Nonsymmetric unified field theory. III. Solution for a point charge

S. Ragusa*
Departamento de Física e Informática, Instituto de Física de São Carlos, Universidade de São Paulo - Caixa Postal 369, 13560-970 - São Carlos - São Paulo, Brazil

(Received 7 December 2000; published 28 August 2001)

The field equations of a proposed nonsymmetric unified theory of gravitation and electromagnetism are solved for a static and spherically symmetric point charge. The generalized electric field is nonlinearly related to the Reissner-Nordström, Coulomb, field. It turns out to be finite everywhere, going to zero at the origin and to a Coulomb value at large distances. The modification of Coulomb's law is analyzed.

DOI: 10.1103/PhysRevD.64.067501 PACS number(s): 04.50.+h

Recently [1,2] we developed a metric nonsymmetric unified theory of gravitation and electromagnetism. The theory provides a version of Einstein's unified theory [3] by modifying the Bonnor [4] and Moffat-Boal [5] unified theories. The formulation is based on a modification of the Einstein part [3] of the Bonnor Lagrangian [4]. It is done in such a way that the antisymmetric part $g_{[\mu\nu]}$ of the metric tensor describes a propagating spin-1 field obeying Maxwell's vacuum equations in the flat space linear approximation, supporting its identification with a generalized electromagnetic field strength $F_{\mu\nu}$ to within a constant, as proposed in Ref. [4]; $g_{[\mu\nu]}=pF_{\mu\nu,}$ where p is a universal constant. The Einstein-Maxwell theory appears to lowest order about a general relativity curved space. By having the Maxwellian behavior satisfied, the theory is shown [1] to be free of negative energy radiative modes even when expanded about a Riemannian background space. It is also free [2] of ghost-negative energy particles, in the sense of elementary particle theory, and of propagating tachyons.

In this paper we solve the field equations for a static and spherically symmetric point charge, and analyze the equation of motion of a charged test particle in this field. The vacuum field equations are [1]

$$U_{\mu\nu}-K_{(\mu\nu)}=0, \tag{1}$$

$$K_{[\mu\nu],\alpha}+\text{c.p.}=0, \tag{2}$$

where c.p. stands for the cyclic permutation of the indices μ,ν, and α, and

$$(\sqrt{-g}g^{[\mu\nu]})_{,\nu}=0. \tag{3}$$

Here

$$U_{\mu\nu}=\Gamma^{\sigma}_{(\mu\nu),\sigma}-\Gamma^{\sigma}_{(\mu\sigma),\nu}+\Gamma^{\sigma}_{(\mu\nu)}\Gamma^{\rho}_{(\sigma\rho)}-\Gamma^{\sigma}_{(\mu\rho)}\Gamma^{\rho}_{(\sigma\nu)} \tag{4}$$

contains only the symmetric part of the connection, and

$$K_{\mu\nu}=\frac{1}{p^2}\left(g_{[\mu\nu]}+g_{\mu\alpha}g^{[\alpha\beta]}g_{\beta\nu}+\frac{1}{2}g_{\mu\nu}g^{[\alpha\beta]}g_{[\alpha\beta]}\right) \tag{5}$$

is the term [4] responsible for the electromagnetic effects. $g^{\alpha\beta}$ is the inverse of $g_{\alpha\beta}$ defined by $g^{\alpha\beta}g_{\alpha\sigma}=\delta^{\beta}_{\sigma}$. We also have the relation

$$\Gamma^{\sigma}_{(\mu\nu)}=\frac{1}{2}g^{(\sigma\alpha)}(s_{\mu\alpha,\nu}+s_{\nu\alpha,\mu}-s_{\mu\nu,\alpha})+\frac{1}{4}(g^{(\lambda\sigma)}s_{\mu\nu}-\delta^{\lambda}_{\mu}\delta^{\sigma}_{\nu}-\delta^{\lambda}_{\nu}\delta^{\sigma}_{\mu})\left(\ln\frac{s}{g}\right)_{,\lambda}, \tag{6}$$

where $s_{\mu\nu}$, symmetric and with determinant s, is the inverse of $g^{(\mu\nu)}$, defined by $g^{(\mu\nu)}s_{\mu\beta}=\delta^{\nu}_{\beta}$.

The static and spherically symmetric metric tensor in polar coordinates is of the form

$$\begin{aligned} &g_{00}=\gamma(r),\quad g_{11}=-\alpha(r),\\ &g_{22}=-r^2,\quad g_{33}=-r^2\sin^2\theta,\\ &g_{01}=\omega(r)=-g_{10}, \end{aligned} \tag{7}$$

and all other components are equal to zero. In this coordinate system, for the electric field we have the identification $E=F_{01}=p^{-1}\omega$. The nonzero elements of the inverse matrix are $g^{00}=A\alpha$, $g^{11}=-A\gamma$, $g^{22}=-r^{-2}=g^{33}\sin^2\theta$, and $g^{01}=-A\omega=-g^{10}$, with $A=(\alpha\gamma-\omega^2)^{-1}$. The nonzero components of Eq. (5) are $K_{00}=p^{-2}\gamma\omega^2A$, $K_{11}=-(\alpha/\gamma)K_{00}$, $K_{22}=K_{33}\sin^2\theta=-p^{-2}r^2\omega^2A$, and $K_{[01]}=p^{-2}\omega(1+A\alpha\gamma)=-K_{[10]}$. From these last relations, we see that Eq. (2) is identically satisfied. From the previous relations Eq. (3) can be integrated to give $\omega r^2(\alpha\gamma-\omega^2)^{-1/2}=pQ$, where the constant of integration has been sent equal to p times the charge of the source particle, for a reason to be discussed below. Then

$$\omega^2=\alpha\gamma\frac{p^2Q^2}{p^2Q^2+r^4}. \tag{8}$$

We have calculated the U part of Eq. (1) before [6], in the context of a nonsymmetric theory of gravitation (pure gravitation with no identification of $g_{[\mu\nu]}$ to the electromagnetic field tensor), in terms of a parameter F that has to be replaced here by pQ. From the combination $(\alpha/\gamma)U_{00}+U_{11}=0$, which results from Eq. (1) together with the relation

*Email address: ragusa@if.sc.usp.br

$(\alpha/\gamma)K_{00}+K_{11}=0$ quoted just above Eq. (8), we obtain $[\ln(\alpha\gamma)]'+2p^2Q^2(r^5+rp^2Q^2)^{-1}=0$. This can be integrated to give

$$\alpha\gamma=\frac{1}{\left(1+\frac{p^2Q^2}{r^4}\right)^{1/2}}, \tag{9}$$

where the constant of integration has been fixed in such a way that the relation goes into the Reissner-Nordström (RN) result, $\alpha\gamma$ equal to 1, when p vanishes. Substituting Eq. (9) into Eq. (8), one obtains an expression for ω^2, which gives, with a proper sign,

$$\omega=\frac{pQr}{(r^4+p^2Q^2)^{3/4}}. \tag{10}$$

This means that the electric field $E=F_{01}=p^{-1}\omega$ is

$$E=\frac{Qr}{(r^4+p^2Q^2)^{3/4}}, \tag{11}$$

with the sign of ω in Eq. (10) chosen in such a way that the Coulomb field of the RN solution results when p vanishes. This is also the reason for the choice of the constant of integration made in obtaining Eq. (8).

With the relation written just above Eq. (9), Eq. (1) for $\mu=\nu=2$ can readily be integrated for α. Choosing the constant of integration to yield the RN solution when p vanishes, one obtains

$$\frac{1}{\alpha}=1+\frac{p^2Q^2}{r^4}-\left(\frac{2M}{r}+\frac{Q^2}{r}H(r)\right)\left(1+\frac{p^2Q^2}{r^4}\right)^{3/4}, \tag{12}$$

where M is the mass of the charged particle and

$$H(r)=\int\frac{rdr}{(r^4+p^2Q^2)^{3/4}}, \tag{13}$$

which goes to $-r^{-1}$ when p vanishes. From Eqs. (9) and (12) we finally obtain

$$\gamma=\left(1+\frac{p^2Q^2}{r^4}\right)^{1/2}-\left(\frac{2M}{r}+\frac{Q^2}{r}H(r)\right)\left(1+\frac{p^2Q^2}{r^4}\right)^{1/4}. \tag{14}$$

From Eq. (11) we see that the generalized electric field is finite in all space, vanishing at the origin and going to the RN (Coulomb) value at large distances. As anticipated in Ref. [2] for the general Einstein-Maxwell field, it is here related nonlinearly to the RN (Coulomb) field $E_c=Q/r^2$. In fact, $E=E_c(1+p^2E_c^2)^{-3/4}$.

Let us now analyze the modification of Coulomb's law in the theory. The equation of motion of a test charge e and mass m was derived in Ref. [2], Eq. (5.5),

$$\frac{du^\alpha}{d\tau}+C^\alpha_{\beta\gamma}u^\beta u^\gamma=\frac{ep}{2m}a^{\alpha\beta}K_{[\beta\gamma]}u^\gamma, \tag{15}$$

where $u^\alpha=dX^\alpha/d\tau$ is the velocity of the particle, $a^{\alpha\beta}$ is the inverse of $g_{(\alpha\sigma)}$ as defined by $a^{\alpha\beta}g_{(\sigma\beta)}=\delta^\alpha_\beta$, and $C^\alpha_{\beta\gamma}=\frac{1}{2}a^{\alpha\sigma}(g_{(\beta\sigma),\gamma}+g_{(\gamma\sigma),\beta}-g_{(\beta\gamma),\sigma})$ is the Christoffel symbol formed with the symmetric part of the metric, with $g_{(\alpha\beta)}$ referring to the background non-Riemannian field where the test particle moves. Neglecting velocities and considering the spherically symmetric field of our point charge Q, the equation for $x^1=r$ reduces to

$$\frac{d^2r}{dt^2}+\frac{\gamma'}{2\alpha}=\frac{eQ}{mr^2}\frac{\sqrt{\gamma}}{\alpha}\left(1+\frac{p^2Q^2}{2r^4}\right)\left(1+\frac{p^2Q^2}{r^4}\right)^{-3/4}. \tag{16}$$

Here use has been made of the relation $u^0=\gamma^{-1/2}$, for null velocities, and of the value of $K_{[10]}$, with the help of Eqs. (8) and (9). The second term on the left-hand side of Eq. (16) contains the Newtonian force and corrections to it. To obtain the situation of Coulomb's law we shall neglect this term, and for the effective electrical force on the right-hand side we shall neglect the curvature contributions due to M and Q, keeping only the pQ terms in Eqs. (12) and (14); that is, we shall set

$$\frac{1}{\alpha}=1+\frac{p^2Q^2}{r^4}=\gamma^2. \tag{17}$$

Then one can see that the modified Coulomb equation of motion is

$$\frac{d^2r}{dt^2}=\frac{eQ}{mr^2}\left(1+\frac{p^2Q^2}{r^4}\right)^{1/2}\left(1+\frac{p^2Q^2}{2r^4}\right). \tag{18}$$

This could probably be used to determine p to some degree.

[1] S. Ragusa, Phys. Rev. D **62**, 024026 (2000); **64**, 029901(E) (2001).
[2] S. Ragusa, Phys. Rev. D **63**, 084019 (2001).
[3] A. Einstein, *The Meaning of Relativity*, 3rd ed. (Princeton University Press, Princeton, NJ, 1950); 5th ed. (Princeton University Press, Princeton, NJ, 1955), Appendix II; Ann. Math. **46**, 578 (1945); A. Einstein and E.G. Straus, *ibid.* **47**, 731 (1946).
[4] W.B. Bonnor, Proc. R. Soc. London **A226**, 366 (1954).
[5] J.W. Moffat and D.H. Boal, Phys. Rev. D **11**, 1375 (1975).
[6] S. Ragusa, Gen. Relativ. Gravit. **30**, 275 (1999).