High-energy neutrino conversion and the lepton asymmetry in the universe

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We study the matter effects on oscillations of high-energy neutrinos in the Universe. A substantial effect can be produced by scattering of the neutrinos from cosmological sources ($z \gtrsim 1$) on the relic neutrino background, provided that the latter has large CP asymmetry: $\eta \equiv (n_\nu - n_{\bar{\nu}})/n_\gamma \gtrsim 1$, where n_ν, $n_{\bar{\nu}}$ and n_γ are the concentrations of neutrinos, antineutrinos, and photons. We consider in detail the dynamics of conversion in the expanding neutrino background. Applications are given to the diffuse fluxes of neutrinos from gamma ray bursters, active galactic nuclei, and the decay of superheavy relics. We find that the vacuum oscillation probability can be modified by $\sim 10-20\%$, and in extreme cases allowed by present bounds on η the effect can reach $\sim 100\%$. The signatures of matter effects would consist (i) for both active-active and active-sterile conversion, in a deviation of the numbers of events produced in a detector by neutrinos of different flavors, N_α ($\alpha = e, \mu, \tau$), and of their ratios from the values given by vacuum oscillations; such deviations can reach $\sim 5-15\%$, and (ii) for active-sterile conversion, in a characteristic energy dependence of the ratios $N_e/N_\mu, N_e/N_\tau, N_\mu/N_\tau$. Searches for these matter effects will probe large CP and lepton asymmetries in the Universe.

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I. INTRODUCTION

The detection of high-energy cosmic neutrinos and detailed studies of their properties are among the main challenges in astrophysics and cosmology. They will give unique information about the structure of the Universe, mechanisms of particle acceleration, sources of cosmic rays, and properties of the galactic and intergalactic media. They will also have important implications for neutrino properties (masses, mixings, etc.) and for particle physics in general.

Intense fluxes of neutrinos, with energies up to $\sim 10^{21}$ eV, are supposed to be produced by cosmological objects such as gamma ray bursters (GRBs) and active galactic nuclei (AGN) [1]. It was suggested that neutrinos of energies as high as $10^{22}-10^{24}$ eV could be produced by topological defects such as cosmic strings, necklaces, and domain walls [2]. Furthermore, neutrinos produced by the decay of superheavy particles have been considered in connection to the problem of ultrahigh-energy cosmic rays exceeding the Greisen-Zatsepin-Kuz'min (GZK) cutoff [3].

The detection of high-energy neutrinos from cosmological sources is challenging for neutrino telescopes. The existing large water, ice, or air shower experiments [4] open up some possibility of detection; however, a detailed study requires larger detectors to be realized in the future.

The properties of high-energy neutrino fluxes can be modified by oscillations on the way from the source to the Earth. In particular, it was remarked [5] that oscillations lead to the appearance of tau neutrinos in the high-energy neutrino flux. Moreover, the study of oscillation effects opens up the possibility to probe neutrino mixings and distinguish between different mass spectra [6]. In all these studies, only vacuum oscillations have been considered. In this connection, we address here two questions: (i) Are matter effects important for high-energy neutrinos propagating in the Universe? (ii) Which information on the properties of the interstellar and intergalactic medium can be obtained from the study of these effects?

During their travel from the production point to the detector, the neutrinos cross large amounts of matter, which could induce significant refraction and conversion. In Ref. [7], we considered the interaction of neutrinos with the matter of the source for neutrinos produced in GRBs and AGN. The effects of matter on vacuum oscillations appeared to be small. It was also found [7] that the neutrino-neutrino interaction in the dark matter halos of galaxies does not affect the vacuum oscillations significantly. Conversely, strong matter effects are not excluded for neutrinos crossing media of larger size, such as the halos of clusters of galaxies. Furthermore, neutrinos from cosmological sources travel for such large distances in the intergalactic space that the Universe itself, with its particle content, can be considered as a medium producing refraction effects. In [7], we found that significant conversion can be realized for neutrinos crossing cosmological distances in the Universe with strongly CP-asymmetric neutrino background.

In this paper, we analyze this possibility in detail. We discuss the refraction and conversion effects of the background on high-energy neutrinos from cosmological sources and on neutrinos of the background itself.

Let us describe the relic neutrino gas by the number densities of the various flavors, n_α ($\alpha = e, \bar{e}, \mu$, etc.), and by the CP asymmetry η_ν defined as

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$$\eta_\nu \equiv (n_\alpha - n_{\bar{\alpha}})/n_\gamma, \tag{1}$$

where n_γ is the concentration of photons.

The big bang nucleosynthesis (BBN) and structure formation [8,9] admit large CP asymmetries for muon and tau neutrinos, while the asymmetry for the electron neutrino is strongly constrained:

$$|\eta_{\mu,\tau}| \lesssim 10, \quad -0.01 \lesssim \eta_e \lesssim 0.3. \tag{2}$$

Large asymmetries have also important implications on the properties of the spectrum of the cosmic microwave background radiation (CMBR) [9]. The recent results on the second acoustic peak of the CMBR from BOOMERANG and MAXIMA-1 experiments [10] seem to favor a large lepton asymmetry, $\eta_\nu \sim 1$ [11–13]. In particular, a satisfactory interpretation of the data requires [12]

$$|\eta_{e,\mu,\tau}| \lesssim 2.2, \tag{3}$$

thus providing a stronger restriction of the allowed η_μ and η_τ values with respect to (2). In our discussion, we will consider asymmetries η_μ and η_τ as large as $|\eta_{\mu,\tau}| \sim 1-2$, according to the upper limit (3); however, results will be given also for larger values, allowed by the less stringent bound (2).

We want to underline here that the realization of large CP asymmetries in the individual lepton flavors is consistent with zero lepton asymmetry. This corresponds to

$$\eta_e + \eta_\mu + \eta_\tau = 0, \tag{4}$$

that is, to zero total lepton number. Large lepton asymmetry, in contrast, implies large CP asymmetry.

The paper is organized as follows. In Sec. II, the properties of the relic neutrino background are discussed. We show that significant matter effects on high-energy neutrinos require large CP asymmetry of the background, and we study the refraction and conversion effects in the background itself. In Secs. III and IV, we study matter effects on high-energy neutrinos produced at cosmological distances. Applications are given in Sec. V to the diffuse fluxes of neutrinos from AGN, GRBs, and from the decay of heavy relics. In Sec. VI, we discuss the experimental signatures of matter effects. Conclusions follow in Sec. VII.

II. ACROSS THE UNIVERSE

Let us consider the interactions of high-energy neutrinos propagating from cosmological sources to the Earth. These neutrinos cross layers of matter in the source itself, then interact with particles in the interstellar and intergalactic media, and finally interact in the matter of our cluster of galaxies and of our galaxy.

In what follows, we will discuss interactions in the intergalactic medium. The effects of the matter of the sources can be neglected [7]. As we will show later, for neutrino oscillation parameters and energies relevant for this discussion, the effects of the galactic halo and of the halo of the cluster of galaxies are also very small [7].

A. Minimum width condition

The necessary condition for significant matter effect is the minimum width condition [7]. We define the width of the medium as the integrated concentration of the particle background of the Universe along the path traveled by the neutrino beam:

$$d \equiv \int_t^{t_0} n(t')dt'. \tag{5}$$

Here t is the epoch of production of the neutrinos and $t_0 \sim 10^{18}$ s is the present epoch. Using the scaling of the concentration $n(t) \propto t^{-2}$, one finds

$$d(t) = d_U\left[\frac{t_0}{t} - 1\right] = d_U[(1+z)^{3/2} - 1], \tag{6}$$

$$d_U \equiv t_0 n_0, \tag{7}$$

where n_0 is the present concentration of the background and we have introduced the redshift $z \equiv (t_0/t)^{2/3} - 1$.

The minimum width condition can be written as [7]

$$r \equiv \frac{d}{d_0} \geqslant 1, \tag{8}$$

where d_0 is the refraction width of the medium:

$$d_0 \equiv \frac{\pi}{2}\frac{n}{V}, \tag{9}$$

with V being the effective matter potential in a given neutrino conversion channel. The width d_0 corresponds to the distance in matter at which the oscillation phase induced by matter equals $\pi/2$. Notice that, since $V \propto n$, d_0 does not depend on the density of the medium and is determined by the properties of the interaction. The usual weak interaction gives $d_0 \sim 1/G_F$, where G_F is the Fermi constant.

Let us consider the fulfillment of the condition (8) for different components of the intergalactic medium.

(i) Because of the very small concentration of nucleons and electrons, the width of these components is extremely small, $d_B/d_0 \ll 1$, even for neutrinos produced at cosmological distances. Indeed, the baryon concentration can be estimated as $n_B = n_\gamma \eta_B$, where $\eta_B = 10^{-10} - 10^{-9}$ is the baryon asymmetry of the Universe and n_γ is the photon concentration. At present, $n_\gamma = n_\gamma^0 \simeq 412$ cm^{-3}. Taking, e.g., production epoch $z \simeq 1$, we find from Eqs. (6)–(9) that for baryons $r \equiv r_B \sim 10^{-11}$.

(ii) The scattering on the electromagnetic background has a negligible effect due to the smallness of the interaction. The neutrino-photon potential is of the second order in the Fermi constant and depends on the energy of the neutrino beam and on the temperature and concentration of the photon gas [14,15]. Using the results of Ref. [15] (see also the discussion in [16]), we find from Eqs. (6)–(9) $r_\gamma \lesssim 10^{-8}$ for neutrino energy $E \lesssim 10^{21}$ eV and production epoch $z \simeq 1$.

(iii) The effect of the scattering on the neutrino background can produce significant effect if the background has large CP asymmetry.[1] Indeed, for asymmetry $\eta_\nu \sim 1$ we get $d_\nu \sim d_0$. Clearly, if the lepton asymmetry is of the order of the baryon one, $\eta_\nu \simeq \eta_B$, the width is negligibly small: $d_\nu \sim d_B \ll d_0$.

Let us consider the minimum width condition for the neutrino background in more detail. The effective potential due to the scattering of neutrinos on the relic neutrino background can be written as

$$V = F \eta_\nu \sqrt{2} G_F n_\gamma, \tag{10}$$

where F is a constant of order 1 which depends on the specific conversion channel (see Secs. III A and IV A). With the potential (10), Eqs. (6), (8), and (9) give the condition

$$r(z) = 1.6 \times 10^{-2} |F| \eta_\nu [(z+1)^{3/2} - 1] \geqslant 1. \tag{11}$$

For neutrinos produced in the present epoch, $z \simeq 0$, and values of η_ν allowed by the bounds (2) and (3), the condition (11) is not satisfied: $r(z \sim 0) \ll 1$. From Eq. (11), we can define the epoch z_d which corresponds to $r = 1$:

$$1 + z_d = \left[1 + \frac{1}{1.6 \times 10^{-2} |F| \eta_\nu} \right]^{2/3}, \tag{12}$$

so that for neutrinos produced at $z \geqslant z_d$ the minimum width condition is fulfilled. Taking, for instance, $\eta_\nu = 1$ and $F = 2$, we get $z_d \simeq 9$; by requiring $r \simeq 0.3$ (which corresponds to 10% matter effect [7]), we find $z_d \simeq 3.7$.

The following remark is in order. The condition (8) is necessary but not sufficient to have significant matter effects. In particular, for the case of oscillations in uniform medium and small mixing angles ($\sin 2\theta \lesssim 0.3$), we have found in Ref. [7] that the width $d_{\min}$ needed to have conversion probability larger than 1/2 equals

$$d_{\min} = \frac{d_0}{\tan 2\theta} > d_0. \tag{13}$$

This quantity represents an absolute minimum. For media with varying density, the required width is larger than $d_{\min}$ [7].

We conclude, then, that the only component of the intergalactic medium which can produce a significant matter effect is a strongly CP-asymmetric neutrino background, with $\eta_\nu \gtrsim 1$. Moreover, cosmological epochs of neutrino production are required: $z \gtrsim 3$.

B. Properties of the relic neutrino background

Neutrino mixing and oscillations modify the flavor composition of the neutrino background, so that one expects the present values of the CP asymmetries in the various flavors to be different from those at the epoch of BBN; the latter are constrained by the bounds (2).

In this section, we assume that large CP asymmetries are produced at some epoch before the BBN, i.e., at temperature $T \gtrsim T_{\mathrm{BBN}} \simeq 1$ MeV, and study how they evolve with time. The evolution of the flavor densities n_e, n_μ, and n_τ is a nonlinear many-body problem, which, in general, requires a numerical treatment [18,19]. In some specific cases, however, an analytical description is possible [20] and conclusions can be obtained on general grounds.

1. Three-neutrino system evolution

Let us first consider the case of mixing between three active neutrinos ν_e, ν_μ, and ν_τ. In Refs. [18,20], it has been shown that the evolution of the flavor densities has peculiar aspects for the ideal case of a monoenergetic gas of neutrinos (with no antineutrinos, $n_{\bar{\nu}} = 0$) initially produced in flavor states. For this specific ensemble of neutrinos, the potential due to neutrino-neutrino interaction cancels in the evolution equation, so that the collective behavior of the system is described by vacuum oscillations. This result holds with a good approximation [18] also for realistic neutrino energy spectra and in the presence of a small component of antineutrinos. For this reason, it can be applied to our case of interest, in which the background is strongly CP-asymmetric, $n_\nu \gg n_{\bar{\nu}}$, and neutrinos have a thermal spectrum. In what follows, we approximate the neutrino energies with the average thermal energy of the gas: $E \simeq \langle E_\nu \rangle = \alpha T_\nu$, where T_ν denotes the temperature of the neutrino gas. The numerical factor α depends on the CP asymmetry of the background: we have $\alpha \simeq 3.15$ in the absence of asymmetry, $\eta \simeq 0$, and $\alpha \simeq 3.78$ for $\eta \simeq 1$.

The length scale of flavor conversion is given by the vacuum oscillation length:

$$\begin{aligned} l_\nu &= \frac{4\pi E}{\Delta m^2} \\ &\simeq \frac{4\pi \alpha \beta T}{\Delta m^2} \\ &= 2.48 \times 10^5 \text{ cm } \alpha\beta \left(\frac{T}{1 \text{ MeV}} \right) \left(\frac{10^{-3} \text{ eV}^2}{\Delta m^2} \right), \end{aligned} \tag{14}$$

where β is the ratio between the temperature of the neutrino background and the temperature T of the electromagnetic radiation: $\beta \equiv T_\nu / T$. We have $\beta = 1$ before the electron-positron recombination epoch, $T \gtrsim 0.5$ MeV, and $\beta = (4/11)^{1/3}$ after this epoch.

Besides oscillations, for temperatures $T \gtrsim 1$ MeV, other phenomena, and therefore other length scales, are relevant.

(i) Inelastic collisions. Let us consider a system of two mixed neutrinos, $\nu_a, \nu_b (a,b = e, \nu, \tau)$. After its production as a flavor state, e.g., $\nu = \nu_a$, a neutrino oscillates in vacuum until a collision occurs with a particle X of the background. At the time of the collision, the quantum state of the neutrino is a coherent mixture of the two flavors: $\nu = A \nu_a + B \nu_b$. The effects of the collision depend on the specific reactions that

[1] For CP-symmetric background, significant effects can appear at large temperature, $T \gtrsim 1$ MeV, due to thermal effects [17], or at extremely high neutrino energies, due to neutrino-antineutrino scattering in the resonant Z^0 channel [7].

take place [21–23] (see also the discussion in [16]). If the reaction is a scattering, $\nu X \to \nu X$, and the interaction is flavor-blind, i.e., it is the same for the two flavors a and b, after the collision the neutrino continues to propagate as a coherent superposition of ν_a and ν_b and the collision does not affect oscillations. For scattering with flavor-sensitive interaction or for absorption processes,[2] $\nu X \to any$, the effect of the collision is to break the coherence between ν_a and ν_b, so that after the collision the two flavors evolve independently, developing vacuum oscillations until the next collision happens.

As a result, one easily obtains that for a beam of neutrinos propagating in a medium, oscillations are damped according to the expression

$$n_a(L) = \frac{1}{2} + \left(n_a^0 - \frac{1}{2}\right)\exp\left[-\frac{L}{l_c}\ln\left(\frac{1}{1-2P_c}\right)\right], \qquad (15)$$

where n_a^0 and $n_a(L)$ are the fractions of ν_a in the neutrino beam at the production time and at distance L from the production point. Here l_c is the coherence length, which represents the distance between two collisions, and P_c is the vacuum oscillation probability between two collisions: $P_c \simeq \sin^2 2\theta \sin^2(\pi l_c/l_v)$. From Eq. (15) we see that, if $P_c \neq 0$, with the increase of L (i.e., of the number of collisions) the interplay of oscillations and collisions leads to the equilibration of the flavor densities: $n_a(L\to\infty) = n_b(L\to\infty) = 1/2$. The convergence to this limit is determined by the equilibration length $l_{\mathrm{eq}} \equiv l_c/\ln(1/|1-2P_c|)$. For small conversion probability, $P_c \ll 1$, the length l_{eq} is much larger than l_c: $l_{\mathrm{eq}} \simeq l_c/(2P_c) \gg l_c$. This is the case if the vacuum mixing is small and/or the collisions are much more efficient than oscillations, $l_c \ll l_v$, so that the vacuum oscillation phase $\Phi = 2\pi l_c/l_v$ is small. Thus, equilibration of the flavor densities can be obtained only after a large number of collisions: $n_{\mathrm{coll}} = L/l_c \gtrsim 1/(2P_c)$. Conversely, if $P_c \sim 1$, equilibration is achieved rapidly after few collisions: $n_{\mathrm{coll}} \gtrsim L/l_c$. This circumstance is realized if $l_c \gtrsim l_v$ and the mixing is large, $\sin^2 2\theta \sim 1$.

Let us consider the coherence length, l_c, in more detail. According to Refs. [21,22], l_c can be written as

$$l_c(a,b)^{-1} = \frac{1}{2}[\Gamma^{\mathrm{abs}}(a) + \Gamma^{\mathrm{abs}}(b) + \Gamma^{\mathrm{fs}}(a,b)], \qquad (16)$$

where $\Gamma^{\mathrm{abs}}(x)$ is the rate of absorption processes for the neutrino of flavor x and $\Gamma^{\mathrm{fs}}(a,b)$ is the contribution of the flavor-sensitive scatterings $\nu X \to \nu X$. This quantity is determined by the square of the difference of the ν_a-X and ν_b-X scattering amplitudes [21,22]. In terms of the total scattering rates $\Gamma(a)$ and $\Gamma(b)$, one gets [21]

$$\Gamma^{\mathrm{fs}}(a,b) \simeq \Gamma(a) + \Gamma(b) - 2\sqrt{\Gamma(a)\Gamma(b)}. \qquad (17)$$

[2]In the situation we are considering, the neutrinos are in thermodynamical equilibrium, so that their disappearance through a given reaction is balanced by their production through the inverse process.

From Eqs. (16)–(17), using the rates given in Ref. [24], we find

$$l_c(a,b) = [k(a,b) G_F^2 \alpha^2 \beta^5 T^5]^{-1}, \qquad (18)$$

where for $T \gtrsim 0.5$ MeV,

$$k(e,\mu) \simeq 6.5\times 10^{-3}\{16 + 0.5[r(\xi_e) + r(\xi_\mu)] + 5.3 r(-\xi_e) + 3.5 r(-\xi_\mu)\}, \qquad (19)$$

$$k(\mu,\tau) \simeq 6.5\times 10^{-3}\{0.5[r(\xi_\mu) + r(\xi_\tau)] + 3.5[r(-\xi_\mu) + r(-\xi_\tau)]\}. \qquad (20)$$

Here $r(\xi) \equiv I(\xi)/I(0)$ and $I(\xi) \equiv \int_0^\infty x^2 dx/[1+\exp(x-\xi)]$. We define $\xi \equiv \mu/T$ with μ the chemical potential of the neutrino gas.[3] At $T \simeq 0.5$ MeV, electrons and positrons annihilate, so that for $T < 0.5$ MeV their contribution to the scattering and absorption rates becomes negligibly small and we get

$$k(e,\mu) \simeq 6.5\times 10^{-3}\{0.5[r(\xi_e) + r(\xi_\mu)] + 3[r(-\xi_e) + r(-\xi_\mu)]\}, \qquad (21)$$

$$k(\mu,\tau) \simeq 6.5\times 10^{-3}\{0.5[r(\xi_\mu) + r(\xi_\tau)] + 3[r(-\xi_\mu) + r(-\xi_\tau)]\}. \qquad (22)$$

Numerically, Eq. (18) gives

$$l_c(a,b) \simeq 1.45\times 10^{11}\ \mathrm{cm}\,\frac{1}{k(a,b)\alpha^2\beta^5}\left(\frac{1\ \mathrm{MeV}}{T}\right)^5. \qquad (23)$$

(ii) The expansion of the Universe. Oscillations and collisions are ineffective if their scale lengths, l_v and l_c, are larger than the inverse expansion rate of the Universe, l_H. In the radiation-dominating regime, l_H is expressed as

$$l_H \simeq \frac{M_p}{1.66\sqrt{g}T^2} \simeq 1.45\times 10^{11}\ \mathrm{cm}\frac{1}{\sqrt{g}}\left(\frac{1\ \mathrm{MeV}}{T}\right)^2, \qquad (24)$$

where M_p is the Planck mass and g represents the number of relativistic degrees of freedom. We have $g = 10.75$ for 1 MeV $\lesssim T \lesssim 100$ MeV and $g = 3.36$ for $T \ll 1$ MeV.

Figure 1 shows the lengths l_v, l_c, and l_H as functions of the temperature T for $\eta_\mu \simeq 1$, $\eta_e \simeq \eta_\tau \simeq 0$, and various values of Δm^2.

According to Fig. 1, for $\Delta m^2 \simeq 10^{-7}$ eV^2 we have $l_v \sim l_c \sim l_H$ at $T \sim 2$ MeV. Before this epoch $l_c \lesssim l_H \lesssim l_v$, so that collisions are much more efficient than oscillations. The inequality $l_c \ll l_v$ implies that the vacuum oscillation probabil-

[3]Let us recall the relation between the quantity ξ and the CP asymmetry η: $\eta = (\xi^3 + \pi^2\xi)\beta^3/[12\zeta(3)]$. In Eqs. (18)–(22), we considered the factor α to have the same value for all the particle species (neutrinos, electrons, positrons). We checked that this is a good approximation even for the large asymmetries we are considering.

ity, P_c, is suppressed by collisions, as we discussed in this section. As a consequence, for small mixings, $\sin^2 2\theta \ll 1$, the flavor composition of the background remains unchanged until $T \sim 2$ MeV; a partial equilibration of the flavors can be realized for large mixings: $\sin^2 2\theta \gtrsim 0.5$. For $T < 2$ MeV, collisions are ineffective, since $l_c \gtrsim l_H$, and vacuum oscillations develop.

With the decrease of Δm^2, $\Delta m^2 \ll 10^{-7}$ eV^2, the oscillation length l_v increases and, as a consequence, for $T \gtrsim 2$ MeV the suppression of oscillations due to collisions is stronger. Even for large mixings, the flavor densities are preserved until the neutrino decoupling, $T \sim 2$ MeV. After this epoch, oscillations are still suppressed by the expansion rate of the Universe and become effective, thus changing the flavor composition of the background only when the oscillation length is smaller than the horizon, $l_v \lesssim l_H$.

For $\Delta m^2 \gtrsim 10^{-7}$ eV^2, the inequality $l_v \lesssim l_c \lesssim l_H$ is realized before the decoupling. In this circumstance, the conversion probability, P_c, is not suppressed, in contrast with the case $\Delta m^2 \gtrsim 10^{-7}$ eV^2. Collisions are effective, thus leading to the equilibration of the flavor densities even for small mixing angles. Taking, for instance, $\Delta m^2 \simeq 10^{-3}$ eV^2, we find that equilibration can be achieved for $\sin^2 2\theta \gtrsim 10^{-3}$. Again, after neutrino decoupling, the conversion is determined by vacuum oscillations.

For a different choice of the asymmetries at production, e.g., $\eta_\mu \simeq \eta_\tau \simeq 1$ and $\eta_e \simeq 0$, the results are similar to those in Fig. 1 and we come to analogous conclusions.

Let us now find the present flavor asymmetries,[4] η_e^0, η_μ^0, η_τ^0, for specific neutrino mixings and mass spectra motivated by the oscillation interpretation of the solar and atmospheric neutrino anomalies.

We consider the mixing matrix

$$U = \begin{pmatrix} c_\theta & -s_\theta & 0 \\ s_\theta c_\Theta & c_\theta c_\Theta & -s_\Theta \\ s_\theta s_\Theta & c_\theta s_\Theta & c_\Theta \end{pmatrix}, \qquad (25)$$

where $c_\theta \equiv \cos\theta$ and $s_\theta \equiv \sin\theta$ and analogous definitions hold for s_Θ and c_Θ. The mass eigenstates ν_1, ν_2, and ν_3 are related to the flavor ones by the rotation: $\nu_\alpha = \Sigma_i U_{\alpha,i} \nu_i$. The mass-squared differences $\Delta m_{ji}^2 \equiv m_j^2 - m_i^2$ are taken to be $\Delta m_{32}^2 = \Delta m_{\rm atm}^2 \sim 10^{-3}$ eV^2 and $\Delta m_{21}^2 = \Delta m_\odot^2 \lesssim 10^{-5}$ eV^2 according to the currently favored solutions of the solar neutrino problem. Let us first consider $\Delta m_{21}^2 < 10^{-7}$ eV^2, as predicted by the low Δm^2 (LOW) and vacuum oscillation (VO) solutions. This range of Δm_{21}^2 is the most relevant to the conversion of ultraenergetic neutrinos (see Sec. VI).

According to the results of this section, we identify the following scenario. Suppose that before the neutrino decoupling epoch, at $T > 2$ MeV, a large asymmetry has been produced in one flavor while the other asymmetries are initially small, e.g., $\eta_\mu = 2\eta$ and $\eta_e \simeq \eta_\tau \simeq 0$, with $\eta \sim 1$. As the Universe evolves down to $T \sim 2$ MeV, the muon and tau asymmetries will be equilibrated by the combined effect of oscillations and collisions. In the same epochs, ν_e oscillations are still suppressed by collisions. Therefore, the electron neutrino asymmetry, η_e, remains unchanged and, at $T \simeq T_{\rm BBN}$, we have $\eta_\mu \simeq \eta_\tau = \eta$ and $\eta_e \simeq 0$. After this epoch, neutrinos decouple from the thermal bath; collisions become ineffective and the system evolves according to vacuum oscillations. During the evolution, decoherence occurs due to the spread of the wave packets. Therefore, at the present epoch the background neutrinos are in mass eigenstates. With the mixing (25), we find the present asymmetries η_i^0 for these states:

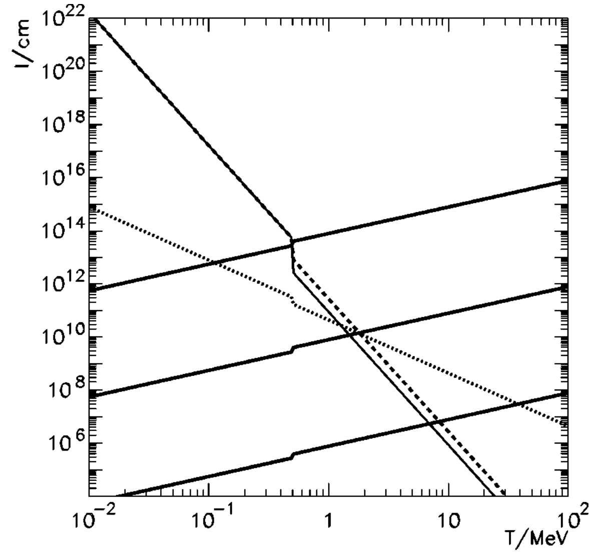

FIG. 1. The length scales l_v, l_c, and l_H as functions of the temperature T of the electromagnetic radiation in the Universe. The three thick solid lines represent the vacuum oscillation length l_v and correspond, from the upper to the lower, to $\Delta m^2 = 10^{-11}$, 10^{-7}, and 10^{-3} eV^2, respectively. The narrow solid line and the dashed line represent the coherence length, l_c, for the $\nu_e - \nu_\mu$ and the $\nu_\mu - \nu_\tau$ channels, respectively. We have taken $\eta_\mu \simeq 1$, $\eta_e \simeq \eta_\tau \simeq 0$. The dotted line represents the inverse expansion rate of the Universe, l_H.

$$\eta_1^0 \simeq \eta \sin^2\theta, \quad \eta_2^0 \simeq \eta \cos^2\theta, \quad \eta_3^0 \simeq \eta. \qquad (26)$$

The corresponding flavor asymmetries equal

$$\eta_e^0 \simeq \eta \frac{1}{2} \sin^2 2\theta,$$
$$\eta_\mu^0 \simeq \eta \left(1 - \frac{1}{2} \sin^2 2\theta \cos^2\Theta\right), \qquad (27)$$
$$\eta_\tau^0 \simeq \eta \left(1 - \frac{1}{2} \sin^2 2\theta \sin^2\Theta\right).$$

One can see that a large asymmetry is produced in the electron flavor provided that the mixing of the electron neutrino

[4]In the following sections, we will consider relatively recent epochs ($z \lesssim 50$) for which the flavor composition of the background is well approximated by the present one (see Secs. III and IV).

is large: $\sin^2 2\theta \sim 1$. Thus, the electron neutrino asymmetry at the present epoch can be much larger than the upper bound (2). Notice also that $\eta_\mu^0 \simeq \eta_\tau^0$ for $\Theta \simeq \pi/4$.

If equally large asymmetries are initially produced in the muon and tau flavors, $\eta_\mu = \eta_\tau = \eta \sim 1$ and $\eta_e \simeq 0$, the equality $\eta_\mu = \eta_\tau$ is preserved until the decoupling epoch, $T \sim 2$ MeV, due to the combined effect of oscillations and collisions. The evolution of η_e is blocked by collisions for $T \gtrsim 2$ MeV. After the neutrino decoupling, vacuum oscillations take place; with the mixing matrix (25), we get the same results as in Eqs. (26) and (27).

For $\Delta m_{21}^2 \simeq 10^{-6}$ eV^2 and small mixing, $\sin^2 2\theta \simeq 10^{-4} - 10^{-3}$, according to the small mixing angle (SMA) solution of the solar neutrino problem, the equilibration length $l_{\rm eq}$ is very large. Therefore, the electron asymmetry η_e is not equilibrated with the muon and tau asymmetries. Again, the present asymmetries are determined by vacuum oscillations which occur after the neutrino decoupling and lead to the result (27).

Conversely, for a large mixing angle, $\sin^2 2\theta \sim 1$ and $\Delta m_{21}^2 \simeq 10^{-7} - 10^{-5}$ eV^2, as given by part of the LOW solution and by the LMA solution regions, equilibration is rapidly realized and one gets

$$\eta_e^0 \simeq \eta_\mu^0 \simeq \eta_\tau^0 \simeq \frac{2}{3}\eta. \tag{28}$$

Notice, however, that the results (26)–(28) depend on the epoch we considered for the production of the large CP asymmetry η: the equilibration effect of collisions does not take place if the neutrino asymmetries are generated at epochs close to the neutrino decoupling epoch, at $T \sim 1$ MeV.

2. Evolution in the presence of a sterile state

If a sterile state, ν_s, is mixed with the three active ones, a general description of the evolution of the neutrino gas is complicated and would deserve a detailed study.

We consider here the specific case in which the sterile neutrino is mixed mainly with one active state only, e.g., ν_e, and the admixture of ν_s with ν_μ and ν_τ is negligible. In other words, we consider the mass states $\nu_0 \simeq \cos\theta\nu_e + \sin\theta\nu_s$ and the orthogonal combination $\nu_1 \simeq -\sin\theta\nu_e + \cos\theta\nu_s$. Similarly, we take $\nu_2 \simeq \cos\varphi\nu_\mu + \sin\varphi\nu_\tau$ and $\nu_3 \simeq -\sin\varphi\nu_\mu + \cos\varphi\nu_\tau$. This would correspond to a $\nu_e - \nu_s$ solution of the solar neutrino problem and a $\nu_\mu - \nu_\tau$ solution of the atmospheric neutrino anomaly.

Let us consider the evolution of the $\nu_e - \nu_s$ system. The effective mixing angle in matter, θ_m, can be written as

$$\tan 2\theta_m = \frac{\sin 2\theta}{\cos 2\theta - 2EV/\Delta m^2}, \tag{29}$$

where V is given in Eq. (10) and E is the average thermal energy of the neutrinos: $E \simeq \alpha\beta T$. Numerically, from Eqs. (10) and (29) we get

$$\tan 2\theta_m = \frac{\sin 2\theta}{\cos 2\theta - 0.8\times 10^4 \alpha\beta F \eta_\nu (T/1\ \text{MeV})^4 (10^{-3}\ \text{eV}^2/\Delta m^2)}, \tag{30}$$

where we used the expression $n_\gamma(T) = 2\zeta(3)T^3/\pi^2$ for the concentration of photons at the temperature T, with the value $\zeta(3) \simeq 1.202$ for the Riemann zeta function.

From Eq. (30), it follows that, for $\eta_\nu \gtrsim 1$, $T \gtrsim T_{\rm BBN}$, and $\Delta m^2 \lesssim 1$ eV^2, the mixing is strongly suppressed, $\tan 2\theta_m \ll 1$, corresponding to $\theta \simeq \pi/2$ ($\theta \simeq 0$) if $F\eta_\nu > 0$ ($F\eta_\nu < 0$). Thus, no level crossing is realized before the BBN epoch. At $T \gtrsim 2$ MeV, collisions are effective (see Fig. 1); however, they do not modify η_e significantly due to the very small value of the mixing and consequently of the conversion probability P_c [see Sec. II B 1, Eq. (15)]. Thus, we conclude that no significant flavor conversion occurs and the original value of η_e is preserved at least until the BBN epoch, even in the case of large vacuum mixing angles.

As the temperature decreases, $T < T_{\rm BBN}$, the mixing angle θ_m approaches rapidly its vacuum value. Taking $\eta_\nu \simeq 1$, $F = 2$, and $\Delta m^2 \simeq 10^{-3}$ eV^2, we get $\tan 2\theta_m - \tan 2\theta \lesssim 10^{-2}$ for $T \lesssim 10$ keV.

Considering that the propagation of the neutrino states is adiabatic (see Sec. IV B), we find the present concentrations of ν_e and ν_s in terms of the initial density n_e:[5]

$$n_s^0 = n_e \cos^2\theta, \quad n_e^0 = n_e \sin^2\theta \quad \text{if } F\eta_\nu > 0, \tag{31}$$

$$n_s^0 = n_e \sin^2\theta, \quad n_e^0 = n_e \cos^2\theta \quad \text{if } F\eta_\nu < 0. \tag{32}$$

If $n_e \gg n_{\bar{e}}$, the concentrations of $\overline{\nu_e}$ can be neglected and relations analogous to Eqs. (31) and (32) hold for the CP asymmetries η_e^0 and η_e.

The present ν_μ and ν_τ asymmetries can be found according to the discussion in Sec. II B 1. The effect of collisions leads to equilibration of η_μ and η_τ at $T \simeq T_{\rm BBN}$: $\eta_\mu \simeq \eta_\tau = \eta$. At later epochs, vacuum oscillations develop, leaving this equality unchanged. Thus, we can summarize the present CP symmetries for the four flavors as follows:

[5]We assume that only active states are initially produced before the BBN epoch, thus $n_s = 0$.

$$\eta_s^0 = \eta_e \cos^2\theta, \quad \eta_e^0 = \eta_e \sin^2\theta \quad \text{if } F\eta_\nu > 0, \tag{33}$$

$$\eta_s^0 = \eta_e \sin^2\theta, \quad \eta_e^0 = \eta_e \cos^2\theta \quad \text{if } F\eta_\nu < 0, \tag{34}$$

$$\eta_\mu^0 \simeq \eta_\tau^0 \simeq \eta. \tag{35}$$

Having neglected any mixing between ν_e and the other (active) flavors, we find that the present value of the electron neutrino asymmetry is smaller than the one at the BBN epoch, $\eta_e^0 \leqslant \eta_e$, thus remaining within the bound given in (2).

III. HIGH-ENERGY NEUTRINO CONVERSION: THE ACTIVE-ACTIVE CASE

Let us consider three mixed active neutrinos—ν_e, ν_μ, and ν_τ—and find the potential for a beam of high energy neutrinos (“beam neutrinos”) due to the interaction with the relic neutrino background (“background neutrinos”). As discussed in Sec. II B, the flavor composition of the neutrino background changes with time due to the neutrino mixing. However, we will focus on neutrinos produced in relatively recent epochs, $z \lesssim 50$, when the flavor content of the relic neutrino gas has already settled down and does not change with time.

A. The refraction potential

According to Sec. II B, decoherence due to the spread of wave packets implies that the background neutrinos are in mass eigenstates. As a consequence, the matrix of potentials, V_ν, for the beam neutrinos propagating in this background is not diagonal in the flavor basis $(\nu_e, \nu_\mu, \nu_\tau)$. It is possible to check [25] that V_ν becomes diagonal in the basis of the mass eigenstates (ν_1, ν_2, ν_3), where it can be written as

$$V_\nu = \sqrt{2}G_F[(n_1 - n_{\bar{1}}) + (n_2 - n_{\bar{2}}) + (n_3 - n_{\bar{3}})] + \sqrt{2}G_F \begin{pmatrix} n_1 f(-s_Z^{(1)}) - n_{\bar{1}} f(s_Z^{(1)}) & 0 & 0 \\ 0 & n_2 f(-s_Z^{(2)}) - n_{\bar{2}} f(s_Z^{(2)}) & 0 \\ 0 & 0 & n_3 f(-s_Z^{(3)}) - n_{\bar{3}} f(s_Z^{(3)}) \end{pmatrix}, \tag{36}$$

where $n_i(n_{\bar{i}})$ denotes the concentration of the mass state $\nu_i(\overline{\nu_i})$ in the background and $f(s_Z^{(i)})$ is the Z-boson propagator function:

$$f(s_Z^{(i)}) \equiv \frac{1 - s_Z^{(i)}}{(1 - s_Z^{(i)})^2 + \gamma_Z^2}. \tag{37}$$

Here γ_Z and $s_Z^{(i)}$ are the normalized width of the Z boson and total energy squared in the ν_i-ν_i center of mass for nonrelativistic background neutrinos:

$$s_Z^{(i)} \simeq \frac{2Em_i}{M_Z^2} \simeq 2.4 \times 10^{-2}\left(\frac{E}{10^{20}\ \text{eV}}\right)\left(\frac{m_i}{1\ \text{eV}}\right), \qquad \gamma_Z \equiv \frac{\Gamma_Z}{M_Z}, \tag{38}$$

with m_i being the mass of the neutrino ν_i and E the energy of the beam neutrino.

The terms in the first line of Eq. (36) are due to neutral current ν-ν scattering in the t channel. The terms in the second line of Eq. (36) represent the contributions of ν_i-ν_i scattering with Z-boson exchange in the u channel, and of ν_i-$\overline{\nu_i}$ annihilation processes.

For $E \lesssim 10^{20}$ eV and $m_i \lesssim 1$ eV, the energy in the ν_i-ν_i center of mass is much below the Z-boson resonance: $s_Z \lesssim 0.03$. In this case, the propagator function (37) reduces to unity: $f(s_Z^{(i)}) \simeq f(-s_Z^{(i)}) \simeq 1$, and the neutrino-neutrino potential (36) becomes energy-independent.

For extremely high energies, $E \simeq 10^{21} - 10^{22}$ eV, and neutrino mass of order 1 eV, the propagator corrections become important. However, in this range of energies the absorption effects of the neutrino background are strong [7,26]. Therefore, the neutrino fluxes at Earth are largely suppressed. In what follows, we will concentrate on the low-energy limit, $s_Z^{(i)} \ll 1$, which is mainly relevant for applications.

For a beam of antineutrinos propagating in a neutrino background, the potential $V_{\bar{\nu}}$ is given by Eq. (36) with the replacement $n_i \rightarrow n_{\bar{i}}$ and vice versa for all the ν_i states.

The fact that the neutrino-neutrino potential matrix, Eq. (36), is diagonal in the basis of mass eigenstates has a straightforward consequence: the effect of refraction consists in a modification of the neutrino effective masses only. In terms of the present CP asymmetries η_i^0 for the mass states ν_i of the background, we find (for $s_Z^{(i)} \ll 1$) the following corrections:

$$\begin{aligned} \frac{\Delta m_{21}^2}{2E} &\rightarrow \mathcal{E}_{21} \equiv \frac{\Delta m_{21}^2}{2E} + \sqrt{2}G_F n_\gamma (\eta_2^0 - \eta_1^0), \\ \frac{\Delta m_{32}^2}{2E} &\rightarrow \mathcal{E}_{32} \equiv \frac{\Delta m_{32}^2}{2E} + \sqrt{2}G_F n_\gamma (\eta_3^0 - \eta_2^0). \end{aligned} \tag{39}$$

As we discussed in Sec. II B, the present composition of the neutrino background is determined by the initial flavor asymmetries, η_e, η_μ, and η_τ, and by the mixing matrix U of the neutrino system. The expressions of $\mathcal{E}_{21}$ and $\mathcal{E}_{32}$ in terms of these quantities can be found from the results of Sec. II B 2. In particular, with the asymmetries (26), Eq. (39) gives

$$\mathcal{E}_{ji}=\frac{\Delta m_{ji}^2}{2E}+V_{ji}, \tag{40}$$

$$V_{ji}\equiv F_{ji}\eta\sqrt{2}G_F n_\gamma, \tag{41}$$

$$F_{21}=\cos 2\theta, \quad F_{32}=\sin^2\theta. \tag{42}$$

Here we denote as η the maximal flavor asymmetry, $\eta \equiv \max\{\eta_\mu,\eta_\tau,\eta_e\}$, which is realized in the background at the epoch of nucleosynthesis, $T\simeq T_{\rm BBN}$; thus η is constrained by the bounds (2).

In Eqs. (40) and (41), the information on the specific mixing matrix and initial composition of the background is encoded in the F factors. The dependence of F_{ji} on the mixing angle in Eq. (42) is a consequence of expressing the potential (41) is terms of the flavor asymmetry η, while the background neutrinos are in mass eigenstates. For simplicity, in what follows we will drop the indexes j,i from the quantities $\mathcal{E}$, V, and F in the expressions (40)–(42).

B. The conversion probability

From the fact that the potential V modifies the effective-mass eigenvalues, Eqs. (40)–(42), it follows that the interaction with the neutrino background does not change the mixing matrix of the neutrino system, which remains the same as in vacuum. Conversely, the phase of oscillations is affected by the medium, so that the dynamics of the neutrino propagation consists in oscillations with constant depth, given by the vacuum mixing angle, and varying oscillation length. The probability P of conversion between two active neutrinos ν_α and ν_β with mixing angle θ equals

$$P(t,t_i)=\sin^2 2\theta\sin^2\left(\frac{\Phi}{2}\right), \tag{43}$$

and the oscillation phase Φ is given by

$$\Phi(t,t_i)=\int_{t_i}^{t}\mathcal{E}(\tau)d\tau. \tag{44}$$

We denote as t_i,t the initial and final time of the evolution of the system; $\mathcal{E}$ is given in Eq. (40). Using the scaling relations

$$E=E_0(t_0/t)^{2/3}=E_0(1+z), \quad V=V_0(t_0/t)^2=V_0(1+z)^3, \tag{45}$$

where E_0 and V_0 are the energy and the potential at the present epoch, $z=0$, we get

$$\Phi=\Phi_{\rm vac}+\Phi_{\rm matt}, \tag{46}$$

with the following expressions for the vacuum oscillation phase, $\Phi_{\rm vac}$, and the matter contribution $\Phi_{\rm matt}$:

$$\Phi_{\rm vac}(x,x_i)=\frac{3}{10}\frac{\Delta m^2 t_0}{E_0}(x^{5/3}-x_i^{5/3}), \tag{47}$$

$$\Phi_{\rm matt}(x,x_i)=V_0t_0\left(\frac{1}{x_i}-\frac{1}{x}\right). \tag{48}$$

We defined $x\equiv t/t_0$ and $x_i\equiv t_i/t_0$.

The matter-induced phase, $\Phi_{\rm matt}$, depends only on the characteristics of the background and on the initial and final moments of time. In particular, for early production epochs, $x_i\ll 1$, one gets

$$\Phi_{\rm matt}\simeq V_0t_0\frac{1}{x_i}, \tag{49}$$

which shows that the phase $\Phi_{\rm matt}$ is accumulated mainly at the production time.

Being independent of $E_0/\Delta m^2$, the phase $\Phi_{\rm matt}$ becomes comparable to or even larger than the vacuum oscillation phase, $\Phi_{\rm vac}$, at very high energies, $E_0/\Delta m^2 \gtrsim 10^{32}$ eV^{-1}. Taking $x_i=0.125$, corresponding to production at redshift $z\simeq 3$, and $x=1$, we have $\Phi_{\rm vac}\simeq 0.23\pi$ for $E_0/\Delta m^2\simeq 10^{32}$ eV^{-1}. This is comparable to the matter phase, $\Phi_{\rm matt}\simeq 0.29\pi$ given by Eqs. (48) and (41) with $F\eta\simeq 10$. As $E_0/\Delta m^2$ increases, the vacuum phase $\Phi_{\rm vac}$ decreases and the total oscillation phase is dominated by the matter contribution $\Phi_{\rm matt}$. From Eqs. (43) and (48), we find the asymptotic value of the conversion probability:[6]

$$P(E_0/\Delta m^2\to\infty)=\sin^2 2\theta\sin^2\left[\frac{1}{2}V_0t_0\left(\frac{1}{x_i}-\frac{1}{x}\right)\right]. \tag{50}$$

Notice that the expression (50) is insensitive to the change of sign of the potential V_0 (i.e., of the product $F\eta$): this implies that in the limit of very high energies, a beam of neutrinos and one of antineutrinos will experience the same matter effect.

In Fig. 2, we show the survival probability, $1-P$, as a function of $E_0/\Delta m^2$ for neutrinos produced at $z=3$ and arriving at Earth at the present epoch, with $\sin^2 2\theta=0.5$ and various values of the product $F\eta$. The figure was produced by averaging the conversion probability, Eq. (43), over the interval $\Delta E_0\simeq E_0$, keeping in mind the finite accuracy in the reconstruction of the neutrino energy in the detector:

$$P(E_0)=\frac{1}{\Delta E_0}\int_{E_0/2}^{3E_0/2}dE'P(E'). \tag{51}$$

Let us comment on Fig. 2. In the absence of asymmetry, $F\eta=0$, the conversion is given by vacuum oscillations

[6]Even though Eq. (50) gives a nonzero value for the conversion probability in the limit $\Delta m^2\to 0$, the matter effect we are describing requires massive nondegenerate neutrinos: $\Delta m^2\neq 0$. This condition is necessary for our starting point [Sec. III A, see Eq. (36)] that the neutrinos in the background are in mass eigenstates (different from the flavor ones), produced from flavor states by the spread of the wave packets during the evolution of the Universe. Thus, the expression (50) should be intended as the high-energy limit of the conversion probability for a given (nonzero) value of Δm^2.

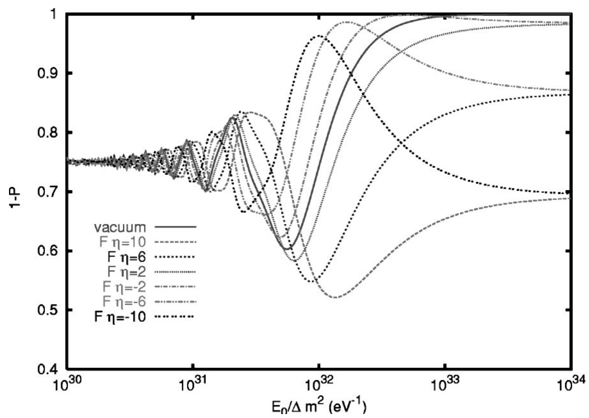

FIG. 2. The survival probability $1-P(\nu_\alpha\to\nu_\beta)$ as a function of the ratio $E_0/\Delta m^2$ for various values of $F\eta$. We have taken $\sin^2 2\theta=0.5$ and production epoch $z=3$.

$(\Phi_{\rm matt}=0)$. For $E_0/\Delta m^2 \gtrsim 10^{32}$ eV^{-1}, the vacuum oscillation phase $\Phi_{\rm vac}$ is very small, thus the conversion probability approaches unity.

For strongly CP-asymmetric neutrino background, the matter-induced phase $\Phi_{\rm matt}$ is sizeable. As a consequence, the deviation of the survival probability $1-P$ from the value given by vacuum oscillations can be as large as $\sim 30\%$. Clearly, as follows from Eq. (43), a strong effect requires a large mixing angle: if $\sin^2 2\theta \ll 1$, the effect of matter on the oscillation phase will be unobservable due to the very small amplitude of oscillations. At extremely high energies, $E_0/\Delta m^2 \gtrsim 10^{33}$ eV^{-1}, the deviation is constant and independent of the sign of $F\eta$, according to Eq. (50).

IV. HIGH-ENERGY NEUTRINO CONVERSION: THE ACTIVE-STERILE CASE

Let us consider the case in which a sterile neutrino ν_s is mixed with the active flavors, ν_α. We discuss here two-neutrino mixing; a generalization to a four-neutrino framework will be given in Sec. VI.

A. The refraction potential

In contrast to the active-active conversion studied in Sec. III, for an active-sterile neutrino system the matrix of the refraction potentials is diagonal in the flavor basis (ν_α,ν_s):

$$V_\nu=\begin{pmatrix} V_\alpha & 0\\ 0 & 0\end{pmatrix}. \tag{52}$$

In the low-energy limit, $s_Z \ll 1$, the potential V_α depends on the flavor asymmetries η_e^0, η_μ^0, and η_τ^0 as follows:

$$V_\alpha=\sqrt{2}G_F n_\gamma\left[\eta_\alpha^0+\sum_{\beta=e,\mu,\tau}\eta_\beta^0\right]. \tag{53}$$

The potential (53) can be written in the same general form as Eq. (41):

$$V=F\eta\sqrt{2}G_F n_\gamma, \tag{54}$$

with the same definition of η as the maximal flavor asymmetry in the background at the epoch of nucleosynthesis, $\eta\equiv\max\{\eta_\mu,\eta_\tau,\eta_e\}$. The factor F depends on the specific conversion channel and the flavor content of the background:

$$F\equiv\frac{1}{\eta}\left[\eta_\alpha^0+\sum_{\beta=e,\mu,\tau}\eta_\beta^0\right]\quad \text{for the}\quad \nu_\alpha-\nu_s \ \text{channel.} \tag{55}$$

Let us consider for instance the conversion of ν_e to ν_s. If the ν_e-ν_s conversion in the background occurred in the resonant channel, the present electron neutrino asymmetry is given by Eq. (33). Using this result and Eq. (35) for η_μ^0 and η_τ^0 we get

$$F=2\left(1+\frac{\eta_e}{\eta}\sin^2\theta\right), \tag{56}$$

where we considered $\eta_\mu,\eta_\tau \gg \eta_e$. If the $\nu_e-\nu_s$ conversion in the background proceeded in the nonresonant channel, η_e^0 is given in Eq. (34). With this expression one finds a form for the factor F analogous to Eq. (56) with the replacement $\sin^2\theta\to\cos^2\theta$.

For conversion of antineutrinos, the $\bar{\nu}-\nu$ potential has opposite sign: $V_{\bar\nu}=-V_\nu$. Thus the expression (54) holds with the replacement $F\to -F$.

B. The dynamics of neutrino conversion

Due to the expansion of the Universe, the cosmological neutrinos experience a potential which changes with time. In contrast with the active-active case, the effect of medium changes both the oscillation length and the mixing, Eq. (29). The dynamics of the flavor transition is determined by the resonance and adiabaticity conditions.

Consider the resonance condition

$$\frac{2EV}{\Delta m^2}\frac{1}{\cos 2\theta}=1. \tag{57}$$

Using Eq. (54) and the scaling relations (45), from Eq. (57) we get the following relations.

The present energy of neutrinos which cross the resonance at the epoch z equals

$$E_0=10^{20}\ \text{eV}\left(\frac{\Delta m^2}{10^{-10}\ \text{eV}^2}\right)\frac{10^4\cos 2\theta}{F\eta(1+z)^4}. \tag{58}$$

For a given E_0 and Δm^2, the redshift z_R at which the resonance condition was realized is given by

$$1+z_R=10\left[\frac{\cos 2\theta}{F\eta(10^{-10}\ \text{eV}^2/\Delta m^2)(E_0/10^{20}\ \text{eV})}\right]^{1/4}. \tag{59}$$

Neutrinos produced at a distance z undergo resonance if their present energy is in the interval

$$E_0 = 10^{20}\ \mathrm{eV}\left(\frac{\Delta m^2}{10^{-10}\ \mathrm{eV}^2}\right)\frac{10^4\cos 2\theta}{F\eta}\left[\frac{1}{(1+z)^4},1\right]. \tag{60}$$

Taking $F\eta=10$, $\cos 2\theta \simeq 1$, $\Delta m^2 = 10^{-10}\mathrm{eV}^2$, and $z=3$ from Eq. (60), we find $E_0 \simeq 4\times 10^{20} - 10^{23}$ eV. With the same values of the parameters and $E_0 = 10^{20}$ eV, we get that the resonance condition (58) is satisfied at $z_R \simeq 4.6$.

The adiabaticity condition involves the time variation of both the neutrino energy and the concentration of the neutrino background. It can be expressed in terms of the adiabaticity parameter at resonance, χ_R, as

$$\chi_R \gg 1, \qquad \chi_R \equiv \frac{(\Delta m^2)^2}{4E}\sin^2 2\theta\left[\frac{d}{dt}(EV)\right]^{-1}\Bigg|_{\mathrm{res}}, \tag{61}$$

where the subscript "res" indicates that the various quantities are evaluated at resonance, i.e., when the condition (57) is fulfilled. With the potential (54), using the scalings (45) and the resonance condition (57), we find

$$\chi_R \simeq 10^{-2}F\eta\tan^2 2\theta(1+z_R)^{3/2}. \tag{62}$$

For $F\eta \lesssim 10$, $\tan^2 2\theta = 1$, and $z_R \lesssim 5$, one finds $\chi_R \lesssim 1.4$. Thus, for neutrinos produced at epochs $z<5$, we expect breaking of the adiabaticity. Notice that χ_R does not depend explicitly on the neutrino energy and mass-squared difference; it increases with η and z_R.

From Eq. (62), we get the redshift z_a corresponding to $\chi_R = 2\pi \gg 1$:

$$1+z_a \equiv \left[\frac{2\pi\times 10^2}{F\eta\tan^2 2\theta}\right]^{2/3}. \tag{63}$$

If the resonance condition is fulfilled at $z \gg z_a$, the level crossing (resonance) proceeds adiabatically. Taking $F\eta=10$ and $\tan 2\theta = 1$, we find $z_a \simeq 15$.

For $\eta \gtrsim 1$ and $\tan 2\theta \lesssim 1$, we have $z_a \gg z_d$. Thus, we can define three epochs of neutrino production, corresponding to different characters of the evolution of the neutrino beam.

(i) Earlier epoch: $z>z_a$, when both adiabaticity and the minimum width conditions are satisfied. If also the resonance condition is satisfied at $z_R > z_a$, the neutrinos will undergo strong resonance conversion. Otherwise, if the resonance condition is not realized (e.g., due to a large value of $\Delta m^2/E$), the matter effect can be small.

(ii) Intermediate epoch: $z_a > z > z_d$. The adiabaticity at resonance is not satisfied (if $z_a > z_R > z_d$). At the same time, the matter width can be large enough to induce significant matter effect.

Two remarks are in order. (i) The propagation can still be adiabatic in the part of the interval $[z_d, z_a]$ outside the resonance, and in the whole of it if the resonance condition is never satisfied in this time interval. (ii) In monotonously varying density, the condition for strong matter effect reduces to the adiabaticity condition [7]. Therefore, in spite of the fulfillment of the minimum width condition, the matter effect can be small for neutrinos produced in the largest part of the interval $[z_d, z_a]$.

(iii) Later epoch: $z<z_d$. For neutrinos produced in this epoch, the matter effects are expected to be small.

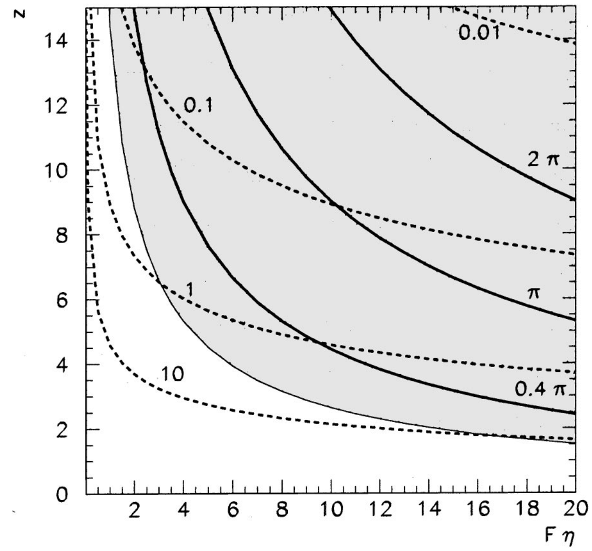

FIG. 3. The minimum width, resonance, and adiabaticity conditions in the z-$F\eta$ plane for $\nu_\alpha - \nu_s$ conversion. The solid lines are iso-contours of adiabaticity, i.e., of the quantity $\chi_R/\tan^2 2\theta$ (numbers on the curves). The dashed lines are iso-contours of resonance, i.e., of the ratio $E_0/(\Delta m^2\cos 2\theta)$; the values are given on the curves in units of 10^{30} eV^{-1}. The minimum width condition is satisfied in the shadowed region.

Figure 3 shows the minimum width, resonance, and adiabaticity conditions in the z-$F\eta$ plane. The minimum width condition (11) is satisfied in the shadowed region. The lower border of this area corresponds to the curve $z = z_d(F\eta)$ [Eq. (12)]. For values of $F\eta$ and of z in this region, one may expect significant matter effect.

The dashed lines show the values of z and $F\eta$ for which the resonance condition (57) is satisfied for neutrinos with a given $E_0/(\Delta m^2\cos 2\theta)$ (isocontours of resonance).

The solid lines are isocontours of the adiabaticity parameter: they are contours of constant ratio $\chi_R/\tan^2 2\theta$ [see Eq. (62)]. The upper curve corresponds to $\chi_R/\tan^2 2\theta = 2\pi$, that is, to $z=z_a$ for $\tan^2 2\theta = 1$. For values of neutrino production epoch z and $F\eta$ above this contour, one would expect resonant adiabatic conversion to be the dominating mechanism of neutrino transformation. For a given $F\eta$ and z, the adiabaticity isocontour gives the value of $\chi_R/\tan^2 2\theta$ for neutrinos produced at the epoch $z_i \gg z$ and having the resonance at z. In turn, the resonance at z and $F\eta$ can be satisfied for certain values of $E_0/(\Delta m^2\cos 2\theta)$. It is clear from the figure that strong adiabatic conversion occurs for large production epochs ($z \gtrsim 10$), large asymmetry, ($F\eta \gtrsim 10$), and large mixing [$\tan^2 2\theta \sim O(1)$]. For $F\eta \sim 2$, the minimal width condition is fulfilled for large production epochs, $z \gtrsim 8$, and some effects of adiabatic conversion may be seen at $z \gtrsim 15$.

From the above considerations, it appears that for realistic parameters a flavor transition of neutrinos occurs either due

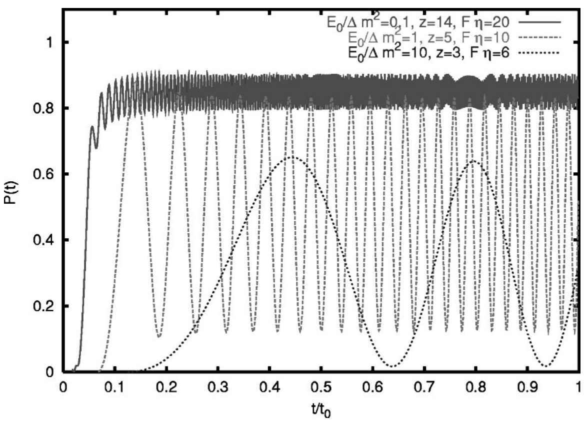

FIG. 4. The $\nu_\alpha - \nu_s$ conversion probability $P(t)$ as a function of time. We have taken $\sin^2 2\theta = 0.5$ and three different choices of $E_0/\Delta m^2$ (in units of 10^{30} eV^{-1}), production epoch z, and $F\eta$. The time t is given in units of the age of the Universe, t_0.

to vacuum oscillations modified by matter effect or by an interplay of oscillations and nonadiabatic conversion.

C. The conversion probability

Let us consider neutrinos produced at a given epoch z with a certain flavor ν_α and propagating in the expanding Universe with a given constant asymmetry η.

We find the $\nu_\alpha - \nu_s$ conversion probability by numerical solution of the evolution equation for two neutrino species with the Hamiltonian in the flavor basis (ν_α, ν_s):

$$H = \begin{pmatrix} -\dfrac{\Delta m^2}{2E(z)}\cos 2\theta + V(z) & \dfrac{\Delta m^2}{4E(z)}\sin 2\theta \\ \dfrac{\Delta m^2}{4E(z)}\sin 2\theta & 0 \end{pmatrix}, \quad (64)$$

where $E(z)$ and $V(z)$ scale according to Eq. (45).

As discussed in Sec. IV B (see Fig. 3), the dynamics of flavor transformation depends on the production epoch z, the resonance epoch, z_R, which depends on $E_0/\Delta m^2$, and on the value of the adiabaticity parameter at resonance, χ_R. Figure 4 illustrates the real time evolution of the neutrino states for $\sin^2 2\theta = 0.5$ and different z, z_R, χ_R, which represent different regimes of conversion.

The solid curve corresponds to production much before the resonance epoch: $z > z_R = 6.8$ and weak adiabaticity breaking in the resonance, $\chi_R \simeq 4.3$. The dominating process is the adiabatic conversion which occurs in the resonance epoch, $t_R/t_0 \simeq 0.05$. The averaged transition probability is close to what one would expect for the pure adiabatic case: $P_{\mathrm{ad}} = 1 - \sin^2\theta = 0.85$. Weak adiabaticity violation leads to the appearance of oscillations at $t > t_R$.

The dashed curve corresponds to production close to resonance, $z \gtrsim z_R = 4.2$, and strong adiabaticity violation in the resonance: $\chi_R \simeq 1.2$. The dominating process is oscillations in matter with resonance density. At production the mixing is almost maximally enhanced, $\sin^2 2\theta_m \simeq 1$. The change of matter density leads to a slight increase of the average conversion probability with respect to $\sin^2 2\theta_m/2$. The decrease of density is fast: the typical scale of density change is smaller than the oscillation length, so that maximal depth oscillations do not have time to develop.

The dotted line shows the same type of regime with stronger adiabaticity violation in resonance. The depth D of oscillations is smaller, and the average conversion probability is close to $D/2$.

For further illustration, in Fig. 5 we show the evolution in the case of good adiabaticity. Different curves correspond to different production epochs: (i) before resonance, $z > z_R$; (ii) at resonance, $z = z_R$; (iii) after resonance, $z < z_R$.

Figures 6 and 7 show similar sets of curves in the cases of moderate and strong violation of adiabaticity.

Let us consider the properties of the conversion probability $P(\nu_\alpha \to \nu_s)$ for neutrinos produced at epoch z and arriving at Earth at the present epoch, $z = 0$. The probability P depends on z, on the product $F\eta$, on the energy and mass-squared difference in the ratio $E_0/\Delta m^2$, and on the mixing

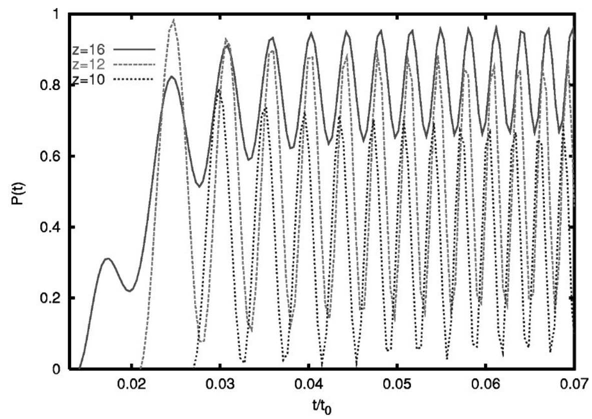

FIG. 5. The $\nu_\alpha - \nu_s$ conversion probability $P(t)$ as a function of time in the regime of good adiabaticity (see Fig. 3). We have taken production epochs z earlier, simultaneous and later than the resonance epoch z_R. Here $\sin^2 2\theta = 0.5$, $F\eta = 14$, and $E_0/\Delta m^2 = 1.8 \times 10^{28}$ eV^{-1}. The time t is given in units of the age of the Universe, t_0.

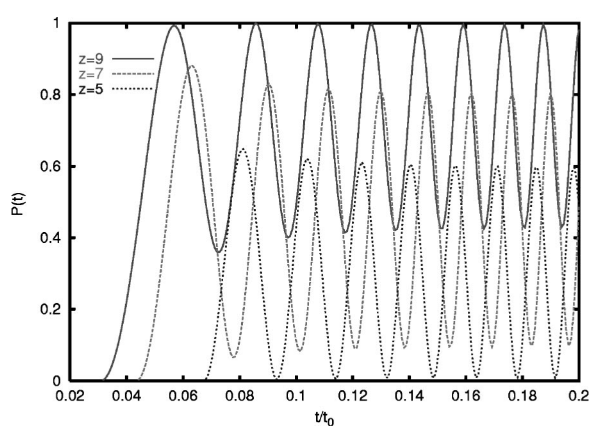

FIG. 6. The same as Fig. 5 for the regime of moderate breaking of adiabaticity. Here $\sin^2 2\theta = 0.5$, $F\eta = 10$, and $E_0/\Delta m^2 = 1.73 \times 10^{29}$ eV^{-1}.

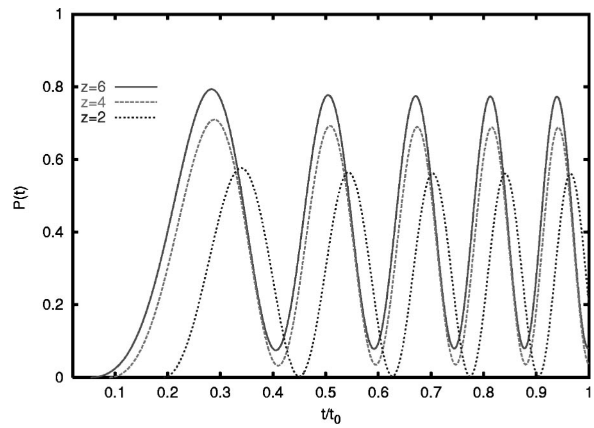

FIG. 7. The same as Fig. 5 for the regime of strong breaking of adiabaticity. Here $\sin^2 2\theta = 0.5$, $F\eta = 6$, and $E_0/\Delta m^2 = 4.6\times 10^{30}$ eV^{-1}.

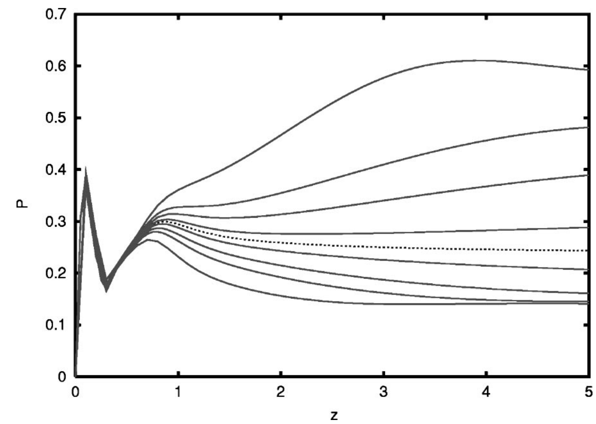

FIG. 8. The ν_α-ν_s conversion probability P as a function of the production epoch z for various values of $F\eta$. From the upper to the lower curve: $F\eta = 20,10,6,2,0,-2,-6,-10,-20$; the dotted line represents the vacuum oscillations probability $(F\eta=0)$. We have taken $\sin^2 2\theta = 0.5$ and $E_0/\Delta m^2 = 10^{31}$ eV^{-1}.

angle θ: $P = P(z, F\eta, E_0/\Delta m^2, \theta)$. As follows from Figs. 4–7, the probability is a rapidly oscillating function of z, and also of $E_0/\Delta m^2$. We averaged P over the energy resolution interval $\Delta E_0 \simeq E_0$ according to Eq. (51). The interpretation of the numerical results can be easily given using the z-$F\eta$ diagram of Fig. 3.

In Fig. 8, we show the dependence of the conversion probability on the production epoch z for different values of $F\eta$ and fixed $E_0/\Delta m^2$ and $\sin^2 2\theta$. The curves with $F\eta > 0$ represent the resonance channel. For $z \lesssim 1$, both vacuum oscillations and matter conversion probabilities have oscillating behavior. For $z \gtrsim 2$, oscillations are averaged out, so that the vacuum oscillation probability converges to $\sin^2 2\theta/2$.[7] A substantial $(\sim 10\%)$ deviation from the vacuum oscillation probability due to matter effect starts at $z \simeq 1$ for $F\eta \simeq 10$ and at $z \simeq 3$ for $F\eta \simeq 2$.

For $F\eta \simeq 6-10$ and $z \simeq 4-5$, neutrinos are produced at densities much higher than the resonance density and they cross the resonance at $z = 2-2.5$. The adiabaticity is broken in the resonance, however above the resonance the propagation can be adiabatic. For higher asymmetry, $F\eta \gtrsim 10$, the adiabaticity starts to be broken near the resonance, so that the original flavor state $\nu_\alpha \simeq \nu_{2m}$ will evolve to $\nu_{2m}^R \simeq (\nu_\alpha + \nu_s)/\sqrt{2}$. Thus, we have $P \simeq \frac{1}{2}$. With the decrease of z, the initial state will deviate from ν_{2m} and the conversion probability becomes smaller. With the decrease of $F\eta$, the adiabaticity starts to be violated earlier (before resonance), so that the transition probability decreases.

For negative values of $F\eta$ (or for antineutrinos), the matter effect suppresses the mixing and, as a consequence, the conversion effect. However, the suppression effect is weaker than the enhancement in the resonant channel.

Notice that for $F\eta \simeq 10$ and $z \simeq 5$ the matter effect can change the vacuum oscillation probability P_v by a factor of 2:

$$(P - P_v)/P_v \simeq 1. \tag{65}$$

For $z \simeq 2$ and $F\eta \simeq 10$, the deviation can reach $\sim 40\%$ and it equals $\sim 20\%$ for $F\eta \simeq 2$.

In Fig. 9, we show the dependence of the survival probability, $1-P$, on $E_0/\Delta m^2$ for production epoch $z=3$, $\sin^2 2\theta = 0.5$, and various values of $F\eta$. Oscillations are averaged for $E_0/\Delta m^2 \lesssim 3\times 10^{30}$ eV^{-1}; the averaging disappears at $E_0/\Delta m^2 \sim 10^{32}$ eV^{-1}, when the oscillation length approaches the size of the horizon (see also Sec. III B and Fig. 2).

The matter effect increases with $E_0/\Delta m^2$. For $E_0/\Delta m^2 \lesssim 5\times 10^{30}$ eV^{-1}, the resonance epoch z_R [see Eq. (59)] is earlier than the production epoch of the neutrinos. Thus the neutrinos do not cross the resonance and the matter effect is realized mainly in the epoch of neutrino production, when the potential (54) was larger (see Fig. 4). The value of the effect is determined by the mixing in matter at the production time. With the increase of $E_0/\Delta m^2$, the resonance epoch z_R

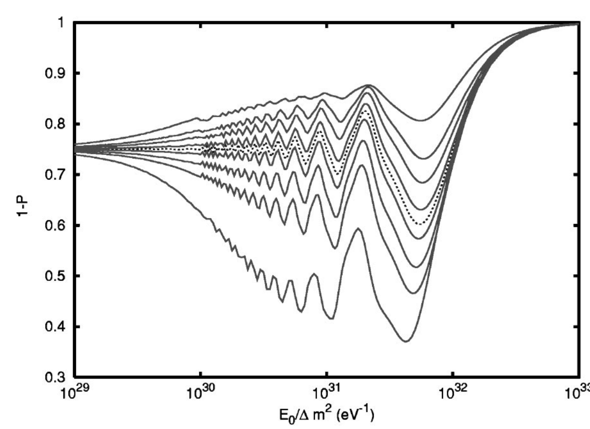

FIG. 9. The survival probability $1-P(\nu_\alpha - \nu_s)$ as a function of the ratio $E_0/\Delta m^2$ for various values of $F\eta$. From the upper to the lower curve: $F\eta = -20,-10,-6,-2,0,2,6,10,20$; the dotted line represents the effect of vacuum oscillations $(F\eta=0)$. We have taken $\sin^2 2\theta = 0.5$ and production epoch $z=3$.

[7]Notice that partial averaging exists already at small z due to our integration over ΔE_0. For this reason P does not reach its maximal possible value $P_{\max} = \sin^2 2\theta$.

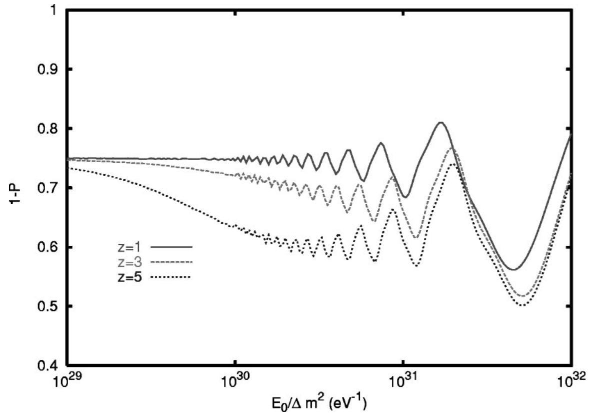

FIG. 10. The survival probability $1-P(\nu_\alpha-\nu_s)$ as a function of the ratio $E_0/\Delta m^2$ for various values of the production epoch z. We have taken $\sin^2 2\theta$=0.5 and $F\eta=6$.

approaches the production epoch (see Fig. 3). As a consequence, the mixing at production, and therefore the matter effect, increase. The maximal matter effect is achieved at energies for which the resonance condition is fulfilled at the production epoch or slightly later (notice that the adiabaticity is strongly broken at resonance). For $z\simeq 3$, this occurs in the interval $E_0/\Delta m^2 \simeq 10^{31}-10^{32}$ eV^{-1}. For $z\simeq 5$, maximal matter effect is realized at $E_0/\Delta m^2 \simeq (5-7)\times 10^{30}$ eV^{-1} (Fig. 10).

In Fig. 11, we show the dependence of the matter effect, i.e., the difference $P-P_v$, on the quantity $F\eta$ for various values of the mixing angle. For the parameters used in the plot the neutrinos are produced close to the resonance and the adiabaticity is strongly violated in the resonance. The matter effect can be estimated as the deviation of the jump probability from 1:

$$1-P_{LZ}\simeq 1-\exp(-\pi\chi_R/2). \tag{66}$$

In our case $\chi_R \ll 1$, so that the matter effect is proportional to $F\eta$:

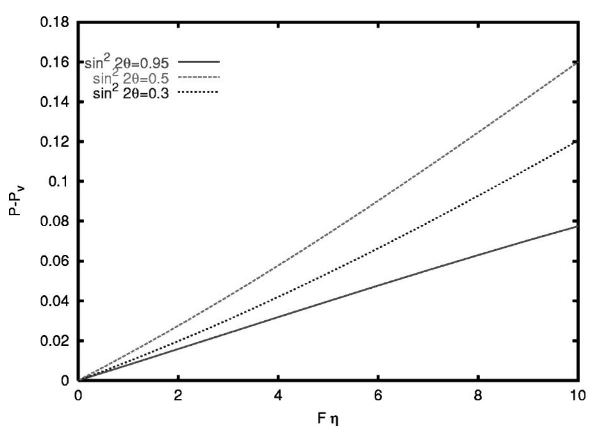

FIG. 11. The deviation with respect to the vacuum oscillation probability, $P(\nu_\alpha-\nu_s)-P_v$ as a function of the product $F\eta$ for various values of $\sin^2 2\theta$. We have taken production epoch $z=3$ and $E_0/\Delta m^2 = 10^{31}$ eV^{-1}.

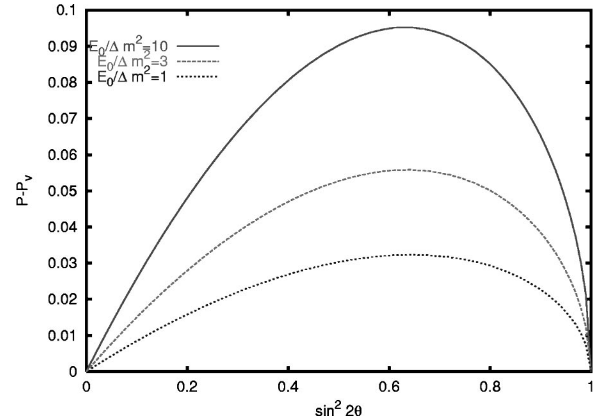

FIG. 12. The deviation with respect to the vacuum oscillation probability, $P(\nu_\alpha-\nu_s)-P_v$, as a function of $\sin^2 2\theta$, for various values of $E_0/\Delta m^2$ (in units of 10^{30} eV^{-1}). We have taken production epoch $z=3$ and $F\eta=6$.

$$P-P_v \simeq \frac{\pi}{2}\chi_R \propto F\eta \tan^2 2\theta, \tag{67}$$

according to Eq. (62). This explains the linear increase of the matter effect with η and F.

In Fig. 12 we show the dependence of $P-P_v$ on the mixing parameter $\sin^2 2\theta$ for different values of the ratio $E_0/\Delta m^2$ and fixed production epoch $z=3$ and $F\eta=6$. The neutrinos are produced in the resonance epoch or after it depending on their energy. For small mixing, the matter effect is proportional to the mixing parameter $\sin^2 2\theta_m$ at the production time. This explains the linear increase of the effect with $\sin^2 2\theta$ ($\sin^2 2\theta_m \propto \sin^2 2\theta$) and with $E_0/\Delta m^2$ (for $E_0/\Delta m^2 \sim 10^{31}$ eV^{-1}, the production epoch coincides with the resonance one). For maximal mixing, $\sin^2 2\theta$=1, the average probability takes the value $P=\frac{1}{2}$ independently of adiabaticity violation [27]. Therefore, in this case $P-P_v=0$. The maximum deviation from the vacuum oscillation effect is realized at $\sin^2 2\theta \simeq 0.65$.

V. CONVERSION EFFECTS ON DIFFUSE NEUTRINO FLUXES

The results we have discussed in Secs. III B and IV C describe the conversion effect for a beam of neutrinos produced by a single source at a certain epoch z. Presently, the possibilities of detection of neutrinos from single sources are limited to objects with redshift $z \ll 1$. For these neutrinos, no substantial effect is expected.[8] There is a hope, however, to detect the diffuse (integrated) neutrino flux which is produced by all the cosmological sources. For this flux, matter effects can be observable.

In what follows, we will calculate the ratio $F^\alpha(E_0)/F_0^\alpha(E_0)$, where $F^\alpha(E_0)$ and $F_0^\alpha(E_0)$ are the present diffuse fluxes of neutrinos of given flavor, ν_α, and a given

[8]Some effect can appear due to conversion in halos of galaxies and of clusters of galaxies [28].

energy, E_0, with and without conversion. The ratio can be written as

$$\frac{F^{\alpha}(E_0)}{F_0^{\alpha}(E_0)}=1-\overline{P}_{\alpha}(E_0), \tag{68}$$

where $\overline{P}_{\alpha}$ is the averaged transition probability:

$$\overline{P}_{\alpha}(E_0)\equiv\frac{1}{F_0^{\alpha}(E_0)}\int_0^{z_{\max}}\frac{dF_0^{\alpha}(E_0,z)}{dz}P_{\alpha}(E_0,z)dz. \tag{69}$$

Here $P_{\alpha}(E_0,z)$ is the transition probability for neutrinos produced in the epoch z, which has been discussed in Secs. III B and IV C. The quantity $dF_0^{\alpha}(E_0,z)$ is the contribution of the neutrinos ν_{α} produced in the interval $[z,z+dz]$ to the present flux in the absence of oscillations.

We first derive the general expression for the differential flux $dF_0^{\alpha}(E_0,z)$. Let $f(E)$ be the flux of neutrinos generated by a single source. Then the total number of neutrinos produced in the unit volume in the time interval $[t,t+dt]$ with energy in the interval $[E,E+dE]$ can be written as

$$f(E)n(t)dEdt, \tag{70}$$

where $n(t)$ is the concentration of sources in the epoch t. The contribution of these neutrinos to the present flux equals

$$dF_0^{\alpha}(E_0,z)=\frac{c}{4\pi}f(E)n(t)(1+z)^{-3}\frac{dE}{dE_0}dt, \tag{71}$$

where c is the speed of light and the factor $(1+z)^{-3}$ accounts for the expanding volume of the Universe. Transferring from t to z variable, we get

$$dF_0^{\alpha}(E_0,z)=\frac{3ct_0}{8\pi}f(E)n(z)(1+z)^{-11/2}\frac{dE}{dE_0}dz. \tag{72}$$

The relation between the energy E and the present neutrino energy E_0 includes, in general, effects of energy losses and of redshift. Neglecting absorption, we have $dE/dE_0=(1+z)$. The density of sources, $n(z)$, can be expressed in terms of the comoving density n_c as $n(z)=(1+z)^3n_c(z)$. Notice that $n_c=$const if the number of sources in the Universe is constant in time. Thus, the evolution of sources is described by the dependence of n_c on the redshift z.

In terms of n_c and E_0, we get finally

$$dF_0^{\alpha}(E_0,z)=\frac{3ct_0}{8\pi}f(E_0(1+z))n_c(z)(1+z)^{-3/2}dz. \tag{73}$$

Inserting $dF_0^{\alpha}(E_0,z)$ in Eq. (69), we find

$$\overline{P}_{\alpha}(E_0)=\frac{1}{F_0^{\alpha}(E_0)}\frac{3ct_0}{8\pi}\int f(E_0(1+z))n_c(z)(1+z)^{-3/2}\times P_{\alpha}(E_0,z)dz, \tag{74}$$

and $F_0^{\alpha}(E_0)$ is given by the same expression with $P_{\alpha}=1$.

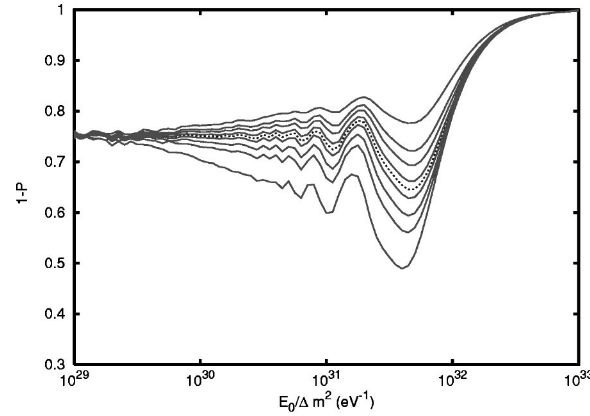

FIG. 13. The averaged survival probability for ν_{α}-ν_s channel, $1-\overline{P}_{\alpha}$, as a function of the ratio $E_0/\Delta m^2$ for the diffuse flux of neutrinos from GRBs. The curves correspond to various values of $F\eta$. From the upper to the lower curve: $F\eta=-20,-10,-6,-2,0,2,6,10,20$; the dotted line represents the effect of vacuum oscillations ($F\eta=0$). We have taken $\sin^2 2\theta=0.5$.

In what follows, we will calculate the survival probability $1-\overline{P}_{\alpha}$ for various possible sources of high-energy neutrinos, assuming certain forms for the produced flux $f(E)$ and the concentration of sources n_c.

A. Conversion of neutrinos from AGN and GRBs

There is evidence that cosmological sources such as GRBs and AGN were more numerous in the past. In particular, the density of GRBs evolved as [29]

$$n_c(z)\propto\begin{cases}(1+z)^3, & z\leq z_p\\ (1+z_p)^3, & z_p<z\leq z_{\max}\\ \sim 0, & z>z_{\max},\end{cases} \tag{75}$$

where z_p is estimated to be $z_p\simeq 1-2$ [29]. The energy spectrum of neutrinos from GRBs scales as a power law [30]:

$$f(E)\propto\frac{1}{E^2}=\frac{1}{E_0^2(1+z)^2}. \tag{76}$$

Combining Eqs. (75) and (76) with Eq. (74), we find the averaged probability,

$$\overline{P}_{\alpha}(E_0)=\frac{1}{N_p}\left[\int_0^{z_p}(1+z)^{-1/2}P_{\alpha}(E_0,z)dz+\int_{z_p}^{z_{\max}}(1+z)^{-7/2}P_{\alpha}(E_0,z)dz\right], \tag{77}$$

where the normalization factor N_p is given by the expression in square brackets with $P_{\alpha}=1$. According to Eq. (77), the contribution of the recent epochs to the present flux is enhanced in spite of the larger number of sources in the past. This leads to suppression of the matter effects, which are more important at large z.

Figure 13 shows the averaged survival probability, 1

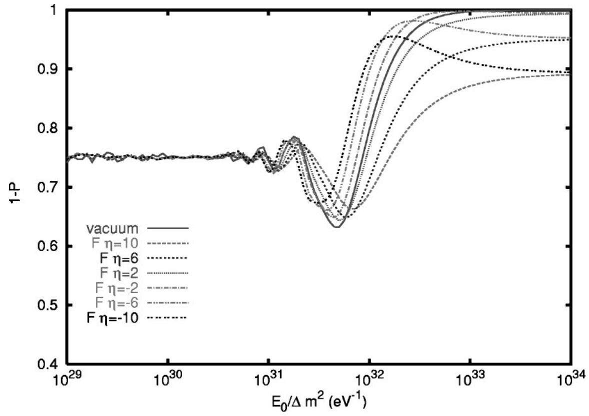

FIG. 14. The averaged survival probability for ν_α-ν_β oscillations, $1-\overline{P}_\alpha$, as a function of the ratio $E_0/\Delta m^2$ for the diffuse flux of neutrinos from GRBs. The curves correspond to various values of $F\eta$. We have taken $\sin^2 2\theta = 0.5$.

$-\overline{P}_\alpha$, for the ν_α-ν_s conversion channel, as a function of $E_0/\Delta m^2$ for different values of $F\eta$. We have taken $z_p=2$ and $z_{\max}=5$. The averaged probability is rather close to the nonaveraged one (see Fig. 8) for neutrinos produced at $z \simeq z_p = 2$. Indeed, the contribution to the flux from the earlier epochs, $z \gtrsim z_p$, is strongly suppressed, according to Eq. (77). The integration over z leads to some smoothing of the oscillatory behavior of the probability. The deviation of the ratio $F^\alpha(E_0)/F_0^\alpha(E_0)$ from its vacuum oscillation value can reach $\sim 25\%$. Maximal effect is realized for $F\eta \simeq 20$ in the resonance interval $E_0/\Delta m^2 \sim (1-5)\times 10^{31}$ eV^{-1}. For $F\eta \simeq 2$, the effect is about 3–4 %.

For conversion between active flavors, the results are shown in Fig. 14: the deviation of the survival probability from the value given by vacuum oscillations can be as large as $\sim 10\%$ for large asymmetry, $F\eta \gtrsim 10$, and high energies, $E_0/\Delta m^2 \gtrsim 10^{32}$ eV^{-1}, for which the matter-induced oscillation phase $\Phi_{\rm matt}$ dominates over the vacuum oscillation phase $\Phi_{\rm vac}$.

The astrophysical data about AGN indicate that the distribution of these objects has a maximum at $z \sim 2$ [31], with a rapid decrease of the concentration with z. The power law $f(E) \propto E^{-2}$ is a good approximation for the most energetic part of the spectrum [32]. For these reasons, in the case of AGN the results are similar to those discussed here for neutrinos from GRBs.

B. Conversion of neutrinos from heavy particle decay

Very heavy particles, with mass of the order of the grand unification scale, are supposed to be produced in the Universe by topological defects, e.g., in monopole-antimonopole annihilation, cosmic strings evaporation, etc. [2]. These particles would then decay very quickly, with lifetime $\tau \ll t_0$, into leptons and hadrons. Neutrinos may be produced directly, as primary decay products, and/or as secondary products from decays of hadrons.

Let us calculate the contribution of the neutrinos produced in the epoch z to the present flux: $dF_0^\alpha(E_0,z)/dz$. In assumption of very fast decay of the heavy particle, X (so that the production epochs of X and of the neutrinos coincide), we can write the total number of neutrinos produced in the unit volume in the time interval $[t,t+dt]$ with energy in the interval $[E,E+dE]$ as

$$\frac{dn_X(t)}{dt}\frac{dN_\nu}{dE}dtdE, \tag{78}$$

where $dn_X(t)$ is the number of X particles produced in the interval $[t,t+dt]$ in the unit volume, and dN_ν is the number of neutrinos in the energy interval $[E,E+dE]$ produced by a single particle X. The contribution of the neutrinos produced in the epoch t, Eq. (78), to the present neutrino flux is

$$dF_0^\alpha(E_0,z) = \frac{c}{4\pi}\frac{dn_X(t)}{dt}\frac{dN_\nu}{dE}(1+z)^{-3}\frac{dE}{dE_0}dt, \tag{79}$$

where we have taken into account the expansion of the Universe. In terms of the redshift z, we get

$$dF_0^\alpha(E_0,z) = \frac{3ct_0}{8\pi}\frac{dn_X}{dt}(z)\frac{dN_\nu}{dE}(E_0(1+z))(1+z)^{-9/2}dz, \tag{80}$$

where we used also the relation $E=E_0(1+z)$.

The production rate of the X particles can be written as

$$\frac{dn_X(t)}{dt} \propto t^{-4+p} \propto (1+z)^{6-(3/2)p}, \tag{81}$$

where $p=1$ for monopole-antimonopole annihilation and cosmic strings [33] and $p=2$ for a constant comoving production rate.

For the fragmentation function of neutrinos, we take a power law:

$$\frac{dN_\nu}{dE} \propto E^\alpha = E_0^\alpha(1+z)^\alpha. \tag{82}$$

If the neutrinos are produced mainly by hadronic decays, the fragmentation function has a polynomial form [34]. The leading term of the polynome gives the expression (82) with $\alpha = -\frac{3}{2}$.

Inserting the expressions from Eqs. (80), (81), and (82) into Eq. (69), we get

$$\overline{P}_\alpha(E_0) = \frac{1}{N_p}\int (1+z)^{6-(3/2)p+\alpha}P_\alpha(E_0,z)dz, \tag{83}$$

and for $p=1$ and $\alpha = -\frac{3}{2}$,

$$\overline{P}_\alpha(E_0) = \frac{1}{N_p}\int (1+z)^{-3/2}P_\alpha(E_0,z)dz. \tag{84}$$

Here N_p is a normalization factor.

We perform the integration (84) starting from the absorption epoch $z_{\rm abs}$. The contribution of the neutrino flux pro-

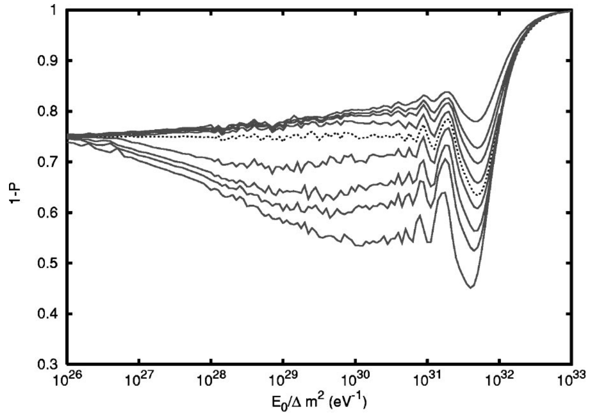

FIG. 15. The averaged survival probability for $\nu_\alpha - \nu_s$ channel, $1-\overline{P}_\alpha$, as a function of the ratio $E_0/\Delta m^2$ for neutrinos from the decay of heavy relics. The curves correspond to various values of $F\eta$. From the upper to the lower curve: $F\eta = -20, -10, -6, -2, 0, 2, 6, 10, 20$; the dotted line represents the effect of vacuum oscillations ($F\eta = 0$). We have taken $\sin^2 2\theta = 0.5$.

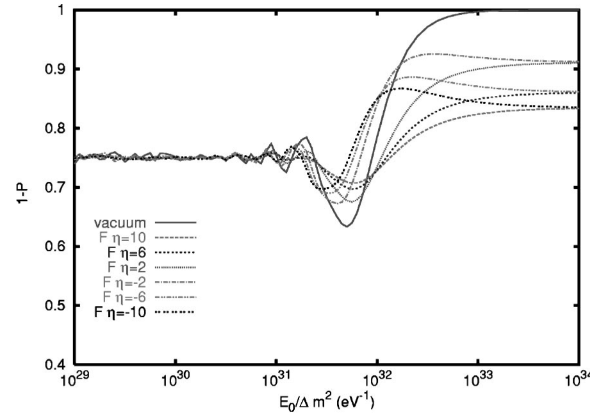

FIG. 16. The averaged survival probability for $\nu_\alpha - \nu_\beta$ oscillations, $1-\overline{P}_\alpha$, as a function of the ratio $E_0/\Delta m^2$ for the diffuse flux of neutrinos from the decay of heavy relics. The curves correspond to various values of $F\eta$. We have taken $\sin^2 2\theta = 0.5$.

duced at $z \gtrsim z_{\rm abs}$ is very small due to absorption.[9] The dominant absorption processes are ν-ν and ν-$\overline{\nu}$ interaction with the neutrino background. The absorption epoch is given by

$$1 + z_{\rm abs} = \left[\frac{d_{\rm abs}}{d_U} + 1 \right]^{2/3}, \tag{85}$$

where d_U is given in Eq. (7) and $d_{\rm abs}$ is the absorption width, which depends on the $\nu - \nu$ energy squared in the center of mass, s_Z [see Eq. (38)]. Taking, for instance, $E \lesssim 10^{22}$ eV and $m_\nu \lesssim 0.05$ eV, we have $s_Z \lesssim 0.1$, and the corresponding absorption width is $d_{\rm abs} \gtrsim 1.5 \times 10^{34}$ cm^{-2} [26]. With this value and $\eta \simeq 10$, Eq. (85) gives $z_{\rm abs} \simeq 50$.

In Fig. 15, we show the averaged survival probability $1 - \overline{P}_\alpha$ for $\nu_\alpha - \nu_s$ conversion channel, as a function of $E_0/\Delta m^2$ for different values of $F\eta$. One can see that, in contrast with the case of neutrinos from GRBs, the deviation of the ratio $F^\alpha(E_0)/F_0^\alpha(E_0)$ from the value given by vacuum oscillation is significant (larger than $\sim 10\%$) in a wide range of energies: $E_0/\Delta m^2 \simeq 10^{26} - 10^{32}$ eV^{-1}.

For a given value of $E_0/\Delta m^2$, the matter effects are determined by the corresponding resonance epoch, z_R, and adiabaticity in resonance. For $E_0/\Delta m^2 \lesssim 10^{29}$ eV^{-1}, the resonance was realized at $z_R \gtrsim 10$, when the adiabaticity condition was fulfilled (see Fig. 3). Therefore, the matter effects are dominated by resonant adiabatic conversion, which occurs for neutrinos produced at $z > z_R \sim 10$. As discussed in Sec. IV C, these neutrinos undergo almost total conversion (see Fig. 4), however their contribution to $\overline{P}_\alpha$ is suppressed according to Eq. (84).

For $E_0/\Delta m^2 \gtrsim 10^{29}$ eV^{-1}, the resonance is realized at $z_R \lesssim 10$, when the adiabaticity is broken (Fig. 3), so that the matter effect is mostly due to nonadiabatic conversion and oscillations in the production epoch.

The maximal effect is realized in the interval $E_0/\Delta m^2 \simeq 10^{29} - 5 \times 10^{31}$ eV^{-1}; the relative deviation of $F^\alpha(E_0)/F_0^\alpha(E_0)$ with respect to the vacuum oscillations value equals $\sim 10\%$ for $F\eta = 2$ and can be as large as 50% for $F\eta = 20$.

Figure 16 shows the average survival probability, $1 - \overline{P}_\alpha$, for active-active conversion. We see that, similarly to what was discussed for neutrinos from GRBs, a substantial ($\sim 15\%$) matter effect requires large asymmetry, $F\eta \gtrsim 10$, and very high energies, $E_0/\Delta m^2 \gtrsim 10^{32}$ eV^{-1}, for which the matter contribution to the oscillation phase is dominant.

VI. OBSERVABLE EFFECTS

Let us consider the experimental signatures of matter effects on neutrino propagation. The observable effects depend on the specific scheme of neutrino masses and mixings and on the initial flavor composition of the neutrino flux.

A. Conversion of cosmic neutrinos and neutrino mass schemes

As follows from the analysis of Secs. V A and V B, a significant matter effect on active-active oscillations of high-energy neutrinos requires

$$\frac{E_0}{\Delta m^2} \gtrsim 10^{32} \ \ {\rm eV}^{-1}. \tag{86}$$

This is the condition for which the matter-induced oscillation phase, $\Phi_{\rm matt}$, dominates over the vacuum one, $\Phi_{\rm vac}$ (see Sec. III B). For conversion into a sterile neutrino, the matter effect is substantial in the ranges

[9]Clearly, the energy of the neutrinos at production cannot exceed the mass of the parent particle, X. This gives a further constraint on the upper integration limit: $1 + z_{\rm max} \lesssim m_X/E_0$. Stronger bounds can be found in some specific production mechanisms: taking, for instance, $X \to \pi^+ \pi^-$ and subsequent production of neutrinos by pion decay, one gets $1 + z_{\rm max} \lesssim 0.2134 m_X/E_0$ [35]. For $E_0 \lesssim 10^{22}$ eV and $m_X \sim 10^{16}$ GeV, this gives the constraint $z_{\rm max} \lesssim 200$, which is weaker than the one given by absorption.

$$\frac{E_0}{\Delta m^2} \gtrsim \begin{cases} 10^{30}\ \ \text{eV}^{-1} & \text{for AGN, GRBs} \\ 10^{28}\ \ \text{eV}^{-1} & \text{for heavy relics decay.} \end{cases} \tag{87}$$

For $E_0 \lesssim 10^{21}$ eV, the conditions (86) and (87) imply

$$\Delta m^2 \lesssim \begin{cases} 10^{-11}\ \ \text{eV}^2 & \text{for} \quad \nu_\alpha - \nu_\beta \\ 10^{-7}\ \ \text{eV}^2 & \text{for} \quad \nu_\alpha - \nu_s\,. \end{cases} \tag{88}$$

For both the active-active and active-sterile channels, the mixing angle should be large enough and, for $\nu_\alpha - \nu_s$, not too close to maximal (see Fig. 12):

$$0.1 \lesssim \sin^2 2\theta \lesssim 0.95. \tag{89}$$

In the three neutrino schemes which explain the solar and atmospheric neutrino data, the effect can be realized for ν_e-ν_τ/ν_μ mixing and the vacuum oscillation (VO) solution of the solar neutrino problem. If the LMA, the SMA, or the LOW solution are confirmed, the effect of the medium on vacuum oscillations of cosmic neutrinos can be neglected.

In the presence of a sterile neutrino, the conditions (87)–(89) can be realized in a number of situations. Oscillations of electron neutrinos into a sterile state, $\nu_e - \nu_s$, with $\Delta m^2 \lesssim 10^{-11}$ eV^2 and mixing close to maximal represent a possible solution of the solar neutrino problem [36]. Another possibility is to consider, e.g., the hierarchical mass spectrum with $m_3 \sim \sqrt{\Delta m^2_{\text{atm}}}$, $m_2 \sim \sqrt{\Delta m^2_{\odot}}$, $m_1 \lesssim 10^{-4}$ eV, and $m_0 < m_1$ so that $\Delta m^2_{10} \simeq m_1^2 \lesssim 10^{-7}$ eV^2. In the simplest case, the sterile state is mixed only in the lightest mass eigenstates ν_1 and ν_0. The mixing angle is only weakly restricted by the solar neutrino data.[10]

B. Flavor composition of detected fluxes

Let us consider the numbers of events N_α and N_α^0 induced in a detector by neutrinos of different flavors α with and without conversion, respectively. These quantities are determined by the present fluxes F^α and F_0^α (see Sec. V); if the detector provides total energy reconstruction and optimal event selection, the flavor composition of the numbers of detected events coincides with that of the fluxes.

In what follows, we consider two possible types of flavor composition for the numbers of events in absence of conversion.

(i) CP-symmetric: $N_\alpha^0 = N_{\bar\alpha}^0$ $(\alpha = e,\mu,\tau)$. As far as flavor content is concerned, we take the normalized numbers of events:

$$\begin{aligned} (N_e^0, N_\mu^0, N_\tau^0) &= (1,2,0), \\ (N_{\bar e}^0, N_{\bar\mu}^0, N_{\bar\tau}^0) &= (1,2,0). \end{aligned} \tag{90}$$

Such a flavor composition is expected for neutrinos produced by the decays of π^+ and π^- mesons, which in turn appear in the process $X \to \pi^+\pi^-$ (see Sec. V B).

(ii) CP-asymmetric: $N_\alpha^0 \neq N_{\bar\alpha}^0$. We consider

$$\begin{aligned} (N_e^0, N_\mu^0, N_\tau^0) &= (1,1,0), \\ (N_{\bar e}^0, N_{\bar\mu}^0, N_{\bar\tau}^0) &= (0,1,0). \end{aligned} \tag{91}$$

This flavor composition is realized for neutrinos produced by the scattering of highly energetic protons on a photon background, where the π^+ decay gives the dominant contribution. The p-γ interaction is supposed to be the main mechanism of neutrino production in GRBs [30].

Neutrinos of different flavors produced in the same decay reaction (X or π decay) share the energy of the parent particle equally with good approximation. Therefore, the produced fluxes of neutrinos and antineutrinos of different flavors have the same energy dependence, and, in the absence of conversion, the ratios $N_e^0/N_\mu^0, N_e^0/N_\tau^0, N_\mu^0/N_\tau^0$ are expected to be energy-independent.

In the presence of vacuum oscillations the ratios of numbers of events are approximately independent of energy in two intervals: (i) $E_0/\Delta m^2 \lesssim 5\times 10^{30}$ eV^{-1} (see Figs. 13–15), where oscillations are averaged out; (ii) $E_0/\Delta m^2 \gtrsim 5\times 10^{32}$ eV^{-1}, where the vacuum oscillation phase is very small or, equivalently, the vacuum oscillation length exceeds the size of the horizon. In this case, the conversion probability is negligibly small and the ratios of numbers of events approach their values in the absence of oscillations.

The effects of vacuum oscillations are modified by the interaction with the neutrino background. According to the results of Secs. V A and V B we find that (a) the energy dependence of the ratios of numbers of events in the interval (i) would be a signal of active-sterile conversion with matter effects,[11] and (b) the deviation of the ratios of numbers of events in the interval (ii) from the values expected in the absence of conversion would indicate matter-affected active-active oscillations. Two elements, however, will make the identification of the effect difficult: its appearance at very high energies, close to the end of the predicted spectra of ultrahigh-energy neutrinos, and the uncertainties on the flavor composition of the neutrino fluxes at production.

We notice an interesting aspect: the interaction with the neutrino background produces strongly different effects on active-active and active-sterile oscillations. Thus the observation of such effects would neatly distinguish between the two channels. In particular, the observation of the characteristics described in (a) would give an indication of the existence of a sterile neutrino.

[10]No restriction exists for $\Delta m^2_{10} \ll 10^{-11}$ eV^2; bounds follow from the solar neutrino data for $\Delta m^2_{10} \gtrsim 10^{-11}$ eV^2 (in this case the solar neutrino data should be treated in a three-neutrino context).

[11]If some difference exists in the energy dependences of the original fluxes of neutrinos of different flavors, this would appear in the total number of events $N^0_{\text{tot}} = \Sigma_\alpha N_\alpha^0$, in contrast with the effect of neutrino conversion. Thus, the energy dependence of ratios of numbers of events due to matter effects can be distinguished.

C. Ratios of numbers of events: Active neutrino mixing

For three neutrino flavors, ν_e, ν_μ, ν_τ, the relation between the numbers of events N_α and N_α^0 can be expressed as

$$\vec{N}_\nu = \mathcal{P}\vec{N}_\nu^0, \tag{92}$$

where

$$\vec{N}_\nu^0 = (N_e^0, N_\mu^0, N_\tau^0),$$
$$\vec{N}_\nu = (N_e, N_\mu, N_\tau), \tag{93}$$

and $\mathcal{P}$ is the matrix of conversion probabilities: $\mathcal{P}_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta)$, $(\alpha, \beta = e, \mu, \tau)$.

As an example, we consider the scenario introduced in Sec. II B 1, in which the solar neutrino problem is solved by $\nu_e - \nu_\mu/\nu_\tau$ vacuum oscillations with $\Delta m_\odot^2 = \Delta m_{21}^2 \simeq 10^{-11}$ eV^2 and the atmospheric neutrino anomaly is explained by $\nu_\mu - \nu_\tau$ oscillations with $\Delta m_{\rm atm}^2 = \Delta m_{32}^2 \simeq 10^{-3}$ eV^2. The mixing matrix is given by Eq. (25).

Since the values of Δm_{32}^2 and Δm_{31}^2 ($\Delta m_{31}^2 \simeq \Delta m_{32}^2$) are out of the range of sensitivity to matter effects (see Sec. VI A), the oscillations due to Δm_{31}^2 and Δm_{32}^2 are described by the average vacuum oscillation probability. The neutrino background influences the $\nu_1 - \nu_2$ system only. In these specific circumstances, matter effects show up in the conversion of $\nu_e = \cos\theta\nu_1 + \sin\theta\nu_2$ into the orthogonal state $\nu' = -\sin\theta\nu_1 + \cos\theta\nu_2$. We denote by P the corresponding two-neutrino conversion probability. Taking the maximal mixing $\Theta = \pi/4$ in the matrix (25), we find the conversion matrix:

$$\mathcal{P} = \begin{pmatrix} 1-P & P/2 & P/2 \\ P/2 & 1/2 - P/4 & 1/2 - P/4 \\ P/2 & 1/2 - P/4 & 1/2 - P/4 \end{pmatrix}, \tag{94}$$

and an analogous expression for the matrix of probabilities for antineutrinos with the replacement $P \to \bar{P}$, where $\bar{P}$ represents the $\bar{\nu}_e \to \bar{\nu}'$ conversion probability.

Taking the CP-symmetric flavor composition (90), from Eqs. (94) and (92) we find that the conversion probability P cancels in the expression of the numbers of events, N_α. Equal numbers of events for the three flavors are predicted independently of matter effects: $\vec{N}_\nu = \vec{N}_{\bar{\nu}} = (1,1,1)$.

For the CP-asymmetric composition (91) we obtain

$$\vec{N}_\nu = (1 - P/2, 1/2 + P/4, 1/2 + P/4),$$
$$\vec{N}_{\bar{\nu}} = (\bar{P}/2, 1/2 - \bar{P}/4, 1/2 - \bar{P}/4). \tag{95}$$

Since the present detectors do not distinguish neutrinos from antineutrinos, we consider the sums of the events induced by ν and $\bar{\nu}$. From Eqs. (95), we find

$$\vec{N}_\nu + \vec{N}_{\bar{\nu}} = (1 - P/2 + \bar{P}/2, 1 + P/4 - \bar{P}/4, 1 + P/4 - \bar{P}/4). \tag{96}$$

Two comments are in order. First, equal numbers of events induced by the muon and tau neutrinos are expected, with no dependence of ratios on matter effects: $(N_\mu + N_{\bar{\mu}})/(N_\tau + N_{\bar{\tau}}) = 1$. Conversely, matter effects are present in ratios involving the electron neutrino. Second, if $P = \bar{P}$, the conversion probability cancels in Eq. (96) and one gets $\vec{N}_\nu + \vec{N}_{\bar{\nu}} = (1,1,1)$. This circumstance is realized in the absence of matter effects ($F\eta = 0$) or in the extremely high-energy limit, $E_0/\Delta m^2 \gtrsim 10^{33}$ eV^{-1}, in which the asymptotic value (50) for the conversion probability is realized (see also Fig. 2). Therefore, the matter effect could be revealed by a deviation from the equality of the numbers of events for the three flavors in the narrow interval $E_0/\Delta m^2 \simeq 10^{32} - 10^{33}$ eV^{-1} in which P and $\bar{P}$ are unequal and have significant deviation from the vacuum oscillation probability.

Considering, for instance, the ratio of e-like over non-e-like events, we find

$$R \equiv \frac{N_e + N_{\bar{e}}}{N_\mu + N_{\bar{\mu}} + N_\tau + N_{\bar{\tau}}} = \frac{1 - P/2 + \bar{P}/2}{2 + P/2 - \bar{P}/2}. \tag{97}$$

The deviation of R from its value $R_p = \frac{1}{2}$ without oscillations is entirely due to matter effects and equals

$$\frac{R - R_p}{R_p} \simeq -\frac{3}{2}\Delta, \tag{98}$$

where $\Delta \equiv (P - \bar{P})/2$. The relative deviation (98) amounts to $\sim 15\%$ for $\Delta \simeq 0.1$. Similar conclusions are obtained for other ratios of numbers of events.

Results are different if the mixing angle Θ in the matrix (25) is not maximal. In the extreme case $\Theta = 0$, the problem reduces to two-neutrino conversion. In the limit $E_0/\Delta m^2 \gtrsim 10^{33}$ eV^{-1}, for both the compositions (90) and (91) we get

$$\frac{R - R_p}{R_p} \simeq \frac{3}{2}P, \tag{99}$$

where we considered $P \simeq \bar{P}$. Taking $P \simeq 0.1$, the deviation (99) equals $\sim 15\%$.

D. Extension to four neutrinos

An example of a four-neutrino scheme with sterile neutrino, ν_s, was introduced in Sec. VI A: the sterile state is present in the two light mass eigenstates, ν_0 and ν_1, so that in the bases $\vec{\nu}_\alpha = (\nu_s, \nu_e, \nu_\mu, \nu_\tau)$, $\vec{\nu}_i = (\nu_0, \nu_1, \nu_2, \nu_3)$ the mixing matrix takes the form

$$U^0 = \begin{pmatrix} c_\phi & s_\phi & 0 & 0 \\ -c_\theta s_\phi & c_\theta c_\phi & -s_\theta & 0 \\ -s_\theta s_\phi/\sqrt{2} & s_\theta c_\phi/\sqrt{2} & c_\theta/\sqrt{2} & -1/\sqrt{2} \\ -s_\theta s_\phi/\sqrt{2} & s_\theta c_\phi/\sqrt{2} & c_\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \tag{100}$$

where $c_\phi \equiv \cos\phi$, $s_\phi \equiv \sin\phi$, $c_\theta \equiv \cos\theta$, $s_\theta \equiv \sin\theta$. The angle ϕ describes the mixing between ν_s and the state $\tilde{\nu} \equiv c_\theta \nu_e$

$+s_\theta\nu_\mu/\sqrt{2}+s_\theta\nu_\tau/\sqrt{2}$. Analogously to the previous case, we consider $\Delta m_{10}^2 \lesssim 10^{-7}$ eV^2 and all the other mass splittings to be much larger than this value, so that the interaction with the neutrino background affects the propagation of the $\nu_0-\nu_1$ system only. As a consequence, the matter effect modifies the angle ϕ only; the changes of θ are negligibly small. Again, the dynamics of the four-neutrino system is reduced to the evolution of the two states ν_s and $\tilde{\nu}$. Introducing the conversion probability $P\equiv P(\nu_s\to\tilde{\nu})$, we find the matrix of probabilities [see Eq. (92)]:

$$\mathcal{P}=\begin{pmatrix} 1-P & c_\theta^2 P & s_\theta^2 P/2 & s_\theta^2 P/2 \\ c_\theta^2 P & s_\theta^4+c_\theta^4(1-P) & s_\theta^2 c_\theta^2(1-P/2) & s_\theta^2 c_\theta^2(1-P/2) \\ s_\theta^2 P/2 & s_\theta^2 c_\theta^2(1-P/2) & [1+c_\theta^4+s_\theta^4(1-P)]/4 & [1+c_\theta^4+s_\theta^4(1-P)]/4 \\ s_\theta^2 P/2 & s_\theta^2 c_\theta^2(1-P/2) & [1+c_\theta^4+s_\theta^4(1-P)]/4 & [1+c_\theta^4+s_\theta^4(1-P)]/4 \end{pmatrix}. \tag{101}$$

Taking the CP-symmetric composition (90) and assuming that no sterile neutrinos are produced, $N_s^0=0$, from Eqs. (101) and (92) one gets the numbers of events:

$$\vec{N}_\nu=(P,1-c_\theta^2 P,1-s_\theta^2 P/2,1-s_\theta^2 P/2),$$
$$\vec{N}_{\bar{\nu}}=(\bar{P},1-c_\theta^2\bar{P},1-s_\theta^2\bar{P}/2,1-s_\theta^2\bar{P}/2). \tag{102}$$

As in the three-neutrino case, we have $(N_\mu+N_{\bar{\mu}})/(N_\tau+N_{\bar{\tau}})=1$ independently of matter effects. Notice that in the total numbers of events $\vec{N}_\nu+\vec{N}_{\bar{\nu}}$ the conversion probabilities appear in the combination $P+\bar{P}$: since the matter effects have opposite signs for neutrinos and antineutrinos, they partially cancel in this quantity.

Introducing the deviation from the averaged vacuum oscillation probability, $\delta_P\equiv P+\bar{P}-2P_v$, we compute the ratio

$$R\equiv\frac{N_e+N_{\bar{e}}}{N_\mu+N_{\bar{\mu}}+N_\tau+N_{\bar{\tau}}}=\frac{1-c_\theta^2(P_v+\delta_P/2)}{2-s_\theta^2(P_v+\delta_P/2)}. \tag{103}$$

The relative deviation of this ratio from the value given by vacuum oscillations equals

$$\frac{R-R_v}{R_v}\simeq-\frac{\delta_P}{2}\frac{c_\theta^2-s_\theta^2/2}{(1-s_\theta^2P_v/2)(1-c_\theta^2P_v)}. \tag{104}$$

Taking $\delta_P\simeq 0.1$, $s_\theta^2\simeq c_\theta^2\simeq 1/2$, and $P_v\simeq 0.4$, Eq. (104) gives a deviation of $\sim 2\%$; the effect is larger, $\sim 10\%$, for small θ: $c_\theta^2\simeq 1$, $s_\theta^2\simeq 0$.

For the CP-asymmetric composition (91), we get

$$\vec{N}_\nu^0=(P(c_\theta^2+s_\theta^2/2),1-s_\theta^2c_\theta^2-c_\theta^2P(c_\theta^2+s_\theta^2/2),$$
$$(1+s_\theta^2c_\theta^2)/2-s_\theta^2P(c_\theta^2+s_\theta^2/2)/2,(1+s_\theta^2c_\theta^2)/2$$
$$-s_\theta^2P(c_\theta^2+s_\theta^2/2)/2),$$

$$\vec{N}_{\bar{\nu}}^0=(s_\theta^2\bar{P}/2,s_\theta^2c_\theta^2(1-\bar{P}/2),$$
$$[1+c_\theta^4+s_\theta^4(1-\bar{P})]/4,[1+c_\theta^4+s_\theta^4(1-\bar{P})]/4).$$

For the ratio, R, of the e-like over non-e-like events, one gets

$$\frac{R-R_v}{R_v}\simeq-[\delta(1-s_\theta^2/2)+\bar{\delta}s_\theta^2/2]\left[\frac{c_\theta^2-s_\theta^2/2}{(1-s_\theta^2P_v/2)(1-c_\theta^2P_v)}\right], \tag{106}$$

where $\delta\equiv P-P_v$ and $\bar{\delta}\equiv\bar{P}-P_v$.

With the values $\delta\simeq 0.1$, $\bar{\delta}\simeq -0.05$, $P_v\simeq 0.4$ and small mixing, $c_\theta^2\simeq 1$, and $s_\theta^2\simeq 0$, the deviation (106) equals $\sim 15\%$, similarly to the case of CP-symmetric composition, Eq. (104). The effect is smaller, $\sim 2\%$, for large mixing, $s_\theta^2\simeq c_\theta^2\simeq\frac{1}{2}$.

Our estimation, 10–15 % effect, gives some hope that the discussed phenomenon will be observed in future large-scale experiments with event rates $\sim$1000 events/year.

VII. CONCLUSIONS

We have studied matter effects on oscillations of high-energy cosmic neutrinos. The only known component of the intergalactic medium which can contribute to such an effect is the relic neutrino background, provided that it has large CP (lepton) asymmetry.

The mixing modifies the flavor composition of the relic neutrino background. Considering atmospheric and solar neutrino-motivated mixings and mass-squared differences, we find that, if large asymmetries in the muon and/or tau flavors are produced before the BBN epoch, they are equilibrated by the combined effect of oscillations and inelastic collisions, so that $\eta_\mu\simeq\eta_\tau$. The asymmetry in the electron flavor, η_e, can be equilibrated with η_μ and η_τ for $\Delta m^2\gtrsim 10^{-7}$ eV^2. For $\Delta m^2\lesssim 10^{-7}$ eV^2, ν_e-ν_μ/ν_τ oscillations are suppressed by collisions and/or by the expansion rate of the Universe, thus leaving η_e unchanged, at least until the BBN epoch. At later epochs, oscillations develop and large asymmetries in the muon and/or tau flavors can be efficiently converted into ν_e asymmetry. Therefore, at present the values of the asymmetries for the three flavors can be comparable. This allows one to reconcile possible large lepton asymmetry in the ν_e flavor at present, $\eta_e\sim 1$, with strong constraint on η_e from nucleosynthesis. Active-sterile conversion is ineffective until the BBN or later, due the matter-induced suppression of the $\nu_\alpha - \nu_s$ mixing.

The dynamics of high-energy neutrino conversion in the CP-asymmetric neutrino background has been considered. For conversion between active neutrinos, the matter effects consist in a modification of the vacuum oscillation length. The effect is significant for large mixing angle, $\sin^2 2\theta \gtrsim 0.3$, and high energies, $E_0/\Delta m^2 \gtrsim 5\times 10^{32}$ eV^{-1}, for which the matter contribution to the oscillation phase dominates over the vacuum oscillation one. In these circumstances, the conversion probability can differ by $\sim$30% from the vacuum oscillations value.

For active-sterile conversion, the matter effects can be important in the interval $E_0/\Delta m^2 \gtrsim 10^{28} - 10^{32}$ eV^{-1}, for which the resonance condition is satisfied. For the majority of realistic situations (with $z \lesssim 10$, $\eta \lesssim 10$), the adiabaticity condition is broken. This implies that the matter effect is reduced to nonadiabatic level crossing or enhancement (suppression) of mixing and therefore of the depth of oscillations in the production epoch. The relative change of the conversion probability can be as large as 20–50 %. For extreme values of the asymmetry and large redshift of production, the adiabatic conversion can take place with almost maximal conversion probability.

We calculated the effect of conversion on the diffuse fluxes of neutrinos produced by GRBs, AGN, and the decay of superheavy relics. For neutrinos from GRBs and AGN, the relative deviation of the flux due to matter effects with respect to vacuum oscillations can reach 20%. For neutrinos from heavy particle decay, the effect can be larger, up to $\sim$40%.

Possible signatures of matter effects consist in the deviation of ratios of numbers of observed events—$N_e/N_\mu, N_e/N_\tau, N_\mu/N_\tau$—from the values predicted by pure vacuum oscillations. Presumably, neutrino mixings and masses will be measured in laboratory experiments and vacuum oscillation effects will be reliably predicted.

For conversion into a sterile state, one expects also a characteristic energy dependence of the ratios which in principle will allow us to distinguish matter effects from the uncertainties in the flavor content of original neutrino fluxes.

For illustration purposes, we estimated observable effects for two possible schemes of neutrino masses and two different flavor compositions of the detected fluxes in the absence of conversion. In a scheme with three flavor states only and parameters in the region of VO solution of the solar neutrino problem, we found that the deviation of ratios of numbers of events from their vacuum oscillation values can be $\sim$10%. A similar conclusion is obtained for schemes with an additional sterile neutrino.

Clearly, more work is needed to clarify the possibilities of observing the effect under consideration. In any case, new large-scale detectors with relatively high statistics ($\sim$1000 events/year) are required.

The detection of matter effects on fluxes of high-energy neutrinos would be evidence of large CP (lepton) asymmetry in the Universe. As follows from our analysis, asymmetries of order $\eta \sim 1$ can be probed in these studies. Clearly, the observation of such a large asymmetry will have far-reaching consequences for our understanding of the evolution of the Universe.

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