Gravitational rainbow

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It is shown that, unlike Einstein's gravity, quadratic gravity produces dispersive photon propagation. The energy-dependent contribution to the deflection of photons passing by the Sun is computed and subsequently the angle at which the visible spectrum would be spread over is plotted as a function of the $R_{\mu\nu}^2$-sector mass.

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I. INTRODUCTION

According to the equivalence principle, photons follow lightlike geodesics in curved spacetime. In addition, they are deflected in a gravitational potential by the same angle independently of their energy or polarization. Both Einstein's gravity [1] and $R+R^2$ gravity [2,3] are examples of gravitational theories that obey this principle, which implies that dispersive photon propagation cannot take place within the context of the same. This is not true, however, as far as gravity with higher derivatives is concerned. Our aim here is precisely to show that quadratic gravity produces energy-dependent photon scattering. An interesting consequence of this fact is that gravity's rainbows and higher-derivative gravity can coexist without conflict. In this sense quadratic gravity is closer to quantum electrodynamics than any currently known gravitational theory. In fact, dispersive photon propagation is a trivial phenomenon in the context of QED. It is worth mentioning that Lafrance and Meyers [4] have shown that energy-dependent light scattering can also be produced within the context of a low-energy effective action for the electromagnetic field in curved spacetime.

On the other hand, we ought to expect a tiny value for the angle at which the visible spectrum would be spread over. Indeed, the prediction of general relativity for the deflection of a light ray passing close to the Sun, namely, 1.75″, is in good agreement with measured values for visible light. We shall also address this question here.

We use natural units ($\hbar=c=1$) throughout. Our conventions are $R^{\alpha}_{\beta\gamma\delta}=-\partial_{\delta}\Gamma^{\alpha}_{\beta\gamma}+\dots$, $R_{\mu\nu}=R^{\alpha}_{\mu\nu\alpha}$, $R=g^{\mu\nu}R_{\mu\nu}$, where $g_{\mu\nu}$ is the metric tensor, and signature $(+---)$.

II. SOLUTION TO THE LINEARIZED FIELD EQUATIONS

The theory of gravity with higher derivatives is defined by the action

$$S=\int d^4x\sqrt{-g}\left\{\frac{2R}{\kappa^2}+\frac{\alpha}{2}R^2+\frac{\beta}{2}R^2_{\mu\nu}-\mathcal{L}_M\right\},\qquad(1)$$

where $\kappa^2=32\pi G$, with G being Newton's constant, is the Einstein's constant, α and β are dimensionless parameters, and $\mathcal{L}_M$ is the Lagrangian density for the usual matter. This theory gained importance when Stelle showed that it is renormalizable, along with its matter couplings [5]. In the weak field approximation, i.e., $g_{\mu\nu}=\eta_{\mu\nu}+\kappa h_{\mu\nu}$, with $\eta_{\mu\nu}=\mathrm{diag}(1-1-1-1)$ and in the Teyssandier gauge [6], namely,

$$0=\Gamma_{\mu}\equiv\left(1-\frac{\beta\kappa^2}{4}\Box\right)\gamma_{\mu\lambda}{}^{,\lambda}-\left(\alpha+\frac{\beta}{2}\right)\frac{\kappa^2}{2}\overline{R}_{,\mu},\qquad(2)$$

with $\gamma_{\mu\nu}\equiv h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h$ and $\overline{R}=\frac{1}{2}\Box h-\gamma^{\mu\nu}{}_{,\mu\nu}$, the field equations related to the action above turn out to be

$$\left(1-\frac{\beta\kappa^2}{4}\Box\right)\left(-\frac{1}{2}\Box h_{\mu\nu}+\frac{1}{6}\eta_{\mu\nu}\overline{R}\right)=\frac{\kappa}{4}\left[T_{\mu\nu}-\frac{1}{3}T\eta_{\mu\nu}\right],\qquad(3)$$

where $T_{\mu\nu}$ is the matter tensor which describes the physical system under consideration in special relativity, i.e., disregarding the gravitational field. The general solution of Eq. (3) for a point particle of mass M located at $\mathbf{r}=\mathbf{0}$ is given by [7,8,6,9]

$$h_{00}(r)=\frac{Mk}{16\pi}\left[-\frac{1}{r}-\frac{1}{3}\frac{e^{-m_0r}}{r}+\frac{4}{3}\frac{e^{-m_1r}}{r}\right],$$

$$h_{11}(r)=h_{22}(r)=h_{33}(r)=\frac{Mk}{16\pi}\left[-\frac{1}{r}+\frac{1}{3}\frac{e^{-m_0r}}{r}+\frac{2}{3}\frac{e^{-m_1r}}{r}\right],$$

where we have assumed $2/[k^2(3\alpha+\beta)]\equiv m_0^2>0(3\alpha+\beta)>0$ and $-4/k^2\beta\equiv m_1^2>0(-\beta>0)$, which corresponds to the absence of tachyons (both positive and negative energy) in the dynamical field. Of course, the potential for quadratic gravity is given by the expression

$$\kappa h_{00}(r)/2\equiv V=MG\left[-\frac{1}{r}-\frac{1}{3}\frac{e^{-m_0r}}{r}+\frac{4}{3}\frac{e^{-m_1r}}{r}\right],\qquad(4)$$

which agrees asymptotically with Newton's law. At the origin, Eq. (4) tends to the finite value $MG[(m_0-4m_1)/3]$.

III. ENERGY-DEPENDENT PHOTON PROPAGATION

Let us now consider the scattering of a photon by a static gravitational field generated by a localized source such as the

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Sun, treated as an external field. The photon-static-external-gravitational-field vertex is given by

$$\mathcal{M}_{\mu\nu}(p,p')=\frac{1}{2}\kappa h^{\lambda\rho}(\mathbf{k})[-\eta_{\mu\nu}\eta_{\lambda\rho}pp'+\eta_{\lambda\rho}p_\nu p'_\mu+2(\eta_{\mu\nu}p_\lambda p'_\rho-\eta_{\nu\rho}p_\lambda p'_\mu-\eta\mu\lambda p_\nu p'_\rho+\eta_{\mu\lambda}pp')],$$

where $h_{\mu\nu}(\mathbf{k})\equiv\int d^3r\, e^{-i\mathbf{k}\cdot\mathbf{r}}h_{\mu\nu}(\mathbf{r})$ is the momentum space gravitational field, and p (p') is the momentum of the incoming (outgoing) photon. Here $|\mathbf{p}|=|\mathbf{p}'|$. The momentum space gravitational field can be written as

$$h_{\mu\nu}(\mathbf{k})=h^{(E)}_{\mu\nu}(\mathbf{k})+h^{(R^2)}_{\mu\nu}(\mathbf{k})+h^{(R^2_{\mu\nu})}_{\mu\nu}(\mathbf{k}),$$

whereupon

$$h^{(E)}_{\mu\nu}(\mathbf{k})=\frac{\kappa M}{4\mathbf{k}^2}\eta_{\mu\nu}-\frac{\kappa M}{2}\frac{\eta_{\mu 0}\eta_{\nu 0}}{\mathbf{k}^2},$$

$$h^{(R^2)}_{\mu\nu}(\mathbf{k})=-\frac{\kappa M}{12}\frac{\eta_{\mu\nu}}{m_0^2+\mathbf{k}^2}$$

and

$$h^{(R^2_{\mu\nu})}_{\mu\nu}(\mathbf{k})=-\frac{\kappa M}{6}\frac{\eta_{\mu\nu}}{m_1^2+\mathbf{k}^2}+\frac{\kappa M}{2}\frac{\eta_{\mu 0}\eta_{\nu 0}}{m_1^2+\mathbf{k}^2},$$

where $h^{(E)}_{\mu\nu}(\mathbf{r})$ is the solution of the linearized Einstein's equations supplemented by the usual harmonic coordinate condition, namely, $\gamma^{(E)}_{\mu\nu}{}^{,\nu}=0$, $\gamma^{(E)}_{\mu\nu}\equiv h^{(E)}_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h^{(E)}$.

The unpolarized cross-section for the process at hand is

$$\frac{d\sigma}{d\Omega}=\frac{1}{32\pi^2}\sum_{r=1}^{2}\sum_{r'=1}^{2}\mathcal{M}^2_{rr'},$$

where $\mathcal{M}_{rr'}=\epsilon^\mu_r(\mathbf{p})\epsilon^\nu_{r'}(\mathbf{p}')\mathcal{M}_{\mu\nu}$, and $\epsilon^\mu_r(\mathbf{p})$ and $\epsilon^\nu_{r'}(\mathbf{p}')$ are the polarization vectors for the initial and final photons, respectively. Before moving on we call attention to the fact that the Feynman amplitude can be recast in the form $\mathcal{M}_{\mu\nu}=\mathcal{M}^{(E)}_{\mu\nu}+\mathcal{M}^{(R^2)}_{\mu\nu}+\mathcal{M}^{R^2_{\mu\nu}}_{\mu\nu}$, where

$$\mathcal{M}^{(R^2)}_{\mu\nu}=\frac{\kappa}{2}\left[-\frac{\kappa M}{12}\frac{\eta^{\lambda\rho}}{m_0^2+\mathbf{k}^2}\right][-\eta_{\mu\nu}\eta_{\lambda\rho}pp'+\eta_{\lambda\rho}p_\nu p'_\mu+2(\eta_{\mu\nu}p_\lambda p'_\rho-\eta_{\nu\rho}p_\lambda p'_\mu-\eta_{\mu\lambda}p_\nu p'_\rho+\eta_{\mu\lambda}\eta_{\nu\rho}pp')].$$

Of course, $\mathcal{M}^{(R^2)}_{\mu\nu}\equiv 0$, which implies that the R^2 sector of the theory of gravitation with higher derivatives does not contribute anything to the photon scattering. After this little digression we come back to the computation of the cross-section for the scattering of a photon by a localized source on the basis of higher-derivative gravity. Performing the calculation yields

$$\frac{d\sigma}{d\Omega}=\frac{\kappa^4E^4M^2}{256\pi^2}(1+\cos\phi)^2\left[-\frac{1}{\mathbf{k}^2}+\frac{1}{m_1^2+\mathbf{k}^2}\right]^2,$$

where E is the energy of the incident photon and ϕ is the scattering angle. For small angles this expression reduces to

$$\frac{d\sigma}{d\Omega}=16G^2M^2\left[-\frac{1}{\phi^2}+\frac{E^2}{m_1^2+E^2\phi^2}\right]^2. \tag{5}$$

On the other hand, for small angles

$$\frac{d\sigma}{d\Omega}=\left|\frac{r\,dr}{\phi\, d\phi}\right|. \tag{6}$$

From Eqs. (5) and (6), we obtain at once

$$r^2=16G^2M^2\left[\frac{1}{\phi^2}+\frac{E^2}{m_1^2+E^2\phi^2}+\frac{2E^2}{m_1^2}\ln\frac{\phi^2E^2}{m_1^2+E^2\phi^2}\right]. \tag{7}$$

Thus, in the framework of higher-derivative gravity the deflection of a photon by a localized source is a function of the energy of the incoming photon. For a photon passing by the Sun, Eq. (7) can be cast in the form

$$\left(\frac{\phi}{\phi_E}\right)^2-1=\frac{1}{1+a^2}+\frac{2}{a^2}\ln\frac{1}{1+a^2}, \tag{8}$$

where $\phi_E\equiv 4GM/R$, with R being the Sun's radius, and $a^2\equiv m_1^2/E^2\phi^2$. Note that the right hand side of Eq. (8) tends to zero as $m_1\to+\infty$ and, as a result, $\phi\to\phi_E$ (as expected). It follows from Eq. (8) that for a photon just grazing the Sun's surface ϕ ranges from 0^+ to 1.75^- arcsec [10].

Of course, for light rays passing close to the Sun the deflection ϕ must lie in the interval $0<\phi<1.75''$. Let us show that this is indeed the case. Consider, in this vein, the interaction between the Sun, treated as a fixed source, and a light ray. The associated energy-momentum tensors will be designated respectively as $T^{\mu\nu}$ and $F^{\mu\nu}$. The current-current amplitude for this process is given by

$$A=g^2T^{\mu\nu}O_{\mu\nu,\rho\sigma}F^{\rho\sigma},$$

where g is the effective coupling constant of the theory and $O_{\mu\nu,\rho\sigma}$ is the propagator for quadratic gravity. In the de Donder gauge, the propagator is given by [11]

$$O=\frac{1}{k^2}P^1+\frac{m_1^2}{k^2(m_1^2-k^2)}P^2+\frac{m_0^2}{2k^2(k^2-m_0^2)}P^0+\left[\frac{2}{\lambda k^2}+\frac{3m_0^2}{2k^2(k^2-m_0^2)}\right]\overline{P^0}+\frac{m_0^2}{2k^2(k^2-m_0^2)}\overline{\overline{P^0}},$$

where P^1, P^2, P^0, $\overline{P^0}$ and $\overline{\overline{P^0}}$ are the Barnes-Rivers operators [12], and λ is a gauge-parameter. But, on mass shell, $k_\mu T^{\mu\nu}=0$ and $k_\mu F^{\mu\nu}=0$, which implies that only P^2 and P^0 will give a non-null contribution to the current-current amplitude. Thus,

$$A=g^2 T^{\mu\nu}F^{\rho\sigma}\left[\frac{m_1^2}{k^2(m_1^2-k^2)}P^2+\frac{m_0^2}{2k^2(k^2-m_0^2)}P^0\right]_{\mu\nu,\rho\sigma}.$$

Now, taking into account that [11]

$$P^2_{\mu\nu,\rho\sigma}=\frac{1}{2}[\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}]-\frac{1}{3}\eta_{\mu\nu}\eta_{\rho\sigma}-\left[P^1+\frac{2}{3}\overline{P^0}-\frac{1}{3}\overline{\overline{P^0}}\right]_{\mu\nu,\rho\sigma},$$

$$P^0_{\mu\nu,\rho\sigma}=\frac{1}{3}\eta_{\mu\nu}\eta_{\rho\sigma}-\frac{1}{3}[\overline{P^0}+\overline{\overline{P^0}}]_{\mu\nu,\rho\sigma},$$

and recalling that the energy-momentum tensor for light (electromagnetic radiation) is traceless, while $T^{\mu\nu}=\eta^{\mu 0}\eta^{\nu 0}T^{00}$ for a static source, we promptly obtain

$$A=g^2 T^{00}F^{00}\left[\frac{1}{k^2}-\frac{1}{k^2-m_1^2}\right].$$

Since $g^2T^{00}F^{00}/k^2$ is precisely the current-current amplitude for the interaction between the Sun and a light ray in the context of general relativity, we come to the conclusion that the gravitational deflection predicted by quadratic gravity is always smaller than that predicted by Einstein's theory. As a result, for light grazing the Sun, $0<\phi<1.75''$. In a sense, this result is a major test of our semiclassical computation.

IV. GRAVITATIONAL RAINBOW

The dispersive-photon propagation we have just found represents a tree-level violation of the equivalence principle. In addition, it tells us that the visible spectrum, whose wavelength ranges from 4000 to 7000 (Å), would always be spread over an angle, say $\Delta\phi$, where $\Delta\phi\equiv(\phi_{\rm red}-\phi_{\rm violet})$, giving rise to a gravitational rainbow. Such a gravity's rainbow is an intrinsic characteristic of quadratic gravity since the vacuum concerning higher-derivative gravity, like that related to QED, is a dispersive medium. This poses an interesting question: Is the aforementioned gravitational rainbow observable? Of course, we ought to expect a tiny value for $\Delta\phi$ at the Sun's limb in order not to conflict with well established results of general relativity. Indeed, Einstein's theory tells us that the deflection angle for a light ray passing close to the Sun is totally independent of the energy of the incident radiation and has a value equal to $1.75''$, a result that is in good agreement with the measured values for visible light for which there is perhaps $10-20\,\%$ uncertainty. Let us then try to answer the question raised above. To do that we evaluate $\Delta\phi$ for different values of β, using Eq. (8) and taking into account that experiment determines $|\beta|\leqslant 10^{74}$ [13]. The result is shown in Fig. 1. A cursory inspection of this graph allows us to conclude that $\Delta\phi$ has a maximum value at $|\beta|=10^{64}$. For $61\leqslant\log_{10}|\beta|\leqslant 69$ the rainbow would be in principle observable. If $|\beta|\leqslant 10^{60}$ gravity's rainbow would be practically imperceptible. Therefore, we come to the conclusion that in order to agree with the currently measured values for visible light, $|\beta|\leqslant 10^{60}$.

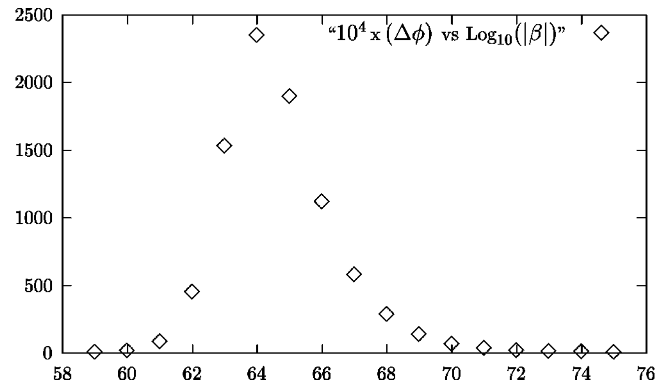

FIG. 1. $\Delta\phi$ as a function of $\log_{10}|\beta|$ for light rays passing by the Sun. Note that $\Delta\phi$ has a maximum at $\log_{10}\beta=64$ which is equal to $0.234''$.

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[1] D. Boccaletti, V. de Sabbata, C. Gualdi, and P. Fortini, Nuovo Cimento A **48**, 58 (1967).
[2] A. Accioly, A.D. Azeredo, E.C. de Rey Neto, and H. Mukai, Braz. J. Phys. **28**, 496 (1998).
[3] A. Accioly, S. Ragusa, E.C. de Rey Neto, and H. Mukai, Nuovo Cimento B **114**, 595 (1999).
[4] R. Lafrance and R.C. Meyers, Phys. Rev. D **51**, 2584 (1995).
[5] K.S. Stelle, Phys. Rev. D **16**, 953 (1977).
[6] P. Teyssandier, Class. Quantum Grav. **6**, 219 (1989).
[7] K.S. Stelle, Gen. Relativ. Gravit. **9**, 353 (1978).
[8] H.-J. Schmidt, Astron. Nachr. **306**, 231 (1985).
[9] A. Accioly, A. Azeredo, H. Mukai, and E.C. de Rey Neto, Prog. Theor. Phys. **104**, 103 (2000).
[10] A. Accioly, H. Mukai, and H. Blas, Nuovo Cimento B **115**, 1235 (2000).
[11] A. Accioly, S. Ragusa, H. Mukai, and E. de Rey Neto, Int. J. Theor. Phys. **39**, 1599 (2000).
[12] I. Antoniadis and E.T. Tomboulis, Phys. Rev. D **33**, 2756 (1986).
[13] J.F. Donoghue, Phys. Rev. Lett. **72**, 2996 (1994).