Relics of cosmological quark-hadron phase transition

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We propose that the density fluctuations amplified by the vanishing sound velocity effect during the cosmological quark-hadron phase transition lead to quark-gluon plasma lumps decoupled from the expansion of the universe, which rapidly transform to quark nuggets (QNs) before they disperse out. Assuming a power-law spectrum of density fluctuations, we investigate the parameter ranges for the QNs to play the role of baryonic dark matter and give inhomogeneities that could affect big-bang nucleosynthesis within the observational bounds of CMBR anisotropy.

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As the temperature of the universe cools down to the critical temperature $T_Q \sim 150$ MeV, the quark-hadron phase transition (QHT) occurs. If the QHT is first order, it is described by the nucleation of hadron bubbles and their growth in the quark-gluon plasma (QGP) sea. As hadron bubbles occupy more than half of the whole space, the picture becomes that of QGP bubbles shrinking in the sea of hadrons. Witten pointed out that the shrinking QGP bubbles in the QHT may evolve to quark nuggets (QNs) which can play the role of baryonic dark matter [1]. The baryon concentration in the shrinking QGP bubbles also produces inhomogeneities that could affect big-bang nucleosynthesis (BBN) [2]. Hydrodynamical analyses of these shrinking QGP bubbles have been extensively studied in a numerical way [3]. Although it is still controversial, recent lattice calculations show that the QHT is weakly first order [4]. Based on these calculations, it was estimated that the mean bubble separation $l_b \sim 1$ cm [5], which is too small for the QHT to achieve the effects mentioned above.[1]

Recently, it was shown that the growth of subhorizon scale fluctuations in the QHT is amplified because the sound velocity during the QHT vanishes [hereafter, we will call this the sound velocity transition (SVT)] [7,8]. The sound velocity would not decrease much with the assumption of frozen volume fraction of the two phases [9]. However, this assumption contradicts the rapid equilibration of pressure and temperature between the two phases and conflicts with the rapid reheating scenario [10]. With a negligible chemical potential, the pressure of the cosmological fluid depends only on the temperature. As the QHT proceeds, the pressure remains constant in phase equilibrium with T_Q, while the energy density of the mixed phase decreases. Thus the isentropic sound velocity vanishes for wavelengths much larger than l_b. The isentropic condition holds for fluctuations with wave number $k \lesssim 10^4 k_H$ where k_H is the wave number of the fluctuation that enters the horizon at the QHT, while smaller scales are damped away before the QHT [10] by neutrino diffusion [11]. With the vanishing sound velocity, the preexisting fluctuations grow without any pressure gradient and restoring force so that their amplitudes are amplified. Amplified subhorizon scale fluctuations could result in clumps of cold dark matter such as axions and primordial black holes (PBHs) [7]. Although it needs fine-tuned initial conditions [12], the formation of PBHs of $1 M_\odot \sim 10^{33}$ g, which can be related to massive compact halo objects (MACHOs) [8], is also possible. It has been shown that the amplified fluctuations in the hadronic phase would be washed out by neutrino damping before the big-bang nucleosynthesis era [7,10] so they could not have any significant effect on the dark matter problem or on the IBBN. In this Brief Report, we propose that the amplified fluctuations produce QGP lumps, which rapidly transform to QNs that could survive until today. With a power-law assumption on the density fluctuation spectrum, we find possible parameter ranges for which the QNs formed can play the role of dark matter and produce baryon number inhomogeneities for the IBBN within the observational bounds of CMBR anisotropy.

During the coexistence phase with duration $\Delta t_Q \approx (0.1\text{–}0.3) t_Q$, where t_Q is the cosmological time at the transition, the SVT amplifies the amplitudes of the subhorizon fluctuations with horizon crossing amplitude δ_H and wave number k up to $\delta_H^{amp} = (k/k_H)^\zeta \delta_H$ for $k_H \lesssim k \lesssim 10^4 k_H$ where $\zeta \approx 3/4$ from the fit to the quenched lattice QCD data and $\zeta \approx 1$ for the bag model [7]. The exponent ζ is obtained for $k \gg k_H$ but we simply regard it as valid for $k \gtrsim k_H$. Due to the SVT, the fluctuations with amplitude larger than the critical amplitude $\delta_H^c = (k_H/k)^\zeta$ will enter the nonlinear regime ($\delta_H^{amp} \gtrsim 1$) during the transition. Since this condition is far

[1]The inhomogeneous nucleation caused by the density fluctuation might produce enough baryon inhomogeneities to affect the inhomogeneous big-bang nucleosynthesis (IBBN) [6].

weaker than $\delta_H^c \sim 1$ in the ordinary case without the SVT, even small fluctuations with $\delta_H \sim (10^{-4})^{\zeta}$ could be nonlinearized.

What will happen as the density fluctuations grow and become nonlinear? The nonlinearization of fluctuations with scale λ can be studied by the evolution of overdense regions of size λ. For simplicity, we will restrict our discussion only to the fluctuations with $\delta_H^c(k) \ll 1$, i.e., those with size far smaller than the horizon at the transition. With small amplitudes, the overdense regions will be approximately in thermal equilibrium with temperature T_Q at the first stage of the transition. Overdense regions with large δ_H^c have a large temperature deviation and this could result in complicated situations but the number of these regions is negligible compared to the small fluctuations. As the fluctuation amplitudes grow due to the SVT, the density of the overdense regions will increase but the temperature of the two phases will not. For the relevant scale of the phase conversion $l_b \sim 1$ cm $\sim 10^{-6} d_H$, nonvanishing sound velocity $v_s = 1/3$ would create equilibrium but superheating is impossible in the fluctuating system. Instead, the density will grow by converting the hadronic phase to the high density QGP phase, maintaining the pressure equilibrium and zero sound velocity for $\lambda \gg l_b$. When the overdense regions with $\sim \delta_c$ become nonlinear, they recover the QGP phase at the end of the transition. Note that the degrees of freedom for the QGP phase and hadronic phase are 51.25 and 17.25, respectively. Thus a little larger δ_c is required but we will not consider this difference seriously. In fact, even though a small change of δ_c causes a great difference in number densities, it causes a slight change to the minimum scale of the QGP lumps and the QNs, etc. The nonlinearized overdense region will collapse, further increasing the temperature as well as the energy density of the QGP lumps for the collapsing time $t_{coll} \sim (G\rho)^{-1/2} \sim t_Q$. Also note that the nonlinearized QGP lumps are decoupled from the cosmic expansion, keeping their temperature $T_{QGP} \sim T_Q$ while the outside temperature decreases with the cosmic expansion. Except for black hole formation, which requires larger amplitudes and so is negligible in amount, the QGP lumps will eventually disperse out. Before the dispersion, however, they could transform rapidly to QNs. The QGP lumps are surrounded by the hadronic sea after the QHT and they evaporate mainly mesons at their surfaces. From the chromoelectric flux tube model (CFTM), the meson evaporation rate at the surface increases with increasing temperature and is $10^{23}(10^{24})$ g cm^{-3}sec^{-1} for $T_{QGP} \simeq 150$ (200) MeV [13]. Hence the time needed for the QGP lumps to be evaporated away for $T_{QGP} \sim 150$ MeV is $t_{evap} \sim 10^{-1}(1)$ μsec$\sim 10^{-2} t_Q (10^{-1} t_Q)$ for $10^4 k_H (10^3) k_H$, faster than t_{coll}. For $T_{QGP} \sim 200$ MeV, t_{evap} is reduced by one more order of magnitude. Meanwhile, the baryon penetrability turns out to be very low [14] so that, as the QGP lumps evaporate, the baryon to entropy ratio inside the lumps, which initially equals the background value $(n_B/s)_{bg} \sim 10^{-10}$, could increase up to the value of 1 needed for the transformation to QNs. Furthermore, when the QGP lumps shrink to the size of the neutrino mean free path $l_\nu \sim 10(T/100\ \text{MeV})^{-5}$ cm, the transformation is accelerated by the rapid entropy loss by the neutrinos. So we can conclude that the overdense regions with $\delta \gtrsim \delta_c$ will rapidly transform to QNs.

The mass and size of the QGP lump can be specified by the baryon number in the lump $N_B \propto k^{-3}$ as $M_Q = (N_B/N_{B\text{-}H}) M_H^Q$ where $N_{B\text{-}H} = 10^{49}(T_Q/100\ \text{MeV})^{-3}$ is the total baryon number contained in the horizon at the QHT. The minimum scale relevant to the SVT is $N_B \approx 10^{37}$. If one takes the quark matter energy density $\simeq$ the QGP energy density at the QHT, then the resulting QN mass $M_{QN} \sim \kappa M_Q$ where κ is the QN to QGP lump volume ratio. The QN mass and hence κ can be estimated qualitatively as follows. We simply take $l_\nu \sim 10$ cm. For large QGP lumps whose sizes $\gg l_\nu$ during their evolution, there are no efficient ways to get rid of entropy and the QN will form solely by surface hadronization (only by the increase of n_B). Then $\kappa \sim (n_B/s)_{bg}$ and n_B should be increased to about the baryon number density of a nucleon ~ 0.3 fm^{-3}. For these large QGP lumps, $M_{QN} \sim 10^{23}(N_B/N_{B\text{-}H})$ g. The QGP lumps with $N_B \lesssim (n_B/s)_{bg}^{-1}(l_\nu/d_H)^3 N_{B\text{-}H} \sim 10^{44}$, however, become smaller than l_ν during their evolution, thus causing very rapid entropy loss. Since the time scale for entropy loss $\sim l_\nu/c \sim 10^{-9}$sec, the QGP lumps rapidly transform to QNs when they shrink to a size $\sim l_\nu$, resulting in $\kappa \sim (10^{34}/N_B)$ and $M_{QN} \sim 10^{18}$ g. The entropy loss by neutrinos also occurs at distances $\lesssim l_\nu$ from the surface and it needs more detailed calculations to find the conditions for QN formation. In models generating large entropy before the QHT [15], the QNs can be formed easily with $\kappa \sim 1$. This situation is not relevant to our work, however, because there can be no SVT.

Once the QNs form, they are unstable to surface evaporation [16] and boiling [17]. Only the QNs with baryon number larger than the critical baryon number N_B^c could survive until today. With small baryon penetrability in the CFTM, N_B^c is lowered and two results of $N_B^c = 10^{39}, 10^{44}$ are known [18,19]. However, as claimed by the authors, their work overestimated the baryon evaporation. In particular, they assumed flavor equilibrium in their calculations. Flavor nonequilibrium will reduce N_B^c. So we take N_B^c as a free parameter bearing in mind the above values. The baryon number of the QNs needed to affect the IBBN is smaller than N_B^c, but the difference will not be large considering the rapid evaporation rate below N_B^c [18,19].

Now, we have found the conditions for QN formation, $\delta_H \gtrsim \delta_H^c$ and $N_B \gtrsim N_B^c$. To estimate the number of QNs, we just count how many overdense regions exist satisfying the conditions. We would like to emphasize that the number density of the QNs depends greatly on the details of the density fluctuation. Although the details of the QHT may rule out QN formation from the shrinking QGP bubbles, whether QNs can be produced sufficiently from the SVT is another problem. The SVT relaxes the condition for nonlinearization and enables more copious QN formation.

We assume a simple power-law spectrum of density fluctuation with $|\delta_k|^2 \propto k^n$, where δ_k is the Fourier transform of $\delta(\mathbf{x}) \equiv [\rho(\mathbf{x}) - \rho_b]/\rho_b$ and the spectral index n is a constant. The initial power spectrum of the fluctuation amplitude is defined by the rms amplitude for a given logarithmic interval in k, $\overline{\delta^2}(k) \equiv k^3|\delta_k|^2/2\pi^2$. From linear analyses, the rms amplitude $\overline{\delta_i}(k)$ at t_i, the time when the fluctuations develop, is related to the rms horizon crossing amplitude $\overline{\delta_H}(k)$ as follows: $\overline{\delta_H}(k) = (k/k_{Hi})^{-2}\overline{\delta_i}(k) = (k/k_0)^{(n-1)/2}\overline{\delta_H}(k_0)$ where k_{Hi} is the wave number of the horizon scale at t_i and the subscript 0 denotes the values at present. From the Cosmic Background Explorer (COBE) measurement $\overline{\delta_H}(k_0) \approx 10^{-5}$ [20], and $n \simeq 1.01^{+0.09}_{-0.07}(^{+0.17}_{-0.14})$ with 68% (95%) confidence limits from combined analyses of the MAXIMA-1, BOOMERANG, and COBE/DMR observations [21]. The number density of overdense regions with $\delta_H \gtrsim \delta_H^c$ can be found by the Press-Schechter method [22]. The difference here is in the scale dependence of the critical amplitude $\delta_H^c \propto k^{-\zeta}$. The initial mass spectrum of the QGP lump in the range $(M_H, M_H + dM_H)$, where $M_H \propto M_Q^{2/3}$ is the horizon mass, $n(M_H)dM_H \propto M_H{}^{\alpha/4}\exp[-(\delta_H^c/\sqrt{2}\sigma_H)^2]dM_H$ where $\alpha = n + 2\zeta - 11$, $\sigma_H = \sigma_{H0}(M_H/M_{H0})^{(1-n)/4}$ is the filtered amplitude of $\overline{\delta_H}(k)$, and $\sigma_{H0} = 9\times 10^{-5}$ [23]. Due to the scale dependence of δ_H^c, the mass spectrum has an exponential cutoff even with the $n=1$ Harrison-Zel'dovich spectrum. For comparison, $n(M_H) \propto M_H{}^{(n-11)/4}\exp(-M_H{}^{(n-1)/2})$ without the sound velocity transition [24], so if $n=1$ the mass spectrum could have very broad mass ranges with $n(M_H) \propto M_H{}^{-5/2}$. Since the mass spectrum decreases rapidly, QNs with only $N_B \simeq N_B^c$ form significantly. Then the initial number fraction of QNs with N_B^c can be approximated as

$$\beta_i(M_H^c) = \sigma_H(M_H^c)\exp\left[-\left(\frac{\delta_H^c}{\sqrt{2}\sigma_H(M_H^c)}\right)^2\right].$$

Since the QNs can be regarded as pressureless dust, the density fraction at present is

$$\Omega_{QN}(t_0) = \Omega_{QN}(t_{eq}) = \frac{\rho_i}{\rho_{eq}}\left(\frac{M_{QN}^c}{M_c}\right)\left(\frac{a(t_{eq})}{a(t_i)}\right)^{-3}\beta_i$$

where the subscript eq represents the values at matter-radiation equal time. M_c is the mass contained in the overdense region with k_c at t_i. The density fraction can be arranged to $\Omega_{QN}(t_0) = \kappa(T_Q/T_{eq})\beta_i$. Figure 1 shows the relation between κ and N_B^c to satisfy $\Omega_{QN}=1$ for some n. Here κ is regarded as an undetermined free parameter. It seems impossible for QNs to be formed solely by surface hadronization $[\kappa \simeq (n_B/s)_{bg}]$. The QGP lumps become smaller than l_ν when $\kappa = (N_B^c/10^{34})^{-1}$ (the bold line in Fig. 1). So, on the left side of the bold line, the QGP lumps can evolve to QNs ($n_B/s \sim 1$) by further losing their entropy via neutrinos. Within the observational limits of n, QN formation is relevant only for $N_B^c \lesssim 10^{40}$. For $n \approx 1$, it needs $N_B^c \lesssim 10^{38}$ for $\zeta = 1$, which is smaller than the CFTM results [18,19]. With $\zeta = 0.75$, QN formation is possible if $n \gtrsim 1.1$. Assuming $\kappa = (N_B^c/10^{34})^{-1}$, the upper limits on n are found from the condition that Ω_{QN} should not exceed unity ($\Omega_{QN}=1$ are the bold lines in Fig. 2). If $N_B^c \lesssim 10^{38}$, the QN can give strong constraints on $n_{upper} \simeq 1$. With smaller Ω_{QN}, even $n_{upper} < 1$ is possible. PBHs could give at best $n_{upper} \gtrsim 1.23$, and

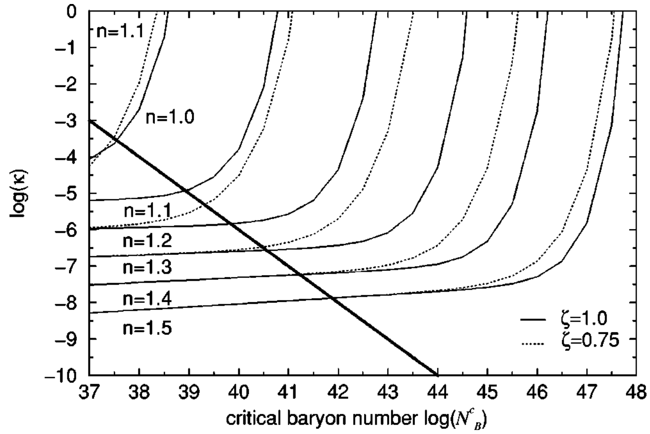

FIG. 1. The parameter ranges for κ and N_B^c for $T_Q = 150$ MeV.

even weaker as the reheating temperature increases [23,25]. Note that the upper limits on n do not depend on t_i or the reheating temperature in inflationary models.

Since QNs have positive electric surface potential of order ~MeV, they absorb only neutrons. This reduces the neutron to proton ratio, so lowering the helium production. Not to violate the observations, the QNs should have size $\gtrsim 10^{-6}$ cm assuming $\Omega_{QN}=1$ and perfect baryon penetrability (even smaller with smaller penetrability) [26]. The corresponding $N_B^c \simeq 10^{13}$, so the QNs considered here easily satisfy the condition. More significant effects on the IBBN can be induced by evaporation of the QNs. The minimal requisition for the IBBN is that the mean separation between QNs should be larger than the proton diffusion length when BBN starts. This requires the mean separation of QNs at the QHT to be $l_{QN} \gtrsim 3$ m [27], ruling out the QNs from shrinking QGP bubbles ($l_b \approx 1$ cm). We find $l_{QN} \approx n_{QN}^{-1/3} \approx 10^8(N_B^c/N_{B\text{-}H})^{1/3}\beta_i^{-1/3}$ m. The upper values of n for the IBBN ($l_{QN} \approx 3$ m) are shown in Fig. 2. It can be seen that the minimal condition is easily satisfied. The closure condition $\Omega_{QN} \leqslant 1$ (the bold line in Fig. 2) corresponds to demanding $l_{QN} \gtrsim 50$ m. It goes to $l_{QN} \gtrsim 100$ m, demanding

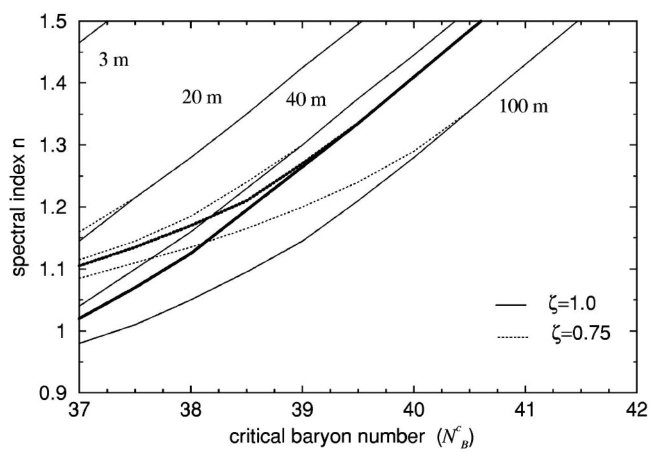

FIG. 2. The relation between the spectral index and the inhomogeneity scale l_{QN} for $T_Q = 150$ MeV. The bold lines are the upper limits on n found from $\Omega_{QN}=1$ with $\kappa = (10^{34}/N_B^c)$.

$\Omega_{QN} \lesssim 0.1$ (not shown in Fig. 2). Interestingly, demanding $0.1 \lesssim \Omega_{QN} \lesssim 1$, l_{QN} lies in the range for the IBBN to be effective [27,28]. So the QNs can contribute to the density of the universe in the form of baryonic dark matter as well as affecting IBBN. Enhanced heavy element formation can be the signature for QNs [28].

In summary, we have proposed that QNs can be formed by the SVT during the QHT. Their formation depends greatly on details of the density fluctuation rather than those of the QHT. We have found that they could be a dark matter candidate and affect IBBN, unlike the QNs from shrinking QGP bubbles whose formation is severely constrained by the recent lattice data. Also, QNs from the SVT could strongly constrain the spectral index. Our analyses are so far rather qualitative and include undetermined parameters such as κ and N_B^c. Further systematic analyses are needed to get more quantitative results. Recent balloon experiments on the CMBR suggest a large baryonic content in the universe $\Omega_b \simeq 0.03$ [29], which is significantly higher than the standard BBN result. This is very inspiring to IBBN models and the QN formation proposed in this work.

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