New fit to solar neutrino data in models with large extra dimensions

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String inspired models with millimeter scale extra dimensions provide a natural way to understand an ultralight sterile neutrino needed for a simultaneous explanation of the solar, atmospheric and Liquid Scintillation Neutrino Detector neutrino oscillation results. The sterile neutrino is the bulk neutrino (ν_B) postulated to exist in these models, and it becomes ultralight in theories that prevent the appearance of its direct mass terms. Its Kaluza-Klein (KK) states then add new oscillation channels for the electron neutrino emitted from the solar core. We show that successive Mikheyev-Smirnov-Wolfenstein transitions of solar ν_e to the lower lying KK modes of ν_B in conjunction with vacuum oscillations between the ν_e and the zero mode of ν_B provide a new way to fit the solar neutrino data. The characteristic predictions can be tested by solar neutrino observations and forthcoming solar neutrino experiments. We discuss both intermediate and low string scale models where the desired spectrum can emerge naturally.

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I. INTRODUCTION

At present there appear to be three classes of experiments that provide evidence for neutrino oscillations: solar neutrino searches [1], atmospheric neutrino data [2] and an accelerator search for oscillations Liquid Scintillation Neutrino Detector by the (LSND) experiment [3]. A simultaneous understanding of all these data seems to require the existence of an ultralight neutrino (beyond the three known ones: ν_e, ν_μ, ν_τ), which must be sterile with respect to weak interactions. Within this four-neutrino scheme [4], the solar ν_e deficit is explained by $\nu_e \to \nu_s$ (where ν_s is a sterile neutrino), the atmospheric ν_μ/ν_e anomaly is attributed to $\nu_\mu \to \nu_\tau$, and the LSND [3] results are explained by the $\nu_e - \nu_\mu$ oscillation predicted in the model. The heavier near-degenerate ν_μ and ν_τ are required by the LSND results to be in the eV range and can therefore share the role of hot dark matter. Exactly this same pattern of neutrino masses and mixings appears necessary to allow production of heavy elements ($A \gtrsim 100$) by type II supernovae [5]. While qualitatively this neutrino scheme [6] seems to explain all existing neutrino phenomena, solar neutrino observations are now sufficiently constraining that the small-angle Mikheyev-Smirnov-Wolfenstein (MSW) $\nu_e \to \nu_s$ explanation appears to be in some difficulty, and seemingly one must go to some length [7] in order to try to rescue this scheme. Although providing better fits to the solar data, even active-active transitions in a three-neutrino scheme do not give a quantitatively good explanation of all those data [8]. The theoretical and phenomenological challenge then is to find a scheme which has an ultralight sterile neutrino and at the same time provides a fit to the solar neutrino data.

Recently, motivated by string inspired brane models with large extra dimensions [9], we pointed out that [10] there appears to be a way to achieve an excellent fit and rescue the apparently needed four-neutrino scheme by including a singlet, sterile neutrino in the bulk. The method provides nearly maximal ν_e vacuum oscillation with the lightest pair of Kaluza-Klein (KK) modes, and also has MSW transitions to several of the other modes. We also pointed out that the model has its characteristic predictions for the ν_e survival probability and can be tested as new solar neutrino data on the neutrino energy distribution accumulates. It is the goal of this paper to elaborate on this proposal and discuss theoretical schemes that can lead to the desired parameters for the neutrinos.

In Sec. II we discuss the examples of brane-bulk models which lead to light neutrinos and the questions of naturalness of an ultralight sterile neutrino; in Secs. III and IV we consider two examples of models which have neutrino spectra desirable from the point of view of understanding neutrino data. In Sec. V, we provide a fit to solar neutrino observations in the context of these models, taking matter effects into account exactly. We present our predictions for the recoil energy distribution for the Super-K data and annual variation of the flux. In Sec. VI we summarize our conclusions. In Appendixes A and B we provide some more details on the TeV scale as well as the local $B-L$ models, and in Appendix C we comment further on the naturalness of ultralight sterile neutrinos in bulk-brane models.

II. NEUTRINO MASSES IN MODELS WITH LARGE EXTRA DIMENSIONS

One of the important predictions of string theories is the existence of more than three space dimensions. For a long time, it was believed that these extra dimensions are small and are therefore practically inconsequential as far as low energy physics is concerned. However, recent progress in the understanding of the nonperturbative aspects of string theories has opened up the possibility that some of these extra dimensions could be large [11,9] without contradicting observations. In particular, models where some of the extra

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dimensions have sizes as large as a millimeter and where the string scale is in the few TeV range have attracted a great deal of phenomenological attention in the past two years [9]. The basic assumption of these models, inspired by the D-branes in string theories, is that the space-time has a brane-bulk structure, where the brane is the familiar (3+1) dimensional space-time, with the standard model particles and forces residing in it, and the bulk consists of all space dimensions where gravity and other possible gauge singlet particles live. One could of course envision (3+d+1) dimensional D-branes where d-space dimensions have miniscule ($\leq$TeV^{-1}) size. The main interest in these models is due to the fact that the low string scale provides an opportunity to test them using existing collider facilities.

A major challenge to these theories comes from the neutrino sector, the first problem being how one understands the small neutrino masses in a natural manner. The conventional seesaw [12] explanation which is believed to provide the most satisfactory way to understand this, requires that the new physics scale [or the scale of $SU(2)_R\times U(1)_{B-L}$ symmetry] be around 10^9 to 10^{12} GeV or higher, depending on the Dirac masses of the neutrinos whose magnitudes are not known. If the highest scale of the theory is a TeV, clearly the seesaw mechanism does not work, so one must look for alternatives. The second problem is that if one considers only the standard model group in the brane, operators such as $LHLH/M_*$ could be induced by string theory in the low energy effective Lagrangian. For TeV scale strings this would obviously lead to unacceptable neutrino masses.

One mechanism suggested in Ref. [13] is to postulate the existence of one or more gauge singlet neutrinos, ν_B, in the bulk which couple to the lepton doublets in the brane. After electroweak symmetry breaking, this coupling can lead to neutrino Dirac masses, which are suppressed by the ratio M_*/M_{Pl}, where M_{Pl} is the Planck mass and M_* is the string scale. This is sufficient to explain small neutrino masses and owes its origin to the large bulk volume that suppresses the effective Yukawa couplings of the Kaluza-Klein (KK) modes of the bulk neutrino to the brane fields. In this class of models, naturalness of small neutrino mass requires that one must assume the existence of a global $B-L$ symmetry in the theory, since that will exclude the undesirable higher dimensional operators from the theory.

An alternative possibility [14] is to consider the brane theory to have an extended gauge symmetry which contains $B-L$ symmetry as a subgroup. Phenomenological considerations, however, require that the local $B-L$ scale and hence the string scale be of order of 10^9 GeV or so. The extra dimensions in these models could also be large. Indeed, it is interesting that if there were only one large extra dimension, R (and all small extra dimensions are $\sim M_*^{-1}$), the formula

$$M_{Pl}^2\simeq M_*^3R \tag{1}$$

leads to $M_*\simeq 10^9$ GeV if $R\sim$mm [14]. While in these models, there is no strict need to introduce the bulk neutrinos to understand the small masses of known neutrinos, if we wanted to include the sterile neutrinos, one must add the ν_B. The high scale models may also have certain other advantages which we will see as we proceed.

Regardless of which path one chooses for understanding small neutrino masses, a very desirable feature of these models is that if the size of extra dimensions is of order a millimeter, the KK excitations of the bulk neutrino have masses of order 10^{-3} eV, which is in the range needed for a unified understanding of oscillation data [4], as already noted.

III. TeV SCALE MODELS

To discuss the mechanisms in a concrete setting, let us first focus on TeV scale models. Here, one postulates a bulk neutrino, which is a singlet under the electroweak gauge group. Let us denote the bulk neutrino by $\nu_B(x^\mu,y)$. The bulk neutrino is represented by a four-component spinor and can be split into two chiral Weyl 2-component spinors as $\nu_B^T=(\chi^T,-i\phi^\dagger\sigma_2)$. The 2-component spinors χ and ϕ can be decomposed in terms of 4-dimensional Fourier components as follows:

$$\chi(x,y)=\frac{1}{\sqrt{2R}}\chi_{+,0}+\frac{1}{\sqrt{R}}\sum_{n=1}^{\infty}\left(\chi_{+,n}\cos\frac{n\pi y}{R}+i\chi_{-,n}\sin\frac{n\pi y}{R}\right). \tag{2}$$

There is a similar expression for ϕ. It has a five dimensional kinetic energy term and a coupling to the brane field $L(x^\mu)$. The full Lagrangian involving the ν_B is

$$\mathcal{L}=i\overline{\nu_B}\gamma_\mu\partial^\mu\nu_B+\kappa\overline{L}H\nu_{BR}(x,y=0)+i\int dy\,\overline{\nu_{BL}}(x,y)\partial_5\nu_{BR}(x,y)+\text{H.c.}, \tag{3}$$

where H denotes the Higgs doublet, and $\kappa=h(M_*/M_{Pl})$ is the suppressed Yukawa coupling. This leads to a Dirac mass for the neutrino [13] given by

$$m=\frac{hv_{wk}M_*}{M_{Pl}}, \tag{4}$$

where v_{wk} is the scale of $SU(2)_L$ breaking. In terms of the 2-component fields, the mass term coming from the fifth component of the kinetic energy connects the fields χ_+ with ϕ_- and χ_- with ϕ_+, whereas it is only the ϕ_+ (or $\nu_{B,R,+}$) which couples to the brane neutrino $\nu_{e,L}$. Thus as far as the standard model particles and forces go, the fields ϕ_- and χ_+ are totally decoupled, and we will not consider them here. The mass matrix that we will write below therefore connects only ν_{eL}, $\phi_{+,n}$ and $\chi_{-,n}$.

From Eq. (3), we conclude that for $M_*\sim 10$ TeV, this leads to $m\simeq 10^{-4}h$ eV. It is encouraging that this number is in the right range to be of interest in the discussion of solar neutrino oscillation if the Yukawa coupling h is appropriately chosen. Furthermore, this neutrino is mixed with all the KK modes of the bulk neutrino, with a mixing mass $\sim\sqrt{2}m$; since the nth KK mode has a mass $nR^{-1}\equiv n\mu$, the mixing angle is given by $\sqrt{2}mR/n$. Note that for $R\sim 0.1$ mm, this mixing angle is of the right order to be important in MSW transitions of solar neutrinos.

It is worth pointing out that this suppression of m is independent of the number and radius hierarchy of the extra dimensions, provided that our bulk neutrino propagates in the whole bulk. For simplicity, we will assume that there is only one extra dimension with radius of compactification as large as a millimiter, and the rest with much smaller compactification radii. The smaller dimensions will only contribute to the relationship between the Planck and the string scale, but their KK excitations will be very heavy and decouple from the neutrino spectrum. Thus, all the analysis can be done as in five dimensions.

In order to make this model applicable to resolving the observed oscillation phenomena, we have to extend the model as has been noted in [15]. Even if one wanted to understand the solar neutrino oscillation using this picture, one would have difficulty fitting all the rates in Gallium, Chlorine and the water Cherenkov data while simultaneously explaining the recoil energy distribution in the Super-Kamiokande data.

One way to do this would be to include new physics in the brane. We parametrize this in terms of an effective Majorana neutrino mass matrix in the brane:

$$\mathcal{M}=\begin{pmatrix} \delta_{ee} & \delta_{e\mu} & \delta_{e\tau} \\ \delta_{e\mu} & \delta_{\mu\mu} & m_0 \\ \delta_{e\tau} & m_0 & \delta_{\tau\tau} \end{pmatrix}. \tag{5}$$

The origin of this pattern of brane neutrino masses will be discussed in Appendix A. In this section we concentrate on the effect of this matrix on the mixing of the bulk neutrinos with the brane ones. For this we will assume that $m_0 \gg \delta_{ij}$; as a result, the $\nu_{\mu,\tau}$ decouple and do not affect the mixing between the bulk neutrino modes and the ν_e, and in the subsequent analysis we consider the remaining modes. Their mass matrix in the basis $(\nu_e, \nu_{BR,+}^{(0)}, \nu_{BL,-}^{(1)}, \nu_{BR,+}^{(1)}, \nu_{BL,-}^{(2)}, \nu_{BR,+}^{(2)}, \cdots)$ is given by:

$$\mathcal{M}=\mathcal{M}_{\mathrm{TeV}}$$
$$\equiv\begin{pmatrix} \delta_{ee} & m & 0 & \sqrt{2}m & 0 & \sqrt{2}m & \cdot & \cdot \\ m & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot \\ 0 & 0 & 0 & \mu & 0 & 0 & \cdot & \cdot \\ \sqrt{2}m & 0 & \mu & 0 & 0 & 0 & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 2\mu & \cdot & \cdot \\ \sqrt{2}m & 0 & 0 & 0 & 2\mu & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}. \tag{6}$$

One can evaluate the eigenvalues and the eigenstates of this matrix. The former are the solutions of the transcendental equation:

$$m_n \;=\; \delta_{ee}+\frac{\pi m^2}{\mu}\cot\left(\frac{\pi m_n}{\mu}\right). \tag{7}$$

The equation for eigenstates is

$$\tilde{\nu}_n=\frac{1}{N_n}\left[\nu_e+\frac{m}{m_n}\nu_{B,+}^{(0)}+\sum_k \sqrt{2}m\left(\frac{m_n}{m_n^2-k^2\mu^2}\nu_{B,-}^{(k)} + \frac{k\mu}{m_n^2-k^2\mu^2}\nu_{B,+}^{(k)}\right)\right], \tag{8}$$

where we have used the notation $\pm$ for the left- and right-handed parts of the KK modes of the bulk neutrino in the two-component notation and dropped the L,R subscripts, where the sum over k runs through the KK modes, and N_n is the normalization factor given by

$$N_n^2=1+m^2\pi^2R^2+\frac{(m_n-\delta_{ee})^2}{m^2}. \tag{9}$$

From Eq. (7) we see that there are two eigenvalues $m_\pm$ near m when $\delta_{ee}\ll m \ll \mu$, and these are given by $m_\pm \simeq (\delta_{ee}/2))\pm m$. These are the lowest two levels and their mass difference square is given by $\sim 2m\delta_{ee}$ eV^2. From Eq. (8), we see that they are maximally mixed. Therefore, if we choose $\delta_{ee}\sim 10^{-7}$ eV, then the transition between these levels can lead to vacuum oscillation (VO) of the solar neutrinos. This will be one of the ingredients of our new solution, as we see below.

IV. LOCAL $B-L$ SYMMETRY MODELS

A second way to achieve the same phenomenology is possible using a much higher string scale. In this class of models [14], one postulates that the theory in the brane is left-right symmetric so that it contains the $B-L$ as a local symmetry. The gauge group of the model is $SU(2)_L\times SU(2)_R \times U(1)_{B-L}$ with field content for leptons given by left and right doublets $\psi_{L,R}\equiv(\nu,l)_{L,R}$ and in the Higgs sector bidoublet $\phi=(2,2,0)$, doublets $\chi_{L,R}$. As in the previous case we choose a single bulk neutrino. We then impose the following Z_2 symmetry on the Lagrangian under which $\chi_L\to-\chi_L$, $\nu_{B,R}\to-\nu_{B,R}$ and all other fields are invariant. Note that under this symmetry, the 5th coordinate $y\to -y$. The invariant Lagrangian is

$$\mathcal{L}=h_l\frac{(\psi_R\chi_R)^2+(\psi_L\chi_L)^2}{M_*}+\bar{\psi}_L\phi\psi_R+\frac{f}{M_*^{1/2}}[\bar{\psi}_{eR}\tilde{\chi}_R + \bar{\psi}_{eL}\tilde{\chi}_L]\nu_B(x,y=0) \tag{10}$$

$$+i\int dy\,\overline{\nu}_{BL}(x,y)\partial_5\nu_{BR}(x,y)+\text{H.c.}, \tag{11}$$

where f and h_l are Yukawa couplings and $\tilde{\chi}_{L,R}=i\tau_2\chi_{L,R}^*$. We then break the right-handed symmetry with $\langle\chi_R^0\rangle=v_R$, while at the same time keeping $\langle\chi_L\rangle=0$. We expect v_R to be of order of the string scale M_*. The Lagrangian involving the electron neutrino and the bulk neutrinos then becomes

$$\mathcal{L}=f\frac{v_R^2}{M_*}\nu_R\nu_R+m\overline{\nu_{eL}}\nu_{eR}+\alpha\overline{\nu_{eR}}\nu_{BL}(x,y=0)$$
$$+i\int dy\overline{\nu_{BL}}(x,y)\partial_5\nu_{BR}(x,y)+\text{H.c.}, \quad (12)$$

where $\alpha\simeq h_l M_* v_R/M_{Pl}$ and $m=h_l v_{wk}$. In this section we discuss only the $\nu_e-\nu_B$ sector and address the full three generation mixing in Appendix B. In what follows, the Majorana mass of the ν_R is denoted by M, i.e., $M=f(v_R^2/M_*)$. This leads to the mass matrix of Eq. (13) which mixes the brane neutrinos with the KK modes of ν_B. For $\langle\chi_R^0\rangle=v_R\gg\langle\phi\rangle,\mu$, which we assume, ν_{eR} has a Majorana mass, M, which is much bigger than any other masses in the theory (ignoring real superheavy KK modes), and ν_{eR} decouples. One can then write an effective Lagrangian at low energies (i.e., $E\ll v_R$) using the seesaw mechanism. The effective mass matrix for the light modes can be written down using the same notation for the KK modes of ν_B as in the previous section. In the basis $(\nu_{eL},\nu_{BL,+}^{(0)},\nu_{BL,+}^{(1)},\nu_{BR,-}^{(1)},\nu_{BL,+}^{(2)},\nu_{BR,-}^{(2)},\cdot,\cdot)$ it is given by

$$\mathcal{M}=\mathcal{M}_{B-L}\equiv\frac{1}{M}\begin{pmatrix} m^2 & \alpha m & \sqrt{2}m\alpha & 0 & \sqrt{2}m\alpha & 0 & \cdot & \cdot \\ \alpha m & \alpha^2 & \sqrt{2}\alpha^2 & 0 & \sqrt{2}\alpha^2 & 0 & \cdot & \cdot \\ \sqrt{2}\alpha m & \sqrt{2}\alpha^2 & 2\alpha^2 & M\mu & 2\alpha^2 & 0 & \cdot & \cdot \\ 0 & 0 & M\mu & 0 & 0 & 0 & \cdot & \cdot \\ \sqrt{2}\alpha m & \sqrt{2}\alpha^2 & 2\alpha^2 & 0 & 2\alpha^2 & 2M\mu & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 2M\mu & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}. \quad (13)$$

It is easy to see that the lowest eigenvalue of this matrix is zero. The transcendental equation describing the rest of the eigenvalues is

$$\frac{m^2}{M}+\frac{\alpha^2}{M}\frac{\pi m_n}{\mu}\cot\frac{\pi m_n}{\mu}=m_n. \quad (14)$$

The next lowest eigenvalue solution of this equation is $m_1\simeq(m^2+\alpha^2)/M$. For $m,\alpha\sim1-5$ MeV (similar to the first generation fermion mass) and $M\simeq10^9$ GeV, we get this eigenvalue to be of order $(10^{-6}-2.5)\times10^{-5}$ eV. Its square is therefore in the range where the VO solution to the solar neutrino puzzle can be applied. Also for $m\simeq\alpha$, the mixing angle between the zero eigenvalue mode and this mode is maximal.

In the diagonalization, we have ignored the radiative corrections that allow us to extrapolate to the weak scale the above mass matrix which is valid at the string scale. Going to the weak scale, $M_Z=$ mass of the Z^0 meson, there are two kinds of contributions that dominate the radiative corrections: one arising from the top quark coupling to the standard model Higgs doublet in the effective *LHLH* operator induced by the seesaw mechanism [16], and a second one which can arise from self couplings of the Higgs fields (for the nonsupersymmetric version of the model). The top quark contribution replaces the parameter m in the matrix in Eq. (13) by $m[1+(6h_t^2/16\pi^2)\ln(M_*/M_Z)]$ in the off-diagonal terms, whereas the self scalar coupling contributes only to the m^2 term. Thus the m^2 term in Eq. (13) is replaced by $m^2[1+(6h_t^2/16\pi^2)\ln(M_*/M_Z)+(2\lambda/16\pi^2)\ln(M_*/M_Z)]^2$. (This expression will have more terms involving extra scalar self couplings if there is more than one Higgs doublet in the low energy theory.) Let us denote the rest of the radiative corrections by ϵ in the m^2 term and ϵ' in the off-diagonal ones. Such radiative corrections could come from one loop contributions at the scale M itself. The magnitude of the radiative corrections also could increase if the low energy theory below M_* has more than one Higgs doublets, as noted. It is therefore not implausible to assume that the radiative corrections are significant. With redefinition of the off-diagonal m terms, $\mathcal{M}$ takes the same form as in Eq. (13), except the m^2 term is replaced with $m^2+\Delta$, with $\Delta=(m^2/16\pi^2)(24h_t^2\lambda+4\lambda^2+\epsilon-\epsilon')\ln(M_*/M_Z)$. The characteristic equation for the m_n becomes

$$\frac{m^2+\Delta}{M}+\frac{\alpha^2\pi(m_n-\Delta/M)}{M\mu}\cot\frac{\pi m_n}{\mu}=m_n. \quad (15)$$

It is now easy to see that if $\Delta\ll(\alpha^2+m^2)$, the zero eigenvalue is replaced by $\sim\alpha^2\Delta/[M(\alpha^2+m^2)]$. Clearly, we want Δ closer to m^2,α^2 to obtain our desired parameters. For this purpose, we choose parameters λ and the other radiative corrections (i.e., ϵ,ϵ') appropriately, so that the two lowest eigenvalues are of almost equal magnitude, and the parameters of the next section can be reproduced. This situation can be realized more easily if the theory is nonsupersymmetric all the way to the string scale. If there is supersymmetry all the way down to the TeV scale, then the scalar self coupling contributions "kick" in only below M_{SUSY} and one has to stretch parameters and perhaps require a two-Higgs doublet model below the SUSY scale to realize the parameters used below.

Let us now turn to the determination of the mixing angles. For this we need the explicit form of the eigenvector Ψ_n for the nth mass eigenstate:

$$\Psi_n = \frac{1}{N_n}\begin{pmatrix} 1 \\ a_n \\ b_1^n \\ b_1^{n\prime} \\ \cdot \\ \cdot \\ b_k^n \\ b_k^{n\prime} \\ \cdot \end{pmatrix}, \tag{16}$$

where

$$a_n = \frac{\alpha}{m}\left(1 - \frac{\mu\Delta}{m_n}\right), \tag{17}$$

$$b_k^n = \frac{\sqrt{2}a_n}{1 - \dfrac{k^2\mu^2}{m_n^2}}, \tag{18}$$

and

$$b_k^{n\prime} = \frac{k\mu}{m_n} b_k^n. \tag{19}$$

The normalization of the state is given by

$$N_n^2 = 1 + \left(\frac{\pi m_n a_n}{\mu \sin \dfrac{\pi m_n}{\mu}}\right)^2. \tag{20}$$

The mixing of the ν_e with the bulk modes is essentially given by the $1/N_n$, which for the nth eigenmode (with $m_n \sim n\mu$) is $\approx m\alpha/Mn\mu$. For $m\alpha/M \sim 10^{-5}$ eV and $\mu \sim 10^{-3}$ eV, this mixing is of order of a percent and is therefore in the interesting range for a possible MSW transition of the solar neutrinos.

V. SOLAR NEUTRINO DATA FIT BY A COMBINATION OF VACUUM AND MSW OSCILLATIONS

In this section, we discuss the question of how to understand the solar neutrino data in these models, while at the same time explaining the atmospheric as well as the LSND data. There have been several recent papers that have addressed the issue of explaining observed oscillation data in the models with large extra dimensions [10,17,18,15,19–22]. In particular, in Ref. [15] it has been shown that while the overall features of the solar and atmospheric data can be accommodated in minimal versions of these models with three bulk neutrinos, it is not possible to explain simultaneously the LSND observation for the $\nu_\mu - \nu_e$ oscillation probability, and one must incorporate new physics in the brane.

In the models presented here, the $\nu_\mu - \nu_\tau$ mass difference responsible for atmospheric oscillation data is generated via the radiative corrections in the TeV scale models, and the seesaw mechanism in the local $B-L$ models. Since we arrange the models so that the mass of the $\nu_{\mu,\tau}$ pair is in the eV range, this provides a way to accommodate the LSND results. Let us therefore focus on the solar data. We will present our discussions using the parameters of the TeV scale model. The discussion also applies to the local $B-L$ models, with only the labels of the parameters changed.

A first glance at the values of the parameters of the model such as m in Eq. (3) and $R^{-1} \sim 10^{-3}$ eV suggests that perhaps one should seek a solution of the solar neutrino data in these models using the small-angle MSW mechanism [17]. However, the present Super-Kamiokande recoil energy distribution seems to disfavor such an interpretation, although any definitive conclusion should perhaps wait until more data accumulate [8]. In any case if the present trend of the data near the higher energy region of the solar neutrino spectrum from Super-Kamiokande persists, it is likely to disfavor the small-angle MSW solution and tend to favor a vacuum oscillation. However, the latter does not give correct rates for the three types of solar neutrino experiments.

As discussed in [10], our solution to the solar neutrino data consists of two components: one involving the vacuum oscillation of ν_e to ν_B^0 and the second one involving the MSW transition of the higher energy ν_e's to higher KK modes of the ν_B. The vacuum oscillation part is straightforward, and in order to get a better fit we have to adjust the $\Delta m^2_{\nu_e - \nu_B^0}$. On the other hand, to discuss the MSW effect for the case of bulk neutrinos, we need to include the matter effects in the diagonalization of the infinite dimensional neutrino mass matrix.

To have a physical understanding of our strategy, note that the simplest way to reconcile the rates for the three classes of solar experiments is to "kill" the ^{7}Be neutrinos, reduce the ^{8}B neutrinos by half and leave the pp neutrinos alone. To achieve this using pure vacuum oscillation, one may put a node of the survival probability function P_{ee} around 0.86 MeV. However, for an arbitrary node number, the oscillatory behavior of P_{ee} before and after 0.86 MeV cannot in general satisfy the other two requirements mentioned above; specifically, if there are more nodes prior to 0.8 MeV, the Gallium pp neutrinos get suppressed. If one uses the first node to "kill ^{7}Be," then for ^{8}B neutrino energies the P_{ee} is close to one and not half as would be desirable. The strategy generally employed is not to "shoot" for a node at the precise ^{7}Be energy but rather somewhat away so that it reduces ^{7}Be to a value above zero. This requires that one must reduce the ^{8}B neutrinos by much more than 50%, so one can fit Chlorine data. The water data then require an additional contribution, which, in the case of active vacuum oscillation (VO), is provided by the neutral current cross section amounting to about 16% of the charged current one. Thus in a pure two-neutrino oscillation picture, VO comes close to working for oscillation to active neutrinos but certainly does not work for active to sterile oscillation. It is here that the large extra dimensions come to the rescue.

In our model, both vacuum oscillations and MSW oscillations are important. This is because the lowest mass pair of neutrinos is split by a very small mass difference, whereas the KK states have to be separated by $>10^{-3}$ eV because of the limits from gravity experiments. We can then use the first node of P_{ee} to suppress the ^{7}Be. Going up in energy toward ^{8}B neutrinos, the survival probability, which in the VO case would have risen to very near 1, is now suppressed by the small-angle MSW transitions to the different KK excitations of the bulk neutrino. This is the essence of our new way to fit the solar neutrino observations [10].

Eigenvectors of the neutrino mass matrix, $\mathcal{M}$, evolve according to $e^{i(px-Et)}$, where for a state of mass m, $px-Et \approx E(x-t)-tm^2/2E$. Neutrino oscillations happen because different mass states interfere with each other. When the neutrino of a particular energy is detected at a particular point, $E(x-t)$ is independent of which mass state contributes. The common phase factor $e^{iE(x-t)}$ can be factored out of all states, because we are only concerned with relative phases when considering interference terms. Each mass state evolves separately in a vacuum according to

$$i\frac{d\vec{a}}{dt}=\frac{m^2}{2E}\vec{a}. \tag{21}$$

The electron neutrino is an eigenstate of neutrino interactions with matter, but is not an eigenstate of the mass matrix, $\mathcal{M}$. In the basis of eigenstates of neutrino interaction with matter, an arbitrary state evolves through a vacuum according to

$$i\frac{d\vec{a}}{dt}=\frac{\mathcal{M}^\dagger\mathcal{M}}{2E}\vec{a}. \tag{22}$$

In matter the squared mass matrix, $\mathcal{M}^\dagger\mathcal{M}$, is replaced by $\mathcal{M}^\dagger\mathcal{M}+2EH_1$, where H_1 is $\rho_e=\sqrt{2}G_F(n_e-0.5n_n)$ when acting on ν_e and is zero on sterile neutrinos. The survival probability, P_{ee}, is the probability that an initially pure ν_e is still a ν_e after the neutrino state propagates from its origin in the Sun to its detection on the Earth. For the TeV scenario, P_{ee} depends on E and the parameters used in $\mathcal{M}_{\rm TeV}$ of Eq. (6): $P_{ee}=P_{ee}^{\rm TeV}(E,\delta_{ee},m,\mu)$. But since the propagation depends only on $\mathcal{M}^\dagger\mathcal{M}/2E+H_1$, the survival probability must be unchanged whenever $\mathcal{M}/\sqrt{E}$ is unchanged. We therefore have the scaling rule

$$P_{ee}^{\rm TeV}(E,\delta_{ee},m,\mu)=P_{ee}^{\rm TeV}\left(1,\frac{\delta_{ee}}{\sqrt{E}},\frac{m}{\sqrt{E}},\frac{\mu}{\sqrt{E}}\right). \tag{23}$$

Similarly, there is a scaling rule for $P_{ee}=P_{ee}^{B-L}(E,M,m,\Delta,\alpha,\mu)$ of the local $B-L$ scenario:

$$P_{ee}^{B-L}(E,M,m,\Delta,\alpha,\mu)=P_{ee}^{B-L}\left(1,1,\frac{m}{\sqrt{M\sqrt{E}}},\frac{\Delta}{M\sqrt{E}},\frac{\alpha}{\sqrt{M\sqrt{E}}},\frac{\mu}{\sqrt{E}}\right). \tag{24}$$

To discuss the MSW effect for an infinite component system, we first diagonalize the matrix $\mathcal{M}^\dagger\mathcal{M}+2EH_1$ for both models. We first give the results for the TeV scale model, and in a subsequent paragraph present the result for the local $B-L$ case.

TeV scenario and matter effect

When $\mathcal{M}=\mathcal{M}_{\rm TeV}$, to express the eigenvectors and eigenvalues of $\mathcal{M}^\dagger\mathcal{M}+2EH_1$, define

$$w_k=\frac{E\rho_e}{\tilde{m}_k\delta_{ee}}+\sqrt{1+\left(\frac{E\rho_e}{\tilde{m}_k\delta_{ee}}\right)^2}; \tag{25}$$

$w_k=1$ in vacuum. The characteristic equation becomes

$$\tilde{m}_k=w_k\delta_{ee}+\frac{\pi m^2}{\mu}\cot\frac{\pi\tilde{m}_k}{\mu}. \tag{26}$$

The eigenvectors are as in Eq. (8), except the coefficients of ν_{0B} and $\nu'_{B,-}$ acquire an additional factor $1/w_n$, the m_k^2 is replaced by $\tilde{m}_k^2$, and the normalization becomes

$$N_n^2=1+\frac{\left(1+\frac{1}{w_n^2}\right)}{2}\left(\pi^2m^2R^2+\frac{(m_n-w_n\delta_{ee})^2}{m^2}\right)-\frac{\left(1-\frac{1}{w_n^2}\right)}{2}\frac{(m_n-w_n\delta_{ee})}{m}. \tag{27}$$

Note that in the presence of a dense medium, very crudely speaking, the ρ_e term in Eq. (25) will dominate over the rest of the terms, and when $\tilde{m}_n^2\simeq k^2\mu^2$ for any of the KK levels, that particular coefficient in Eq. (8) dominates, and the MSW resonant condition is satisfied.

Matter effect in the local $B-L$ case

Following the same procedure, we get for the local $B-L$ case the following eigenvalue equation in the presence of matter effects (where we will ignore the radiative corrections since they do not effect the results materially):

$$\tilde{m}_k^2=\frac{m^2}{M^2}\left\{(m^2+\rho_e)+\frac{\alpha^2\pi\tilde{m}_k}{\mu}\cot\frac{\pi\tilde{m}_k}{\mu}\left[2-\frac{2\rho_e}{\tilde{m}_k^2}+\frac{\rho_e}{m^2}\left(1-\frac{\rho_e}{\tilde{m}_k^2}\right)\frac{\pi\tilde{m}_k}{\mu}\cot\frac{\pi\tilde{m}_k}{\mu}\right]\right\}. \tag{28}$$

If we denote the eigenvector of the matter-affected mass matrix as

$$\tilde{\Psi}_k = \begin{pmatrix} 1 \\ \tilde{b} \\ \tilde{b}_1 \\ \tilde{b}_1' \\ \cdot \\ \cdot \end{pmatrix}, \tag{29}$$

we find that

$$\tilde{b} = \frac{\alpha(\tilde{m}_k^2 - \rho_e)}{m\tilde{m}_k^2}$$

$$\tilde{b}_1 = \frac{\sqrt{2}\alpha/m}{\tilde{m}_k^2 - \mu^2}(\tilde{m}_k^2 - \rho_e)$$

$$\tilde{b}_1' = \frac{\sqrt{2}\alpha m\mu/M}{\tilde{m}_k^2 - \mu^2}\left[1 + \frac{\alpha^2}{m^2}\left(1 - \frac{\rho_e}{\tilde{m}_k^2}\right)\frac{\pi\tilde{m}_k}{\mu}cot\frac{\pi\tilde{m}_k}{\mu}\right]$$

$$\tilde{b}_n = \frac{\tilde{m}_k^2 - \mu^2}{\tilde{m}_k^2 - n^2\mu^2}\tilde{b}_1$$

$$\tilde{b}_n' = \frac{\tilde{m}_k^2 - \mu^2}{\tilde{m}_k^2 - n^2\mu^2}n\tilde{b}_1'. \tag{30}$$

Here again we see that when the density term dominates $\tilde{m}_k^2$, there can be an MSW transition to the nth level when $\tilde{m}_k^2 \simeq n^2\mu^2$.

To carry out the fit, we studied the time evolution of the ν_e state using a program that evolved from one supplied by Haxton [23]. The program was updated to use the solar model of Bahcall, Basee, and Pinsonneault (BBP98) [24] and modified to do all neutrino transport within the Sun numerically. For example, no adiabatic approximation was used. Changes were also necessary for oscillations into sterile neutrinos and to generalize beyond the two-neutrino model. Up to 16 neutrinos were allowed, but no more than 14 contribute for the solutions we considered.

For comparison with experimental results, tables of detector sensitivity for the Chlorine and Gallium experiments were taken from Bahcall's web site [24]. The Super-Kamiokande (Super-K) detector sensitivity was modeled using [25], where the percent resolution in the signal from Cherenkov light, averaged over the detector for various total electron energies, is provided. For details see [10].

Calculations of electron neutrino survival probability, averaged over the response of detectors, were compared with measurements. While theoretical uncertainties in the solar model and detector response were included in the computation of χ^2 as described in Ref. [26], the measurement results given here include only experimental statistical and systematic errors added in quadrature. The Chlorine survival probability, from Homestake [27], is 0.332 ± 0.030. Gallium results [28] for SAGE, GALLEX and GNO were combined to give a survival probability of 0.579 ± 0.039. The $5.5-20$ MeV 1117 day Super-K experimental survival probability [28] is 0.465 ± 0.015. The best fits were with $R \approx 58$ μm, mR around 0.0093, and $\delta_{ee} \sim 0.84 \times 10^{-7}$ eV, corresponding to $\delta m^2 \sim 0.53 \times 10^{-11}$ eV^2. These parameters give average survival probabilities for Chlorine, Gallium, and water of 0.386, 0.533, and 0.460, respectively. They give a ν_e survival probability whose energy dependence is shown in Fig. 1. For two-neutrino oscillations, the coupling between ν_e and the higher mass neutrino eigenstate is given by $\sin^2 2\theta$, whereas here the coupling between ν_e and the first KK excitation replaces $\sin^2 2\theta$ by $4m^2R^2 = 0.00035$.

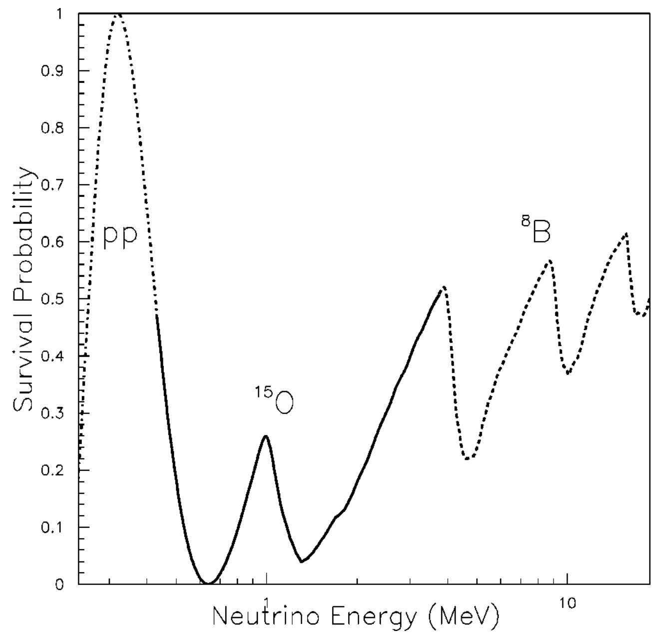

FIG. 1. Energy dependence of the ν_e survival probability when $R = 58$ μm, $mR = 0.0093$, $\delta_{ee} = 0.84 \times 10^{-7}$ eV. The dot-dashed part of the curve assumes the radial dependence in the Sun for neutrinos from the *pp* reaction, the solid part assumes ^{15}O radial dependence, and the dashed part assumes ^{8}B radial dependence.

Vacuum oscillations between the lowest two mass eigenstates nearly eliminate electron neutrinos with energies of .63 MeV/(2n+1) for $n = 0,1,2, \ldots$. Thus Fig. 1 shows nearly zero ν_e survival near 0.63 MeV, partly eliminating the ^{7}Be contribution at 0.862 MeV, and giving a dip at the lowest neutrino energy. Increasing δ_{ee} moves the low energy dip to the right into Gallium's most sensitive *pp* energy range, making the fit worse. Decreasing δ_{ee} increases Gallium, but hurts the Chlorine fit by moving the higher energy vacuum oscillation dip further to the left of the ^{7}Be peak. MSW resonances start causing the third and fourth eigenstates to be significantly occupied above ~ 0.8 MeV, the fifth and sixth eigenstates above ~ 3.7 MeV, the 7th and 8th above ~ 8.6 MeV, and the 9th and 10th above ~ 15.2 MeV. Figure 1 shows dips in survival probability just above these energy thresholds. The typical values of the survival probability within the ^{8}B region (~ 6 to ~ 14 MeV) are quite sensitive

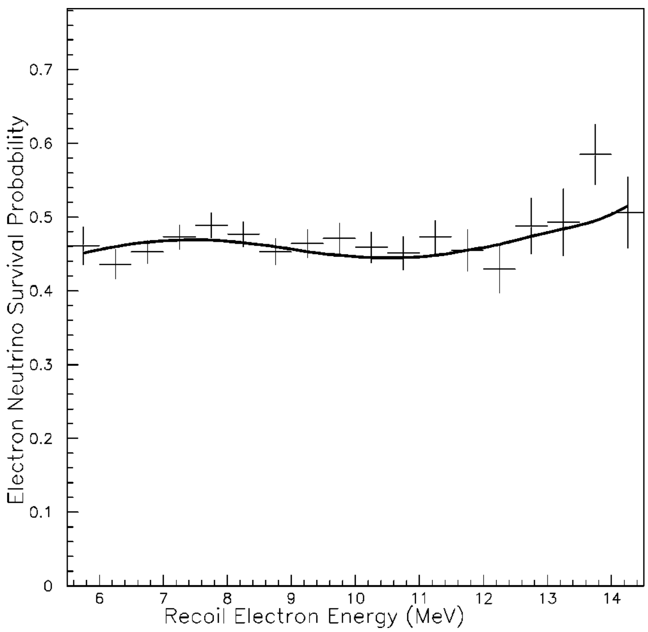

FIG. 2. Super-Kamiokande energy spectrum: measured [19] preliminary results based on 1117 days (error bars) and predicted (curve) for the same parameters as in Fig. 1. The curve is not a fit to these data.

to the value of mR. As can be seen from Eq. (9), higher mR increases $1/N_n \approx m/m_n \approx mR/n$ for various n, and thereby increases ν_e coupling to higher mass eigenstates, strengthens MSW resonances, and lowers ν_e survival probability.

The expected energy dependence of the ν_e survival probability is compared with preliminary Super-K data [28] in Fig. 2. The uncertainties are statistical only. The parameters used in making Fig. 2 were chosen to provide a good fit to the total rates only; they were not adjusted to fit this spectrum. But combining spectrum data with rates using the method described in Ref. [29], with numbers supplied by Super-K [30], gives $\chi^2 = 15.6$ (''probability'' 74%) for the spectrum predicted from the fit to total rates. It is incorrect to calculate probability as if there were (number of data points) −(number of parameters)=17 degrees of freedom, but if we do, χ^2 corresponds to a ''probability'' of 55%.

One may also seek fits with δ_{ee} constrained to be very small, thereby eliminating vacuum oscillations. The best such fit had $\chi^2 = 5.5$ (''probability'' 14%). The same parameters then used with the Super-K spectrum gave $\chi^2 = 18.7$ (''probability'' 35%).

The seasonal effect was computed for a few points on the Earth's orbit. If r is the distance between the Earth and the Sun,

$$\frac{r_0}{r} = 1 + \epsilon \cos(\theta - \theta_0), \tag{31}$$

where r_0 is one astronomical unit, $\epsilon = 0.0167$ is the orbital eccentricity, and $\theta - \theta_0 \approx 2\pi(t - t_0)$, with t in years and t_0 = January 2, 4 h 52 min. Table I shows very small seasonal variation.

TABLE I. Predicted seasonal variations in ν_e fluxes, excluding the $1/r^2$ variation. The model assumed $\mu_0 = 0.32 \times 10^{-2}$ eV, $m_0 = 0.34 \times 10^{-4}$ eV, and $\delta_{ee} = 0.78 \times 10^{-7}$ eV.

$\theta - \theta_0$ in Eq. (31)	Chlorine	Gallium	Water
0 (January 2)	0.3787	0.5144	0.4635
$\pm \pi/2$	0.3762	0.5121	0.4633
π (July 4)	0.3747	0.5082	0.4631

VI. CONCLUSIONS

Finally, models with bulk sterile neutrinos lead to new contributions to low energy weak processes, which have been addressed in several papers [31]. In the domain of astrophysics, they lead to new contributions to supernova energy loss, as well as to the energy density in the early Universe, which can influence the evolution of the Universe. Currently these issues are under discussion [32,18,33], and if neutrino oscillation data favor these models, any cosmological constraints must be addressed. Note, however, that this model is less sensitive to these issues than other fits because for VO Δm^2 is an order of magnitude smaller, for MSW $\sin^2 2\theta$ is more than an order of magnitude smaller, and there is only a single KK tower starting from a very small mass [32]. Finally, we note that in a recent paper [34], it has been argued that due to the gravitational potential of the brane fields in the 5th direction, there is a new contribution to the bulk neutrinos in the neutrino mass matrix. This can effect the mixing angle of the brane to bulk neutrinos. If we take the estimate of this effect given in Ref. [34], we find that this effect for the zero mode of the bulk neutrino dominates the usual matter effect in the solar core by a factor $10^5(\text{TeV}/M_*)^3$. Clearly for $M_* \leqslant 50$ TeV, it dominates. It is then clear that the effect is completely negligible in the local $B-L$ model.

In summary, we have presented a new way to provide a simultaneous fit to solar, atmospheric and LSND data using the idea that the sterile neutrino is in a large extra dimensional bulk. We have presented two classes of models where our basic idea for the fit can be accommodated in a natural way.

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APPENDIX A: MODEL FOR THE MAJORANA MASS MATRIX FOR THE BRANE NEUTRINOS

In this appendix, we seek a possible theoretical origin of the neutrino mass pattern used in the TeV scale model. We will keep the standard model gauge group and attempt to extend the Higgs sector in such a way that one generates the neutrino masses at loop levels. The basic idea will be to consider $L_e + L_\mu - L_\tau$ symmetry for neutrinos, with $L_e = 1$ for ν_B. On general symmetry grounds, the allowed mass

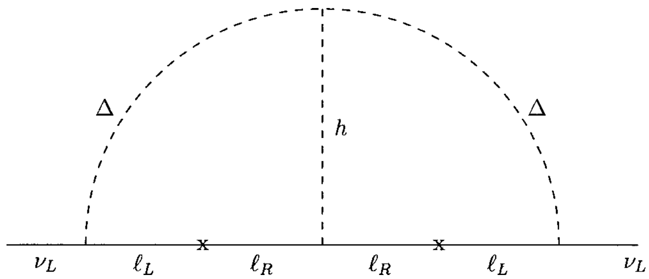

FIG. 3. Typical two loop diagram for neutrino mass in the TeV string scale model.

matrix for $(\nu_e, \nu_\mu, \nu_\tau)$ can be written as:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & \delta_{e\tau} \\ 0 & 0 & m_0 \\ \delta_{e\tau} & m_0 & 0 \end{pmatrix}. \tag{A1}$$

The remaining terms are assumed to arise after we turn on the symmetry breaking so that those elements are small.

For an explicit realization, we augment the standard model by the singlet charged Higgs field h^{++} which is blind with respect to lepton number, h'_e, which carries $L_e = -2$ and $SU(2)_L$ triplet fields, $\Delta_{e,\mu,\tau}$, with $Y=2$ which carry two negative units of lepton numbers $L_{e,\mu,\tau}$, respectively. The Lagrangian involving these fields consists of two parts: one $\mathcal{L}_0$ which is invariant under $(L_e + L_\mu - L_\tau)$ number and is given by:

$$\mathcal{L}_0 = f_{e\tau} h^{++} e_R^- \tau_R^- + f_{\mu\tau} h^{++} \mu_R^- \tau_R^- + f'_{ee} h^{++\prime} e_R^- e_R^- + g_e L_e L_e \Delta_e + g_\mu L_\mu L_\mu \Delta_\mu + g_\tau L_\tau L_\tau \Delta_\tau + \text{H.c.} \tag{A2}$$

We assume the Higgs potential to contain the $L_e + L_\mu - L_\tau$ invariant terms

$$V' = \sum_{a=e,\mu,\tau} M_0 h^{++} (\Delta_a^2). \tag{A3}$$

The second part of the Lagrangian consists of terms that break the symmetry $L_e + L_\mu - L_\tau$ and is

$$\mathcal{L}_1 = h^{++} \left(\sum_{i=e,\mu,\tau} M_{ii} \Delta_i^2 + M_{0\mu} \Delta_\mu \Delta_\tau + M_{0\tau} \Delta_e \Delta_\tau \right) + \mu^2 h^{++\dagger} h^{++\prime} + \text{H.c.} \tag{A4}$$

With these couplings, the neutrino Majorana masses arise from two-loop effects (similar to the mechanism of Ref. [35]), and we have the $L_e + L_\mu - L_\tau$ violating entries $\delta_{ij} \sim c m_{e_i} m_{e_j}$. Using $\delta_{ee} \sim 10^{-8}$ eV, then $\delta_{\tau\tau} \sim 10^{-2}$, as would be required to understand the atmospheric neutrino data. In Fig. 3, we give a typical two loop diagram that leads to neutrino masses.

APPENDIX B: NEUTRINO MIXINGS IN THE LOCAL B−L MODEL

In this section, we discuss the details of neutrino mixing in the local B-L model described in the text (Sec. III). In the text we considered only the $\nu_e - \nu_B$ sector. However, in order to explain LSND results, we have to generate the $\nu_e - \nu_\mu$ mixing and make sure that its presence does not effect the considerations in the text.

For this purpose, we consider a model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with quark $Q \equiv (u,d)$ and lepton $L \equiv (\nu_e, e)$ doublets assigned as usual in a left-right symmetric manner and Higgs fields belonging to bidoublet field $\phi(2,2,0)$ and $B-L=1$ doublets $\chi_{L,R}$. In addition, we require the theory to respect a softly broken global $L' = L_e - L_\mu + L_\tau$ symmetry. The $U(1)_{L'}$ invariant part of the Lagrangian can be written as

$$\mathcal{L} = h_{ab} \frac{(\psi_{a,R}\chi_R)(\psi_{b,R}\chi_R) + (R \to L)}{M_*} + h_{ee} \frac{[(\psi_{e,R}\chi_R)(\psi_{e,R}\chi_R) + (R\to L)]\delta}{M_*^2} + \sum_a \overline{\psi}_{a,L} \phi \psi_{a,R} + \frac{f}{M_*^{1/2}} [\overline{\psi}_{eR}\chi_R + \overline{\psi}_{eL}\chi_L] \nu_B(x, y=0) + \int dy \; \overline{\nu}_{BL}(x,y) \partial_5 \nu_{BR}(x,y) + \text{H.c.}, \tag{B1}$$

where the form of the matrix h_{ab} is determined by the $U(1)_{L'}$ symmetry. After symmetry breaking, i.e. $\langle \delta \rangle \simeq \langle \chi_R \rangle \neq 0$, Eq. (B1) leads to a right-handed Majorana neutrino mass matrix of the form:

$$M_R = \begin{pmatrix} M_{ee} & M_{e\mu} & 0 \\ M_{e\mu} & 0 & M_{\mu\tau} \\ 0 & M_{\mu\tau} & 0 \end{pmatrix}. \tag{B2}$$

The complete mass matrix for $(\nu_e, \nu_\mu, \nu_\tau, N_e, N_\mu, N_\tau, \nu_B)$ is given by

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & m_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_3 & 0 & 0 \\ m_1 & 0 & 0 & M_{ee} & M_{e\mu} & 0 & m & 0 \\ 0 & m_2 & 0 & M_{e\mu} & 0 & M_{\mu\tau} & 0 & 0 \\ 0 & 0 & m_3 & 0 & M_{\mu\tau} & 0 & 0 & 0 \\ 0 & 0 & 0 & m & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . \end{pmatrix}. \tag{B3}$$

Let us now consider a hierarchical form for the Dirac masses i.e. $m_1 \ll m_2 \ll m_3$. Looking at the "non-bulk" part of this mass matrix, one can see that if we choose $M_{e\mu} \simeq M_{ee} \simeq m_2 M_{\mu\tau}/10 m_3$ and $m_2 \simeq m_3/10$–100 MeV, then one gets the right value for the Δm^2_{ATMOS} and the $\nu_e-\nu_\mu$ mixing angle needed for understanding the LSND results. Furthermore, one can decouple all the heavy neutrinos as well as the $\nu_{\mu,\tau}$ from the spectrum and obtain the same equation as in Eq. (14) and all the considerations in the $B-L$ model described in the text go through.

APPENDIX C: NATURALNESS OF ULTRALIGHT STERILE NEUTRINOS IN BRANE-BULK MODELS

The bulk neutrino ''self mass'' terms are constrained by the geometry of the bulk and could therefore under certain circumstances be zero. If that happens, the only mass of the KK states of the ν_B will arise from the kinetic energy terms such as $\overline{\nu_B}\Gamma^I\partial_I\nu_B$, where $I=5,6\ldots$ and will be given by n/R for R the radius of the extra dimensions. In such a situation, an ultralight $\nu_{B,KK}$ arises naturally.

The key to naturalness of the ultralight bulk neutrino is the geometry that forbids both Dirac and Majorana mass terms. Let us give a few examples. In five dimensions, if we impose the Z_2 orbifold symmetry $(y\rightarrow -y)$, then it follows that the Dirac mass vanishes. Now if we impose lepton number symmetry in the brane, the Majorana mass vanishes, leaving us with no mass term for the bulk neutrino in 5-dimensions.

Another interesting example is the 10-dimensional bulk, where the bulk neutrino is a 16-component spinor which, when reduced to four dimensions, leads to eight 2-component spinors. The interesting point is that for a 16-dimensional spinor, one cannot write a Dirac or Majorana mass term consistent with 10-dimensional Lorentz invariance. In this case, there is no need for assuming lepton number symmetry to get an ultralight sterile neutrino. A similar situation is also expected in six dimensions if we choose the bulk neutrino to be a 4-component complex chiral spinor.

[1] Super-Kamiokande Collaboration, Y. Suzuki *et al.*, Nucl. Phys. B (Proc. Suppl.) **77**, 35 (1999); B. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998); SAGE Collaboration, J. N. Abdurashitov *et al.*, Phys. Rev. C **60**, 055801 (1999); GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. B **447**, 127 (1999); GNO Collaboration, M. Altman *et al.*, *ibid.* **490**, 16 (2000).

[2] Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).

[3] C. Athanassopoulos *et al.*, Phys. Rev. C **54**, 2685 (1996); *ibid.* **58**, 2489 (1998).

[4] D. O. Caldwell and R. N. Mohapatra, Phys. Rev. D **48**, 3259 (1993); J. Peltoniemi and J. W. F. Valle, Nucl. Phys. **B406**, 409 (1993).

[5] D. O. Caldwell, G. M. Fuller, and Y-Z. Qian, Phys. Rev. D **61**, 123005 (2000).

[6] For review and detailed analysis of the four neutrino models, see S. M. Bilenky, C. Giunti, and W. Grimus, Prog. Part. Nucl. Phys. **43**, 1 (1999).

[7] S. M. Bilenky, C. Giunti, W. Grmus, and T. Scwetz, Phys. Rev. D **60**, 073007 (1999); V. Barger, B. Kayser, J. Learned, T. Weiler, and K. Whisnant, Phys. Lett. B **489**, 345 (2000); O. Peres and A. Y. Smirnov, Nucl. Phys. **B599**, 3 (2001).

[8] C. Gonzalez-Garcia and C. Peña-Garay, Nucl. Phys. B (Proc. Suppl.) **91**, 80 (2000).

[9] I. Antoniadis, Phys. Lett. B **246**, 377 (1990); I. Antoniadis, K. Benakli, and M. Quirós, *ibid.* **331**, 313 (1994); J. Lykken, Phys. Rev. D **54**, 3693 (1996); K. R. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. **B436**, 55 (1998); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **B429**, 263 (1998); Phys. Rev. D **59**, 086004 (1999); I. Antoniadis, S. Dimopoulos, and G. Dvali, Nucl. Phys. **B516**, 70 (1998); N. Arkani-Hamed, S. Dimopoulos, and J. March-Russell, Phys. Rev. D **63**, 064020 (2001).

[10] D. O. Caldwell, R. N. Mohapatra, and S. Yellin, Phys. Rev. Lett. **87**, 041601 (2001).

[11] P. Horava and E. Witten, Nucl. Phys. **B460**, 506 (1996); **B475**, 94 (1996).

[12] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in ''Proceedings of Workshop on Unified Theory and Baryon Number in the Universe,'' edited by O. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba, 1979; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[13] K.R. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. **B557**, 25 (1999); N. Arkani-Hamed, S. Dimopoulos, G. Dvali, and J. March-Russell, hep-ph/9811448.

[14] R. N. Mohapatra, S. Nandi, and A. Perez-Lorenzana, Phys. Lett. B **466**, 115 (1999); R. N. Mohapatra and A. Perez-Lorenzana, Nucl. Phys. **B576**, 466 (2000).

[15] R. N. Mohapatra and A. Perez-Lorenzana, Nucl. Phys. **B593**, 451 (2001).

[16] K. S. Babu, C. N. Leung, and J. Pantaleone, Phys. Lett. B **319**, 191 (1993); P. H. Chankowski and Z. Pluciennik, *ibid.* **316**, 312 (1993).

[17] G. Dvali and A. Yu. Smirnov, Nucl. Phys. **B563**, 63 (1999).

[18] R. Barbieri, P. Creminelli, and A. Strumia, Nucl. Phys. **B585**, 28 (2000).

[19] A. Lukas, P. Ramond, A. Romanino, and G. Ross, Phys. Lett. B **495**, 136 (2000); J. High Energy Phys. **04**, 010 (2001).

[20] K. Dienes and I. Sarcevic, Phys. Lett. B **500**, 133 (2001).

[21] A. Ionissian, and J. W. F. Valle, Phys. Rev. D **63**, 073002 (2001).

[22] Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000).

[23] W. Haxton, Phys. Rev. D **35**, 2352 (1987).

[24] J. Bahcall, S. Basu, and M. Pinsonneault, Phys. Lett. B **433**, 1 (1998), with detailed tables given at Bahcall's web site, http://www.sns.ias.edu/~jnb

[25] M. Nakahata *et al.*, Nucl. Instrum. Methods Phys. Res. A **421**, 113 (1999).

[26] G. L. Fogli, E. Lisi, D. Montanino, and A. Palazzo, Phys. Rev. D **62**, 013002 (2000).

[27] B. T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998).

[28] *Proceedings of the 19th International Conference on Neutrino Physics and Astrophysics at Sudbury, 2000* [Nucl. Phys. B (Proc. Suppl.) **91** (2000)].

[29] M. C. Gonzalez-Garcia, P. C. de Holanda, C. Penña-Garay, and

J. W. F. Valle, Nucl. Phys. **B573**, 3 (2000).
[30] Y. Totsuka (private communication).
[31] A. Faraggi and M. Pospelov, Phys. Lett. B **458**, 237 (1999); G. C. McLaughlin and J. N. Ng, *ibid.* **470**, 157 (1999); Phys. Rev. D **63**, 053002 (2001); A. Ioannisian and A. Pilaftsis, *ibid.* **62**, 066001 (2000).
[32] K. Abazajian, George Fuller, and M. Patel, hep-ph/0011048.
[33] K. Aghase and G. H. Wu, Phys. Lett. B **498**, 230 (2001).
[34] M. Fabrichese, M. Piai and G. Tasinato, hep-ph/0012227.
[35] K. S. Babu, Phys. Lett. B **203**, 132 (1988).