Localized gravity and mass hierarchy in $D=6$ with a Gauss-Bonnet term

Jihn E. Kim,* Bumseok Kyae,† and Hyun Min Lee‡
Department of Physics and Center for Theoretical Physics, Seoul National University, Seoul 151-747, Korea
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We obtain a 3-brane solution with localized gravity in $D=6$ in the presence of the Gauss-Bonnet term. If the extra dimensions are compactified with the $T^2/(Z_2\times Z_2)$ orbifold symmetry, the mass hierarchy between the Planck scale and the weak scale can be explained by putting our Universe at the positive tension TeV brane located at the orbifold fixed point.

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I. INTRODUCTION

The recent proposals for fundamental TeV scale physics [1,2] have been a great surprise in high energy physics, which has not been noted for a long period of superstring research. Of particular interest is the first Randall-Sundrum (RSI) model [2] in which the warp factor geometry in the extra direction in five-dimensional (5D) spacetime introduces a large exponential suppression factor, enabling one to introduce a TeV scale from the Planck scale with an $O(10)$ ratio of the input parameters, through the compactification of the extra dimension y on S_1/Z_2. There are two branes in the RSI model: brane 1 (B1) located at $y=0$ and brane 2 (B2) located at $y=y_2$.

Probably a more interesting proposal is the second Randall-Sundrum (RSII) model [3] in which only one brane (B1) located at $y=0$ is introduced. Thus, the fifth dimension is not compactified, but still this model can describe meaningful effective four-dimensional (4D) physics since gravity is localized around B1. It is an alternative to the compactification idea of the extra dimension(s). Both Randall-Sundrum models need AdS spacetime in the bulk.

Subsequently, extensions of the RS type models were proposed toward the hierarchy solution [4–7], for the study of localization of gravity [8,10–13,7], and for other aspects [14–16]. In particular, the RSII model has been studied with the aim of finding a self-tuning solution of the cosmological constant problem. This is because, from the beginning of these proposals the solution of the cosmological constant was sought for in the RS models since the Einstein equations can choose a flat space even with a negative nonvanishing bulk cosmological constant and nonvanishing brane tension(s). But in the first proposals, the nonvanishing parameters need to be fine tuned for the Universe to be flat in the model [2,3]. It has been suggested that introduction of a bulk real scalar field with a coupling to the brane may give a self-tuning of the cosmological constant but this retains the serious fine-tuning problem due to a naked singularity [17]. There exists an example of a self-tuning solution with an unconventional interaction of a bulk antisymmetric tensor field [18] which may shed more light toward a final solution of the cosmological constant problem. We note that the final solution must allow inflation, which seems to be needed for the explanation of homegeneity and isotropy of the observed universe [19].

In the intersecting brane world scenarios in higher than five dimensions [8,9], our Universe is regarded as a 3-brane with higher codimensions given by the common intersection of higher dimensional objects with lower codimensions. However, when we consider discrete sources of higher dimensional objects in the bulk space, no additional contribution is allowed from the brane-brane interaction to the tension of their intersection corresponding to the 3-brane tension, since the Einstein tensor just gives rise to a one-dimensional delta function from the intersecting branes. This behavior is well understood in the smooth limit of intersecting branes. For instance, for n orthogonal $(n+2)$-branes in $D=4+n$, each $(n+2)$-brane has the tension $T_{n+2}=M^{n+4}L$ while the tension of their intersection is $T_3=M^{n+4}L^n$ by dimensional analysis. [M is the $(4+n)$-dimensional fundamental scale and L is the brane thickness.] Therefore, the 3-brane tension shows up with a higher power of L, so it becomes suppressed in the thin brane limit, which means that higher curvature terms should be taken into account for better resolution to see such a thin 3-brane. Without nonzero 3-brane tension, it is difficult to discuss the generation of vacuum energy after a phase transition on the intersection as in our world. Because the corresponding nonzero tension of the intersection is not allowed, the vacuum energy induced by a phase transition has no way but, at most, to leak away along the intersecting branes, whose tensions are allowed to be nonzero. In this context, it is necessary that the nonzero brane-brane interaction or the nonzero tension of the intersection should appear in a natural way.

In this paper we consider a RS type solution for the case of two orthogonally intersecting *nonsolitonic* 4-branes[1] and one 3-brane (or string) on their intersection in $D=6$ when the Gauss-Bonnet term is added in the bulk action. In that case, we can regard our world as a common intersection of

*Electronic address: jekim@phyp.snu.ac.kr
†Electronic address: kyae@fire.snu.ac.kr
‡Electronic address: minlee@phya.snu.ac.kr

[1]Here, *solitonic* means supported by gravity only while *nonsolitonic* means supported by sources.

two 4-branes where the localization of gravity arises. In the existence of the Gauss-Bonnet term, in particular, a string tension should be introduced at the beginning to match an additional boundary condition on the intersection. So our solution with two 4-branes and one string is based on two fine-tuning conditions between input parameters, but there is the possibility for naturally regarding the vacuum energy in our world as the string tension in the intersecting brane world scenario, which it was not been possible to get without the Gauss-Bonnet term. Thus, it seems that the higher curvature terms are affected by the inner structure of the intersecting branes while the Einstein-Hilbert term has lower resolution.

For the special relation between the bulk cosmological constant and the Gauss-Bonnet coupling in our model, it is shown that there exists a string solution with codimension 2 by considering the $Z_2\times Z_2$ symmetry of the extra dimensions as usually imposed in the case of the two orthogonal 4-branes. In that case, the bulk space is found to be a discrete patch of the pure AdS_6 space to make the bulk symmetry manifest and the resultant discontinuities of the derivative of the metric across the symmetry axes are shown to be automatically canceled between those derived from the Einstein-Hilbert term and from the Gauss-Bonnet term in the equations of motion without the need to introduce 4-branes along the symmetry axes. In other words, it is shown that the Einstein-Gauss-Bonnet (EGB) gravity itself is able to support singularities produced on orbifolding without the need to introduce additional *nonsolitonic* singular sources. From the point of view of Einstein's gravity, however, the singularities are interpreted as the so called *solitonic* 4-branes [16], of which the tensions are determined by the Gauss-Bonnet coupling and the 3-brane tension. Nonetheless, since the solitonic 4-branes are supported by gravity only without sources in $D=6$, they do not give any fine-tuning conditions. Therefore, on patching the AdS_6 bulk in the $Z_2\times Z_2$ invariant way, there exists a solution of a string residing on the intersection of two solitonic 4-branes, which is based on one fine tuning condition between bulk parameters but for which the 3-brane cosmological constant Λ_1 can take any positive value without being involved in any fine-tuning. In particular, it is interesting to see that the confinement of gravity arises for the solitonic 4-branes, which results in exactly two copies of the 5D RSII model in $D=6$.

For the string solution with codimension 2 in $D=6$, it has been shown that the singular global string solution is possible with a massless scalar field in the flat bulk by the unitarity boundary condition at the singularity [4]. Later, it was pointed out that there exist regular global string solutions obtained by introducing a bulk cosmological constant [11,12,7]. One more interesting observation is that the local string defects were shown to have localized gravity with no fine tuning of the bulk cosmological constant, but here the components of the string tension are required to satisfy a certain relation [13], which is a fine tuning. Pertinent to our study in this paper, we note the work of Corradini and Kakushadze, in which it has been argued that it is possible to have localized gravity on a solitonic 3-brane with the Gauss-Bonnet term while freely choosing the brane cosmological constant equivalent to a deficit angle in the extra polar coordinate in the fifth and the sixth spaces [20]. (Note that a similar result is known in the case with 3-brane sources in 6D Einstein gravity without a bulk cosmological constant [21] and with a positive bulk cosmological constant [22].) This solution has one fine-tuning condition between bulk parameters and there exists a conical singularity corresponding to the brane tension [20].

Based on our string solution in the intersecting brane scenario, we can compactify the extra dimensions with the $T^2/(Z_2\times Z_2)$ orbifold symmetry. Then, we can show that the hierarchy problem can be solved if we put the branes at the four fixed points of the orbifold $T^2/(Z_2\times Z_2)$ and the two neighboring 3-branes are connected to each other by one 4-brane. In this case, the positive tension brane diagonally far away from the origin of the extra dimension is regarded as our Universe and some other three 3-branes as the hidden branes.

In Sec. II, we obtain a 3-brane (or string) solution in EGB theory. It is the most relevant generalization of the RSII model. *Solitonic* 4-brane solutions appears. In Sec. III, we consider the metric perturbation near the background geometry and ensure that there is no tachyonic mode of the graviton. Then, in Sec. IV, we discuss the gravity confinement of the solitonic 4-branes. In Sec. V, we compactify 6D with the $T^2/(Z_2\times Z_2)$ orbifold symmetry and obtain four fixed points where 3-brane sources can be placed. It is the most relevant generalization of the RSI model in which a TeV 3-brane can occur naturally. Section VI is a conclusion.

II. LOCALIZED GRAVITY ON A 3-BRANE IN 6D

If we impose the Z_2 symmetry on each extra dimension in a $(D=n+4)$-dimensional generalization of the RS model, we should have $(n+2)$-branes orthogonally intersecting each other to match the boundary conditions of the metric [8,9]. Therefore, the 3-brane as our Universe appears only as the common intersection of all the $(n+2)$-branes [8], but without its tension. However, in the presence of the Gauss-Bonnet term, from which no higher than second derivatives are derived in the equations of motion, the intersection of two orthogonal 4-branes in $D=6$ is required to have a nonzero tension; this will be shown below.

When the Gauss-Bonnet term is added as the next leading-order ghost-free interaction to the Einstein-Hilbert term in 6 dimensions (6D) with two spacelike extra dimensions, we start with the Einstein-Gauss-Bonnet 6D action with singular brane sources,

$$S_6=\int d^4x dz_1 dz_2\sqrt{-g}\left[\frac{M^4}{2}R-\Lambda_b+\frac{1}{2}\alpha M^2(R^2 -4R_{MN}R^{MN}+R_{MNPQ}R^{MNPQ})\right]+\int d^4x dz_2 \times\sqrt{-g^{(z_1=0)}}(-\Lambda_{z_1})+\int d^4x dz_1\sqrt{-g^{(z_2=0)}}(-\Lambda_{z_2}) +\int d^4x\sqrt{-g^{(z_1=0,z_2=0)}}(-\Lambda_1) \tag{1}$$

where $g, g^{(z_1=0)}, g^{(z_2=0)}$, and $g^{(z_1=0,z_2=0)}$ are the determinants of the metrics in the bulk, orthogonally intersecting 4-branes and a 3-brane, M is the six-dimensional gravitational constant, and Λ_b, $\Lambda_{z_1}, \Lambda_{z_2}$, and Λ_1 are the bulk and the brane cosmological constants, α is the effective coupling. We considered the 4-branes to write down general equations of motion, but we will see later that there is a possibility of getting the string solution without these 4-brane sources by imposing the $Z_2 \times Z_2$ symmetry in the bulk.

The equations of motion in this EGB theory are

$$G_{MN}+H_{MN}=M^{-4}T_{MN}. \tag{2}$$

The tensors in the above equation are

$$G_{MN}\equiv R_{MN}-\frac{1}{2}g_{MN}R, \tag{3}$$

$$H_{MN}\equiv \frac{\alpha}{M^2}\bigg[-\frac{1}{2}g_{MN}(R^2-4R^2_{PQ}+R_{PQST}R^{PQST})+2RR_{MN} -4R_{MP}R_N{}^P-4R^K{}_{MPN}R_K{}^P+2R_{MQSP}R_N{}^{QSP}\bigg], \tag{4}$$

$$T_{MN}\equiv -\Lambda_b g_{MN}-\frac{\sqrt{-g^{(z_1=0)}}}{\sqrt{-g}}\Lambda_{z_1}\delta(z_1)\,\delta^p_M\delta^q_N g^{(z_1=0)}_{pq} -\frac{\sqrt{-g^{(z_2=0)}}}{\sqrt{-g}}\Lambda_{z_2}\delta(z_2)\,\delta^a_M\delta^b_N g^{(z_2=0)}_{ab} -\frac{\sqrt{-g^{(z_1=0,z_2=0)}}}{\sqrt{-g}}\Lambda_1\delta(z_1)\,\delta(z_2)\,\delta^\mu_M\delta^\nu_N g^{(z_1=0,z_2=0)}_{\mu\nu}, \tag{5}$$

where the indices $M,N=(0,1,2,3,5,6)$, $p,q=(0,1,2,3,6)$, $a,b=(0,1,2,3,5)$, and $\mu,\nu=(0,1,2,3)$.

Taking the metric ansatz as a conformally flat one in 6D, which is manifestly 4D Poincaré invariant,

$$ds_6^2=A^2(z_1,z_2)(\eta_{\mu\nu}dx^\mu dx^\nu+dz_1^2+dz_2^2), \tag{6}$$

where $(\eta_{\mu\nu})=\mathrm{diag}(-1,+1,+1,+1)$, we obtain the tensor components G_{MN} and H_{MN} as follows:

$$G_\mu{}^\nu=\frac{2}{A^2}\left[\left(\frac{A'}{A}\right)^2+\left(\frac{\dot A}{A}\right)^2+2\frac{A''}{A}+2\frac{\ddot A}{A}\right]\delta^\nu_\mu, \tag{7}$$

$$G_5{}^5=\frac{2}{A^2}\left[5\left(\frac{A'}{A}\right)^2+\left(\frac{\dot A}{A}\right)^2+2\frac{\ddot A}{A}\right], \tag{8}$$

$$G_5{}^6=\frac{4}{A^2}\left[-\frac{\dot A'}{A}+2\frac{\dot A A'}{A^2}\right], \tag{9}$$

$$G_6{}^6=\frac{2}{A^2}\left[5\left(\frac{\dot A}{A}\right)^2+\left(\frac{A'}{A}\right)^2+2\frac{A''}{A}\right], \tag{10}$$

and

$$H_\mu{}^\nu=-\frac{12\alpha}{M^2}\frac{1}{A^4}\bigg\{-3\left[\left(\frac{A'}{A}\right)^2+\left(\frac{\dot A}{A}\right)^2\right]^2+4\left(\frac{A'}{A}\right)^2\frac{A''}{A} +4\left(\frac{\dot A}{A}\right)^2\frac{\ddot A}{A}+2\frac{A''\ddot A}{A^2}-2\frac{\dot A'}{A}\left(\frac{\dot A'}{A}-4\frac{\dot A A'}{A^2}\right)\bigg\}\delta^\nu_\mu, \tag{11}$$

$$H_5{}^5=\frac{12\alpha}{M^2}\frac{1}{A^4}\bigg\{-2\left(\frac{A'}{A}\right)^2\left(\frac{\dot A}{A}\right)^2-5\left(\frac{A'}{A}\right)^4+3\left(\frac{\dot A}{A}\right)^4 -4\left[\left(\frac{A'}{A}\right)^2+\left(\frac{\dot A}{A}\right)^2\right]\frac{\ddot A}{A}\bigg\}, \tag{12}$$

$$H_5{}^6=-\frac{48\alpha}{M^2}\frac{1}{A^4}\left[\left(\frac{A'}{A}\right)^2+\left(\frac{\dot A}{A}\right)^2\right]\left(-\frac{\dot A'}{A}+2\frac{\dot A A'}{A^2}\right), \tag{13}$$

$$H_6{}^6=\frac{12\alpha}{M^2}\frac{1}{A^4}\bigg\{-2\left(\frac{A'}{A}\right)^2\left(\frac{\dot A}{A}\right)^2-5\left(\frac{\dot A}{A}\right)^4+3\left(\frac{A'}{A}\right)^4 -4\left[\left(\frac{A'}{A}\right)^2+\left(\frac{\dot A}{A}\right)^2\right]\frac{A''}{A}\bigg\} \tag{14}$$

where the prime and the overdot denote the derivatives with respect to z_1 and z_2, respectively. The energy momentum tensor T_{MN} is given by

$$T_M{}^N=-\Lambda_b\delta^N_M-\frac{1}{A}\Lambda_{z_1}\delta(z_1)\,\delta^p_M\delta^N_q\delta^q_p-\frac{1}{A}\Lambda_{z_2}\delta(z_2)\,\delta^a_M\delta^N_b\delta^b_a -\frac{1}{A^2}\Lambda_1\delta(z_1)\,\delta(z_2)\,\delta^\mu_M\delta^N_\nu\delta^\nu_\mu. \tag{15}$$

Then, the (56) component of the modified Einstein's equations is

$$\frac{4}{A^2}\left\{1-\frac{12\alpha}{M^2}\frac{1}{A^2}\left[\left(\frac{A'}{A}\right)^2+\left(\frac{\dot A}{A}\right)^2\right]\right\}\left(-\frac{\dot A'}{A}+2\frac{\dot A A'}{A^2}\right)=0. \tag{16}$$

Therefore, to ensure that the above equation is satisfied, we require that the second factor vanishes,

$$-\frac{\dot A'}{A}+2\frac{\dot A A'}{A^2}=0; \tag{17}$$

i.e., the general solution of the metric is given by

$$A(z_1,z_2)\propto\frac{1}{[F(z_1)+G(z_2)]} \tag{18}$$

where F and G are undetermined functions of z_1 and z_2, respectively. Note that in the case of a vanishing first factor in Eq. (16), Eq. (17) is automatically satisfied. To determine the exact solution of the above type, we can rewrite the (00) [or (ii)], (55), and (66) components under the condition Eq. (17), respectively:

$$E+e_1+e_2+e_3=M^{-4}\left[-\Lambda_b-\frac{1}{A}\Lambda_{z_1}\delta(z_1)-\frac{1}{A}\Lambda_{z_2}\delta(z_2)-\frac{1}{A^2}\Lambda_1\delta(z_1)\delta(z_2)\right], \tag{19}$$

$$E+e_2=M^{-4}\left[-\Lambda_b-\frac{1}{A}\Lambda_{z_2}\delta(z_2)\right], \tag{20}$$

$$E+e_1=M^{-4}\left[-\Lambda_b-\frac{1}{A}\Lambda_{z_1}\delta(z_1)\right], \tag{21}$$

where

$$E=10\left\{1-\frac{6\alpha}{M^2}\frac{1}{A^2}\left[\left(\frac{A'}{A}\right)^2+\left(\frac{\dot{A}}{A}\right)^2\right]\right\}\frac{1}{A^2}\left[\left(\frac{A'}{A}\right)^2+\left(\frac{\dot{A}}{A}\right)^2\right], \tag{22}$$

$$e_1=\frac{4}{A}\left(\frac{A'}{A^2}\right)'\left\{1-\frac{12\alpha}{M^2}\frac{1}{A^2}\left[\left(\frac{A'}{A}\right)^2+\left(\frac{\dot{A}}{A}\right)^2\right]\right\}, \tag{23}$$

$$e_2=\frac{4}{A}\left(\frac{\dot{A}}{A^2}\right)^{\cdot}\left\{1-\frac{12\alpha}{M^2}\frac{1}{A^2}\left[\left(\frac{A'}{A}\right)^2+\left(\frac{\dot{A}}{A}\right)^2\right]\right\}, \tag{24}$$

$$e_3=-\frac{24\alpha}{M^2}\frac{1}{A^2}\left(\frac{A'}{A^2}\right)'\left(\frac{\dot{A}}{A^2}\right)^{\cdot}. \tag{25}$$

Thus, the bulk equation in all the above components, $E=-\Lambda_b/M^4$, can be solved only if $F(z_1)=k_1z_1+c_1$ and $G(z_2)=k_2z_2+c_2$ (c_1,c_2 are integration constants), i.e.,

$$A(z_1,z_2)=\frac{1}{(k_1|z_1|+k_2|z_2|+1)}, \tag{26}$$

where the Z_2 symmetry is used along each extra dimension and the integration constants are arbitrarily chosen for A to be 1 at $(z_1,z_2)=(0,0)$. k_1,k_2 are determined by the following relations:

$$k_1^2+k_2^2=\frac{M^2}{12\alpha}\left[1\pm\sqrt{1+\frac{12\alpha\Lambda_b}{5M^6}}\right]\equiv k_\pm^2, \tag{27}$$

$$k_1\left(1-\frac{12\alpha k_\pm^2}{M^2}\right)=\frac{\Lambda_{z_1}}{8M^4}, \tag{28}$$

$$k_2\left(1-\frac{12\alpha k_\pm^2}{M^2}\right)=\frac{\Lambda_{z_2}}{8M^4}, \tag{29}$$

$$\alpha k_1k_2=\frac{\Lambda_1}{96M^2}, \tag{30}$$

where the last three equations are derived from the boundary conditions on the branes in Eqs. (19)–(21). The first and fourth equations determine k_1 and k_2 in terms of α, Λ_b, and Λ_1, and they should be such that $|\Lambda_1|\leqslant 48|\alpha|k_\pm^2M^2$, where the equality implies the existence of exchange symmetry between two extra dimensions, and $\mathrm{sgn}(\Lambda_1)=\mathrm{sgn}(\alpha)$ to give real solutions for k_1 and k_2. Then, the second and third equations give rise to two fine-tuning conditions between input parameters. Note that the Gauss-Bonnet term requires an additional condition, Eq. (30), on the 3-brane other than those the Einstein-Hilbert action imposes on the 4-branes, Eqs. (28) and (29).

However, if we chose a relation between bulk parameters from the beginning,

$$\frac{12\alpha\Lambda_b}{5M^6}=-1, \tag{31}$$

such that $k_\pm^2=M^2/12\alpha$ for $\alpha>0$, *nonsolitonic* 4-brane tensions would not be allowed to exist, viz., Eqs. (28) and (29). Then, the 3-brane tension Λ_1 can take any positive values without being involved in any fine-tuning relations. In this case, the remaining equations (27) and (30) just determine k_1 and k_2 in terms of α and Λ_1. This particular point in the solution space is made possible only with the addition of the Gauss-Bonnet term, but is not possible with the Einstein-Hilbert term alone.[2] In other words, on patching the bulk space in a $Z_2\times Z_2$ symmetric way as shown in the chosen metric, we naturally obtain a string solution via the cancellation between those terms derived from the Einstein-Hilbert term and the Gauss-Bonnet term in the equations of motion. However, from the point of view of Einstein's gravity, singularities on orbifolding should be seen to stem from *solitonic* 4-brane tensions, just as in Iglesias and Kakushadze's method [16]. In our case, the solitonic 4-brane tensions f_1 (f_2) located at $z_1=0$ ($z_2=0$) are determined to be positive as

$$f_1=8k_1M^4, \quad f_2=8k_2M^4 \tag{32}$$

where k_1 and k_2 are given by solving Eqs. (27) and (30) under the condition Eq. (31).

[2]In the extension of the RS model with one extra timelike dimension in $D=6$ [23], it is shown that there exists a 3-brane solution as a common intersection of two 4-branes with no fine tuning of the cosmological constant if the exchanging symmetry $y'\leftrightarrow t'$ is assumed between the extra space and time coordinates. However, in the existence of the Gauss-Bonnet term, there arises a fine tuning from the necessity for the 3-brane to match the boundary condition.

Then, after integrating the extra dimensions with the 4D part of the metric as $\overline{g}_{\mu\nu}(x)=\eta_{\mu\nu}$ in Eq. (6), we obtain the 4D effective action as follows:

$$S_{eff}=\frac{M_{P,eff}^2}{2}\int d^4x\sqrt{-\overline{g^{(4)}}}\left[\overline{R}+\frac{\alpha_{eff}}{M_{P,eff}^2}(\overline{R}^2-4\overline{R}_{\mu\nu}^2+\overline{R}_{\mu\nu\rho\sigma}^2)\right] \tag{33}$$

where the 4D Planck mass and the 4D Gauss-Bonnet coupling are given by

$$\begin{aligned}M_{P,eff}^2&=M^4\int_{-\infty}^{\infty}dz_1\int_{-\infty}^{\infty}dz_2\left(A^4\left\{1+\frac{12\alpha}{M^2}\frac{1}{A^2}\left[\left(\frac{A'}{A}\right)^2+\left(\frac{\dot{A}}{A}\right)^2\right]\right\}-\frac{12\alpha}{M^2}[(AA')'+(A\dot{A})^{\cdot}]\right)\\&=\frac{2M^4}{3k_1k_2}\left(1+\frac{12\alpha k_\pm^2}{M^2}\right)\\&=\frac{64\alpha M^6}{\Lambda_1}\left(1+\frac{12\alpha k_\pm^2}{M^2}\right)\geqslant\frac{4M^4}{3k_\pm^2}\left(1+\frac{12\alpha k_\pm^2}{M^2}\right),\end{aligned} \tag{34}$$

$$\alpha_{eff}=\alpha M^2\int_{-\infty}^{\infty}dz_1\int_{-\infty}^{\infty}dz_2A^2 \tag{35}$$

where the $(AA')'$ and $(A\dot{A})^{\cdot}$ terms in the first equality of Eq. (34) vanish after integration. For a negative Gauss-Bonnet coupling α, the 4D Planck mass will not be positive definite due to the contribution from the Gauss-Bonnet term. Therefore, the positivity condition gives $|\alpha|<M^2/12k_\pm^2$ for $\alpha<0$ and any value for $\alpha>0$. On the other hand, the 4D Gauss-Bonnet coupling is shown to become logarithmically divergent after integration. This seems to be a generic feature of higher curvature terms, which is rephrased as the delocalization of gravity in warped geometry [15]. Nonetheless, no problem arises in our case since the Gauss-Bonnet term is a total derivative in $D=4$ and thus it does not modify the equation of motion for a graviton in 4D spacetime. Therefore, we can drop the 4D Gauss-Bonnet term in Eq. (33) to get the 4D effective Einstein gravity [15].

III. METRIC PERTURBATION NEAR THE BACKGROUND GEOMETRY

Now that we have obtained the background solution, it is of interest to examine the perturbation effects of gravity near the background solution. Since the effects inform us how the gravitational interaction between matter is described at low energy scales under a background geometry, it is indispensable to study the perturbative expansion and compare it with the well-known gravitational interaction. The perturbation in higher dimensional spacetime is usually interpreted as the graviton in the corresponding spacetime dimension, and is, in the six-dimensional case, decomposed into a four-dimensional graviton, two kinds of vector, and three kinds of scalar. In this section, however, we assume that the vector and scalar modes are decoupled by some physics due to their absence at the low energy scale, and we focus on the gravitational interaction mediated by the four-dimensional graviton.

Thus, for the study, let us assume the metric to be the following:

$$ds^2=[A^2(z_1,z_2)\eta_{\mu\nu}+h_{\mu\nu}(x,z_1,z_2)]dx^\mu dx^\nu+A^2(z_1,z_2)\times(dz_1^2+dz_2^2) \tag{36}$$

$$=A^2(z_1,z_2)[(\eta_{\mu\nu}+\tilde{h}_{\mu\nu}(x,z_1,z_2))dx^\mu dx^\nu+dz_1^2+dz_2^2], \tag{37}$$

where x denotes the four-dimensional coordinate, and we would keep the linear parts in $h_{\mu\nu}$ in the full expression of the Einstein equation. Here, $A(z_1,z_2)$ is the background solution given by Eq. (26) and $h_{\mu\nu}$ represents a small perturbation near it. With Eq. (36), the linearized variations for $G_{\mu\nu}$, $H_{\mu\nu}$, and $T_{\mu\nu}$ are given by

$$\delta G_{\mu\nu}=-\frac{1}{2}\left[\frac{1}{A^2}\Box_4+\frac{1}{A^2}(\partial_{z_1}^2+\partial_{z_2}^2)-26(k_1^2+k_2^2)+\frac{20}{A}(k_1\delta(z_1)+k_2\delta(z_2))\right]h_{\mu\nu}, \tag{38}$$

$$\begin{aligned}\delta H_{\mu\nu}=\frac{\alpha}{M^2}\Bigg[&\frac{1}{A^2}\left(6(k_1^2+k_2^2)-\frac{8k_1}{A}\delta(z_1)-\frac{8k_2}{A}\delta(z_2)\right)\Box_4+\frac{1}{A^2}\left(6(k_1^2+k_2^2)-\frac{8k_2}{A}\delta(z_2)\right)\partial_{z_1}^2\\&+\frac{1}{A^2}\left(6(k_1^2+k_2^2)-\frac{8k_1}{A}\delta(z_1)\right)\partial_{z_2}^2+\frac{8k_1}{A}\left(\frac{3k_1}{A}\delta(z_1)-\frac{k_2}{A}\delta(z_2)\right)\mathrm{sgn}(z_1)\partial_{z_1}+\frac{8k_2}{A}\left(\frac{3k_2}{A}\delta(z_2)-\frac{k_1}{A}\delta(z_1)\right)\mathrm{sgn}(z_2)\partial_{z_2}\\&-96(k_1^2+k_2^2)^2+\frac{k_1}{A}\delta(z_1)(168k_1^2+152k_2^2)+\frac{k_2}{A}\delta(z_2)(168k_2^2+152k_1^2)-160\frac{k_1k_2}{A^2}\delta(z_1)\delta(z_2)\Bigg]h_{\mu\nu},\end{aligned} \tag{39}$$

$$\delta T_{\mu\nu}=-\Lambda_b h_{\mu\nu}-\frac{1}{A}\Lambda_{z_1}\delta(z_1)h_{\mu\nu}-\frac{1}{A}\Lambda_{z_2}\delta(z_2)h_{\mu\nu}-\frac{1}{A^2}\Lambda_1\delta(z_1)\delta(z_2)h_{\mu\nu}, \tag{40}$$

where $\Box_4\equiv\eta^{\mu\nu}\partial_\mu\partial_\nu$, and we choose the traceless transverse gauge conditions $\partial^\mu h_{\mu\nu}=h^\mu{}_\mu=0$.

The above expressions lead to the linearized Einstein equation

$$\begin{aligned}&-\frac{1}{2A^2}\left(1-\frac{12\alpha}{M^2}(k_1^2+k_2^2)\right)[\Box_4+\partial_{z_1}^2+\partial_{z_2}^2-6A^2(k_1^2+k_2^2)]h_{\mu\nu}-\delta(z_1)\left[\frac{8\alpha}{M^2}\frac{k_1}{A}\left(\frac{1}{A^2}(\Box_4+\partial_{z_2}^2)+\frac{k_2}{A}\mathrm{sgn}(z_2)\partial_{z_2}\right.\right.\\&\left.\left.-\frac{3k_1}{A}\mathrm{sgn}(z_1)\partial_{z_1}\right)+\frac{k_1}{A}\left(10-\frac{\alpha}{M^2}(168k_1^2+152k_2^2)\right)-\frac{1}{A}\frac{\Lambda_{z_1}}{M^4}\right]h_{\mu\nu}-\delta(z_2)\left[\frac{8\alpha}{M^2}\frac{k_2}{A}\left(\frac{1}{A^2}(\Box_4+\partial_{z_1}^2)+\frac{k_1}{A}\mathrm{sgn}(z_1)\partial_{z_1}\right.\right.\\&\left.\left.-\frac{3k_2}{A}\mathrm{sgn}(z_2)\partial_{z_2}\right)+\frac{k_2}{A}\left(10-\frac{\alpha}{M^2}(168k_2^2+152k_1^2)\right)-\frac{1}{A}\frac{\Lambda_{z_2}}{M^4}\right]h_{\mu\nu}-\delta(z_1)\delta(z_2)\left[\frac{160\alpha}{M^2}\frac{k_1k_2}{A^2}-\frac{1}{A^2}\frac{\Lambda_1}{M^4}\right]h_{\mu\nu}=0,\end{aligned} \tag{41}$$

where we use Eq. (27). The above equation for $h_{\mu\nu}$ is simplified in the conformal coordinate,

$$\begin{aligned}&-\frac{1}{2}\left(1-\frac{12\alpha}{M^2}k_\pm^2\right)[\Box_4+\partial_{z_1}^2+\partial_{z_2}^2-4A\{k_1\,\mathrm{sgn}(z_1)\partial_{z_1}+k_2\,\mathrm{sgn}(z_2)\partial_{z_2}\}]\tilde{h}_{\mu\nu}-\frac{\delta(z_1)}{A}\left[\frac{8\alpha}{M^2}k_1(\Box_4+\partial_{z_2}^2-3A\{k_1\,\mathrm{sgn}(z_1)\partial_{z_1}\right.\\&\left.+k_2\,\mathrm{sgn}(z_2)\partial_{z_2}\})+A^2\left(8k_1\left\{1-\frac{12\alpha}{M^2}k_\pm^2\right\}-\frac{\Lambda_{z_1}}{M^4}\right)\right]\tilde{h}_{\mu\nu}-\frac{\delta(z_2)}{A}\left[\frac{8\alpha}{M^2}k_2[\Box_4+\partial_{z_1}^2-3A\{k_2\,\mathrm{sgn}(z_2)\partial_{z_2}+k_1\,\mathrm{sgn}(z_1)\partial_{z_1}\}]\right.\\&\left.+A^2\left(8k_1\left\{1-\frac{12\alpha}{M^2}k_\pm^2\right\}-\frac{\Lambda_{z_2}}{M^4}\right)\right]\tilde{h}_{\mu\nu}-\delta(z_1)\delta(z_2)\left[\frac{96\alpha}{M^2}k_1k_2-\frac{\Lambda_1}{M^4}\right]\tilde{h}_{\mu\nu}=0,\end{aligned} \tag{42}$$

where $\tilde{h}_{\mu\nu}$ is defined in Eq. (37).

The bulk contribution in the above equation comes only from the first term of Eq. (41). The second and third parts of Eq. (41) and (42) describe the behavior of the graviton on the corresponding 4-brane, and the last part of Eq. (42) just gives a boundary condition of $h_{\mu\nu}$ at the origin (i.e., at the 3-brane), which is consistent with Eq. (30). In general, the bulk equations, the first part of Eq. (41) [or Eq. (42)], cannot be solved easily, but the solution for the massless mode is trivial. If we assume $\partial_{z_1}\tilde{h}_{\mu\nu}=\partial_{z_2}\tilde{h}_{\mu\nu}=0$ and put the background relations Eqs. (28)–(30) into the above equation, we obtain

$$\Box_4\tilde{h}^0_{\mu\nu}(x)=0. \tag{43}$$

Hence, the massless graviton has the following profile in the bulk:

$$h^0_{\mu\nu}(x,z_1,z_2)=A^2(z_1,z_2)\tilde{h}^0_{\mu\nu}(x)=A^2(z_1,z_2)\epsilon_{\mu\nu}e^{ipx}, \tag{44}$$

where ϵ is the polarization tensor of the four-dimensional graviton.

As the effective four-dimensional theory would be described by the massless graviton predominantly, let us calculate approximately the effective four-dimensional Planck mass $M_{P,eff}$. After integrating the extra dimensions with the 4D part of the metric as $\tilde{g}_{\mu\nu}(x)\equiv\eta_{\mu\nu}+\tilde{h}_{\mu\nu}$ in Eq. (6), we obtain the 4D effective action as follows:

$$S_{eff}=\frac{M_{P,eff}^2}{2}\int d^4x\sqrt{-\tilde{g}^{(4)}}[\tilde{R}+\cdots], \tag{45}$$

where $\tilde{R}$ is the 4D Ricci scalar. The 4D Planck mass is calculated by reading off the coefficients of $\Box_4$ in Eq. (41) or Eq. (42) and integrating those with respect to z_1 and z_2,

$$\begin{aligned}M_{P,eff}^2&=M^4\int_{-\infty}^{\infty}dz_1\int_{-\infty}^{\infty}dz_2A^4\left[1-\frac{12\alpha}{M^2}k_\pm^2\right.\\&\left.+\frac{1}{A}\frac{16\alpha}{M^2}[k_1\delta(z_1)+k_2\delta(z_2)]\right]\\&=\frac{2M^4}{3k_1k_2}\left(1+\frac{12\alpha k_\pm^2}{M^2}\right),\end{aligned} \tag{46}$$

which gives a finite value. Therefore, we can explain gravitational interactions consistently even in the noncompact six spacetime dimensions. Note that our effective 4D Planck mass obtained above from the Einstein equation is the same as the one obtained from the action itself by integrating out z_1 and z_2, as given in Eq. (34).

In case of the absence of 4-branes, i.e., $k_1^2+k_2^2=M^2/(12\alpha)$, the bulk kinetic term in Eq. (41) or Eq. (42) does not contribute to the linearized Einstein equation and thus the graviton is not allowed to propagate in the bulk. But through higher order terms in the $h_{\mu\nu}$ expansion a certain "gravity interaction" could exist in the bulk even though the mediating particle cannot be defined as the graviton.

Now let us discuss the Kaluza-Klein (KK) modes of the graviton. We will get a bulk solution first using Eq. (41) or Eq. (42), and then apply the boundary conditions with the delta functions in the above equations. Equation (41) is easier to treat rather than Eq. (42) because the former does not have any first derivative terms in the bulk equation. It is possible to separate the variables $h_{\mu\nu}(x,z_1,z_2)=\psi(z_1,z_2)e^{ip\cdot x}\epsilon_{\mu\nu}$, where x^μ and p^μ are the 4D coordinate and momentum, respectively. Then, the bulk part of Eq. (41), which is a two-dimensional differential equation, is

$$\left[-\partial_{z_1}^2-\partial_{z_2}^2+\frac{6(k_1^2+k_2^2)}{(k_1|z_1|+k_2|z_2|+1)^2}\right]\psi(z_1,z_2)=m^2\psi(z_1,z_2), \tag{47}$$

where $p^2=-m^2$. To separate the bulk variables, let us introduce a new coordinate (s,t),

$$s\equiv k_1|z_1|+k_2|z_2|+1,$$

$$t\equiv k_2|z_1|-k_1|z_2|+1. \tag{48}$$

Then Eq. (47) becomes

$$(k_1^2+k_2^2)\left[-\partial_s^2-\partial_t^2+\frac{6}{s^2}\right]\hat{\psi}(s,t)=m^2\hat{\psi}(s,t), \tag{49}$$

where $\hat{\psi}(s,t)\equiv\psi(z_1,z_2)$. It is separable as

$$\left[-\partial_s^2+\frac{6}{s^2}\right]\phi_s(s)=m_s^2\phi_s(s) \tag{50}$$

$$-\partial_t^2\phi_t(t)=m_t^2\phi_t(t), \tag{51}$$

where $\phi_s(s)$, $\phi_t(t)$, m_s^2, and m_t^2 are defined as

$$\hat{\psi}(s,t)=\phi_s(s)\phi_t(t),$$

$$\frac{m^2}{(k_1^2+k_2^2)}=m_s^2+m_t^2. \tag{52}$$

From Eqs. (50) and (51), we can see that m_s^2, m_t^2, and so m^2 should be positive definite, because they could be regarded as a "Hamiltonian" in quantum mechanics, and have positive and flat "potentials," respectively. Hence, they have positive "energies" or eigenvalues. Thus we conclude that *there do not exist any tachyonic KK modes*.

Equations (50) and (51) are easily solved and have the following solutions:

$$\begin{aligned}\phi_s(s)&=c_1\sqrt{s}J_{5/2}(m_s s)+c_2\sqrt{s}Y_{5/2}(m_s s)\\&=\sqrt{\frac{2}{\pi m_s}}\left\{c_1\left[\left(\frac{3}{(m_s s)^2}-1\right)\sin(m_s s)-\frac{3}{m_s^2 s^2}\cos(m_s s)\right]+c_2\left[\frac{3}{m_s s}\sin(m_s s)+\left(\frac{3}{(m_s s)^2}-1\right)\cos(m_s s)\right]\right\},\end{aligned} \tag{53}$$

$$\phi_t(t)=d_1\sin(m_t t)+d_2\cos(m_t t), \tag{54}$$

where $J_{5/2}$ and $Y_{5/2}$ are Bessel functions. c_1, c_2, d_1, and d_2 are arbitrary constants but should be determined by the boundary conditions. Note that for large $m_s s$ we have

$$\phi_s(s)\approx-\sqrt{\frac{2}{\pi}}[c_1\sin(m_s s)+c_2\cos(m_s s)], \tag{55}$$

i.e., KK modes behave like free particles.

On integrating Eq. (41) near the extra dimension axes and the origin, the boundary conditions for the spin-2 graviton modes are given respectively as follows:

$$\left[\left(1-\frac{12\alpha k_\pm^2}{M^2}\right)\xi+\frac{8\alpha k_1}{M^2A}[-\xi'+A(k_2\eta-k_1\xi)]\right]_{z_1=0+}=0, \tag{56}$$

$$\left[\left(1-\frac{12\alpha k_\pm^2}{M^2}\right)\eta+\frac{8\alpha k_2}{M^2A}[-\dot{\eta}-A(k_2\eta-k_1\xi)]\right]_{z_2=0+}=0, \tag{57}$$

$$\frac{8\alpha}{M^2A^3}(k_1\eta+k_2\xi)|_{(z_1=0+,z_2=0+)}=0 \tag{58}$$

where we used the bulk equation (47) and

$$\xi=\psi'+2k_1A\psi=k_1\left(\frac{\partial}{\partial s}+\frac{2}{s}\right)\hat{\psi}+k_2\frac{\partial\hat{\psi}}{\partial t}, \tag{59}$$

$$\eta=\dot{\psi}+2k_2A\psi=k_2\left(\frac{\partial}{\partial s}+\frac{2}{s}\right)\hat{\psi}-k_1\frac{\partial\hat{\psi}}{\partial t}. \tag{60}$$

The zero mode solution $\hat{\psi}_0=A^2=s^{-2}$ is shown to satisfy all of the above boundary conditions since $\xi=\eta=0$ identically, and it is regarded as the 4D massless graviton since it is a normalizable bound state with its norm being $\|\psi_0\|^2<\infty$. For the KK massive modes, there are two types of bulk solutions since we have to deal with the zero mode separately:

$$\hat{\psi}_m^{(1)}=s^{-2}\phi_t(t)=s^{-2}[d_1\sin(m_t t)+d_2\cos(m_t t)], \tag{61}$$

$$\hat{\psi}_m^{(2)} = \phi_s(s)\phi_t(t) = \sqrt{s}[c_1 J_{5/2}(m_s s) + c_2 Y_{5/2}(m_s s)] \times [d_1 \sin(m_t t) + d_2 \cos(m_t t)] \quad (62)$$

where ϕ_s and ϕ_t are given by Eqs. (53) and (54), respectively, and we note that $m_s^2 = 0$ for the case of $\hat{\psi}_m^{(1)}$.

Thus, $\hat{\psi}_m^{(1)}$ satisfies the boundary condition at the origin automatically for $k_1 = k_2$ but otherwise only with $d_2/d_1 = \cot(m_t)$, and the remaining boundary conditions are rewritten as

$$\left[\left(1 - \frac{12\alpha k_\pm^2}{M^2}\right)\frac{d\phi_t}{dt} + \frac{8\alpha k_1 k_2 m_t^2}{M^2} s\phi_t\right]\Bigg|_{z_1=0+} = 0, \quad (63)$$

$$\left[\left(1 - \frac{12\alpha k_\pm^2}{M^2}\right)\frac{d\phi_t}{dt} - \frac{8\alpha k_1 k_2 m_t^2}{M^2} s\phi_t\right]\Bigg|_{z_2=0+} = 0. \quad (64)$$

There exist no KK massive modes of type $\hat{\psi}_m^{(1)}$ satisfying the above boundary conditions. On the other hand, for the KK massive modes of the other type $\hat{\psi}_m^{(2)}$, the boundary conditions look complicated to solve, but if we assume no t dependence we can obtain the ratio between coefficients of the Bessel functions as

$$\frac{c_1}{c_2} = -\frac{Y_{3/2}(m_s)}{J_{3/2}(m_s)}, \quad (65)$$

and the boundary conditions on the extra dimension axes are simplified to

$$\left[\left(1 - \frac{\alpha}{M^2}(36k_1^2 + 4k_2^2)\right)\left(\frac{d}{ds} + \frac{2}{s}\right)\phi_s + \frac{8\alpha k_1^2 m_s^2}{M^2} s\phi_s\right]\Bigg|_{z_1=0+} = 0, \quad (66)$$

$$\left[\left(1 - \frac{\alpha}{M^2}(4k_1^2 + 36k_2^2)\right)\left(\frac{d}{ds} + \frac{2}{s}\right)\phi_s + \frac{8\alpha k_2^2 m_s^2}{M^2} s\phi_s\right]\Bigg|_{z_2=0+} = 0. \quad (67)$$

However, the above boundary conditions are not satisfied by KK massive modes that are a function of s only except for $m_s^2 = 0$, i.e., the zero mode. Moreover, the situation would not be different for more general KK modes of type $\hat{\psi}_m^{(2)}$. Therefore, even though the bulk equation for 4D massive gravitons is exactly solvable, there will not exist bulk solutions satisfying the boundary conditions along the extra dimension axes with the simple ansatz for separation of variables, Eq. (48). It is shown that this situation does not change even without the Gauss-Bonnet term.

IV. CONFINING GRAVITY TO THE SOLITONIC 4-BRANES

Let us discuss the case with the orthogonal 4-branes regarded as *solitonic* by choosing the relation between bulk parameters Eq. (31), for which there is no six-dimensional bulk propagation of the graviton but the gravity is confined to the solitonic 4-branes as shown in Eq. (41) or Eq. (42). In this case, we can rewrite the linearized equation (42) with $\tilde{h}_{\mu\nu} = A^{-3/2}\tilde{\psi}(z_1,z_2)e^{ip\cdot x}\epsilon_{\mu\nu}$ as

$$\begin{aligned}
&-\delta(z_1)\frac{8\alpha k_1}{M^2}\left[m^2 + \partial_{z_2}^2 - \frac{15}{4}k_2^2A^2 + 3k_2A\delta(z_2)\right]\tilde{\psi} \\
&+ \frac{24\alpha k_1^2}{M^2}\mathrm{sgn}(z_1)A\delta(z_1)\left(\partial_{z_1} + \frac{3}{2}k_1\,\mathrm{sgn}(z_1)A\right)\tilde{\psi} \\
&- \delta(z_2)\frac{8\alpha k_2}{M^2}\left[m^2 + \partial_{z_1}^2 - \frac{15}{4}k_1^2A^2 + 3k_1A\delta(z_1)\right]\tilde{\psi} \\
&+ \frac{24\alpha k_2^2}{M^2}\mathrm{sgn}(z_2)A\delta(z_2)\left(\partial_{z_2} + \frac{3}{2}k_2\,\mathrm{sgn}(z_2)A\right)\tilde{\psi} = 0.
\end{aligned} \quad (68)$$

Then the above equation is decomposed into two five-dimensional bulk equations for the graviton and three boundary conditions:

$$\left(-\partial_{z_1}^2 + \frac{15}{4}k_1^2A^2\right)\tilde{\psi} = m^2\tilde{\psi} \quad (\text{along } z_1 \text{ axis}), \quad (69)$$

$$\left(-\partial_{z_2}^2 + \frac{15}{4}k_2^2A^2\right)\tilde{\psi} = m^2\tilde{\psi} \quad (\text{along } z_2 \text{ axis}), \quad (70)$$

$$\left(\partial_{z_1} + \frac{3}{2}k_1A\right)\tilde{\psi}\Bigg|_{z_1=0+} = 0, \quad (71)$$

$$\left(\partial_{z_2} + \frac{3}{2}k_2A\right)\tilde{\psi}\Bigg|_{z_2=0+} = 0, \quad (72)$$

$$\left[\left(\partial_{z_1} + \frac{3}{2}k_1A\right)\tilde{\psi} + \left(\partial_{z_2} + \frac{3}{2}k_2A\right)\tilde{\psi}\right]\Bigg|_{(z_1=z_2=0+)} = 0 \quad (73)$$

where we note that the last equation is a necessary consequence if the third and fourth ones are satisfied and vice versa for our case, as will be shown later. From Eqs. (69) and (70), the zero mode solution for $m^2 = 0$ becomes the same as in the nonsolitonic case,

$$\tilde{\psi}_0 = (k_1|z_1| + k_2|z_2| + 1)^{-3/2}, \quad (74)$$

which automatically satisfies the boundary conditions Eqs. (71)–(73). Note that the zero mode wave $\tilde{\psi}_0$ is chosen to be nonvanishing only along the solitonic 4-branes.

On the other hand, by solving Eqs. (69) and (70), the KK mode solutions are given as linear combinations of Bessel functions of order 2 as in the RS case, propagating along solitonic 4-branes located at the z_1 and z_2 axes:

$$\tilde{\psi}_m = N_m^{(1)}(|z_1|+1/k_1)^{1/2}[Y_2(m(|z_1|+1/k_1)) + B_m J_2(m(|z_1|+1/k_1))] \quad (\text{along } z_1 \text{ axis}), \tag{75}$$

$$\tilde{\psi}_m = N_m^{(2)}(|z_2|+1/k_2)^{1/2}[Y_2(m(|z_2|+1/k_2)) + C_m J_2(m(|z_2|+1/k_2))] \quad (\text{along } z_2 \text{ axis}), \tag{76}$$

where $N_m^{(1,2)}$, B_m, and C_m are constants to be determined by boundary conditions and normalization. Then, for the KK modes with small masses, i.e., $m(|z_{1,2}|+1/k_{1,2}) \ll 1$, the constants B_m and C_m are determined approximately from the boundary conditions Eqs. (71) and (72), as follows:

$$B_m \simeq \frac{4k_1^2}{\pi m^2}, \quad C_m \simeq \frac{4k_2^2}{\pi m^2}. \tag{77}$$

Furthermore, from plane wave normalization such that

$$1 = \int_0^{z_c} dz_1 |\tilde{\psi}_m|^2 + \int_0^{z_c} dz_2 |\tilde{\psi}_m|^2, \tag{78}$$

we also obtain the normalization constant $N_m^{(1,2)}$ as

$$N_m^{(1)} \sim B_m^{-1} \sqrt{\frac{\pi m}{z_c}} \left(1+\frac{k_2}{k_1}\right)^{-1/2} = \left(\frac{k_2}{k_1}\right)^{3/2} N_m^{(2)}. \tag{79}$$

Therefore, the Newtonian potential for two point sources m_1 and m_2 separated by r on the 3-brane is found in a conventional way to be

$$\begin{aligned} V(r) &\simeq \frac{G_N m_1 m_2}{r} + (16\alpha k_2 M^2)^{-1} \times \int_0^\infty dm \frac{m_1 m_2 e^{-mr}}{r} |\tilde{\psi}_m(0)|^2 \\ &\quad + (16\alpha k_1 M^2)^{-1} \int_0^\infty dm \frac{m_1 m_2 e^{-mr}}{r} |\tilde{\psi}_m(0)|^2 \\ &\simeq \frac{G_N m_1 m_2}{r}\left[1+\left(\frac{k_\pm^2}{k_1 k_2}\right)^2 \frac{1}{(k_\pm r)^2}\right] \end{aligned} \tag{80}$$

where we used $G_N = M_P^{-2} = (3k_1k_2)/(4M^4)$ from Eq. (34), $|\tilde{\psi}_m(0)|^2 \sim m/(k_1+k_2)$, and the effective 5D gravity couplings for KK modes are read off from the coefficients of the 5D kinetic terms in Eq. (68). As a result, corrections due to the KK massive modes are *five dimensional* due to the confinement of gravity to the solitonic 4-branes and suppressed in comparison with the Newton force at larger length scales than the curvature scales. Consequently, the confinement of gravity gives rise to exactly two copies of the five-dimensional RSII model. In addition, since gravity does not propagate into the bulk, one fine-tuning condition between bulk parameters, Eq. (31), remains intact at the quantum level of linearized gravity.

V. THE MASS HIERARCHY WITH THE ORBIFOLD $T^2/(Z_2 \times Z_2)$

We have just shown that there exist two orthogonal 4-brane solutions with nonzero tension of the intersection (or 3-brane) in 6D with the Gauss-Bonnet term. Therefore, it is possible to put another 3-brane in the appropriate position of the bulk as the additional intersection of 4-branes to solve the hierarchy problem as in the RSI case. But it should be guaranteed that the additional brane is located at the fixed point of the orbifold to be stable, i.e., the bulk should end at the position of the additional brane. Thus, we assume that there exist compact extra dimensions with the orbifold $T^2/(Z_2 \times Z_2)$, where Z_2 acts on each extra dimension once. Let us set the range of the extra coordinates as $z_1 \in (-a,a)$ and $z_2 \in (-b,b)$. Here we assumed the periodicity of $2a(2b)$ along the z_1 (z_2) direction. Then, with the $Z_2 \times Z_2$ symmetric solution Eq. (26), we need four 3-branes to match the boundary conditions at the four fixed points of the torus, $(z_1,z_2)=(0,0)$, $(a,0)$, (a,b), and $(0,b)$. Let us denote the 3-brane tensions as Λ_1, Λ_2, Λ_3, and Λ_4 in order, and the neighboring two 3-branes are connected to each other by one 4-brane denoted as Λ_{12}, Λ_{23}, Λ_{34}, and Λ_{41} in cyclic order. If the boundary equations in Eqs. (19)–(21) are changed to the following:

$$e_1 = -M^{-4}\frac{1}{A}[\Lambda_{41}\delta(z_1) + \Lambda_{23}\delta(z_1-a)], \tag{81}$$

$$e_2 = -M^{-4}\frac{1}{A}[\Lambda_{12}\delta(z_2) + \Lambda_{34}\delta(z_2-b)], \tag{82}$$

$$e_3 = -M^{-4}\sum_{i=1}^{4}\frac{1}{A^2}\Lambda_i \delta(z_1 - z_1^{(i)})\,\delta(z_2 - z_2^{(i)}), \tag{83}$$

where $z_1^{(i)}$ and $z_2^{(i)}$ are the positions of the branes, then we obtain the following relations between the 4-brane tensions and similarly for the 3-brane tensions:

$$\Lambda_{41} = -\Lambda_{23} = k_1\left(1 - \frac{12\alpha k_\pm^2}{M^2}\right), \tag{84}$$

$$\Lambda_{12} = -\Lambda_{34} = k_2\left(1 - \frac{12\alpha k_\pm^2}{M^2}\right), \tag{85}$$

$$\Lambda_1 = \Lambda_3 = -\Lambda_2 = -\Lambda_4 = 96\alpha k_1 k_2 M^2. \tag{86}$$

In general, in view of Eqs. (27)–(30), for fixed bulk parameters, two orthogonal 4-brane tensions should be fine tuned with the 3-brane tension on their intersection [e.g., between Λ_{41} (Λ_{12}) and Λ_1, etc.]. When we adopt the string solution with two *solitonic* 4-branes, each 3-brane tension can take an arbitrary value of either sign irrespective of the bulk parameters, as argued in the previous section, but they should be fine tuned to one another as shown in Eq. (86). Then, to explain the large mass hierarchy for both the string solution with *nonsolitonic* 4-branes for $\alpha>0$ and the string solution with *solitonic* 4-branes, we may take the Λ_3 brane with positive tension as the visible brane, whereas the Λ_1 brane can be considered as the hidden brane of the Planck scale. In addition, if the Λ_2 brane and Λ_4 branes are considered as the second and third generation family branes while the Λ_3 brane is interpreted as the first family brane, we may understand the mass hierarchy between families and neutrino oscillation. In this case, the gauge fields are required to exist in the bulk. But we do not digress into this family problem here.

Before considering how the mass hierarchy is generated in this model, let us rewrite the metric as

$$\begin{aligned} ds_6^2 &= A^2(z_1,z_2)(\eta_{\mu\nu}dx^\mu dx^\nu + dz_1^2 + dz_2^2) \\ &= A^2(y_1,y_2)\eta_{\mu\nu}dx^\mu dx^\nu + B^2(y_1,y_2)dy_1^2 \\ &\quad + C^2(y_1,y_2)dy_2^2 \end{aligned} \tag{87}$$

by the following bulk coordinate transformations:

$$dz_1 = \frac{B}{A}dy_1,\ dz_2 = \frac{C}{A}dy_2, \tag{88}$$

i.e., $k_1 z_1 = \mathrm{sgn}(y_1)(e^{k_1|y_1|}-1), k_2 z_2 = \mathrm{sgn}(y_2)(e^{k_2|y_2|}-1)$. Then, we can have the metric functions in the new coordinate: $A=(e^{k_1|y_1|}+e^{k_2|y_2|}-1)^{-1}$, $B=e^{k_1|y_1|}A$, and $C=e^{k_2|y_2|}A$. So the 4D Planck mass becomes

$$\begin{aligned} M_{P,eff}^2 &= M^4\int_{-a}^{a}dz_1\int_{-b}^{b}dz_2\Bigg(A^4\Bigg\{1+\frac{12\alpha}{M^2}\frac{1}{A^2}\Bigg[\left(\frac{A'}{A}\right)^2 \\ &\quad + \left(\frac{\dot A}{A}\right)^2\Bigg]\Bigg\} - \frac{12\alpha}{M^2}[(AA')' + (A\dot A)^{\cdot}]\Bigg) \\ &= M^4\left(1+\frac{12\alpha k_\pm^2}{M^2}\right)\int_{-b_1}^{b_1}dy_1\int_{-b_2}^{b_2}dy_2 A^2BC \\ &= \frac{2M^4}{3k_1k_2}\left(1+\frac{12\alpha k_\pm^2}{M^2}\right)[1+(e^{k_1b_1}+e^{k_2b_2}-1)^{-2} \\ &\quad - e^{-2k_1b_1} - e^{-2k_2b_2}] \end{aligned} \tag{89}$$

where the $(AA')'$ and $(A\dot A)^{\cdot}$ terms in the first equality vanish after integration due to the periodicity of the extra dimensions, b_1 and b_2 are the range of the extra dimensions in the new coordinate, and in the limit of $b_1\to\infty$ and $b_2\to\infty$ Eq. (34) can be reproduced. Note that the 4D Planck mass has a finite value if $k_1k_2\neq 0$, i.e., $\Lambda_i\neq 0$ for all i from Eqs. (30) and (83) and its positiveness is assured for $|\alpha|<M^2/12k_\pm^2$ for $\alpha<0$ and any value for $\alpha>0$. In this new coordinate, let us consider the action for the Higgs scalar field at the Λ_3 brane,

$$\begin{aligned} S_{vis} &\supset \int dx^4\sqrt{-g^{(vis)}}[\bar g^{\mu\nu}\partial_\mu H\partial_\nu H - (H^2-m_0^2)^2] \\ &= \int dx^4\sqrt{-g^{(4)}}A^4[A^{-2}(\partial H)^2 - (H^2-m_0^2)^2], \end{aligned} \tag{90}$$

which becomes of a canonical form by redefining the scalar field as $\tilde H = AH$,

$$\int dx^4\sqrt{-g^{(4)}}[(\partial\tilde H)^2 - (\tilde H^2 - m_3^2)^2], \tag{91}$$

where the Higgs mass parameter on the visible brane is given by

$$m_3 = Am_0 = (e^{k_1b_1}+e^{k_2b_2}-1)^{-1}m_0. \tag{92}$$

Similarly, we obtain the effective mass scales on the other branes Λ_2 and Λ_4, respectively:

$$m_2 = e^{-k_1b_1}m_0, \quad m_4 = e^{-k_2b_2}m_0. \tag{93}$$

Therefore, when we regard the Λ_3 brane as our Universe, we can obtain the hierarchy between the Planck scale (m_0) and the weak scale (m_3) by choosing k_1b_1 and/or k_2b_2 as about 37. It is interesting to see that the mass parameters on the branes are related by

$$\frac{1}{m_2}+\frac{1}{m_4}-\frac{1}{m_3}=\frac{1}{m_0}, \tag{94}$$

where m_0 is the mass scale of order of the Planck mass at the 3-brane located at (0,0). Since the right-hand side of Eq. (94) is negligible, the magnitudes of at least two of m_2, m_3, and m_4 are of the same order, which may allow a deeper understanding of the family structure. Instead of putting different families in the different 3-branes, one can put all the fermions and the Higgs doublet in the (a,b) brane or in the (a,b) and $(0,b)$ branes with $b\gg a$. Then the $(a,0)$ brane can be used for an intermediate scale brane. However, it is not necessarily needed as proposed in [24] for a solution of the μ problem with supersymmetry [25], because the visible sector fields here are already put at the TeV brane. On the other hand, if the visible sector fields with supersymmetric extension are put at the two Planck scale branes at (0,0) and $(a,0)$ with $b\gg a$, then it is necessary to introduce intermediate scale brane(s) at $(0,b)$ and (a,b) [24]. In this case, there can be two intermediate scales in principle due to the two 3-branes at the intermediate scales.

VI. CONCLUSION

In this paper we obtained the localized gravity on the intersection of two orthogonal nonsolitonic or solitonic 4-branes in the Einstein-Gauss-Bonnet theory in 6D. Nonzero 3-brane tension is allowed, which has been made possible due to the presence of the Gauss-Bonnet term. The Gauss-Bonnet term can contain a product of two terms with two derivatives of the metric on each term. Therefore, in the EGB theory 3-brane solutions are not possible beyond 6D.

To have 3-brane solutions beyond 6D, we have to introduce higher derivative gravity than the Gauss-Bonnet term.

The solution has a warp factor that decreases exponentially at large distance from the origin in the extra dimension. If $Z_2 \times Z_2$ symmetry is assumed on the bulk space even without nonsolitonic 4-branes, one can consider a solution of a 3-brane residing on the intersection of two solitonic 4-branes for the localization of gravity and also for a possible solution of the cosmological constant problem as in the RSII model [18]. With this solution, it is interesting to make possible the confinement of gravity to the solitonic 4-branes, which results in nothing but two copies of the 5D RSII model. In addition, the extra dimension can be compactified. The $T^2/(Z_2 \times Z_2)$ orbifold symmetry gives four fixed points where 3-branes reside on intersections of two 4-branes. In this case, the electroweak scale versus the Planck scale hierarchy can be understood. We also pointed out the possibility of understanding the family structure, which will be studied in a future publication.

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[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *ibid.* **436**, 257 (1998).

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).

[4] A. G. Cohen and D. B. Kaplan, Phys. Lett. B **470**, 52 (1999).

[5] Z. Chacko and A. E. Nelson, Phys. Rev. D **62**, 085006 (2000).

[6] J. E. Kim, B. Kyae, and H. M. Lee, Phys. Rev. D **62**, 045013 (2000); Nucl. Phys. **B582**, 296 (2000).

[7] S.-H. Moon, S.-J. Rey, and Y. Kim, Nucl. Phys. **B602**, 467 (2001).

[8] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, and N. Kaloper, Phys. Rev. Lett. **84**, 586 (2000).

[9] C. Csaki and Y. Shifman, Phys. Rev. D **61**, 024008 (2000); A. E. Nelson, *ibid.* **63**, 087503 (2001).

[10] J. E. Kim and H. M. Lee, Nucl. Phys. **B602**, 346 (2001).

[11] R. Gregory, Phys. Rev. Lett. **84**, 2564 (2000).

[12] I. Olasagasti and A. Vilenkin, Phys. Rev. D **62**, 044014 (2000).

[13] T. Gherghetta and M. Shaposhnikov, Phys. Rev. Lett. **85**, 240 (2000); T. Gherghetta, E. Roessl, and M. Shaposhnikov, Phys. Lett. B **491**, 353 (2000).

[14] I. Low and A. Zee, Nucl. Phys. **B585**, 395 (2000); N. E. Mavromatos and J. Rizos, Phys. Rev. D **62**, 124004 (2000); I.P. Neupane, J. High Energy Phys. **09**, 040 (2000); S. Nojiri and S. D. Odintsov, *ibid.* **07**, 049 (2000).

[15] O. Corradini and Z. Kakushadze, Phys. Lett. B **494**, 302 (2000)

[16] A. Iglesias and Z. Kakushadze, hep-th/0011111; hep-th/0012049.

[17] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and R. Sundrum, Phys. Lett. B **480**, 193 (2000); S. Kachru, M. Schulz, and E. Silverstein, Phys. Rev. D **62**, 045021 (2000); S. Forste, Z. Lalak, S. Lavignac, and H. P. Nilles, Phys. Lett. B **481**, 360 (2000); J. High Energy Phys. **09**, 034 (2000); C. Csaki, J. Erlich, C. Grojean, and T. Hollowood, Nucl. Phys. **B584**, 359 (2000); S. H. H. Tye and I. Wasserman, Phys. Rev. Lett. **86**, 1682 (2001); A. Kehagis and K. Tamvakis, hep-th/0011006; E. Flanagan, N. Jones, S. H. H. Tye, and I. Wasserman, Phys. Rev. D **64**, 045007 (2001); C. Csaki, J. Erlich, and C. Grojean, Nucl. Phys. **B604**, 312 (2001).

[18] J. E. Kim, B. Kyae, and H. M. Lee, Phys. Rev. Lett. **86**, 4223 (2001); hep-th/0101027.

[19] A. H. Guth, Phys. Rev. D **23**, 347 (1981).

[20] O. Corradini and Z. Kakushadze, Phys. Lett. B **506**, 167 (2001).

[21] R. Sundrum, Phys. Rev. D **59**, 085010 (1999).

[22] A. Chodos and E. Poppitz, Phys. Lett. B **471**, 119 (1999).

[23] Z. Berezhiani, M. Chaichian, A. B. Kobakhidze, and Z. -H. Yu, hep-th/0102207.

[24] J. E. Kim and B. Kyae, Phys. Lett. B **500**, 313 (2001).

[25] J. E. Kim, Phys. Lett. B **452**, 255 (1999), and references therein.