

Hierarchy stabilization in warped supersymmetry

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We show that exponentially large warp factor hierarchies can be dynamically generated in supersymmetric compactifications. The compactification we consider is the supersymmetric extension of the Randall-Sundrum model. The crucial issue is the stabilization of the radius modulus for large warp factor. The stabilization sector we employ is very simple, consisting of two pure Yang-Mills sectors, one in the bulk and the other localized on a brane. The only fine-tuning required in our model is the cancellation of the cosmological constant, achieved by balancing the stabilization energy against supersymmetry breaking effects. Exponentially large warp factors arise naturally, with no very large or small input parameters. To perform the analysis, we derive the four-dimensional effective theory for the supersymmetric Randall-Sundrum model, with a careful treatment of the radius modulus. The manifestly (off-shell) supersymmetric form of this effective Lagrangian allows a straightforward and systematic treatment of the nonperturbative dynamics of the stabilization sector.

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I. INTRODUCTION

Following the work of Refs. [1,2] there has been a great deal of interest in the phenomenological possibilities of warped higher-dimensional spacetimes of the form

$$ds^2 = \omega^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + h_{mn}(y) dy^m dy^n, \quad (1.1)$$

where x^μ ($\mu=0, \dots, 3$) are the four noncompact spacetime dimensions, and y^m are compactified. In particular, the y -dependent renormalization of effective four-dimensional mass scales implied by the “warp factor” $\omega(y)$ provides a powerful mechanism for generating hierarchies in nature. Randall and Sundrum (RS) [1] presented a very simple warped five-dimensional compactification with an exponential warp factor (the “RS1” model), which exploited this mechanism to explain the hierarchy between the weak and the Planck scales, without appealing to supersymmetry.

Warped spacetimes may also be important in models with supersymmetry (SUSY). One motivation is to allow phenomenological effective field theory approaches to make contact with warped superstring backgrounds [3]. A particularly interesting string background is $\text{AdS}_5 \times \text{S}^5$, which plays a central role in the Maldacena realization of holographic duality [4]. In Ref. [5] it is emphasized that such dualities may have a profound connection to the Randall-Sundrum models, based on the (partial) AdS_5 geometry of these models. Supersymmetry may allow a more precise understanding. Another example is 11-dimensional heterotic M theory compactified on a six-dimensional Calabi-Yau space and an S^1/\mathbb{Z}_2 orbifold [6]. This $\mathcal{N}=1$ supersymmetric theory has been taken as the starting point for phenomenological studies, where the warp factor may play an important role [7]. There are also purely phenomenological motivations; warp

factors can generate hierarchies required in realistic supersymmetric theories [8]. It is also an interesting open question to ask what patterns of SUSY breaking can arise in warped spacetimes. In the future we hope to focus on the effect of warping on higher-dimensional SUSY mediation mechanisms such as anomaly-mediated SUSY breaking [9], gaugino-mediated SUSY breaking [10], and radion-mediated SUSY breaking [11].

In this paper we will study the minimal supersymmetric extension of the simplest warped compactification, namely, RS1. This extension was constructed in Ref. [12]. Our first result is a derivation of the four-dimensional effective theory of the supersymmetric RS1 model valid at long wavelengths, including a careful treatment of the radius modulus.¹ This effective Lagrangian is valid to two-derivative order, but to all orders in the fields, including the radion field. The effective Lagrangian will be presented in terms of off-shell SUSY multiplets, which will greatly simplify the analysis of non-perturbative effects and SUSY breaking.

The other main result of our paper is a dynamical mechanism to stabilize the radius modulus in the supersymmetric RS1 model. This mechanism naturally stabilizes the radius at a sufficiently large value that the warp factor hierarchy across the extra dimension is large. The stabilization sector consists of two super-Yang-Mills (SYM) sectors, one in the bulk and the other localized on one of the four-dimensional boundaries. The radius of the extra dimension is stabilized by the balance between brane and boundary gaugino condensate contributions to the supergravity (SUGRA) potential. We first proposed this mechanism in Ref. [13], where it was shown to stabilize the radius in a supersymmetric compactification with negligible warp factor. We stress that for any value of the warp factor the mechanism is completely natural

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¹A related derivation and discussion of the four-dimensional effective theory by Bagger, Nemeschansky, and Zhang [24] will appear at the same time as the present paper.

(except for the cosmological constant problem) and controlled in an effective field theory expansion. In the nonsupersymmetric RS1 model, a simple classical mechanism that stabilized a large warp factor was presented in Ref. [14]. The supersymmetric mechanism we present here is intrinsically nonperturbative.

We believe that it is an important development to have a supersymmetric model of radius stabilization that is both *complete* and *calculable*. Moduli describing the size and shape of the extra dimensions are a generic feature of higher-dimensional compactifications with supersymmetry, and in particular superstring theory. These moduli must be stabilized both to avoid phenomenological and cosmological problems of light scalars, and also to select an appropriate vacuum. This problem has been extensively discussed in string-inspired contexts; see, e.g., Ref. [15]. The stabilization problem is especially severe because of the constraints of higher-dimensional local supersymmetry. Our model gives a simple stabilization mechanism consistent with these constraints, even if it does not display the full complexity of string compactifications. We hope that some of the tools we have developed can be extended to superstring or M theory.

This paper is organized as follows. In Sec. II we describe the model we will study. In Sec. III we derive the supersymmetric four-dimensional effective field theory of the supersymmetric RS1 model. In Sec. IV we analyze the nonperturbative gauge dynamics needed for stabilization using the effective four-dimensional description. These results are summarized and discussed in Sec. V. In the interest of readability, some details of the derivation of the effective theory in Sec. III are relegated to the Appendix, which, however, gives a self-contained account.

II. THE MODEL

The theory we are interested in is minimal five-dimensional SUGRA, where the fifth dimension is a finite interval realized as an $\mathbf{S}^1/\mathbf{Z}_2$ orbifold. We will also couple this theory to matter and gauge fields in the bulk or localized on the orbifold boundaries.

Our starting point is the on-shell Lagrangian for five-dimensional SUGRA [16]

$$\mathcal{L}_{\text{SUGRA},5} = -M_5^3 \left\{ \sqrt{-G} \left[\frac{1}{2} R(G) + \frac{1}{4} C^{MN} C_{MN} - 6k^2 \right] + \frac{1}{6\sqrt{6}} \epsilon^{MNPQR} B_M C_{NP} C_{QR} + \text{fermion terms} \right\}, \quad (2.1)$$

where $M, N, \dots = 0, \dots, 3, 5$, are five-dimensional spacetime indices, G_{MN} is the five-dimensional metric, $C_{MN} = \partial_M B_N - \partial_N B_M$ is the field strength for the graviphoton B_M , and k is a mass scale defined so that $-6M_5^3 k^2$ is the five-dimensional cosmological constant. Unbroken SUSY requires that the cosmological constant have **AdS** sign ($k^2 > 0$). In order to realize this theory on an $\mathbf{S}^1/\mathbf{Z}_2$ orbifold, the \mathbf{Z}_2 parity assignments of the bosonic fields must be taken as in Table I.

TABLE I. Bosonic fields of five-dimensional SUGRA with their \mathbf{Z}_2 parity assignments.

Field	\mathbf{Z}_2 parity
$G_{\mu\nu}$	+
$G_{5\mu}$	-
G_{55}	+
B_μ	-
B_5	+

We now couple five-dimensional SUGRA to localized energy density on the orbifold boundaries:

$$\Delta \mathcal{L}_5 = -\delta(\vartheta) \sqrt{-g_1} V_1 - \delta(\vartheta - \pi) \sqrt{-g_2} V_2, \quad (2.2)$$

where $g_{1,2}$ are the induced four-dimensional metrics on the boundaries, $V_{1,2}$ are constants, and $-\pi < \vartheta \leq \pi$ parametrizes the fifth dimension. This theory admits the Randall-Sundrum solution [1]

$$ds^2 = e^{-2kr_0|\vartheta|} \eta_{\mu\nu} dx^\mu dx^\nu + r_0^2 d\vartheta^2, \quad (2.3)$$

$$B_\vartheta = b_0, \quad B_\mu = 0,$$

provided that

$$k = \frac{V_1}{6M_5^3} = -\frac{V_2}{6M_5^3}. \quad (2.4)$$

This metric is a slice of **AdS** $_5$. The exponential factor $e^{-2kr_0|\vartheta|}$ is the ‘‘warp factor’’ that gives rise to mass hierarchies across the fifth dimension. The theory including the boundary terms Eq. (2.2) can be made supersymmetric by the addition of suitable fermion terms, and the ‘‘vacuum’’ solution Eq. (2.3) then preserves four real supercharges [12]. The bulk Lagrangian Eq. (2.1) is invariant under eight real supercharges, but half of the supersymmetry is explicitly broken by the orbifold projection and the boundary terms.

Equation (2.3) is a solution for any value of r_0 and b_0 ; r_0 is the radius of the compact \mathbf{S}^1 , while b_0 is the Aharonov-Bohm phase of the graviphoton around the \mathbf{S}^1 . When we consider fluctuations about the solution Eq. (2.3), these integration constants become propagating massless modes. The mode corresponding to r_0 (the radion) is particularly important, since it controls the couplings in the four-dimensional effective theory. In this paper we will show how to stabilize the radion in the interesting case where the warp factor is a large effect.

In addition, we will couple this theory to bulk super-Yang-Mills theory. The minimal five-dimensional SYM multiplet consists of a vector field A_M , a real scalar Φ , and a symplectic Majorana gaugino λ^j ($j=1,2$). The bulk Lagrangian is [17]

TABLE II. Fields of five-dimensional super-Yang-Mills sector with their \mathbf{Z}_2 parity assignments.

Field	\mathbf{Z}_2 parity
A_μ	+
A_5	-
Φ	-
λ^1	+
λ^2	-

$$\begin{aligned}
 \mathcal{L}_5 = & -\sqrt{-G} \frac{1}{2g_5^2} \text{tr} F^{MN} F_{MN} \\
 & - \frac{1}{2\sqrt{6}g_5^2} \epsilon^{MNPQR} B_M \text{tr} F_{NP} F_{QR} \\
 & + \text{scalar and gaugino terms.} \quad (2.5)
 \end{aligned}$$

The SYM fields are taken to transform under the orbifold \mathbf{Z}_2 as shown in Table II. The even fields form an $\mathcal{N}=1$ SYM multiplet.

To obtain realistic models we will couple these bulk fields to fields localized on the orbifold boundaries. Working out these couplings and verifying that they preserve supersymmetry is nontrivial. An off-shell construction of the boundary couplings was given by Ref. [18] using the method of Mirabelli and Peskin [19]. The off-shell couplings of bulk SYM to SUGRA were constructed in Ref. [20]. It is clearly crucial for the results of this paper that these couplings exist and preserve SUSY. However, the results of this paper will be derived using only the on-shell bosonic Lagrangian together with consistency arguments.

We can now summarize the theory that we will analyze in this paper. The theory consists of minimal five-dimensional SUGRA, with a SYM sector in the bulk, and an additional SYM sector on one of the orbifold boundaries, the ‘‘hidden brane.’’ The bulk Lagrangian has dimensionful parameters M_5 and g_5 that we take to be of the order of the Planck scale. Additionally, we assume that there is a SUSY breaking sector also localized on the hidden brane. The SYM multiplets together with the SUSY breaking sector will play the role of the radius stabilization sector, as we will see. For a fully realistic model, one would want to add standard model fields, presumably some or all of them localized on the other boundary, the ‘‘visible brane.’’ These play no role in the stabilization dynamics. We will study complete realistic models in future work.

III. THE FOUR-DIMENSIONAL EFFECTIVE LAGRANGIAN

At sufficiently low energies, the dynamics of the theory above is approximately four dimensional. The matching scale between the five- and four-dimensional effective theories is determined by the mass of the lowest Kaluza-Klein (KK) mode, given by [1,2]

$$m_{\text{KK}}^2 \sim \left(\frac{k}{1 + e^{\pi k r_0}} \right)^2. \quad (3.1)$$

We assume that the theory is weakly interacting at this scale, justifying the use of classical matching. This will be true as long as the radius of compactification r_0 and the radius of curvature $1/k$ are larger than the five-dimensional Planck length.

In this section, we will derive the four-dimensional effective theory below the scale Eq. (3.1). Our strategy is to match enough bosonic terms between the five- and four-dimensional Lagrangians so that we can infer the remaining terms using $\mathcal{N}=1$ SUSY. The justification of some of the steps is relegated to the Appendix. The Appendix gives a complete self-contained derivation, including a discussion of some subtleties of classical matching.

We begin by considering the massless bosonic fields arising from the five-dimensional SUGRA sector. The solution Eq. (2.3) has undetermined integration constants r_0 and b_0 whose long-wavelength fluctuations are massless moduli. Also, unbroken four-dimensional Lorentz invariance implies that there is a massless graviton in the four-dimensional effective theory. These massless four-dimensional fluctuations can be parametrized by making the replacements $\eta_{\mu\nu} \rightarrow g_{\mu\nu}(x)$, $r_0 \rightarrow r(x)$, and $b_0 \rightarrow b(x)$ in Eq. (2.3):

$$\begin{aligned}
 ds^2 = & e^{-2kr(x)|\vartheta|} g_{\mu\nu}(x) dx^\mu dx^\nu + r^2(x) d\vartheta^2, \\
 & -\pi < \vartheta \leq \pi,
 \end{aligned}$$

$$B_{\vartheta}(x, \nu) = b(x), \quad B_\mu(x, \vartheta) = 0. \quad (3.2)$$

If this satisfied the five-dimensional equation of motion, one could obtain the classical four-dimensional effective action by substituting Eq. (3.2) into the five-dimensional action and integrating over the fifth dimension. Equation (3.2) does not satisfy the five-dimensional equations of motion [21]. However, in the Appendix we show that for the metric in Eq. (3.2) this ‘‘naive’’ procedure gives a result that differs from the exact classical effective action only by terms with four or more x derivatives. We can therefore use the metric in Eq. (3.2) to parametrize the radion at leading order in the derivative expansion.² For the graviphoton, the naive procedure does not work; the graviphoton can still be parametrized by $b(x)$ defined by Eq. (3.2), but there is a nontrivial correction to the classical effective Lagrangian that is computed in the Appendix. However, to determine the effective theory it is sufficient to know the terms that depend only on the radion, which can be determined by substituting Eq. (3.2) into the five-dimensional action. The terms depending on the graviphoton can then be inferred from SUSY. Therefore, the calculation of the graviphoton effective Lagrangian carried out in the Appendix serves only as a redundant check on our results.

²Reference [21] gives an alternative parametrization of the radion that satisfies the five-dimensional equations of motion at linear order in fluctuation fields, but to all orders in x derivatives.

We turn to the five-dimensional SYM sector. It is straightforward to verify that

$$A_\mu(x, \vartheta) = a_\mu(x), \quad A_\vartheta(x, \vartheta) = 0 \quad (3.3)$$

is a solution to the five-dimensional equations of motion if $a_\mu(x)$ is a solution to the four-dimensional YM equation of motion. Therefore, $a_\mu(x)$ parametrizes a four-dimensional vector zero mode. The fact that the zero mode is independent of ϑ despite the presence of the warp factor can be traced to the classical conformal invariance of four-dimensional Yang-Mills theory. Note that there are no massless A_ϑ or Φ fluctuations because of the orbifold projection.

We wish to relate the massless bosonic fields defined above (and their fermionic superpartners) to a manifestly $\mathcal{N} = 1$ supersymmetric formulation of the four-dimensional effective theory. The massless bosonic fields are two real scalars $r(x)$ and $b(x)$, a real vector multiplet $a_\mu(x)$, and the metric $g_{\mu\nu}(x)$. Given that these bosonic fluctuations are part of an $\mathcal{N} = 1$ locally supersymmetric theory, they can be parametrized by one chiral superfield T , one non-Abelian vector superfield V , and the minimal SUGRA multiplet. The most general effective Lagrangian at two-derivative order can be written

$$\mathcal{L}_{4,\text{eff}} = \int d^4\theta \phi^\dagger \phi f(T, T^\dagger) + \left[\int d^2\theta S(T) \text{tr}(W^\alpha W_\alpha) + \text{H.c.} \right]. \quad (3.4)$$

There is no superpotential for T because the radion modulus does not have a potential. We are using the superconformal approach to SUGRA [22]. The field ϕ is the superconformal compensator [23,22] that is responsible for breaking the local superconformal symmetry down to local super-Poincaré:

$$\phi = 1 + \theta^2 F_\phi. \quad (3.5)$$

F_ϕ is the scalar auxiliary field of the minimal off-shell $\mathcal{N} = 1$ SUGRA multiplet. We are using superspace notation as a shorthand for expressions that can be rigorously defined using the superconformal tensor calculus approach [22]. In particular, factors of the metric (or vierbein) are implicit in this notation.

We now make a holomorphic field redefinition $S(T) \rightarrow T/g_5^2$ in the effective theory so that the effective Lagrangian has the form

$$\mathcal{L}_{4,\text{eff}} = \int d^4\theta \phi^\dagger \phi f(T, T^\dagger) + \left[\int d^2\theta \frac{T}{g_5^2} \text{tr}(W^\alpha W_\alpha) + \text{H.c.} \right]. \quad (3.6)$$

From this, we have

$$\frac{1}{2g_4^2} = \frac{\text{Re}(T)}{g_5^2}. \quad (3.7)$$

We can also calculate the four-dimensional gauge coupling g_4 by substituting the zero mode gauge field Eq. (3.3) into the five-dimensional action and integrating over the fifth dimension. This yields

$$\frac{1}{g_4^2} = \frac{2\pi r}{g_5^2}. \quad (3.8)$$

Therefore, we see that

$$\text{Re}(T) = \pi r. \quad (3.9)$$

Similarly, from Eq. (3.6) we see that $\text{Im}(T)$ is proportional to the four-dimensional theta angle, which in turn is proportional to B_ϑ from the mixed Chern-Simons term in the five-dimensional theory:

$$\begin{aligned} \Delta \mathcal{L}_5 &= -\frac{1}{2\sqrt{6}g_5^2} \epsilon^{MNPQR} B_M \text{tr}(F_{NP} F_{QR}) \\ &= -\frac{1}{2\sqrt{6}g_5^2} \epsilon^{\mu\nu\rho\sigma} B_\vartheta \text{tr}(F_{\mu\nu} F_{\rho\sigma}) + \dots \end{aligned} \quad (3.10)$$

This determines

$$\text{Im}(T) = \frac{2\pi}{\sqrt{6}} b. \quad (3.11)$$

Thus we have fixed the relation between T and the component fields $r(x)$ and $b(x)$. Note that Eqs. (3.9) and (3.11) are exactly the same as in flat space [13]. This is ultimately due to the classical conformal invariance of Yang-Mills theory in four dimensions.

It still remains to fix the relation between the metric that appears in the four-dimensional $\mathcal{N} = 1$ SUGRA multiplet, and the metric $g_{\mu\nu}$ defined by Eq. (3.2). This is nontrivial because in the four-dimensional effective theory we have the freedom to make field redefinitions $g'_{\mu\nu} = c(r)g_{\mu\nu}$, where $c(r)$ is an arbitrary function. However, such field redefinitions in general do not preserve the property that T transforms independently of the four-dimensional SUGRA multiplet under $\mathcal{N} = 1$ SUSY. In the Appendix, it is shown that imposing this condition implies that the two metrics are identical (as implicitly assumed in the notation used above).

Expanding the four-dimensional SUGRA Lagrangian Eq. (3.4) in component fields, we obtain

$$\begin{aligned} \mathcal{L}_{\text{SUGRA},4} &= \sqrt{-g} \left[-\frac{1}{6} f R(g) \right. \\ &\quad - \frac{1}{4f} (f_T \partial^\mu T - \text{H.c.})(f_T \partial_\mu T - \text{H.c.}) \\ &\quad \left. - f_{T^\dagger T} \partial^\mu T^\dagger \partial_\mu T + \text{fermion terms} \right], \end{aligned} \quad (3.12)$$

where $f_T = \partial f / \partial T$, etc., and $R(g)$ is the four-dimensional Ricci scalar associated with the metric g . As discussed above, the terms depending on the metric and the radion r

can be obtained by substituting Eq. (3.2) into the five-dimensional action and integrating over the fifth dimension. We can use this procedure to determine f by calculating the coefficient of $R(g)$ [see Eq. (3.12)]. One obtains

$$f = \frac{3M_5^3}{k} (e^{-2\pi k r} - 1). \quad (3.13)$$

Note that for $r \rightarrow r_0$, f is the four-dimensional Planck scale computed in Ref. [1]. The graviphoton Aharonov-Bohm phase cannot contribute to the coefficient of $R(g)$. Using Eqs. (3.9) and (3.11) therefore gives

$$f(T, T^\dagger) = + \frac{3M_5^3}{k} (e^{-k(T+T^\dagger)} - 1). \quad (3.14)$$

Having fixed the function f , the other two-derivative terms in Eq. (3.12) that depend on r and b are fixed. In the Appendix we show that these agree with a direct matching calculation, giving a highly nontrivial check of this derivation.

We now turn to fields localized on the boundary. Note that, in terms of the component fields, we have chosen coordinates so that the warp factor is unity at $\vartheta=0$ (the hidden brane). Therefore the radion field (as parametrized above) does not appear in the hidden brane terms in the Lagrangian.³ Correspondingly, it is shown in the Appendix that the terms arising from the hidden brane are independent of T . Therefore the general form of the effective Lagrangian involving the hidden fields is

$$\begin{aligned} \mathcal{L}_{4,\text{hid}} = & \int d^4\theta \phi^\dagger \phi f_{\text{hid}}(\Sigma, \Sigma^\dagger) + \int d^2\theta [S_{\text{hid}}(\Sigma) \text{tr} W'^\alpha W'_\alpha \\ & + \phi^3 \mathcal{W}_{\text{hid}}(\Sigma)] + \text{H.c.}, \end{aligned} \quad (3.15)$$

where Σ are hidden sector chiral multiplets and W'_α is the field strength of the hidden sector gauge multiplets. The terms arising from the visible brane do have couplings to the radion, since by Eq. (3.2) the induced metric on the brane is $e^{-2\pi k r(x)} g_{\mu\nu}(x)$. The unique supersymmetrization of these terms is

$$\begin{aligned} \mathcal{L}_{4,\text{vis}} = & \int d^4\theta \phi^\dagger \phi e^{-k(T+T^\dagger)} f_{\text{vis}}(Q, Q^\dagger) \\ & + \int d^2\theta [S_{\text{vis}}(Q) \text{tr} \tilde{W}^\alpha \tilde{W}_\alpha + \phi^3 e^{-3kT} \mathcal{W}_{\text{vis}}(Q)] \\ & + \text{H.c.}, \end{aligned} \quad (3.16)$$

where Q is a visible sector chiral multiplet, and \tilde{W}_α is the field strength of the visible sector gauge multiplets. Note that Eq. (3.16) has the same form as Eq. (3.15) with ϕ replaced by ϕe^{-kT} . This is not a coincidence. The radion dependence of $\mathcal{L}_{4,\text{vis}}$ is entirely due to the fact that the induced metric is

a Weyl rescaling of $g_{\mu\nu}$, which is precisely equivalent to a rescaling of the conformal compensator ϕ .

Comparing Eqs. (3.15) and (3.16) one readily sees that, relative to fundamental mass parameters, physical mass scales in the visible sector (including UV regulator and renormalization scales) are rescaled by a factor of $e^{-k\pi r_0}$, while scales in the hidden sector are not. For $k>0$, mass scales are suppressed in the visible sector, while for $k<0$ mass scales are enhanced in the visible sector. This is the warp factor effect that can naturally generate exponentially large hierarchies.

It is more conventional to describe the kinetic terms in supergravity in terms of the Kähler potential. This is given by

$$K \equiv -3M_4^2 \ln \left[- \frac{f(T, T^\dagger) + f_{\text{vis}}(Q, Q^\dagger) + f_{\text{hid}}(\Sigma, \Sigma^\dagger)}{3M_4^2} \right]. \quad (3.17)$$

The properties of supersymmetry breaking and renormalization are easier to see in terms of f , but the Kähler potential is more useful for determining the sigma model couplings of the bosonic fields.

IV. THE RADIUS MODULUS EFFECTIVE POTENTIAL

In this section, we construct the effective potential for the model described above and minimize the potential to show that the radius is in fact stabilized. The model was analyzed in Ref. [13] for the case where the warp factor is a small effect, $e^{-kT} \simeq 1$. We will therefore be interested in the case where the warp factor is a large effect.

Just below the KK matching scale Eq. (3.1), the four-dimensional effective theory is

$$\begin{aligned} \mathcal{L}_{4,\text{eff}} = & \frac{3M_5^3}{k} \int d^4\theta \phi^\dagger \phi (e^{-k(T+T^\dagger)} - 1) \\ & + \int d^2\theta \left(\frac{T}{g_5^2} \text{tr} W^\alpha W_\alpha + \frac{1}{2g_4^2} \text{tr} W'^\alpha W'_\alpha \right) \\ & + \text{H.c.} + \mathcal{L}_{\text{SB}}. \end{aligned} \quad (4.1)$$

Here the first gauge kinetic term arises from the bulk SYM sector, while the second arises from the SYM sector localized on the hidden brane. \mathcal{L}_{SB} is the Lagrangian for the SUSY breaking sector, also assumed to be localized on the hidden brane. We are using coordinates where the warp factor is unity on the hidden brane (so that \mathcal{L}_{SB} is independent of T). There are therefore two cases to consider: the warp factor at the visible brane is either smaller or larger than unity. In the formulas above, these cases correspond to $k>0$ and $k<0$, respectively, so we can analyze both cases using Eq. (4.1). Classical matching is justified by assuming that the asymptotically free gauge forces are weak at the KK matching scale, and that the spacetime curvature is also small, $|k| < M_5$.

The SYM sectors become strong in the infrared of the four-dimensional effective theory and give rise to a dynamical superpotential from gaugino condensation. In addition,

³The physical radion mode does couple to the hidden brane via kinetic mixing with the graviton field.

the hidden SUSY breaking sector is assumed to dynamically generate a nonzero vacuum energy. This vacuum energy will be positive provided that SUGRA is a perturbation to the SUSY breaking dynamics. We also assume that the SUSY breaking dynamics has a mass gap, except for the Goldstino. The effective Lagrangian below the scale where these effects occur is then

$$\begin{aligned} \mathcal{L}_{4,\text{eff}} = & \frac{3M_5^3}{k} \int d^4\theta \phi^\dagger \phi (e^{-k(T+T^\dagger)} - 1) \\ & + \left[\int d^2\theta \phi^3 (ae^{-\xi T} + c) + \text{H.c.} \right] \\ & - V_{\text{SB}} + \text{Goldstino terms.} \end{aligned} \quad (4.2)$$

If the bulk SYM gauge group is $SU(N)$, we have

$$a = \mathcal{O}\left(\frac{1}{N^4 g_5^6}\right), \quad \xi = \frac{16\pi^2}{3Ng_5^2}. \quad (4.3)$$

The exact T dependence in the superpotential term of Eq. (4.2) is fixed by holomorphy and the anomalous shift symmetry in T [13].

It is straightforward to integrate out the auxiliary fields for T and ϕ to obtain the effective potential. However, additional insight into the form of the answer is given by writing the Lagrangian in terms of the ‘‘warp factor superfield’’

$$\omega \equiv \phi e^{-kT} \quad (4.4)$$

in place of T . This gives

$$\begin{aligned} \mathcal{L}_{4,\text{eff}} = & \frac{3M_5^3}{k} \int d^4\theta (\omega^\dagger \omega - \phi^\dagger \phi) \\ & + \left[\int d^2\theta (a\phi^{3-n}\omega^n + c\phi^3) + \text{H.c.} \right] \\ & - V_{\text{SB}} + \text{Goldstino terms,} \end{aligned} \quad (4.5)$$

where

$$n \equiv \frac{\xi}{k}. \quad (4.6)$$

From Eq. (4.5) one can immediately read off the potential

$$\begin{aligned} V_{\text{eff}} = & \frac{k}{3M_5^3} [n^2 |a|^2 (\omega^\dagger \omega)^{n-1} - |(3-n)a\omega^n + 3c|^2] + V_{\text{SB}} \\ = & \frac{k}{3M_5^3} [n^2 |a|^2 \omega^{2(n-1)} - (n-3)^2 |a|^2 |\omega|^{2n} - 9|c|^2 \\ & - 6(n-3)|a||c||\omega|^n \cos \gamma] + V_{\text{SB}}, \end{aligned} \quad (4.8)$$

where

$$\gamma \equiv \arg(a) - \arg(c) + n \arg(\omega). \quad (4.9)$$

We now minimize the potential as a function of $|\omega|$ and γ .

We first consider $k > 0$, corresponding to the case where the warp factor is smaller than unity on the visible brane. If the warp factor is an important effect, then $|\omega| \ll 1$ and we can neglect the second term in Eq. (4.8) compared to the first. (We assume that n is not much larger than unity.) Minimizing with respect to γ simply sets $\cos \gamma = \text{sgn}(n-3)$. There is a nontrivial minimum provided that $n > 3$, which is satisfied provided that the bulk SYM sector is weakly coupled at the KK matching scale. We then obtain

$$|\omega| = e^{-\pi k r} = \left(\frac{3(n-3)}{n(n-1)} \frac{|c|}{|a|} \right)^{1/(n-2)}. \quad (4.10)$$

We see that for any given n we can obtain $|\omega| \ll 1$ provided that $|c|/|a|$ is sufficiently small.⁴ This is perfectly natural, since $|c|$ is exponentially small in terms of the fundamental couplings. Thus, if we want to use the small warp factor to explain some mass hierarchy in nature, the small warp factor itself can be explained in terms of order-1 fundamental parameters in this model of stabilization.

To complete our analysis of this case, we find the other extrema of the potential. There is an obvious extremum where $|\omega| \rightarrow 0$. It is easy to check that this has higher energy than the solution Eq. (4.10). We must also look for solutions with $|\omega| \sim 1$. In this case we can neglect the last term of Eq. (4.8) since $|c| \ll |a|$. This gives another extremum

$$|\omega| = \left(\frac{n(n-1)}{(n-3)^2} \right)^{1/2}. \quad (4.11)$$

However, for $n > 1$ this solution has $|\omega| > 1$, which is outside the physical region $r > 0$.⁵ It is therefore unphysical for the values of n we are considering. It is also easy to see that this extremum has higher energy than the solution Eq. (4.10).

Combining the results above, we conclude that Eq. (4.10) is in fact the true (global) minimum. In order to cancel the four-dimensional cosmological constant, we note that the term $-9|c|^2$ in Eq. (4.8) dominates the vacuum energy in the solution, so we must fine-tune

$$V_{\text{SB}} \simeq \frac{3|c|^2}{M_4^2}, \quad (4.12)$$

where $M_4^2 = M_5^2/k$. Note that this gives $V_{\text{SB}} > 0$, as desired. We obtain

$$m_{3/2}^2 = \frac{|c|^2}{M_4^4} \sim \frac{V_{\text{SB}}}{M_4^2}. \quad (4.13)$$

The mass of the radion fields at the minimum of the potential is straightforward to work out using the component Lagrangian given above, or in terms of the standard four-dimensional

⁴This assumes that n is not too large. The regime $n \gg 1$ corresponds to $k \ll \xi$, i.e., small bulk curvature. As shown in Ref. [13], the model also stabilizes the radius in this regime.

⁵We are consistently using the convention that the proper distance in the $+\vartheta$ direction is $+r\vartheta$, which fixes $r > 0$.

supergravity potential. Parametrizing the radion by ω greatly simplifies the calculation. We find

$$m_{\text{scalar}}^2 = \frac{|c|^2 (n-2)(n-3)^2}{M_4^4 (n-1)}, \quad m_{\text{pseudoscalar}}^2 = \frac{|c|^2 n(n-3)^2}{M_4^4 (n-1)}. \quad (4.14)$$

Note that $m_{\text{scalar}} \sim m_{\text{pseudoscalar}} \sim m_{3/2}$. The radion is lighter than the KK matching scale Eq. (3.1) provided that $|c|/M_5^3 \ll |\omega|$, which is guaranteed by Eq. (4.10) since $|c| \ll |a| \ll M_5^3$.

We now consider $k < 0$, corresponding to the case that the warp factor is larger than unity on the visible brane. Note that in this case $n < 0$. We again look for solutions where the warp factor is a large effect, so that $|\omega| \gg 1$. We can therefore neglect the first term of Eq. (4.8) compared to the second. Because the factor in front of Eq. (4.8) is now negative, minimizing with respect to the phase γ now gives $\cos \gamma = -\text{sgn}(n-3)$. We then obtain the solution

$$|\omega| = \left(\frac{|n-3| |a|}{3 |c|} \right)^{1/|n|}. \quad (4.15)$$

We see that $|\omega| \gg 1$ provided that $|c| \ll |a|$. Again, Eq. (4.11) is an extremum, as is $|\omega| \rightarrow +\infty$. As before, Eq. (4.11) is outside the physical region, and both Eq. (4.11) and the ‘‘runaway’’ solution $|\omega| \rightarrow +\infty$ have higher energy than the solution Eq. (4.15).

Together, these results imply that Eq. (4.15) is in fact the true (global) minimum. In order to cancel the four-dimensional cosmological constant, we note that the first term in Eq. (4.8) dominates the vacuum energy, and we must fine-tune

$$V_{\text{SB}} \approx \frac{3|c|^2}{M_4^2} \frac{n^2}{(n-3)^2}, \quad (4.16)$$

where $M_4^2 = M_5^3 |\omega|^2 / |k|$. Again $V_{\text{SB}} > 0$ as desired. We find

$$m_{3/2}^2 = \frac{|c|^2 (n-6)^2}{M_4^4 (n-3)^2} \approx \frac{V_{\text{SB}}}{M_4^2}. \quad (4.17)$$

The radion masses are

$$m_{\text{scalar}}^2 = m_{\text{pseudoscalar}}^2 = \frac{|c|^2}{M_4^4} |n|^2 |\omega|^4. \quad (4.18)$$

Note that $m_{\text{scalar}} = m_{\text{pseudoscalar}} \gg m_{3/2}$ in this case. The radion is lighter than the KK matching scale Eq. (3.1) provided only that $|c|/M_5^3 \ll 1$.

We conclude that the simple model we are considering does in fact stabilize the radius modulus in the regime where the warp factor is large, provided only that $|c| \ll |a|$. This works both where the warp factor is largest on the hidden brane and where it is largest on the visible brane. In both cases, the cosmological constant can be canceled by positive vacuum energy from the SUSY breaking sector.

V. DISCUSSION

Let us summarize what has been accomplished. The four-dimensional effective Lagrangian describing the supersymmetric Randall-Sundrum model at long distances was derived. Like the nonsupersymmetric Randall-Sundrum model, it has a vanishing potential for the radius modulus, now a chiral superfield. We also showed that the mechanism proposed in Ref. [13] stabilizes this modulus in the interesting regime where the warp factor is a large effect.

The stabilizing sector consists of two types of supersymmetric Yang-Mills sector, one in the bulk and the other on one of the boundaries, the ‘‘hidden brane.’’ These two sectors become strongly coupled in the infrared, where the dynamics can be controlled using holomorphy in the four-dimensional description. The two resulting nonperturbative gaugino condensates were shown to provide a stabilizing potential for the radius modulus. In order to cancel the effective four-dimensional cosmological constant a source of spontaneous supersymmetry breaking is required. We analyzed the simplest possibility that a supersymmetry breaking sector is also localized on the hidden brane.

The stabilized radius is in the regime where the warp factor effect is large provided that (i) the hidden brane gaugino condensate is small compared to the five-dimensional Planck scale, and (ii) the bulk radius of curvature $1/k$ is not much larger than the bulk super-Yang-Mills coupling g_5^2 . Neither condition requires any fine-tuning. In particular, the first condition is very natural, since the nonperturbative gaugino condensate is exponentially suppressed in terms of the fundamental gauge coupling.

We emphasize that the fact that the radius potential is dominated by nonperturbative super-Yang-Mills dynamics is crucially dependent on supersymmetry. In a nonsupersymmetric theory, there would be perturbative contributions to the radius potential at the compactification scale from Casimir energy that would dominate the exponentially smaller contribution from nonperturbative bulk Yang-Mills dynamics. In our model, these effects are absent because supersymmetry is unbroken at the compactification scale.

A heuristic understanding of how stabilization is achieved in our model is to note that the infrared confinement of the bulk Yang-Mills theory gives a field-theoretic realization of composite extended states in the bulk, namely, the confined hadrons. The spectrum of such extended states is certainly sensitive to the radius and it is not surprising that their virtual effects can generate a radius potential. It is indeed possible that the stabilization role could instead be played by *fundamental* extended objects, in a string or M theory description. A virtue of our mechanism is that it involves only the infrared dynamics of point particles, and is therefore under full theoretical control.

We hope to use the stabilization mechanism presented in this paper as the basis for further studies of supersymmetric and supersymmetry breaking physics in warped compactifications.

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APPENDIX: DERIVATION OF EFFECTIVE THEORY

In this appendix, we give a complete and self-contained derivation of the four-dimensional effective Lagrangian.

1. Matching and heavy tadpoles

We begin by explaining the formalism we will use to integrate out heavy fields at tree level. We consider a general classical and local theory of some light fields $L(x)$ interacting with some heavy fields $H(x)$. We will truncate the effective Lagrangian at two-derivative order, higher derivatives being subdominant at long wavelengths. While x denotes a point in a spacetime of fixed dimensionality (4 in the case of interest) this spacetime need not be exactly flat but may have small curvatures relative to the heavy masses. We will be interested in the case where the heavy fields are an infinite tower of KK states; however, we will suppress indices on the fields since it will be obvious where they go at the end.

Let $S[H, L]$ denote a local classical action that we start with. We will assume (by shifting the definitions of fields if necessary) that $L=H=0$ is a classical solution, and we will expand our theory about this ‘‘vacuum’’ solution. Expanding the action in heavy fields and x derivatives gives

$$S[H, L] = S_{\text{light}}[L] + \int d^4x [\lambda(L)H - \frac{1}{2}M^2(L)H^2 + \Phi(L) \times (\partial L)H + \mathcal{O}(\partial^2 H) + \mathcal{O}(H\partial H) + \mathcal{O}(H^3)]. \quad (\text{A1})$$

$S_{\text{light}}[L]$ consists of the part of the fundamental action that is independent of H ; by assumption the mass terms in S_{light} are small compared to $M^2(L=0)$, the mass scale of the heavy fields. Note that the remaining terms in the action contain terms linear in H , which we call ‘‘heavy tadpole’’ terms. The first two terms in the integral contain all terms linear and quadratic in H but containing no derivatives. The third term contains all terms linear in H with at least one derivative, which by integration by parts can be taken to act only on light fields. The remaining terms contain terms linear in H with two or more derivatives, terms quadratic in H with one or more derivatives, and terms of cubic and higher order in H .

Without loss of generality we can set $\lambda \equiv 0$, by making the field redefinition

$$H \rightarrow H + \frac{1}{M^2(L)}. \quad (\text{A2})$$

Since the fields H are heavy by assumption we can expand this in powers of L , with higher-order terms suppressed by $M^2(L=0)$, the mass scale of the heavy fields. With this choice, the only heavy tadpoles involve derivatives.

The equations of motion for H then read

$$H = \frac{1}{M^2(L)} [\Phi(L)\partial L + \mathcal{O}(\partial^2) + \mathcal{O}(\partial H) + \mathcal{O}(H\partial L) + \mathcal{O}(H^2)]. \quad (\text{A3})$$

[In the $\mathcal{O}(\partial^2)$ terms, the derivatives act on light fields.] This equation can be solved iteratively by expanding in powers of L , starting with the leading order solution

$$H = \frac{\Phi(L)\partial L}{M^2(L)} + \mathcal{O}(\partial^2). \quad (\text{A4})$$

Subleading terms are suppressed by additional powers of $M^2(L)$.

We now substitute the solution for H back into the fundamental action, thereby obtaining an effective action purely for the light fields. To determine the long-wavelength action up to two-derivative order, only the leading order solution Eq. (A4) for H is required. At this order, we therefore obtain

$$S_{\text{eff}}[L] = S_{\text{light}}[L] + \int d^4x \frac{[\Phi(L)\partial L]^2}{2M^2(L)} + \mathcal{O}(\partial^3). \quad (\text{A5})$$

We see that at two-derivative order there is a correction to the naive effective action $S_{\text{light}}[L]$ when the original action has heavy tadpoles with one derivative.

2. Radion effective field theory

We now apply the ideas above to derive the effective Lagrangian for the radion. We parametrize the light modes by generalizing the solution for the metric Eq. (2.3) by $r_0 \rightarrow r(x)$, $\eta_{\mu\nu} \rightarrow g_{\mu\nu}(x)$:

$$ds^2 = e^{-2kr(x)|\vartheta|} g_{\mu\nu}(x) dx^\mu dx^\nu + r^2(x) d\vartheta^2. \quad (\text{A6})$$

Note that $g_{\mu\nu}(x)$ transforms under four-dimensional general coordinate transformations as a two-index tensor, and therefore its couplings in the four-dimensional action are precisely those of the four-dimensional metric. There are no non-derivative couplings of $g_{\mu\nu}$ provided we cancel the four-dimensional cosmological constant. Also note that $r(x)$ is derivatively coupled, since $r(x) = r_0$ is a solution for any constant r_0 . Therefore the action S_{light} obtained by substituting the metric Eq. (A6) into the five-dimensional action does not contain mass terms for the light fields.

We parametrize the heavy modes in terms of the five-dimensional metric

$$ds^2 = e^{-2kr(x)|\vartheta|} [g_{\mu\nu}(x) + H_{\mu\nu}(x, \vartheta)] dx^\mu dx^\nu + 2H_{\vartheta\mu}(x, \vartheta) d\vartheta dx^\mu + r^2(x) [1 + H_{\vartheta\vartheta}(x, \vartheta)] d\vartheta^2. \quad (\text{A7})$$

This must be supplemented with a restriction on $H_{\mu\nu}$ to ensure that it is ‘‘orthogonal’’ to the zero mode $g_{\mu\nu}$, and we must impose a gauge on the fluctuations H_{MN} . The details of this will not be needed for our discussion.

As explained in Sec. 1 of this appendix, the correct effective action at two-derivative order differs from S_{light} if there

are heavy tadpoles containing a single x derivative. By four-dimensional Lorentz invariance, the only terms of this form involve the metric fluctuation $H_{\vartheta\mu}$, e.g., $\partial^\mu r H_{\vartheta\mu}$. Direct calculation shows that this vanishes in the metric Eq. (A7). Therefore, there are no corrections to the effective action at two-derivative order, and the correct effective action is obtained simply by using the metric Eq. (A6). This gives

$$S_{4,\text{eff}} = -\frac{M_5^3}{k} \int d^4x \sqrt{-g} [(1 - e^{-2\pi k r(x)})R(g) + \dots]. \quad (\text{A8})$$

3. The graviphoton

We now turn to the graviphoton B_M . In this case, there is a classical solution $B_\mu \equiv 0$, $B_\vartheta \equiv b_0$ for constant b_0 . In analogy with the radion, we parametrize the light modes by generalizing this solution by $b_0 \rightarrow b(x)$:

$$B_\mu \equiv 0, \quad B_\vartheta(x, \vartheta) = b(x). \quad (\text{A9})$$

In this case there are $\mathcal{O}(\partial_\mu)$ heavy tadpoles involving the massive modes B_μ :

$$S_5 = -M_5^3 \int d^5X \partial_\vartheta [\sqrt{-G} G^{\mu\nu} G^{\vartheta\vartheta} \partial_\nu B_\vartheta] B_\mu + \mathcal{O}(B_\mu^2) + \mathcal{O}(\partial_\mu^2). \quad (\text{A10})$$

Here G_{MN} is the five-dimensional metric Eq. (A6), which includes the light modes $g_{\mu\nu}(x)$ and $r(x)$.

As explained in Sec. 1 of this appendix, the presence of the tadpole Eq. (A10) means that there are corrections to the effective Lagrangian at $\mathcal{O}(\partial_\mu^2)$. We must therefore integrate out the heavy fields B_μ , including the effects of the tadpole in Eq. (A10). The fields B_μ have nonzero KK masses because they are odd under the orbifold \mathbf{Z}_2 ; the mass terms are contained in the $\mathcal{O}(\partial_\vartheta^2 B_\mu^2)$ terms in the action. Including these mass terms and the B_μ tadpole in Eq. (A10), the B_μ equation of motion is

$$\partial_\vartheta \{ e^{-2k|\vartheta|r(x)} [\partial_\vartheta B_\mu(x, \vartheta) - \partial_\mu b(x)] \} = 0. \quad (\text{A11})$$

The solution is

$$e^{-2k|\vartheta|r(x)} [\partial_\vartheta B_\mu(x, \vartheta) - \partial_\mu b(x)] = c_\mu(x) \quad (\text{A12})$$

where $c_\mu(x)$ is independent of ϑ . The function $c_\mu(x)$ is determined by demanding the periodicity of B_μ in ϑ :

$$c_\mu(x) = -2\pi k r(x) \frac{\partial_\mu b(x)}{e^{2\pi k r(x)} - 1}. \quad (\text{A13})$$

We now substitute this back into the action using the result for the graviphoton field strength

$$C_{\vartheta\mu}(x, \vartheta) = e^{+2k|\vartheta|r(x)} c_\mu(x). \quad (\text{A14})$$

In this way, we obtain

$$\Delta S_{4,\text{eff}} = -2\pi^2 M_5^3 k \int d^4x \sqrt{-g} \frac{\partial^\mu b \partial_\mu b}{e^{2\pi k r} - 1}. \quad (\text{A15})$$

4. The radiation supermultiplet

We have derived the low-energy effective theory in terms of $r(x)$ and $g_{\mu\nu}(x)$ [defined by Eq. (A6)] and $b(x)$ [defined by Eq. (A9)]. In a manifestly supersymmetric description, these degrees of freedom can be parametrized by an $\mathcal{N}=1$ supergravity multiplet and a chiral superfield T . We wish to find the relation between the fields $g_{\mu\nu}(x)$, $r(x)$, and $b(x)$, and the components of the supermultiplets in an off-shell supersymmetric formulation. To do this it is useful to couple the five-dimensional theory to various probes, and track how these probes appear in the four-dimensional effective action. Matching the component and manifestly supersymmetric forms of the four-dimensional action gives the relation between the component fields and superfields.

We first couple the SUGRA theory to a bulk SYM multiplet. The additional massless bosonic fields in the four-dimensional effective theory are then the gauge field A_μ and an adjoint scalar Φ . Because these both transform in the adjoint representation of the gauge group, there is no possibility of mixing between the gauge and gravitational modes in the four-dimensional effective theory. The SYM zero modes form a four-dimensional $\mathcal{N}=1$ SYM multiplet. The vector zero mode is given by

$$A_\mu(x, \vartheta) = a_\mu(x). \quad (\text{A16})$$

The fact that the zero mode is independent of the warp factor is due to the classical conformal invariance of Yang-Mills theory.

In the four-dimensional theory effective theory, the gauge kinetic term can be written in the manifestly supersymmetric form

$$\Delta \mathcal{L}_{4,\text{eff}} = \int d^2\theta S(T) \text{tr}(W^\alpha W_\alpha) + \text{H.c.}, \quad (\text{A17})$$

where $S(T)$ is holomorphic. We will make a holomorphic field redefinition $S(T) \rightarrow T/g_5^2$ so that the action becomes

$$\Delta \mathcal{L}_{4,\text{eff}} = \int d^2\theta \frac{T}{g_5^2} \text{tr}(W^\alpha W_\alpha) + \text{H.c.} \quad (\text{A18})$$

Expanding this in components, we see that

$$\frac{T}{g_5^2} = \frac{1}{2g_4^2} + \frac{i\Theta}{16\pi^2} + \dots, \quad (\text{A19})$$

where g_4 is the gauge coupling and Θ is the gauge theta angle. Substituting Eqs. (A6) and (A16) into the five-dimensional SYM action and integrating over ϑ gives

$$\frac{1}{g_4^2} = \frac{2\pi r}{g_5^2}, \quad (\text{A20})$$

which yields

$$\text{Re}(T) = \pi r. \quad (\text{A21})$$

The gauge theta angle gets a contribution from the graviphoton from the five-dimensional SUGRA coupling [17]

$$\Delta \mathcal{L}_5 = -\frac{1}{2\sqrt{6}g_5^2} \epsilon^{MNPQR} B_M \text{tr} F_{NP} F_{QR}, \quad (\text{A22})$$

which gives

$$\frac{\Theta}{16\pi^2} = \frac{2\pi}{\sqrt{6}} b. \quad (\text{A23})$$

We therefore obtain

$$\text{Im}(T) = \frac{2\pi}{\sqrt{6}} b. \quad (\text{A24})$$

5. Supersymmetry and Weyl rescaling

At two-derivative order, the most general locally $\mathcal{N}=1$ supersymmetric Lagrangian for the radion chiral multiplet T can be written

$$\mathcal{L}_{\text{SUGRA},4} = \int d^4\theta \phi^\dagger \phi f(T, T^\dagger) \quad (\text{A25})$$

$$\begin{aligned} &= \sqrt{-\bar{g}} \left[-\frac{1}{6} fR(\bar{g}) \right. \\ &\quad - \frac{1}{4f} \bar{g}^{\mu\nu} (f_T \partial_\mu T - \text{H.c.})(f_T \partial_\nu T - \text{H.c.}) \\ &\quad \left. - f_{T^\dagger T} \bar{g}^{\mu\nu} \partial_\mu T^\dagger \partial_\nu T + \text{fermion terms} \right]. \quad (\text{A26}) \end{aligned}$$

Note that the metric $\bar{g}_{\mu\nu}$ that appears here is not assumed to be the same as the metric $g_{\mu\nu}$ introduced above. The most general relation between them compatible with general coordinate invariance is⁶

$$\bar{g}_{\mu\nu} = h(r, b) g_{\mu\nu}. \quad (\text{A27})$$

The function h is not well defined until we completely fix the definition of $\bar{g}_{\mu\nu}$ in the manifestly supersymmetric theory. We do this by considering a probe consisting of a superpotential term $\int d^2\theta J$ localized on the hidden brane at $\vartheta=0$. In the four-dimensional effective theory, this gives rise to

$$\Delta \mathcal{L}_{4,\text{eff}} = \int d^2\theta \phi^3 \ell(T) J + \text{H.c.}, \quad (\text{A28})$$

where ϕ is the conformal compensator and $\ell(T)$ is holomorphic.

We can now make a field redefinition to set $\ell(T) \equiv 1$. This can be done by means of a Kähler transformation. In the superconformal formalism, this is a redefinition of the conformal compensator

$$\phi' = [\ell(T)]^{1/3} \phi \quad (\text{A29})$$

that is made *prior* to fixing the superconformal gauge Eq. (3.5). That is, we break the superconformal invariance by the choice

$$\phi' = 1 + \theta^2 F'_\phi. \quad (\text{A30})$$

Note that since the gauge kinetic term is classically scale invariant it is independent of ϕ . Therefore this does not affect the field definition made in Eq. (A18). In components, this field redefinition involves a Weyl rescaling of the metric $\bar{g}_{\mu\nu}$, and fixes its definition completely.

With this choice, we now compare the effective action for the brane superpotential to what is obtained by substituting the metric Eq. (A6) into the component form. In the supersymmetric form, the brane action is independent of T , and in the component form it is independent of r and b . This can only be the case if

$$\bar{g}_{\mu\nu} = g^{\mu\nu}. \quad (\text{A31})$$

Having established this, we can read off the function f from Eq. (A8). Note that the graviphoton Aharonov-Bohm phase does not contribute to the coefficient of $R(g)$ in the effective action. Therefore,

$$f = \frac{3M_5^3}{k} (e^{-\pi k(T+T^\dagger)} - 1). \quad (\text{A32})$$

Having determined f , the remaining terms in Eq. (A26) are fixed. With the identification of T in Eqs. (A21) and (A24), we have checked that these terms agree with the direct component calculation of the $(\partial r)^2$ and $(\partial b)^2$ terms. In particular, both the nontrivial functional form and the coefficient of the graviphoton kinetic term Eq. (A15) agree with Eq. (A26) with f given by Eq. (A32).

6. Brane couplings

We now consider arbitrary couplings localized on the hidden brane. In the four-dimensional effective theory, local $\mathcal{N}=1$ SUSY implies that these take the form

$$\begin{aligned} \mathcal{L}_{4,\text{hid}} &= \int d^4\theta \phi^\dagger \phi f_{\text{hid}}(\Sigma, \Sigma^\dagger, T, T^\dagger) \\ &\quad + \int d^2\theta [S_{\text{hid}}(\Sigma, T) \text{tr}(W'^\alpha W'_\alpha) + \phi^3 \mathcal{W}_{\text{hid}}(\Sigma, T)] \\ &\quad + \text{H.c.} \quad (\text{A33}) \end{aligned}$$

In the coordinates we have chosen, the induced metric on the hidden brane is independent of r [see Eq. (A6)]. Therefore, by locality $\mathcal{L}_{4,\text{hid}}$ is independent of r . Since S_{hid} and \mathcal{W}_{hid} are holomorphic, this immediately implies that they are independent of T . For the nonholomorphic function f_{hid} , the argument requires a few steps. Because there are no derivative couplings involving r , we have

⁶By four-dimensional parity, h must be an even function of b .

$$f_{\text{hid}} = c(T - T^\dagger) + \text{terms independent of } T, \quad (\text{A34})$$

where c is a constant. Because the four-dimensional Planck scale is independent of the Aharonov-Bohm phase b_0 , we have $c=0$. Therefore, f_{hid} is also independent of T , and we have

$$\begin{aligned} \mathcal{L}_{4,\text{hid}} = & \int d^4\theta \phi^\dagger \phi f_{\text{hid}}(\Sigma, \Sigma^\dagger) + \int d^2\theta [S_{\text{hid}}(\Sigma) \text{tr}(W'^\alpha W'_\alpha) \\ & + \phi^3 \mathcal{W}_{\text{hid}}(\Sigma)] + \text{H.c.} \end{aligned} \quad (\text{A35})$$

For couplings localized on the visible brane, the induced metric is $e^{-2\pi k r(x)} g_{\mu\nu}(x)$, and the couplings localized on the

visible brane will depend on T . Using arguments similar to those above, it is easy to see that the result is

$$\begin{aligned} \mathcal{L}_{4,\text{vis}} = & \int d^4\theta \phi^\dagger \phi e^{-k(T+T^\dagger)} f_{\text{vis}}(Q, Q^\dagger) \\ & + \int d^2\theta [S_{\text{vis}}(Q) \text{tr} \tilde{W}^\alpha \tilde{W}_\alpha + \phi^3 e^{-3kT} \mathcal{W}_{\text{vis}}(Q)] \\ & + \text{H.c.} \end{aligned} \quad (\text{A36})$$

Summarizing, the full four-dimensional effective Lagrangian is the sum of Eqs. (A25), (A35), and (A36).

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