higher-order contributions arise from electroweak doublets in both the $\mathbf{10}_H$ and $\mathbf{16}_H$ $SO(10)$ representations, with additional singlet Higgs vacuum expectation values (VEV's) and a $\mathbf{45}_H$ Higgs GUT scale VEV pointing in the B-L direction. Due to the $SU(5)$ structure of the Higgs fields, the diagram appear-

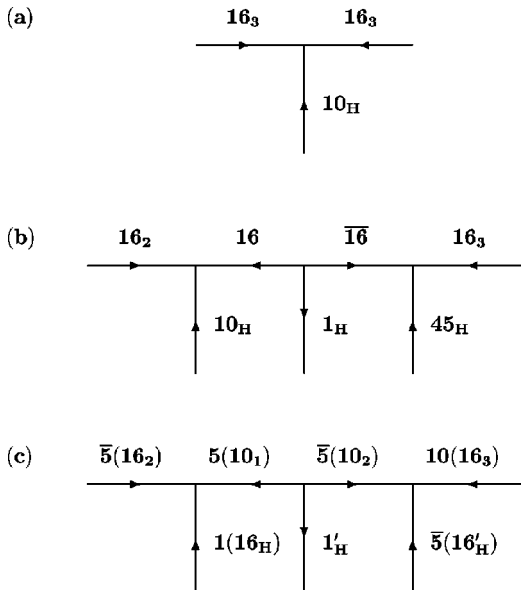

FIG. 1. Diagrams that generate the 33, 23 and 32 elements in the quark and lepton Dirac mass matrices shown in Eqs. (7). (a) The 33 elements denoted "1." (b) Antisymmetric contributions denoted "ϵ," to the 23 and 32 elements where the VEV of the $\mathbf{45}_H$ appears in the *B-L* direction. (c) Asymmetric contributions to the 23 and 32 elements denoted "σ" appearing in the down quark and charged lepton mass matrices arise from this diagram. They do not appear in the up quark and Dirac neutrino mass matrices due to the $SU(5)$ structure of the fields explicitly indicated in the diagram.

ing in Fig. 1(c) contributes only to the D_{23} and L_{32} elements of the down quark and charged lepton mass matrices. Note that the internal superheavy fermions appearing in $\mathbf{16}$, $\overline{\mathbf{16}}$, $\mathbf{10}_1$ and $\mathbf{10}_2$ are integrated out.

In Fig. 2 we show the lower-order diagrams which can contribute to the 2-3 sector of the right-handed Majorana mass matrix. Here a singlet Higgs GUT scale VEV, V_M, couples two superheavy conjugate singlet fermions thus inducing a breaking of lepton number. The VEV's in the $\overline{\mathbf{16}_H}$'s also appear at the GUT scale. The $\overline{\mathbf{16_H}}-\mathbf{1''_H}$ pair appearing in insertion "A" of Fig. 2 serves to lower the heaviest right-handed Majorana neutrino mass down to $\Lambda_R = 2.5 \times 10^{14}$ GeV from the GUT scale value of 2×10^{16} GeV. By making use of the techniques spelled out in detail in [9], one can readily show that the 23 elements of N and M_R are scaled by the same factor ϵ relative to their 33 elements. The factor enters antisymmetrically in N for the 23 and 32 elements due to the $B-L$ nature of the $\mathbf{45}_H$ VEV and the presence of both left-handed neutrino and conjugate neutrino states, while it appears symmetrically in M_R since both states involve conjugate neutrinos. In the Majorana case, both superheavy singlet and **45** fermions must be integrated out. We have checked that these diagrams can be achieved as indicated with proper assignment of the $U(1)\times Z_2 \times Z_2$ flavor quantum numbers for the new heavy fermion fields introduced.

We now turn to the small entries of the first row and column of $M_R^{(LMA)}$ in Eq. (18) with a, b, and c numbers of

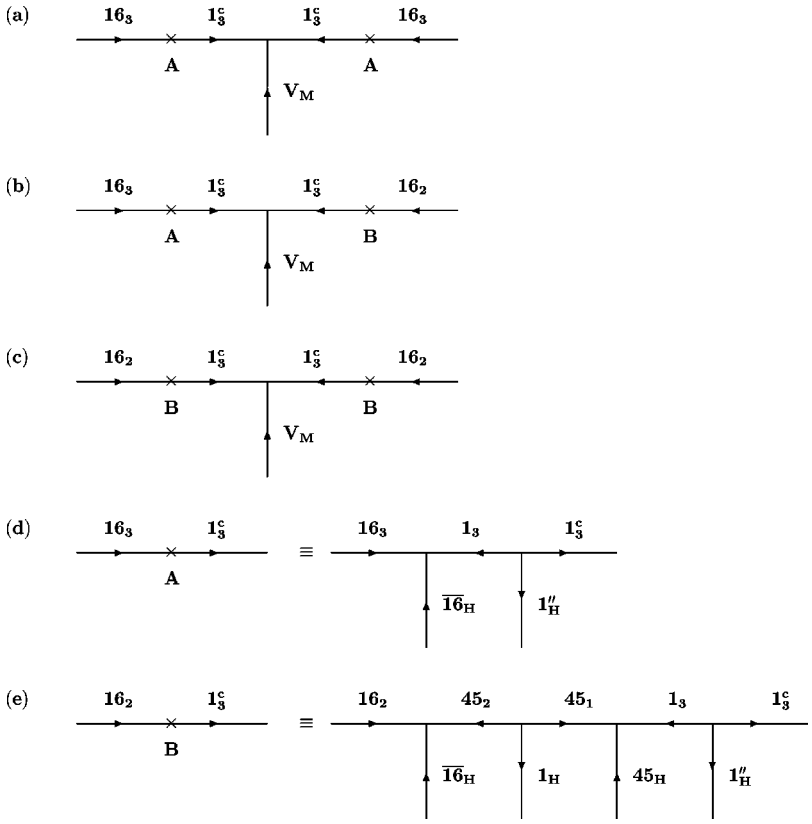

FIG. 2. (a), (b) and (c), respectively, show the diagrams leading to $(M_R)_{33}$, $(M_R)_{32}=(M_R)_{23}$ and $(M_R)_{22}$. Note that these diagrams all come from the same vertex $(1_3^c 1_3^c)V_M$ and so lead to an exact factorized or geometrical form where $(M_R)_{33}(M_R)_{22}=(M_R)_{32}(M_R)_{23}$. The insertions denoted "A" and "B" are defined in (d) and (e). The ratio B/A is proportional to $\langle \mathbf{45}_H\rangle/\langle \mathbf{1''}_H\rangle$ and so is of order $-\epsilon = N_{23}/N_{33}$, as can be seen by inspecting Figs. 1(a) and 1(b).

order unity. The fact that the whole matrix manifests a geometrical hierarchy involving the same small parameters ϵ and η that appear in N reinforces the idea that M_R may be simply obtained by Froggatt-Nielsen-type diagrams involving some of the same VEVs that generate N. If it were the case that $a=b=c$ exactly, then the whole matrix would have rank 1, and thus all its elements could be obtained from a single Yukawa vertex $(\mathbf{1}_3^c\mathbf{1}_3^c)V_M$, in the same way that we illustrated for the 23 block. However, that would, of course, be unrealistic in that two neutrinos would then be massless. However, it is not necessary that the matrix be of rank 1 in order that it arise from simple Froggatt-Nielson diagrams. Thus we have the possibility that a, b, and c are not all equal.

For an especially interesting numerical example, suppose that

$$a=1,\ \ b=c=2,\ \ \Lambda=2.5\times10^{14}\ \ \text{GeV}. \tag{20}$$

This has a simple interpretation in that all elements of the M_R matrix receive contributions from the Yukawa vertex involving V_M, while only the 13 and 31 elements receive contributions from a second $\Delta L=2$ violating Yukawa vertex involving V_M'. This can be realized with the proper choice of flavor indices for V_M'. By the see-saw formula, one then has

$$M_\nu^{(LMA)}=\begin{pmatrix} 0 & -\epsilon & 0 \\ -\epsilon & 0 & 2\epsilon \\ 0 & 2\epsilon & 1 \end{pmatrix} M_U^2/\Lambda_R \tag{21}$$

with three texture zeros from which we obtain

$$m_3=57.4\ \ \text{meV},\quad m_2=9.83\ \ \text{meV},\quad m_1=5.61\ \ \text{meV},$$

$$M_3=2.5\times10^{14}\ \ \text{GeV},\quad M_2=M_1=2.8\times10^{8}\ \ \text{GeV},$$

$$U_{e2}=0.572,\quad U_{e3}=-0.014,\quad U_{\mu3}=0.733,\quad \delta_{CP}'=3.6°,$$

$$\Delta m_{32}^2=3.2\times10^{-3}\ \ \text{eV}^2,\quad \sin^2 2\theta_{atm}=0.994,$$

$$\Delta m_{21}^2=6.5\times10^{-5}\ \ \text{eV}^2,\quad \sin^2 2\theta_{solar}=0.88,$$

$$\Delta m_{31}^2=3.2\times10^{-3}\ \ \text{eV}^2,\quad \sin^2 2\theta_{reac}=0.0008. \tag{22}$$

These results fit both the atmospheric and the LMA solar mixing solutions extremely well and can be considered a success for the model. In fact, the best fit point for the LMA solar mixing solution as given by the Super-Kamiokande Collaboration in their latest analysis of 1258 days of data taking [2] is ($\sin^2 2\theta_{sol}=0.87$, $\Delta m_{21}^2=7\times10^{-5}$ eV^2). We find the whole newly allowed LMA region can be covered with a, b and c varying by factors of $O(1)$ from the values given in Eq. (20). It is noteworthy that the solar neutrino mixing is near maximal, but not actually maximal as that is presently excluded experimentally by the Super-Kamiokande results at more than the 95% confidence level.

How fine-tuned is the form of M_R that we have been discussing? One feature that at least appears fine-tuned is the fact that the 23 and 32 entries in Eq. (18) are not only of order ϵ but actually equal to $-\epsilon$ exactly. One has no right to expect this from the mere fact that the same VEVs come into the diagrams for N and M_R, since as can be seen from Figs. 1 and 2 different Yukawa couplings are involved in the 23 entries of the two matrices. One can test how fine-tuned the form in Eq. (18) is by replacing the 23 and 32 elements by $-d\epsilon$ and the 22 element by $d^2\epsilon^2$. (The fact that the same d enters is due to the factorized structure of the diagrams in Fig. 2, and is therefore not a fine-tuning.) One naturally expects that d is of order unity, but how close must it be to 1 to give a realistic LMA solution? It turns out that the most severe constraint on the value of d comes from the limit on U_{e3}. To satisfy the condition that $|U_{e3}|\leqslant 0.15$ [16], one requires that $0.85\leqslant d\leqslant 1.15$. Thus, the LMA solution does not require an unnatural fine-tuning of parameters.

Finally we note that the upper bound on the lightest heavy Majorana neutrino mass M_1 should be less than or of order 10^9 GeV to prevent overproduction of gravitinos from overclosing the universe after inflation [17]. This bound is satisfied for all four solar solutions as determined in Eq. (17) and, in particular, for the model illustrated above. A second condition for leptogenesis is that the 13 and 31 elements of M_R be suppressed by a factor of at least 10^3 relative to the 33 element to inhibit mixing of the heaviest right-handed neutrino with the lightest one in order to prevent its rapid decay washing out the lepton asymmetry generated. This is satisfied in our model.

V. SUMMARY

We have investigated how an $SO(10)$ SUSY GUT model proposed earlier can be modified in order to obtain solar neutrino solutions other than the vacuum solution. The study revealed that only the right-handed Majorana neutrino mass matrix needed to be modified, with the Dirac matrices for the neutrinos and charged leptons (as well as for the quarks) left unchanged. In short, in this model the maximal atmospheric neutrino mixing is controlled primarily by the structure of the charged lepton mass matrix, while the type of solar neutrino solution is largely determined by the form of the right-handed Majorana mass matrix.

Of particular interest was the finding that the large mixing angle solar solution is readily obtained with a nearly geometrical hierarchy in M_R, where the 2-3 subsector has a close relationship with that for the Dirac neutrino matrix, as seen by study of the Froggatt-Nielsen diagrams. It is precisely this structure which provides the ''fine-tuning'' necessary to achieve the LMA solar solution.

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